yard will usually forecast how many containers would like to loading and plan the appropriate space for them to use in advance.

As containers which would be transported by the same ship will be stacked together, so while containers are preparation for retrieving, if a stack of containers will be transported by the same ship, it would be impossible to stack another new container right after retrieving one container, yet it should wait for all stacks of container retrieval finished. However, if the top of target container will be transported by different ship, compared with the below containers, the new container could be stacked instantly after retrieving. Only the numbers of retrieval containers are greater than the numbers of queuing containers, the queuing phenomenon could be radically eliminated.

Supposed after passing through control station, the averagely handling time spent (retrieving or stacking) for each container would be *T*. Additionally, the ship-loading sequence is starting from around trailer area by near to far. If there are the numbers of *N* containers queuing outside the control station, the total queuing time spent for the *N*-th container would be equal to the time spent for retrieving the number of *Q* containers plus the time spent for stacking the number of (*N*-1) containers. Therefore, the total spending time for retrieving is $T \cdot \sum_{i=1}^{N} Q_i$, and the total spending time for entering container yard is $T \cdot (N-1)$, so the total queuing time spent is $T \cdot \sum_{i=1}^{N} Q_i + T \cdot (N-1) = T \cdot \left(\sum_{i=1}^{N} Q_i + N - 1 \right)$. Where Q_i is defined as the total number of *i*-th row of containers which require retrieving.

During the span of queuing time from the starting point (t_q) to the ending point ($t_{q'}$), there are the number of *N* containers queuing outside control station because of waiting for entry the container yard. Assumed the total queuing time spent would be θ, which could be calculated form the following formula, shown on equation (11).

$$t_{q'} - t_q = T \cdot \left(\sum_{i=1}^{N} Q_i + N - 1 \right) = \theta \tag{11}$$

According to the formula of on-time arrival mode: $\tilde{t} + T_Q(\tilde{t}) + T_Y = t^*$, the following two equations could be obtained from Figure 1.

$$\tilde{t} + \frac{\beta}{\alpha - \beta} \cdot (\tilde{t} - t_q) + T_Y = t^* \tag{12}$$

$$\tilde{t} + \frac{-\gamma}{\alpha + \gamma} \cdot (\tilde{t} - t_{q'}) + T_Y = t^* \tag{13}$$

With regard to the equations from (11) to (13), they could help to calculate the three respective time spots, i.e. $\tilde{t}$, t_q and $t_{q'}$, under the equilibrium situation prior to execution of toll collection. These three time values are shown as below:

$$\tilde{t} = t^* - \frac{\beta\gamma}{\alpha(\beta + \gamma)} \cdot \theta - T_Y \tag{14}$$

$$t_q = t^* - \frac{\gamma}{\beta + \gamma} \cdot \theta - T_Y \tag{15}$$

$$t_{q'} = t^* + \frac{\beta}{\beta+\gamma} \cdot \theta - T_Y \qquad (16)$$

Under the equilibrium status, the total cost for all shippers are equivalent. Substituting the obtained values from the equations (14) to (16) into equations (5) to (7), the equilibrium cost (TC^e) for all shippers can be obtained as $\frac{\beta\gamma}{\beta+\gamma} \cdot \theta$. Figure 2 shows the relation between containers' (shippers') arrival time and their costs. The red thin lines of $(t_q, \tilde{t})$ and $(\tilde{t}, t_{q'})$ represents the queuing time cost ($\alpha \cdot T_Q(t)$) for the early and late arrivals, respectively, while the blue thick line signifies the time cost of early arrival ($\beta \cdot T_E(t)$), and the green thick line symbolises the time cost of late arrival ($\gamma \cdot T_L(t)$). In the light of early arrival mode ($t_q \le t < \tilde{t}$), the sum of red line and blue line costs represent the equilibrium cost (TC^e). On the contrary, in the late arrival mode ($\tilde{t} < t \le t_{q'}$), the equilibrium cost (TC^e) could be calculated by the sum of red line and green line. Moreover, the conditions of on-time arrival mode do not have to count the early arrival cost as well as late arrival cost, yet it has to burden the maximum queuing time spent.

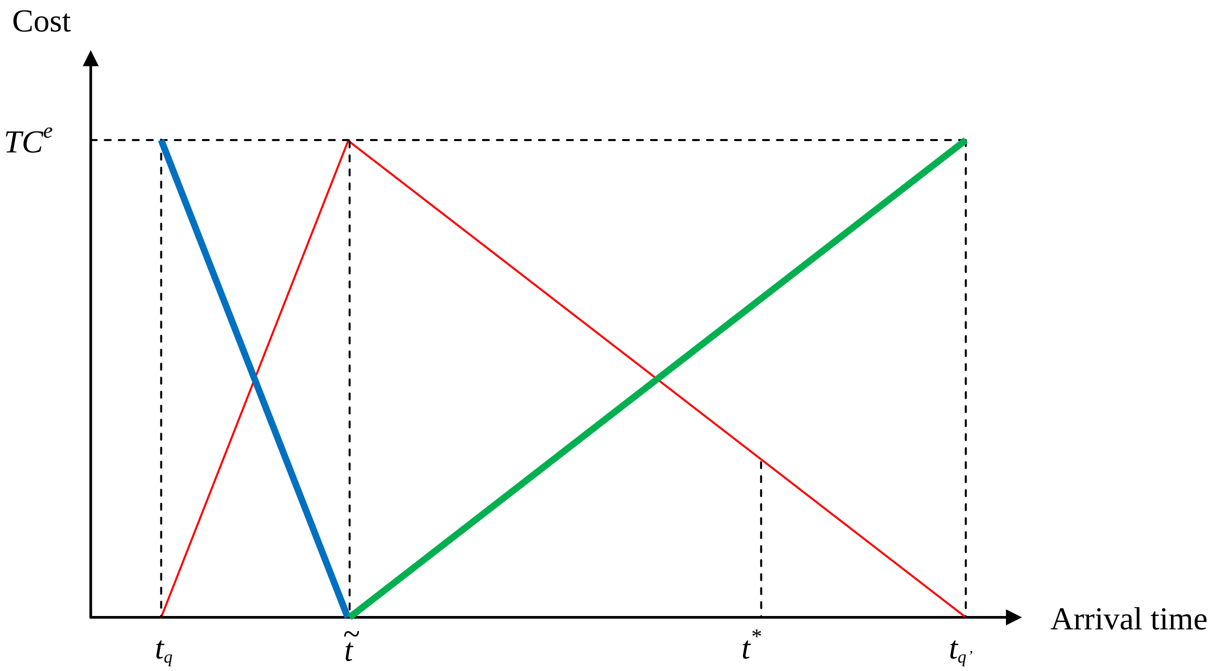

Figure 2. The Queuing Time, Early and Late Costs through the Queuing Period

3.2 The optimal non-queuing toll scheme and arrival rates

The non-queuing pricing model is established under the concept of charging the variable tolls in compliance with the different arrival time so as to completely substitute the queuing time cost of containers and further eliminate all the queuing time spent. Based on the rule of "conservation of equilibrium cost", the equilibrium cost (TC^e) is supposed to remain the equivalent amount whether administering toll collection or not. Therefore, the two important prerequisites of $T_Q(t)=0$ and $TC(t)=TC^e$ have to be fulfilled after execution of non-queuing toll collection. In view of the above, the equations from (5) to (7) could be reworded, and the non-queuing toll collection ($\Omega(t)$) could also be obtained, which are presented as follows:

Early Arrival:

$$TC(t)=\beta\cdot T_E(t)+\Omega(t)=TC^e$$
$$\Omega(t)=TC^e-\beta\cdot(t^*-t-T_Y) \tag{17}$$
$$t_q\le t<t^*$$

On-Time Arrival:

$$TC(t)=\Omega(t)=TC^e$$
$$\Omega(t)=TC^e \tag{18}$$
$$t=t^*$$

Late Arrival:

$$TC(t)=\gamma\cdot T_L(t)+\Omega(t)=TC^e$$
$$\Omega(t)=TC^e-\gamma\cdot(t+T_Y-t^*) \tag{19}$$
$$t^*<t\le t_{q'}$$

As seen in Figure 3, $\triangle t_q a t_{q'}$ would be the queuing time cost generated before tolling, while $\triangle t_q b t_{q'}$ would be the optimal non-queuing toll scheme, which would be continuously variable corresponding to different arrival time (*t*). As the two triangles are congruent, it could be speculated that the optimal non-queuing toll collection could wholly substitute the total queuing time cost before tolling execution. Under the non-queuing toll scheme, the range before point t^* is defined as the early arrival interval after toll collection, and the slope of $\overline{t_q b}$ could be indicated as β from equation (17). Furthermore, the range after t^* could be represented as the late arrival interval after toll collection, and the slope of $\overline{bt_{q'}}$ could be viewed as $-\gamma$ in terms of equation (19). Therefore, t^* could be regarded as on-time arrival, even though the sum costs of early arrival and late arrival could be excluded, the tolling amount required to pay would be the highest among three different modes.

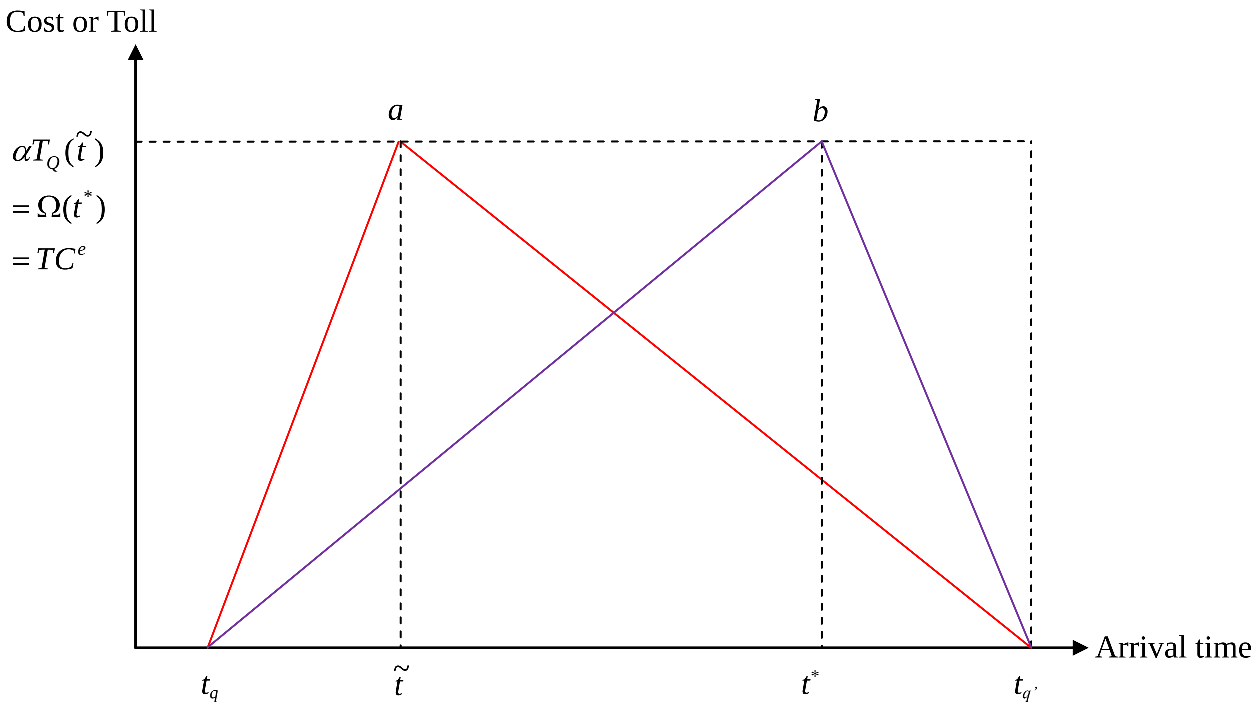

Figure 3. The Queuing Time Cost and Optimal Non-Queuing Toll Scheme

Supposed that during the queuing time periods, the average hourly arrival rate of containers is *s*, then the arrival rate of variable containers could be expressed as the marginal arrival rate $\frac{d(s \cdot T_Q(t))}{dt}$. Thus, before toll collection, the marginal arrival rate for early arrival periods ($t_q \le t < \tilde{t}$) would be $\frac{\beta \cdot s}{\alpha - \beta}$, and its total arrival rate would be indicated as $\frac{\alpha \cdot s}{\alpha - \beta}$ $(= s + \frac{\beta \cdot s}{\alpha - \beta})$. On the other hand, the marginal arrival rate for late arrival periods ($\tilde{t} < t \le t_{q'}$) before tolling is $\frac{-\gamma \cdot s}{\alpha + \gamma}$, so its total arrival rate could be calculated as $\frac{\alpha \cdot s}{\alpha + \gamma}$ $(= s + \frac{-\gamma \cdot s}{\alpha + \gamma})$. Finally, since $T_Q(t)$ is equal to zero through the queuing period ($t_q \le t \le t_{q'}$), the total arrival rate under the optimal non-queuing toll scheme would always be *s*.

4. NUMERICAL EXAMPLE

Assumed that there have 75 containers (*N*) intended to enter the container yard to stack during the same loading periods, while they have to queue outside the control station because of the limited capacity of yard. All of the queuing containers will be handled under the principle of "first come first served". As the capacity of container yard has been saturated, the total number of containers that need to be retrieved is $\sum_{i=1}^{N} Q_i = 75$. Further assumed that the average handling time spent (*T*) for per container in the container yard would be 15 minutes, so the total queuing time spent would be equal to the time spent for retrieving 75 containers plus the time spent for stacking 74 containers. Therefore, the sum of queuing time spent would be $T \cdot \sum_{i=1}^{N} Q_i + T \cdot (N-1) = 15 \cdot 75 + 15 \cdot 74 = 2235$ minutes, i.e., $t_{q'} - t_q = 37.25$ hours.

Take Keelung port container yard, located in Taiwan, for the investigation example, the unit queuing time cost (α) of containers which wait at the outside of control station have included driver salaries, management fees for container car, fuel costs, maintenance and components costs, depreciation of container car, insurances, and taxes upon license and fuel; accordingly, the total estimated amount would be around NT$ 371.97461 per hour (NT$:US$=30:1). If a container has arrived earlier and has earlier entered the container yard for stacking, it would further add some additional fees, involving rental for using yard and insurances against stacking containers in the yard, so its unit time cost (β) of early arrival is estimated about NT$ 65.55 per hour. On the other hand, if containers will be late to the yard which will also lead to delay for entry, the loading cannot be successfully. Therefore, it will need to arrange for transhipment or wait for the next scheduled departure, so its unit time cost of late arrival (γ) have to include freight forwarding for export containers, container demurrage, and the added rental for using container, which total fees could be computed to NT$ 630.44 per hour. In order to facilitate the calculation, we hypothesised that the containers would start to queue at 00:00, and the queuing would be finished at 37:15 after the 75 containers had totally entered the yard. In practice, the actual loading time could be backward adjusted, while the results would not be affected. The queuing pricing model for the port container yard in this research is based on the above values to calculate the following outcomes:

$t_{q'} - t_q = \theta = 37.25$ hours, $t_q = 0 = 00:00$, $t_{q'} = 37.25 = 37:15$.

$$\tilde{t} = \frac{TC^e}{\frac{\alpha\beta}{\alpha-\beta}} = \frac{\frac{\beta\gamma}{\beta+\gamma}\cdot\theta}{\frac{\alpha\beta}{\alpha-\beta}} = 27.7957 \text{ hours.}$$

$$t^* = 37.25 - \frac{TC^e}{\gamma} = 37.25 - \frac{\frac{\beta\gamma}{\beta+\gamma}\cdot\theta}{\gamma} = 33.7417 \text{ hours.}$$

In terms of the above computation, containers would start to queue outside the control station for entry at 00:00 (the first day), and the queuing would be ended at 37:15 (the second day) after all containers entering the container yard. Take the upper half of Figure 4 for example, the blue triangle (with the base from $t_q = 0$ to $t_{q'} = 37.25$, and the apex $TC^e = 2211.7688$ located on $\tilde{t} = 27.7957$) shows the equilibrium queuing time cost before toll execution. The blue lines of $(t_q, \tilde{t})$ and $(\tilde{t}, t_{q'})$ represents the queuing time cost for the early and late arrivals, respectively. On the other hand, the green triangle (with the same base as the blue triangle, and the apex $TC^e = 2211.7688$ located on $t^* = 33.7417$) represents the optimal non-queuing toll scheme. The green lines of (t_q, t^*) and $(t^*, t_{q'})$ represents the tolls that should be paid by the early and late arrivals, respectively. After executing the optimal non-queuing toll scheme, all of the containers' arrival time will be effectively dispersed at the entrance of port container yard, so the phenomenon of queuing will no longer exist. Furthermore, take the lower half of Figure 4 for example, the two blue solid lines indicate the container's arrival rates ($\frac{s\alpha}{\alpha-\beta}$ =2.44410 and $\frac{s\alpha}{\alpha+\gamma}$ =0.74713 for the early and late arrivals, respectively) before tolling. While the green horizontal solid line represents the ship's arrival rate (s=2.0134) after implementing the optimal non-queuing toll scheme. Therefore, the numbers of the total early arrivals and total late arrivals of containers are equal to 67.9355 and 7.0645, respectively.

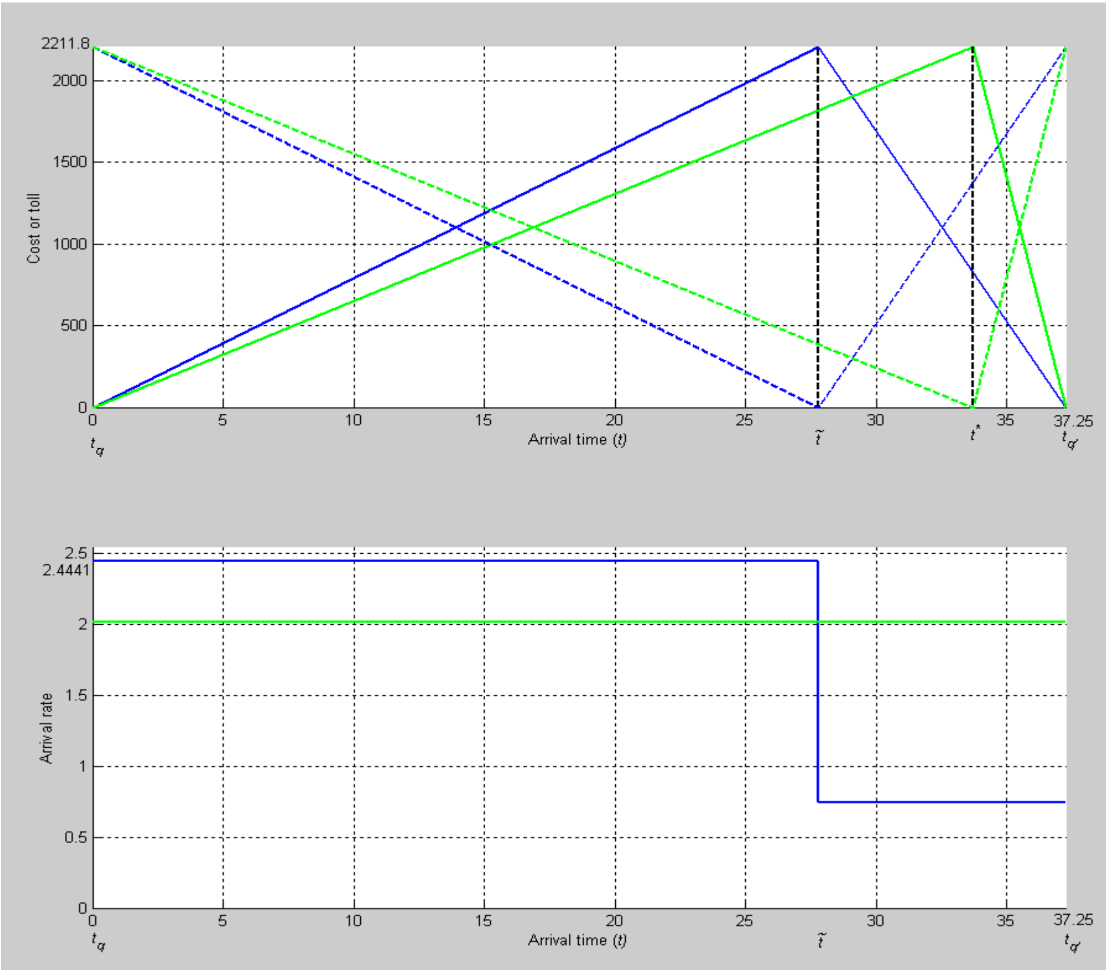

Figure 4. All Results in the Numerical Example

5. CONCLUSION

As the capacity of port container yard has been saturated, there has no any empty space inside the yard to use. Containers arrived the control station at this saturated moment should queue outside in sequence for waiting available space. Such queuing phenomenon would be reasonably eliminated by using the queuing pricing model to disperse containers' arrival time. This research is on the basis of "conservation of equilibrium cost" to compute the optimal non-queuing toll scheme, in compliance with the different arrival time charging for the relative tolls. This model could radically eliminate the queuing phenomenon and facilitate the using efficiency of container yard. With regard to shippers, it could be effective in saving the time cost of transportation; in relation to managers of container yard, it could enhance the operational efficiency toward the container yard in order to attract more shippers who are willing to make use of it. Consequently, the non-queuing toll scheme could help to achieve win-win objective no matter for consignors or for managers. In practice, we suggest that the policy of queuing pricing model would be suitable for using on shipping season. As the overall volume of containers on the peak period will be greater than the dull season, so if the scale of container yard is not large enough, it could easily bring about the long queuing phenomenon outside the yard for export containers to enter due to its restricted capacity.

The numerical example has computed the respective unit time cost of queuing periods, early and late arrival periods. These three significant parameter values have determined each container's equilibrium cost, the optimal non-queuing toll scheme, arrival rates, and the consequence of dispersing containers' arrival time after toll collection. Consequently, the results of numerical example could provide a useful reference for the port container yard manager to consider the implementation of queuing pricing.

REFERENCES

Arnott, R.J., De Palma, A., Lindsey, R. (1990) Economic of a bottleneck. *Journal of Urban Economic*, 27, 111-130.

Braid, M. (1989) Uniform versus peak-load pricing of a bottleneck with elastic demand. *Journal of Urban Economic*, 26, 320-327.

Kim, K.H., Kim, K.Y. (1999) An optimal routing algorithm for a transfer crane in port container terminals. *Transportation Science*, 33(1), 17-33.

Kim, Y.H., Park T., Ryu K.R. (2007) Dynamic weight adjustment for developing a stacking policy for automated container terminals. Paper presented at the international conference on intelligent manufacturing and logistic systems (IML 2007). Kitakyushu, Japan, 26-28.

Laih, C.H. (1994) Queuing at a bottleneck with single-and multi-step tolls. *Transportation Research*, 28(A), 197-208.

Laih, C.H. (2004) Effects of the optimal step toll scheme on equilibrium commuter behavior. *Applied Economics*, 36(1), 59-81.

Laih, C.H. (2007) Effect of the optimal port queuing pricing on arrival decision for container ships. *Applied Economics*, 39, 855-1865.

Laih, C.H., Chen, K.Y. (2008) Economics on the optimal *n*-step toll scheme for a queuing port. *Applied Economics*, 40, 209-228.

Pettitt, T. (2007) Will a national port regulator solve congestion problems in Australian ports? *Maritime Policy and Management*, 34(2), 121-130.

Sen, P. (1980) Optimal priority assignment in queues: application to marine congestion problems. *Maritime Policy and Management*, 7(3), 175-184.

Stahlbock, R., Voβ, S. (2008) Operations research at container terminals: a literature update. *OR Spectrum*, 30, 1-52.

Steenken, D., Voβ, S., Stahlbock, R. (2004) Container terminal operation and operations research—a classification and literature review. *OR Spectrum*, 26, 3-49.

Taleb-Ibrahimi, M., De Castilho, B., Daganzo, C. F. (1993) Storage space vs. handling work in container terminals. *Transportation Research*, 27(B), 13-32.

Vickrey, W.S. (1969) Congestion theory and transport investment. *American Economic Review*, 59(2), 251-260.