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An Equitable, Acceptable and Socially Beneficial Congestion Pricing Design: A Continuum Modeling Approach

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Abstract: Consider a region of an arbitrary shape with multiple Central Business Districts (CBDs) competing for multi-class users that are distributed continuously over the region. Within this region, the road network is represented as a continuum and users patronize in a two-dimensional continuum transportation system to travel to their chosen CBD. A logit-type distribution function is specified to model the probabilistic destination choices made by the different classes of users. In this paper, a cordon-based congestion-pricing model for this continuum network is developed and the optimal location and toll level of the charging cordon is selected based on the equity, acceptability and social benefit of the charging schemes. The cordon-based congestion-pricing model is solved by using the finite element method and a promising Newtonian-based solution algorithm. A numerical example is given to show the usefulness of the proposed model, algorithm and criteria for cordon design and selection.

Key Words: *Equity, acceptability, congestion pricing*

1. INTRODUCTION

Congestion pricing can be used to reduce traffic congestion and raise revenue for the funding of transportation improvements. The principle underlying congestion pricing is that the choice behavior of users should be regulated with a view to approaching a system-optimal travel pattern which maximizes the total benefit of the system as a whole by imposing tolls on users. This is known as the first-best pricing principle and can be implemented by introducing a toll that is equal to the user externality, or the difference between the marginal social cost and the marginal private cost, for each link, such that the user-equilibrium network traffic flow condition maximizes the total benefit of the system as a whole

For the first-best pricing principle, all the links within the modeled network have to be tolled in order to achieve a system optimal travel pattern. But in reality, tolling all the links is not practicable and may not be easily accepted by public. Thus, one of the major directions of research in congestion pricing is to determine the optimal toll locations and levels such that only a subset of links in the network could be tolled. Such optimal toll locations and levels found are known as the second-best solution of congestion pricing. Hearn and Ramana (1998) proposed an approach to determine the minimal number of toll links to achieve a system optimal solution. Verhoef (2002) and Shepherd and Sumalee (2004) examined the selection of individual toll links and the determination of toll levels using sensitivity indicators. Using the mathematical model proposed by Verhoef (2002), Shepherd *et al.* (2001) examined the performance of various pre-specified toll cordons on a simple hypothetical network. In the city of Cambridge, May and Milne (2000) tested and compared four kinds of road pricing schemes with charges based on cordon crossed, distance traveled, time spent traveling and time spent in congestion, using the congested assignment network model, SATURN, and its elastic assignment demand response routine, SATEASY. A series of studies from Sumalee and his colleague (Sumalee, 2007, 2008; Sumalee *et al.*, 2004; Shepherd and Sumalee, 2004) have focused on developing a all-rounded, which considered the benefits from both users and system side, and practicable congestion-pricing design with the corresponding efficient solution algorithms

In most of the congestion-pricing design (Yang and Huang, 2004; Ho *et al.*, 2005), focuses have been placed on maximizing the social benefit of the system and/or revenue of the operator(s). Although these designs are most beneficial to the whole system and/or the operator(s), they are difficult to gain public's support for actual implementation as they have not considered the interest of system users (or travelers). In order to account for this shortcoming in gaining public's support in congestion-pricing implementation, various studies have incorporate equity measures in congestion-pricing design model (Maruyama and Sumalee, 2002; Yang and Zhang, 2002; Connors *et al.*, 2005; Juan *et al.*, 2008) to provide a more socially acceptable and beneficial pricing scheme. Equity issue of congestion-pricing is regularly related to an unequal distribution of the cost and benefit of the pricing scheme among different users within the system. Depending on the classification of users, equity issue could be considered in vertical and horizontal dimensions. The vertical dimension of the equity issue can be seen as an unequal distribution of effects of the scheme across different groups of population categorized by income level, age, socio-economic group, etc. The horizontal equity impact is referred to the uneven distribution of benefits and costs from the pricing scheme across the population from different areas in the region. A congestion-pricing scheme is considered to be inequitable if it only benefits a small number of people from the same geographic areas or social groups (e.g. rich or poor).

System benefit and equity are respectively the measure of benefits and costs of congestion-pricing scheme solely in the system's and users' perspective. However, most of the proposals of road pricing schemes are failed on the public acceptability ground. As congestion-pricing scheme will have an impact on all the members living in the same society, the decision of an individual on whether to support or object the implementation will not only based on their own utility, but also based on the indirect utility of their related groups (Mueller, 1986). This consideration of indirect utility of other groups in the evaluation of congestion-pricing schemes could be explained by the existence of moral or social-norm effect among the individuals.

In the literature, the applications of congestion-pricing models and studies of finding the optimal charging cordons design have been focused on the discrete network (Dafermos and Sparrow, 1971; Yang and Huang, 2004; Gentile *et al.*, 2005; Sumalee, 2007), in which the network is modeled as a set of nodes and links. Although this discrete network approach is relatively simple and has been well-developed in the literature, it may not be suitable for regional/national congestion-pricing studies due to the scale of the issues to be tackled and the amount of data required for setting up such a model. The continuum modeling approach (Wong *et al.*, 1998; Ho *et al.*, 2006), in which the road network is approximated as a continuum in which users are free to choose their routes in a two-dimensional space (Ho and Wong, 2006), offers an alternative strategy for representing the supply side of the traffic equilibrium problem. The fundamental assumption made is that differences in modeling characteristics, such as travel cost and demand pattern, between adjacent areas within a network are relatively small when compared to the degree of variation over the entire network.

Hence, the characteristics of a network, such as flow intensity, demand, and travel cost, can be represented by smooth mathematical functions (Vaughan, 1987).

In this paper, a cordon-based congestion-pricing model for continuum network with multiple user classes and destinations choice is developed. Concentric circles from CBD with different radius and toll levels will be taken as feasible congestion-pricing schemes and an optimal location and toll level will be selected based on the equity, acceptability and social benefit of these schemes. The remainder of this paper is organized as follows. Section 2 defines the notation and specific equations that are used in this study. Section 3 introduces the formulation and solution algorithm of the cordon-based congestion-pricing model for this multi-class multi-destination problem. A numerical example that demonstrates the usefulness of the proposed model, algorithm and criteria in cordon design is given in Section 4. We conclude the paper in Section 5.

2. DEFINITIONS AND NOTATION

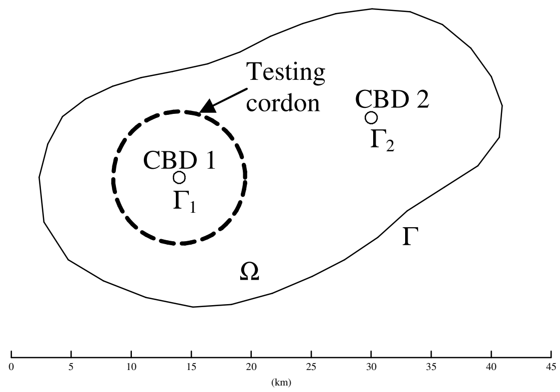

Figure 1 The modeled city and testing cordon

Consider a study region of arbitrary shape as shown in Figure 1. Within this region, there is a number of central business districts (CBDs) which are assumed to be sufficiently compact when compared with the study region as a whole. These CBDs compete for different classes of users who are continuously distributed over space. Each user within the study region makes a choice of CBD based on his utility gain in choosing that CBD, the total cost of patronizing that CBD and the toll incurred along the path to that CBD. The users will travel from their demand locations to the chosen CBD along the least costly route. Let M denote the set of user classes, such as high income, low income, etc., in this study. Let the study region be represented by Ω and the boundary by Γ. Further, let the locations of CBD n be $O_n, \forall n \in N$, where N is the set of CBDs within the region. To avoid singularity among the CBDs, it is assumed that each of the CBDs is of finite size and enclosed by a boundary, Γ_{cn}. The travel cost per unit distance of travel at location $(x, y) \in \Omega$ for class m users is denoted by $c_m(x, y)$, which is location dependent and has the following functional relationship with traffic flows at that location:

$$c_m(x, y) = a_m(x, y) + \sum_r \sum_s b_{mr}(x, y)\left|\mathbf{f}_{rs}(x, y)\right|, \quad \forall (x, y) \in \Omega, m \in M, n \in N, \tag{1}$$

where $a_m(x, y)$ and $b_{mr}(x, y)$, which are strictly positive scalar functions of the cost-flow relationship that reflects the local characteristics of the streets at location $(x, y) \in \Omega$, are respectively the free-flow travel cost of class m users and the congestion-related parameter of class r users on class m. As $b_{mr}(x, y)$ is not necessarily equal to $b_{rm}(x, y)$, the asymmetric congestion effect between different vehicle types can be modeled. $\mathbf{f}_{mn}(x, y) = \left(f_{xmn}(x, y), f_{ymn}(x, y)\right)$ is the flow vector of class m users who are heading to CBD n, and $f_{xmn}(x, y)$ and $f_{ymn}(x, y)$ are the corresponding flow fluxes in the x and y directions, respectively. The flow vector $\mathbf{f}_{mn}(x, y)$ indicates the class m users' movement direction at location $(x, y) \in \Omega$ when heading to CBD n in the two-dimensional plane, and

$$\left|\mathbf{f}_{mn}(x, y)\right| = \sqrt{f_{xmn}^{2}(x, y) + f_{ymn}^{2}(x, y)} \tag{2}$$

is the corresponding flow intensity which measures in a unit time the number of class m users who cross a small segment of unit width perpendicular to the flow direction when heading to CBD n. Equation (1) represents an isotropic cost function because it depends only on flow intensity and not on flow direction. The congestion component includes the flow intensities for all classes of users because all users share the same road space. Each user creates an additional delay cost to all other users as he makes the road more congested.

For a particular class of users, they have the choice of traveling to any of the CBDs within the modeling region, as well as the choice of non-travel. The probabilities of these choices being made depend on the utility gain, the total transportation cost and the total toll paid to travel from the user's demand location to each of the CBDs, and are governed by a logit-type distribution. The demand distributed to any of these choices (including the choice of all CBDs and the choice of non-travel) can be expressed as:

$$q_{m\tilde{n}}(x, y) = q_m(x, y) \frac{\exp(\zeta_m v_{m\tilde{n}})}{\sum_s \exp(\zeta_m v_{ms})}, \quad \forall (x, y) \in \Omega, \tilde{n}, s \in \tilde{N}, m \in M\ , \tag{3}$$

where $v_{ms} = U_{ms} - \bar{u}_{ms}$ is the utility gain by a class m user making a choice s; $\tilde{n}, s \in \tilde{N}$ and $\tilde{N}$ is the enlarged set of N that includes the choice of non-travel; $q_{m\tilde{n}}(x, y)$ is the demand of class m users at location $(x, y) \in \Omega$ who make a choice $\tilde{n}$; $q_m(x, y)$ is the fixed total demand of class m users; $U_{m\tilde{n}}$ is the utility gain for class m users making a choice $\tilde{n}$; $\bar{u}_{m\tilde{n}}(x, y)$ is the total travel cost (including toll, if any) for a class m user at his demand location $(x, y) \in \Omega$ of making a choice $\tilde{n}$ ($\bar{u}_{m\tilde{n}}(x, y) = 0$ if the user chooses not to travel); and ζ_m is a scalar sensitivity parameter of the distribution function for class m users. For each combination of user class and CBD, the following flow conservation equation should be satisfied:

$$\nabla \cdot \mathbf{f}_{mn}(x, y) - q_{mn}(x, y) = 0, \quad \forall (x, y) \in \Omega, n \in N, m \in M\ . \tag{4}$$

Assuming no flow across the boundary of the study area, we have the following boundary condition:

$$\mathbf{f}_{mn}(x, y) = 0, \quad \forall (x, y) \in \Omega, n \in N, m \in M\ . \tag{5}$$

However, it is not difficult to extend the model to represent a given demand pattern that enters or leaves the region at the boundary. In such a case, we need only replace the above constraint with $\mathbf{f}_{mn}(x, y) \cdot \mathbf{n}(x, y) = g_{mn}(x, y)$, where $g_{mn}(x, y)$ is the demand function and $\mathbf{n}(x, y)$ is the unit normal vector pointing away from the modeled region at the boundary point $(x, y) \in \Gamma$. System benefit, which is one of the three indexes used to evaluate the congestion-pricing schemes in this study, is defined by the following equation:

$$SB(\mathbf{u}, \mathbf{q}) = \iint_{\Omega} \sum_m \sum_{\tilde{n}} q_{m\tilde{n}} U_{m\tilde{n}} + \sum_m \frac{1}{\zeta_m} q_m \ln q_m - \sum_m \sum_{\tilde{n}} \frac{1}{\zeta_m} q_{m\tilde{n}} \ln q_{m\tilde{n}} - \sum_m \sum_n u_{mn} q_{mn} \, \mathrm{d}\Omega \tag{6}$$

where u_{mn} is the total travel cost (without toll) for class m users patronizing CBD n. For equation (6), the first term represents the weighted utility gain of making the relevant choice, using demand as the weight; the second and third term represents the utility gain by the users due to the presence of the diversity of choice (Yang, 1999; Oppenheim, 1995); the last term represents the total travel cost (without toll) for the whole system. Apart from the system benefit, equity is another measure that is used to evaluate the congestion-pricing schemes. In this study, both vertical (among different classes of users) and horizontal (over different locations of the modeling region) dimensions of equity are considered. Theil's entropy (Theil, 1967), which is commonly used in socioeconomics for measuring inequality, is adopted in this study as the measure for the equal distribution of user utility. Apart from the Theil entropy, Gini index is the other commonly adopted measure for equity. In this study, as system users are continuously distributed over the modeling region, the evaluation of Gini index, which depends on the utility difference of users between different locations (horizontal dimension), will become too complicated to evaluate. Instead, the Theil entropy, which compares the utility of user from a particular location with the overall average value, is chosen for its simplicity in representing the level of equity in this continuum system. Considering the continuum nature of this study, the Theil's entropy (*Th*) is redefined as:

$$Th(\mathbf{u},\mathbf{q}) = \frac{1}{Q}\sum_{m}\sum_{\tilde{n}}\iint_{\Omega} q_{m\tilde{n}} \frac{v_{m\tilde{n}}}{\overline{V}} \ln\left(\frac{v_{m\tilde{n}}}{\overline{V}}\right) \mathrm{d}\Omega \tag{7}$$

where $Q = \sum_{m}\sum_{\tilde{n}}\iint_{\Omega} q_{m\tilde{n}}\ \mathrm{d}\Omega$ is the total demand of the system; $\overline{V} = \sum_{m}\sum_{\tilde{n}}\iint_{\Omega} q_{m\tilde{n}} v_{m\tilde{n}}\ \mathrm{d}\Omega \Big/ Q$ is the average users utility over the modeling region. As Theil entropy is a measure of difference between the actual and even distribution, system with uneven distribution of utility will have larger Theil entropy. For equity, and also acceptability in the following paragraph, the utility gain v_{ms}, which depends on the total travel cost, is adopted as the major variables for determining these indexes. Utility gain is chosen for the following two reasons: 1) Implementation of congestion-pricing has direct impact on travel cost, as it reduces the congestion delay by imposing a monetary toll; 2) System users are most concern of the change of the travel cost they have to pay (or change of utility they gain) after the implementation of the scheme.

Acceptability is a measure on how well the proposed congestion-pricing scheme is accepted by the public. As discussed in the previous paragraph, utilities of system users are the major variables in evaluating acceptability. But in considering the acceptability of a congestion pricing scheme, Jaensirisak *et al.* (2003) suggested that system users will not only consider their own benefits (i.e. selfish perspective or egoism) but also consider the benefit of the system (i.e. social perspective or altruism). In order to account for the trade-off between users' selfish and social perspective, the utility function that governs the system users' acceptance of a congestion-pricing scheme ($W_{m\tilde{n}}$), will not only consider their own utility, but also the utility of other users (Mueller, 1986; Jaensirisak *et al.*, 2003).

$$W_{m\tilde{n}}(\mathbf{v}) = v_{m\tilde{n}} + \sum_{\substack{r\in M,\\ r\neq m}} w_{mr1} v_{r\tilde{n}} + \sum_{\substack{s\in \tilde{N},\\ s\neq \tilde{n}}} w_{ms2} v_{ms} + \sum_{s\in \tilde{N}} \frac{w_{ms3}}{A} \iint_{\Omega} v_{ms}\ \mathrm{d}\Omega \tag{8}$$

For equation (8), the first term is the direct utility gain for user class *m* making a choice $\tilde{n}$; the second term is the indirect utility gain from the utilities of all users from the same location making the same choice; the third term is the indirect utility gain from the utilities of class *m* users from the same location; the forth term is the indirect utility gain from the utilities of the all the class *m* users in the network. Note that w_{mr1}, w_{ms2} and w_{ms3} are respectively the relative weights given to the indirect utilities of the second, third and forth terms discussed above with respect to the direct utility gain ($v_{m\tilde{n}}$). Assume a Gumbel distribution of the error term in the perception of utility defined by equation (8), the acceptance of a congestion-pricing scheme could be found by the following logit type equation:

$$Acc\left(\mathbf{v}^1, \mathbf{v}^0\right) = \frac{1}{Q}\sum_{m}\sum_{\hat{n}}\iint_{\Omega} q^0_{m\hat{n}} \frac{\exp\left[\xi_m W_{m\tilde{n}}\left(\mathbf{v}^1\right)\right]}{\exp\left[\xi_m W_{m\tilde{n}}\left(\mathbf{v}^1\right)\right] + \exp\left[\xi_m W_{m\tilde{n}}\left(\mathbf{v}^0\right)\right]} \mathrm{d}\Omega \tag{9}$$

where v^1 and v^0 are respectively the utility gains for the toll and no-toll (user-equilibrium) scenario; ξ_m is a scalar sensitivity parameter of the distribution function for class m users; $q^0_{m\hat{n}}$ is the demand of class m user making a choice $\tilde{n}$ in the no-toll scenario.

3 CORDON-BASED CONGESTION-PRICING MODEL

3.1 Model Formulation

Based on a given cordon set $\mathbf{\Lambda}$ and the corresponding toll level $\tilde{\boldsymbol{\tau}}$, the cordon-based congestion-pricing model, which gives users' responses to such cordon set and toll level, is set up. Compared with the cordon-based model introduced in Ho *et al*. (2005), the cordon-based model introduced in this paper is more complicated, as the cordon-toll charged in this model, while path-dependent, is not location-dependent. Thus, a fixed-point model is introduced to solve this path-dependent cordon-toll problem. Two sub-models, named the toll determination sub-model and the flow pattern determination sub-model, are included in this fixed-point problem. In the toll determination sub-model, the paths chosen by system users are traced graphically from their home locations to their chosen CBDs based on the given flow pattern. Based on these traced paths, the total toll ($\tilde{T}_{mn}$) for the system users is determined by adding together the flat tolls for all the cordons crossed along the users' paths. This sub-model can be represented in the following abstract form:

$$\tilde{\mathbf{T}} = P(\mathbf{F}) \tag{10}$$

where $\tilde{\mathbf{T}}$ is the toll calculated on the basis of this cordon-based pricing scheme and $\mathbf{F}$ is the corresponding flow pattern. On the other side of the fixed-point problem, the flow pattern determination sub-model is aimed at finding the user equilibrium flow pattern $\mathbf{F}$ for a given toll pattern $\tilde{\mathbf{T}}$. This user equilibrium flow pattern can be obtained by solving the following set of differential equations:

$$\left(a_m + \sum_{o}\sum_{p} b_{mo}\left|\mathbf{f}_{op}\right|\right)\frac{\mathbf{f}_{mn}}{\left|\mathbf{f}_{mn}\right|} + \nabla u_{mn} = 0 \quad \forall m \in M, n \in N, (x, y) \in \Omega \tag{11a}$$

$$q_m \frac{\exp[\zeta_m (U_{mn} - u_{mn} - \tilde{T}_{mn})]}{\sum_{\tilde{n}} \exp[\zeta_m (U_{m\tilde{n}} - u_{m\tilde{n}} - \tilde{T}_{m\tilde{n}})]} - \nabla\mathbf{f}_{mn} = 0 \quad \forall m \in M, n \in N, \tilde{n} \in \tilde{N}, (x, y) \in \Omega \tag{11b}$$

$$\mathbf{f}_{mn} = 0 \quad \forall m \in M, n \in N, (x, y) \in \Gamma \tag{11c}$$

$$u_{mn} = 0 \quad \forall m \in M, n \in N, (x, y) \in \Gamma_n \tag{11d}$$

where u_{mn} is the total travel cost (excluding tolls) for class m user traveling to CBD n; $\tilde{T}_{m\tilde{n}}$ is the total toll that a class m user has to pay for making choice $\tilde{n}$ and is kept constant in this sub-model. $\tilde{T}_{m\tilde{n}}$ is equal to $\tilde{T}_{mn}$, which is the solution of the toll determination sub-model, where users choose to travel to any of the CBDs. Note that $\tilde{T}_{m\tilde{n}}$ is equal to zero where users choose not to travel. The fraction in equation (11b) is the modified demand distribution function based on this total toll. The total cost incurred by a class m user for any used route p or unused route p' in traveling from the demand location (H) to CBD n (O_n) can be defined as:

$$\tilde{C}_{mnp} = \int_p c_m \mathrm{d}s = \int_p c_m \frac{\mathbf{f}_{mn}}{\left|\mathbf{f}_{mn}\right|} \cdot \mathbf{ds} = -\int_p \nabla u_{mn} \cdot \mathbf{ds} = u_{mn}(\mathrm{H})$$

$$\tilde{C}_{mnp'} = \int_{p'} c_m \mathrm{d}s \geq \int_{p'} c_m \frac{\mathbf{f}_{mn}}{|\mathbf{f}_{mn}|} \cdot \mathbf{ds} = -\int_{p'} \nabla u_{mn} \cdot \mathbf{ds} = u_{mn}(\mathrm{H})$$

Therefore, $\tilde{C}_{mnp'} \geq \tilde{C}_{mnp}$, indicating that this set of differential equations satisfies the user equilibrium condition in path selection. Similar to the toll determination sub-model, this flow pattern determination sub-model can be represented in the following abstract form:

$$\mathbf{F} = S(\tilde{\mathbf{T}}) \tag{12}$$

Combining equations (10) and (12), the cordon-based congestion-pricing problem can be formulated as the following fixed-point problem:

$$\mathbf{F} = S(P(\mathbf{F})) \tag{13}$$

Solving the fixed-point problem (13), the flow pattern **F** that satisfies the user equilibrium condition (the flow pattern determination sub-model) and takes into account the cordon toll charged (the toll determination sub-model) can be found.

3.2 Solution Algorithm

As the toll determination sub-model can be solved exactly by tracing the path from the given flow pattern, we direct our solution algorithm efforts to the other sub-model. For the flow pattern determination sub-model, the finite element method (FEM) is used to approximate the continuous variables in the modeled region (Cheung *et al.*, 1996). As there is no explicit objective function for this multi-class cordon-based congestion-pricing problem, the mixed finite element procedure used in Wong *et al.* (1998) cannot be applied directly. Hence, we adopt the Galerkin formulation of the weighted residual technique (Cheung *et al.*, 1996) in which differential equations (11a) and (11b) are transformed into the following equivalent integral expressions:

$$\iint_{\Omega} \left[\left(a_m + \sum_o \sum_p b_{mo} |\mathbf{f}_{op}| \right) \frac{\mathbf{f}_{mn}}{|\mathbf{f}_{mn}|} + \nabla u_{mn} \right] \psi(x,y)\, \mathrm{d}\Omega = 0 \quad \forall m \in M, n \in N, \psi(x,y) \tag{14}$$

$$\iint_{\Omega} \left[q_m \frac{\exp[\zeta_m (U_{mn} - u_{mn} - \tilde{T}_{mn})]}{\sum_{\tilde{n}} \exp[\zeta_m (U_{m\tilde{n}} - u_{m\tilde{n}} - \tilde{T}_{m\tilde{n}})]} - \nabla \mathbf{f}_{mn} \right] \psi(x,y)\, \mathrm{d}\Omega = 0 \quad \forall m \in M, n \in N, \tilde{n} \in \tilde{N}, \psi(x,y) \tag{15}$$

where $\psi(x, y)$ is the trial (or weight) function in the weighted residual technique. Boundary conditions (11c) and (11d) are enforced by taking a zero weight function (Cheung *et al.*, 1996). In the Galerkin formulation, the local interpolation function of the finite element is used as the trial function. The modeling area is first discretized into a finite element mesh in which the Galerkin formulation is applied at the element level. The governing equations at a particular finite element node s are given as follows:

$$\{\mathbf{r}_{mn}^s(\mathbf{\Phi})\} = \begin{Bmatrix} r_{mn1}^s \\ r_{mn2}^s \\ r_{mn3}^s \end{Bmatrix} = \begin{Bmatrix} \sum_{e \in E_s} \iint_{\Omega_e} \left[\left(a_m + \sum_o \sum_p b_{mo} |\mathbf{f}_{op}| \right) \frac{f_{xmn}}{|\mathbf{f}_{mn}|} + \frac{\partial u_{mn}}{\partial x} \right] N_s(x,y)\, \mathrm{d}\Omega \\ \sum_{e \in E_s} \iint_{\Omega_e} \left[\left(a_m + \sum_o \sum_p b_{mo} |\mathbf{f}_{op}| \right) \frac{f_{ymn}}{|\mathbf{f}_{mn}|} + \frac{\partial u_{mn}}{\partial y} \right] N_s(x,y)\, \mathrm{d}\Omega \\ \sum_{e \in E_s} \iint_{\Omega_e} \left[q_m \frac{\exp[\zeta_m (U_{mn} - u_{mn} - \tilde{T}_{mn})]}{\sum_{\tilde{n} \in \tilde{N}} \exp[\zeta_m (U_{m\tilde{n}} - u_{m\tilde{n}} - \tilde{T}_{m\tilde{n}})]} - \nabla \mathbf{f}_{mn} \right] N_s(x,y)\, \mathrm{d}\Omega \end{Bmatrix} \quad \forall m,n \tag{16}$$

where $\mathbf{\Phi} = \mathrm{Col}(\mathbf{f}, \mathbf{u}, \overline{\mathbf{u}})$ denotes the solution for this cordon-based congestion-pricing problem, Ω_e denotes the domain of the finite element e, E_s is the set of finite elements that connects node s, $N_s(x, y)$ is the local interpolation function of the finite element that corresponds to node s and $\mathbf{r}_{mn}^s$ is the nodal residual vector for class m users traveling to CBD n at node s, which represents how far governing equations (11a) and (11b) are satisfied locally around node s. For the global satisfaction of governing equations (11), we require that $\mathbf{R}(\mathbf{\Phi}) = \mathrm{Col}(\mathbf{r}_{mn}^s(\mathbf{\Phi})) = \mathbf{0}$. For this flow pattern determination sub-model, we apply the Newton-Raphson algorithm with a line search to solve the problem, in which we derive the iterative equation

$$\mathbf{\Phi}_{k+1} = \mathbf{\Phi}_k - \lambda \mathbf{J}(\mathbf{\Phi}_k)^{-1} \mathbf{R}(\mathbf{\Phi}_k) \tag{17}$$

where $\mathbf{J}(\mathbf{\Phi}_k)$ is the Jacobian matrix of vector $\mathbf{R}(\mathbf{\Phi}_k)$ in iteration k and λ is the step size (to be determined using the golden section method). By modifying the Newton-Raphson algorithm adopted for the flow pattern determination sub-model, the solution procedure for this fixed-point cordon-based congestion-pricing problem can be set as follows:

Solution Procedure

Step 1: Find an initial solution $\mathbf{\Phi}_0$. Set $k = 0$.

Step 2: With the solution $\mathbf{\Phi}_k$, solve the total toll $\tilde{\mathbf{T}}_k$ using the toll determination sub-model (10)

Step 3: Evaluate $\mathbf{R}(\mathbf{\Phi}_k)$ and $\mathbf{J}(\mathbf{\Phi}_k)$.

Step 4: Apply the golden section method to determine the step size λ^* which minimizes the norm of the residual vector $\left|\mathbf{R}\left(\mathbf{\Phi}_k - \lambda \mathbf{H}(\mathbf{\Phi}_k)^{-1} \mathbf{R}(\mathbf{\Phi}_k)\right)\right|$. Then, set $\mathbf{\Phi}_{k+1} = \mathbf{\Phi}_k - \lambda^* \mathbf{J}(\mathbf{\Phi}_k)^{-1} \mathbf{R}(\mathbf{\Phi}_k)$.

Step 5: If the relative error $\left|\mathbf{\Phi}_{k+1} - \mathbf{\Phi}_k\right| / \left|\mathbf{\Phi}_k\right|$, is less than an acceptable error ε, then terminate, and $\mathbf{\Phi}_{k+1}$ is the solution.

Step 6: Replace $\mathbf{\Phi}_k$ with $\mathbf{\Phi}_{k+1}$. Set $k = k + 1$ and go to Step 2.

4. NUMERICAL EXAMPLE

We now present a numerical example to illustrate the proposed approach for selecting a cordon location and toll level for a cordon-based congestion-pricing model with multiple user classes within a continuum system. The modeled region is shown in Figure 1. The region spans about 35 km from east to west and 25 km from north to south, and has two CBDs competing for two different classes of users (say, high and low income group). For the sake of simplicity, it is assumed that the users are uniformly distributed over this region. In a real-life application, the demand surface may vary over space according to the zonal population density and other demographic and socio-economic characteristics of the population specified by the modelers. The demands for class 1 and 2 users are respectively taken as 50 users/km^2 and 60 users/km^2, while the corresponding demand distribution functions for the different destination choices are specified as:

Class 1 non-travel: $$q_{10} = 50 \frac{\exp[0.008(100)]}{\exp[0.008(100)] + \exp[0.008(310 - \overline{u}_{11})] + \exp[0.008(150 - \overline{u}_{12})]}$$

Class 1 to CBD 1: $q_{11} = 50 \dfrac{\exp[0.008(310 - \bar{u}_{11})]}{\exp[0.008(100)] + \exp[0.008(310 - \bar{u}_{11})] + \exp[0.008(150 - \bar{u}_{12})]}$

Class 1 to CBD 2: $q_{12} = 50 \dfrac{\exp[0.008(150 - \bar{u}_{12})]}{\exp[0.008(100)] + \exp[0.008(310 - \bar{u}_{11})] + \exp[0.008(150 - \bar{u}_{12})]}$

Class 2 non-travel: $q_{20} = 60 \dfrac{\exp[0.006(150)]}{\exp[0.006(150)] + \exp[0.006(380 - \bar{u}_{21})] + \exp[0.006(200 - \bar{u}_{22})]}$

Class 2 to CBD 1: $q_{21} = 60 \dfrac{\exp[0.006(380 - \bar{u}_{21})]}{\exp[0.006(150)] + \exp[0.006(380 - \bar{u}_{21})] + \exp[0.006(200 - \bar{u}_{22})]}$

Class 2 to CBD 2: $q_{22} = 60 \dfrac{\exp[0.006(200 - \bar{u}_{22})]}{\exp[0.006(150)] + \exp[0.006(380 - \bar{u}_{21})] + \exp[0.006(200 - \bar{u}_{22})]}$

The demand function is dependent on the total travel cost, $\bar{u}_{mn}$ (including toll of any), incurred in traveling from the demand locations continuously dispersed on the 2-dimensional plane to a common and compact destination in the CBD. The unit transportation cost function is specified as:

Class 1 user: $c_1(x, y) = 0.7v_a(x, y) + v_b(x, y)[0.004(|\mathbf{f}_{11}| + |\mathbf{f}_{12}|) + 0.005(|\mathbf{f}_{21}| + |\mathbf{f}_{22}|)]$.

Class 2 user: $c_2(x, y) = 0.8v_a(x, y) + v_b(x, y)[0.006(|\mathbf{f}_{11}| + |\mathbf{f}_{12}|) + 0.007(|\mathbf{f}_{21}| + |\mathbf{f}_{22}|)]$.

where $c_m(x, y)$ is measured in HKD (Hong Kong Dollars) per kilometer. $v_a(x, y) = 1.10 - 0.005\bar{d}(x, y)$ and $v_b(x, y) = 1.20 - 0.007\bar{d}(x, y)$ are the factors that account for the variation in the location-dependent parameters of the unit transportation cost function and $\bar{d}(x, y)$ is the mean distance from the two CBDs to location (x,y). These factors increase when the mean distance from the CBDs decreases, which reveals the network characteristic that junctions are more closely spaced nearer to the CBDs. Hence, the parameters of the unit transportation cost function increase. The cost function is flow-dependent and explicitly takes into account the fact that traffic congestion will lead to a higher travel cost through the opportunity cost of waiting in queues or the payment of a congestion toll. The relative weights for defining the utility function governing the acceptance of congestion-pricing scheme are taken as:

$$(w_{121}, w_{211}, w_{102}, w_{112}, w_{122}, w_{202}, w_{212}, w_{222}, w_{103}, w_{113}, w_{123}, w_{203}, w_{213}, w_{223}) = \\ (0.11, 0.09, 0.11, 0.12, 0.13, 0.09, 0.10, 0.11, 0.12, 0.13, 0.14, 0.10, 0.11, 0.12)$$

where the subscript 0 represents the choice of non-travel; w_{123}, for instant, represents the weight for the indirect utility gain from all the other class 1 users choosing CBD 2 in the network. The sensitivity parameter ξ_m of equation (9) is taken as 0.02 and 0.01 for class 1 and class 2 users respectively. Given the above definitions of the problem, we proceed to apply the continuum approach in solving the cordon-based congestion-pricing models for cordon location and toll level selection based on the measures of social benefit, acceptability and equity. For the sake of simplicity in demonstrating the usefulness of these three indexes and the continuum approach in cordon design, the set of concentric circles of different radius from CBD 1 (as shown in Figure 1) will be taken as the feasible cordon shapes and locations for this numerical example. For each of the concentric circles, different toll levels will be considered for finding the optimal congestion-pricing design. Figure 2 and 3 are respectively the flow pattern and total travel cost of class 1 users traveling to CBD 1 for the no-toll scenario. These figures, which give the spatial variation of the variables over the modeling region, are typical outputs for the continuum modeling approach.

By tracing the flow vectors presented in Figure 2, the chosen paths of class 1 users traveling to CBD 1 could be found. In Figure 2, due to the rapid increase in the level of congestion in the proximity of CBD 1, the total travel cost increases more rapidly than in the rest of the region. Similarly, as the area around CBD 2 is highly congested by the traffic that patronizes it, the travel cost contours in Figure 3 is highly distorted in that area. In this paper, only figures for class 1 user traveling to CBD 1 in the no-toll scenario are included, as the solution for the other combinations of user classes, CBDs and charging schemes are quite similar.

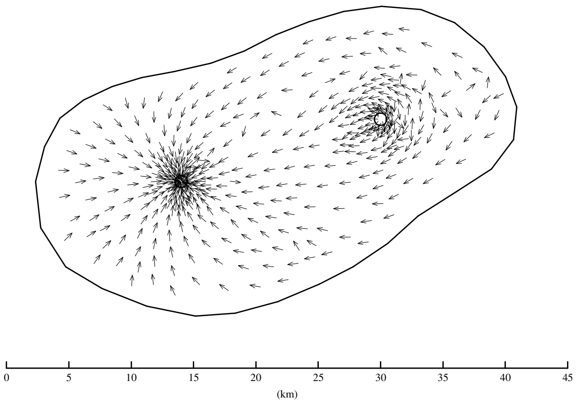

Figure 2 Flow pattern of class 1 users traveling to CBD 1 for no-toll scenario

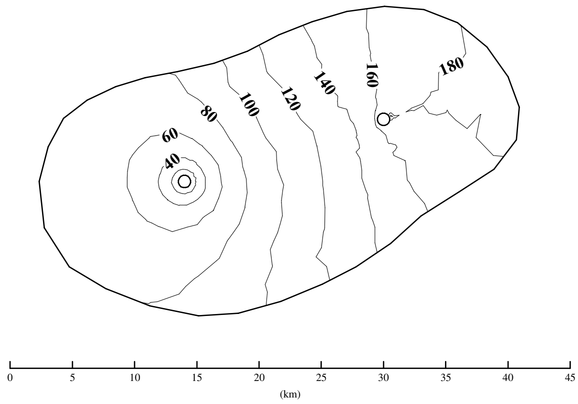

Figure 3 Total travel cost of class 1 users traveling to CBD 1 for no-toll scenario (HKD)

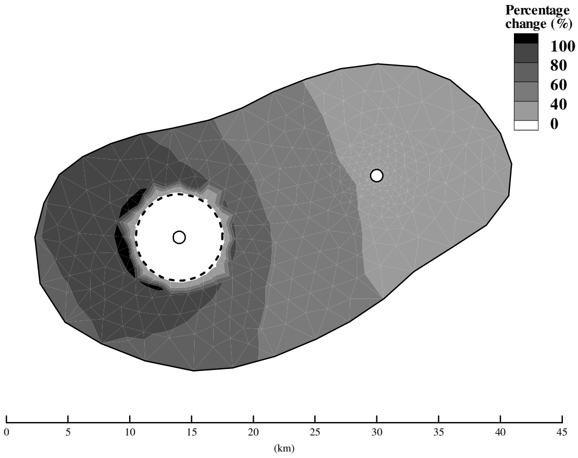

Figure 4 Percentage change of total travel cost for class 1 users traveling to CBD 1 under scheme of 3 km toll cordon radius and 70 HKD toll level (%)

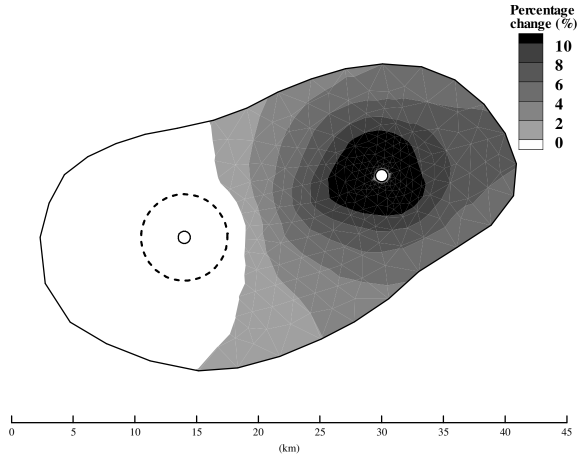

Figure 5 Percentage change of total travel cost for class 1 users traveling to CBD 2 under scheme of 3 km toll cordon radius and 70 HKD toll level (%)

Figure 4 and 5 respectively shows the percentage change of total travel cost, which compares with the no-toll scenario, for class 1 users traveling to CBD 1 and 2 for a cordon scheme of 3 km radius and 70 HKD toll level. Due to the similarities among different user classes and cordon schemes, only the above figures are shown and discussed in this paper. Note that the white contour in Figure 4 and 5 represent a negative percentage change in total travel cost (i.e. reduction in total travel cost). In Figure 4, it could be seen that there is a reduction in the total travel cost for the users located within the charging cordon. It is because users in that region, while enjoying a less congested road as the resulted from the congestion pricing-scheme, do not have to pay any toll for traveling to CBD 1. On the other hand, the total travel cost increases for the region outside of the charging cordon as users have to pay an extra toll for traveling to CBD 1. This percentage increase is diminishing as the distance from CBD 1 increases. It is because the total travel cost is higher for users that are further away from CBD 1 (Figure 3), thus the fixed toll charged will cause a smaller percentage increase in the total travel cost. Figure 5 shows a reduction in the total travel cost in the western part of the modeling region. In that area, since users traveling to CBD 2 have to pass through the vicinity of CBD 1, the implementation of congestion-pricing scheme, which reduces the congestion around CBD 1, could effectively reduces the total travel cost of that group of users. On the other hand, users from the vicinity of CBD 2 experience higher total travel cost due to an increase in the travel demand. The implementation of charging cordon increases the total travel costs to CBD 1, users within the region, especially those are out of the charging cordon, shift to travel to CBD 2.

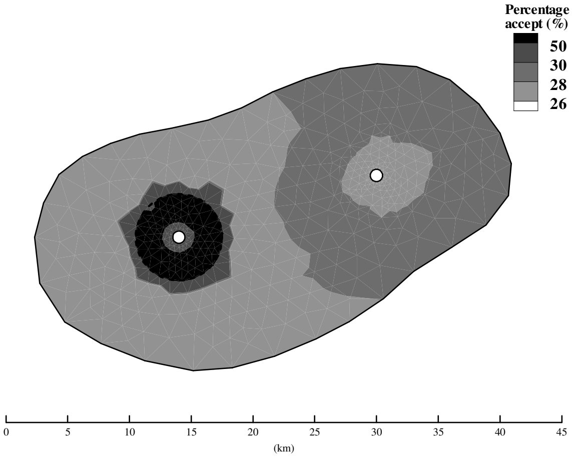

Figure 6 Acceptability of the scheme (3 km radius, 70 HKD toll level) for users choosing CBD 1

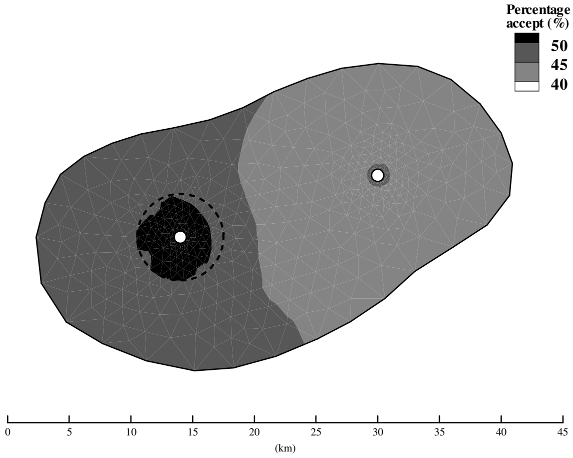

Figure 7 Acceptability of the scheme (3 km radius, 70 HKD toll level) for users choosing CBD 2

Figure 6 and 7 respectively shows the acceptability of users choosing CBD 1 and 2 on the implementation of the cordon scheme: 3 km cordon radius with 70 HKD toll level. In Figure 6, the majority of users located within the charging cordon have the highest level of acceptability (≥ 50%, but less than 53%) as they enjoy the less congested network resulted from the cordon-based pricing scheme without paying any toll charge. For the region just outside CBD 1 (i.e. distance traveled is less than 1 km), the acceptability decreases as the gain in utility from the less congested network is relatively small due to the short distance of travel and could not compensate the loss of utility from other groups of users considered in making the decision (Equation (8)). If the study region is a highly (or purely) egoistic society, this decrease in acceptability will not exist. Outside the charging cordon, the acceptability is relatively low (between 26% and 30%) as users have to pay and an extra toll for traveling from this area to CBD 1. Moving away from the charging cordon, the acceptability increase gently as the proportion of fixed toll (70 HKD) in the total travel cost is continuously decreasing. Lastly, in the vicinity of CBD 2, the acceptability decreases as this area becomes more congested as a result of the shift of destination choice after the implementation of congestion pricing.

In Figure 7, the highest level of acceptability (≥ 50%, but less than 55%) occurs within the charging cordon for the reason similar to users choosing CBD 1. Note that users traveling to CBD 2 are not tolled as they going outward of the charging cordon. Also in this figure, it could be seen that the acceptability gradually increases with the distance from CBD 2. This is due to the change in travel cost as shown in Figure 5. This explanation also applies for the region close to CBD 2 (i.e. distance traveled is less than 0.5 km) as the increase in congestion cost is negligible. The overall acceptability of this scheme (3 km cordon radius and 70 HKD toll level) is 30% for users choosing CBD 1 and 45% for users choosing CBD 2. Although in these two cases, users within the charging cordon have a higher acceptability (≥ 50%), but as they are the minority of the overall population (7.3% for the case of CBD 1 and 2.8% for the case of CBD 2) and their acceptability are not high (< 55%), the overall acceptability is less than 50%. Figure 4 to 7 demonstrate the application of the continuum modeling approach in finding the spatial variation of the change in total travel cost and acceptability upon the implementation of cordon-based congestion-pricing scheme. With the help of these figures decision makers could have a clearer picture on where the resources should be allocated in providing public transport for maintaining the mobility of the high travel cost area and public recognition campaign for improving the acceptability.

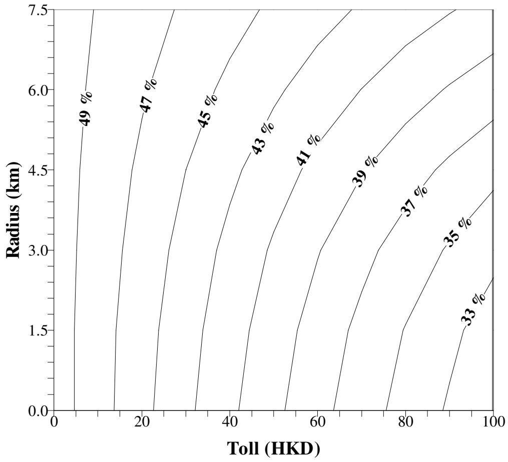

Figure 8 Plot of acceptability for different cordon locations and toll levels

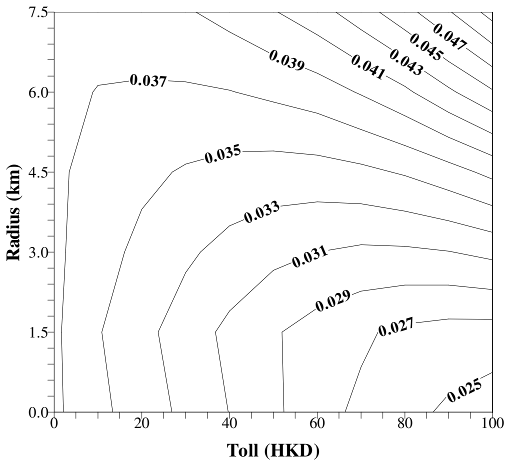

Figure 9 Plot of Theil entropy for different cordon locations and toll levels

As discussed before, by solving the cordon-based congestion-pricing model (11) for each combination of cordon location and toll level, the variations of acceptability, equity and social benefit among these combinations could be drawn. Figure 8 gives a contour plot of variation of acceptability for different cordon locations and toll levels. In Figure 8, the y-axis, which the toll level of the pricing schemes vanished, will have an acceptability of 50%. It is because in this case, the utility gains ($v_{m\hat{n}}$) of the tested pricing schemes are exactly the same as the no-toll scenario, based on the Logit distribution function defined in equation (9) half of the population will support the schemes. Considering the acceptability for a fixed toll level, which is represented by vertical lines in Figure 8, it could be seen that at low toll levels, which between 0 to 10 HKD, the variation of acceptability is quite small. It is because in these cases, the toll level is relatively low as compare to the total travel cost and has very limited effect on the acceptability. On the other hand, for high toll levels the acceptability increases as the radius increase for the same toll level. It is because as the radius of the cordon increases, the area of the non-tolled region will increase and thus the number of tolled users within the system will decrease. As the total travel costs of these non-tolled users will be less than that in the no-toll scenario (due to the travel time saving from the less congested network), their utility gain will be higher and resulted in a higher acceptability. This increase in acceptability becomes more rapid as the toll level increase for its increasing effect in reducing congestion.
Considering the acceptability for a fixed toll cordon radius, represented by horizontal lines in Figure 8, it could be seen that the acceptability decreases as the toll level increase. This decrease is mainly from the higher toll levels the tolled users have to pay. Besides, there will also be a decrease in the acceptability from users traveling to CBD 2 as the vicinity of CBD 2 is more congested due to the demand shift. Also as the cordon radius increase, the rate of

decrease of acceptability along the horizontal line decreases. This is due to the fact that the number of non-tolled users increases as the cordon radius increases.

Figure 9 gives a contour plot of the variation of Theil entropy, which is defined by equation (7) and is taken as a measure of equity, for different cordon locations and toll levels. Considering the Theil entropy for a fixed toll level, represented by vertical lines in Figure 9, the Theil entropy increases with the radius of cordon. This is because as the radius of the cordon increases, the number of non-tolled CBD 1 users also increases. As these non-toll users benefit from the less congested network, which is in the expense of the tolled users, the utility gain is becoming more unevenly distributed which results in a higher Theil entropy, or a less equitable system. Comparing the increase of the Theil entropy along the vertical lines for different toll levels, the increases are more rapid with the high toll levels as compare to that with the low toll levels. Such a difference could be explained by the direct relation between toll level and its inequity impact. For example, a higher toll level will cause a larger difference in the utility gains between tolled and non-tolled users and result in a less equitable system.

Considering the Theil entropy for a fixed toll cordon radius, which is represented by horizontal lines in Figure 9, although the variation is complicated, it could be explained by two conflicting effects of toll level on equity (Theil entropy). The first effect, which is denoted as the negative effect, is the direct relation between toll level and its strength in causing inequity as discussed in the previous paragraph. The second effect, which is denoted as the positive effect, represents the increase in equity resulted from the increase of demand in more equitable choices as toll level increases. In this case, as the toll level increases, some of the users, who originally choosing CBD 1, will shift to more equitable choices: non-travel or CBD 2. This will increase the equity of the system as these choices are less affected by the charging cordon than the choice of traveling to CBD 1. In Figure 9, for cordons with a small radius (≤ 2.0 km), the positive effect dominates as almost all the users traveling to CBD 1 are tolled. As a result, the Theil entropy decreases (increasing equity) as the toll level increase. In this case, the system is more equitable than the no-toll scenario as the overall spatial inequity is reduced due to more users choosing not to travel. For the cordons with a large radius (≥ 6.5 km), the positive effect diminishes as there is the comparable number of tolled and non-tolled users. However, for this case the system is more diverse in terms of utility gain and makes the negative effect to become dominant. As a result, the Theil entropy increases (decreasing equity) as the toll level increase. For cordons with radius between 2.0 km and 6.5 km, neither the positive nor the negative effects dominant. But as the negative effect is more favorable in the high toll level, the Theil entropy is first decrease in the low toll level and follow by an increase in the high toll level.

Figure 10 gives a contour plot of the variation of social benefit, which is defined by equation (6), for different combinations of cordon radius and toll levels. In low toll levels (0 ~ 50 HKD) of Figure 10, it could be seen that the social benefits increase evenly for all radius of cordons. This steady increase in the social benefit comes from the steady increase in toll revenues and savings in travel costs as the toll levels increase. For high toll levels (50 ~ 100 HKD), the system benefits could not be increased efficiently by increasing the toll levels with small cordons. It is because that in these cases the congestion around CBDs is the dominant issue in evaluating the system benefits. The small cordons could not effectively divert the traffic from the proximity of CBDs, system benefits could only be increased by increasing the radius of the cordon.

In this numerical example, the optimal congestion-pricing scheme, which includes radius of cordon and toll level, is selected by maximizing the social benefit subjected to the acceptability and Theil entropy (equity) constraints. For acceptability, a congestion-pricing scheme in this numerical example will only be selected if over 45% of users within the modeling region support the implementation of scheme. For Theil entropy, this numerical example accepts a scheme with more than 10% decrease in the Theil entropy of the no-toll scenario, which is 0.034 in this case. Taking these constraints and with the help of Figure 8 and 9, the feasible region of the congestion-pricing scheme, which satisfies the acceptability and equity constraints, is defined and marked as the shaded area in Figure 10. By confining the toll level to be a whole number, the optimal congestion-pricing scheme of 3 km radius and 26 HKD toll level is selected.

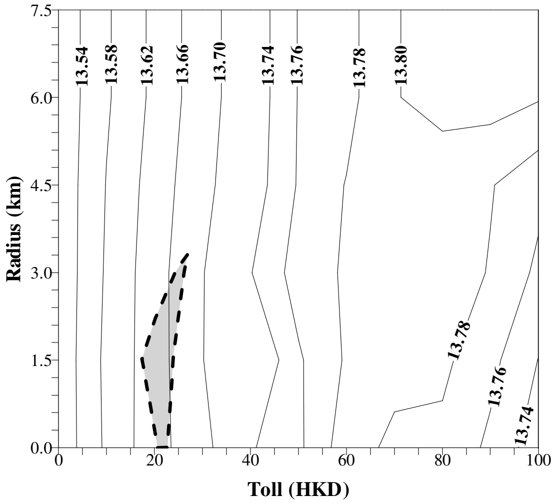

Figure 10 Plot of system benefit for different cordon locations and toll levels (million HKD)

Table 1 shows the comparison of this optimal scheme with the no-toll scenario. From Table 1, it could be seen that by implementing the optimal congestion-pricing scheme there is a 1.2% increase in social benefit and a 0.82 million HKD toll will be collected. Comparing the results of the optimal scheme with the no-toll scenario, it could be seen that both class 1 and 2 users have shifted from traveling to CBD 1 to either traveling to CBD 2 or even not to travel. This could be simply explained by the implementation of charging cordon around CBD 1 for increasing the travel cost for users traveling to this destination.

Table 1 Comparison of the scenario with optimal scheme and the no-toll scenario

	No-toll scenario	**Optimal scheme (Radius = 3.0 km, Toll = 26 HKD)**
Social benefit (HKD)	13,520,810	13,678,356
Acceptability (%)	---	45
Theil entropy	0.0355	0.0338
Toll received (HKD)	---	817,126
Class 1 users		
Percentage choosing CBD 1 (%)	55.0	51.0
Percentage choosing CBD 2 (%)	20.5	22.2
Percentage choosing not travel (%)	24.5	26.8
Class 2 users		
Percentage choosing CBD 1 (%)	47.8	45.1
Percentage choosing CBD 2 (%)	22.1	23.1
Percentage choosing not travel (%)	30.1	31.8

The optimal scheme that found from Figure 10 is correspondence to the equity, acceptability and social benefit indexes defined in section 2. Although, in general, these indexes are the most appropriate/direct measurements of the corresponding area, but owning to the different focuses (e.g. environmental justice) of decision makers and the specific social/economical environment of the study region (e.g. a highly egoistic/altruistic society), different indexes may be adopted. For example, equity measure could be based on environmental improvement instead of utility improvement. By using the same approach described in this paper, such different indexes may result in a different, or even conflicting, optimal scheme. For example, if users put a much larger weight in social perspective than the selfish perspective, the variation of acceptability for different congestion-pricing schemes will be similar to that for the system benefit. As a result, the optimal toll level for this case will be much higher than that of the original one, say 70 HKD.

5. CONCLUSIONS

We have introduced the acceptability and equity measures in cordon design problem for a continuum transportation system with multiple user classes and CBDs. In this study, the acceptability and equity measures serve as constraints for defining the feasible solutions of the cordon-based schemes, while the optimal scheme is found by maximizing the social benefit of the system. A cordon-based congestion-pricing model for multiple user classes and Logit type destination choice has been developed as the base model for evaluating different feasible congestion-pricing schemes. This developed model was solved by using an efficient finite element method and a promising Newtonian-based solution algorithm. Numerical example has been set for finding the optimal radius and toll level from a set of concentric circular cordons from one of the CBD within the modeling region. The numerical example demonstrates the usefulness of the proposed approach in: 1) finding the spatial variation of acceptability and change of travel cost under congestion pricing; 2) finding the variation of equity, acceptability and social benefit for different cordon sizes and toll levels, and; 3) finding an equitable, acceptable and socially beneficial cordon-based congestion-pricing scheme.

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