

## **Integrated Shipment Routing and Container Booking Model**

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**Abstract:** Sea freight forwarders provide transportation services for less-than-container-load shipment customers by simultaneously routing shipments and placing them in containers booked for travel on sea liner networks. A shipment may be (i) sent directly to the destination for better service, (ii) sent via a transfer hub to allow load consideration for cost saving purposes, or (iii) transferred to partners when it is deemed necessary or beneficial from the financial perspective. This research proposes linear mixed integer programming models that integrate shipment routing and container booking decisions. The models take into account the risks of shipment damage during transfer and keep them below a pre-specified threshold. We propose two solution algorithms for solving these large-scale models and find that they are able to solve larger instances of the problem unsolvable by the traditional branch-and-bound algorithm. Our solutions consistently outperform the solutions from sequential planning by a large margin.

*Keywords:* Shipment Routing, Container Booking, Service Planning Optimization

### **1. INTRODUCTION AND RELATED LITERATURE**

Sea freight forwarders provide transportation services for less-than-container-load shipment customers by simultaneously routing shipments and placing them in containers booked for travel on sea liner networks. There are often several routes on which a shipment can be sent. A shipment may be sent directly to the destination for better service or sent via a transfer hub to allow load consideration for cost saving purposes. Furthermore, freight forwarders may decide to transfer shipments to partners when they deem necessary or profitable.

Majority of the works on ocean transportation found in the literature are related to the planning and operation of ship operators (e.g., tramps and liners), which typically focuses on ship routing, ship scheduling, empty container, or network design. Ronen (1982), Ronen (1993), and Christiansen, Fagerholt, and Ronen (2004) give a complete perspective and overview of ship routing and scheduling over a period of three decades. For liner network design, see Agarwal and Ergun (2008). An excellent comprehensive review can be found in Christiansen, et. al. (2007).

Shipment routing problem appears routinely in other modes of transportation. See for example, Root and Cohn (2008) for land transportation, Armacost, Barnhart, and Ware (2002) for air transportation, and Kim and Barnhart (1997) for multimodal transportation.

These are complicate decisions that are often done manually in sequential order by the planner. Sequential planning, however, has a major drawback in that the resulting operational plan may result in inefficient use of container's space and thus higher costs because the routing and consolidation decision is made prior to the container booking decision and without *accurate* knowledge of the cost involved.

Any attempt to incorporate the container booking costs into the shipment routing and consolidation decision is *approximate* at best because of two reasons. First, the costs of each link are not linearly associated with the volume on each link, but are derived by the number of containers required on each link. This observation is further complicated by the second fact that there are interdependencies among available links in the network because individual shipments can be routed in a number of ways to their destinations. Lesser volume on one link will mean higher volume on other links. Thus, making routing decision for each individual shipment is inherently inappropriate. These two reasons make the resulting *approximate* cost function for each link takes on a complicated piecewise-like relationship with the volume of shipments on the link.

To the best of the authors' knowledge, no literature can be found that specifically address the integrated shipment routing and container booking problem for ocean freight forwarders. Ang, Cao, and Ye (2007) present multi-period sea cargo mix problem, which addresses the mixing of cargos on different routes but does not consider discrete container booking decision. Xue and Lai (1997) and Wu (2008) address a relatively similar problem to ours but for air cargo forwarders. The differences from our problem are that the cost structure for air cargo is piece-wise linear, but their scope is only limited to one pair of origin and destination. Thus there is no element of shipment routing in these works. The authors present a method to incorporate this piece-wise linear cost function into the mixed integer program proposed. The authors demonstrate the model with very small data sets to show proof of concept

Further complication may arise when we incorporate other considerations such as risks of damage to shipments during sorting at hub ports.

For these reasons, we propose linear programming network flow models that integrate shipment routing and container booking decisions. The models take into account the risks of shipment damage during transfer and keep them below a pre-specified threshold.

We propose two solution algorithms for solving these large-scale models, namely, a two-phase solution method and an incremental solution method and test the models using real-life data from a freight forwarder in Thailand.

In this paper, the authors make the following contributions:

1. propose the mathematical formulation for the integrated shipment routing and container booking problem,
2. propose two solution algorithms for solving the integrated shipment routing and container booking model,
3. implement and test the model using real data from a major forwarder in Thailand to demonstrate its solvability of the solution algorithms and their practical applicability, and
4. analyze the results and provide insights into the savings achieved by the model.

## 2. PROBLEM DESCRIPTION AND STATEMENT

To give more details and understand the problem clearly, consider an illustrative network for a sample forwarder in Figure 1, in which there are one origin port (O), two transshipment ports ( $H_1$  and  $H_2$ ), and two destination ports ( $P_1$  and  $P_2$ ). Three types of arrow represent three types of service for this network as shown in Figure 2. If there is no arrow between a pair of ports, there is no service between the two ports.

For a shipment from O to  $P_1$ , the forwarder can route the shipment on the transshipment service O- $H_1$ - $P_1$  or send via partner's service. For a shipment originated from O and destined for  $P_2$ , the forwarder can route the shipment on the direct O- $P_2$  route or via any one of the transshipment ports using the O- $H_1$ - $P_2$  or O- $H_2$ - $P_2$  routes. In term of container booking,

the forwarder has to decide how many and what type containers to be used *only* on each direct link

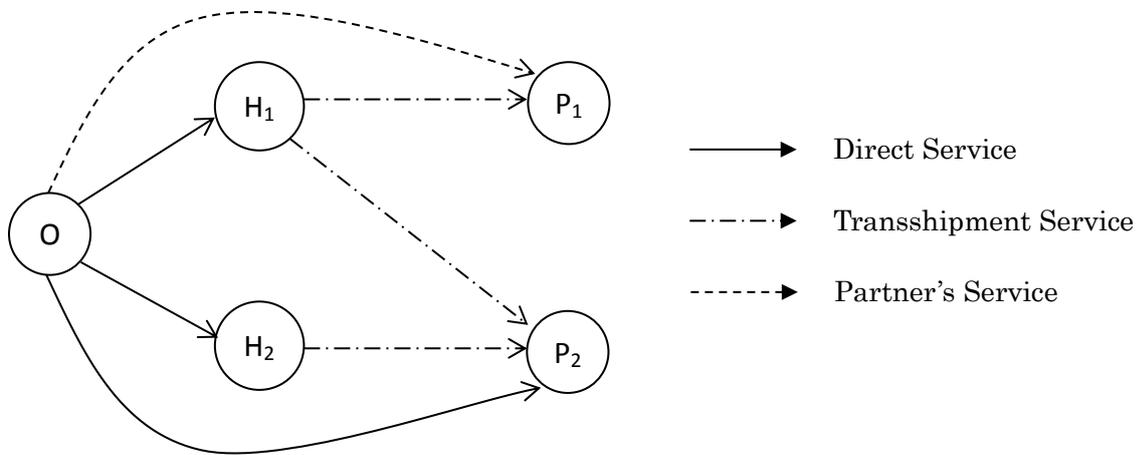


Figure 1. Illustrative Network

to destination. Any shipments that are routed through transshipment ports or sent using partner's services will be charged on a per unit basis (weight or volume depending on shipment characteristics). Note that the costs on transshipment services going through different transshipment ports are not necessarily equal.

From the forwarder's perspective, the planner has to route those shipments to their destinations using the available services but the available capacity on each direct link depends on the number and type of containers booked, which directly affects the costs of operation for the forwarder.

Another aspect to consider while planning the shipment routing is the risk of damages during transportation and transfer. We take this into account by assigning different probability values that the damage may occur to different route types. Specifically, we use parameter  $\beta^x$  and  $\beta^y$  to represent the probabilities of damages from direct shipping and transshipping via hub, respectively. Figure 2 demonstrates this idea with  $\beta^x = 0.02\%$  and  $\beta^y = 0.07\%$ . In the model, we keep the average risk probability to a target value,  $\beta^t$ .

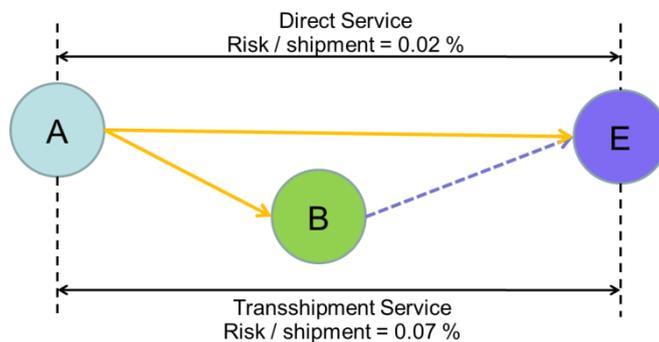


Figure 2. Risk of Damages

We formally state the problem statement for the Integrated Shipment Routing and Container Booking Problem as follows:

*Given a set of shipping orders specifying destination ports and associated revenue, available direct, transshipment, and partner services, associated damage probabilities, and container booking costs for a given period, find the cost minimizing transportation plan detailing routing for individual shipments and number and type of containers to be booked on each direct service, such that all shipping demands are satisfied, damage probability target is kept, and container capacities are honored.*

In our model, we make the following assumptions:

1. Shipping orders are smaller than the largest container size available and shipments are not separable, i.e., forwarders cannot break individual shipment into smaller shipments and shipped them on separate containers.
2. Container loading can be achieved in operation.

The first assumption reflects actual operation where shipments are rarely broken. For shipments larger than one container, shippers would be better off shipping them on a full container load (FCL) service. Assumption 2 has to do with the Container Loading Problem, in which the optimal sorting of shipments in a container is determined. (Readers in interested in this aspect are referred to Vis and Koster, 2003) We do not consider container loading problem in this paper but in order to ensure feasible loading in operation we give certain allowance to the capacity of a container. That is, we consider effective capacity of a container to be some fractions (normally 90-95%) of the full capacity.

### 3. MATHEMATICAL MODEL

#### 3.1 Notations

##### Sets

- $O$  is the set of shipping demands, indexed by  $o$ .  
 $\hat{O}$  is the set of mandatory shipping demands that cannot be rejected,  $\hat{O} \subset O$ .  
 $T$  is the set of transshipment ports, indexed by  $t$ .  
 $D$  is the set of destination ports, indexed by  $d$ .  
 $K$  is the set of container types, indexed by  $k$ .  
 $N_d^k$  is the ordered set of container type  $k$  going to destination port  $d$ , indexed by  $n$ .

##### Parameters

- $c_d^k$  is the cost of booking container of type  $k$  for use in service to destination port  $d$ .  
 $c_o^t$  is the cost of servicing demand  $o$  through transshipment port  $t$ .  
 $c_o$  is the cost of servicing demand  $o$  through partner service.  
 $r_o^x$  is the revenue of servicing demand  $o$  using direct service.  
 $r_o^y$  is the revenue of servicing demand  $o$  using transshipment service.  
 $r_o^z$  is the revenue of servicing demand  $o$  using partner's service.  
 $v_o$  is the volume of demand  $o$ .  
 $w_o$  is the weight of demand  $o$ .  
 $V^k$  is the effective volume capacity of type- $k$  container.  
 $W^k$  is the effective weight capacity of type- $k$  container.  
 $\delta_{do}$  equals to 1 if the destination of demand  $o$  is port  $d$ , 0 otherwise.  
 $\beta^x$  is the damage probability for direct service (0.02% unless specified otherwise).

- $\beta^y$  is the damage probability for transshipment service (0.07% unless specified otherwise).  
 $\beta^t$  is the maximum allowable damage probability (0.05% unless specified otherwise).  
 $\mu_d$  is the total allowable volume of shipments diverted to partner service for a destination  $d$ .  
 $M$  is a sufficiently big number.

### Variables

- $x_{do}^{kn}$  equals to 1 if demand  $o$  is shipped on the  $n^{\text{th}}$  type- $k$  container going to destination port  $d$ , 0 otherwise.  
 $y_o^t$  equals to 1 if a transshipment service is used to ship demand  $o$  through transshipment port  $t$ , 0 otherwise.  
 $z_o$  equals to 1 if a partner service is used to ship demand  $o$ , 0 otherwise.  
 $q_d^{kn}$  equals to 1 if the  $n^{\text{th}}$  type- $k$  container to destination port  $d$  is used, 0 otherwise.

If variable  $x_{do}^{kn}$  equals 1, it specifies that the demand  $o$  is shipped on the  $n^{\text{th}}$  container of type  $k$  going to destination  $d$ . Note, however, that the first leg of the transshipment service will also utilize these variables  $x_{do}^{kn}$  in order to ship from the origin port to the transshipment port on a booked container. Note further that  $n \in N_d^k$  serves as a running index in the ordered set  $N_d^k$ , which specifies container number 1, 2, 3, and so on. In the implementation, we specify greater number of containers than needed, but only necessary number of containers will be utilized. Note also that the index  $d$  specifies the destination port of that leg of transportation. Thus, if it is a transshipment routing, the indices  $d$  and  $o$  (destination port of the demand) can have different values.

The variable  $y_o^t$  likewise performs both the service type assignment (transshipment service) and the transshipment port assignment ( $t$ ) for the demand  $o$ .

### 3.2 Model Formulation

The *Integrated Shipment Routing and Container Booking Problem (ISRCB)* is formulated as follows:

$$\text{Maximize } z = R - C \quad (1)$$

Subject to

$$R = \sum_{o \in O} \left[ \sum_{d \in D} \sum_{k \in K} \sum_{n \in N_d^k} r_o^x \delta_{do} x_{do}^{kn} + \sum_{t \in T} r_o^y y_o^t + r_o^z z_o \right] \quad (2)$$

$$C = \sum_{d \in D} \sum_{k \in K} \sum_{n \in N_d^k} c_d^k q_d^{kn} + \sum_{o \in O} \sum_{t \in T} c_o^t y_o^t + \sum_{o \in O} c_o z_o \quad (3)$$

$$\sum_{d \in D} \sum_{k \in K} \sum_{n \in N_d^k} \delta_{do} x_{do}^{kn} + \sum_{t \in T} y_o^t + c_o z_o = 1 \quad \forall o \in \hat{O} \quad (4)$$

$$\sum_{d \in D} \sum_{k \in K} \sum_{n \in N_d^k} \delta_{do} x_{do}^{kn} + \sum_{t \in T} y_o^t + c_o z_o \leq 1 \quad \forall o \in O \setminus \hat{O} \quad (5)$$

$$\sum_{o \in O} v_o x_{do}^{kn} - V^k q_d^{kn} \leq 0 \quad \forall d \in D, \forall k \in K, \forall n \in N_d^k \quad (6)$$

$$\sum_{o \in O} w_o x_{do}^{kn} - W^k q_d^{kn} \leq 0 \quad \forall d \in D, \forall k \in K, \forall n \in N_d^k \quad (7)$$

$$q_d^{kn} - q_d^{k(n+1)} \geq 0 \quad \forall d \in D, \forall k \in K, \forall n \in N_d^k \quad (8)$$

$$\sum_{k \in K} \sum_{n \in N_d^k} x_{to}^{kn} - y_o^t = 0 \quad \forall o \in O, \forall t \in T \quad (9)$$

$$\sum_{o \in O} \delta_{do} v_o z_o \leq \mu_d \quad \forall d \in D \quad (10)$$

$$\sum_{o \in O} \left[ (\beta^x - \beta^t) \sum_{d \in D} \sum_{k \in K} \sum_{n \in N_d^k} \delta_{do} x_{do}^{kn} + (\beta^y - \beta^t) \sum_{t \in T} y_o^t \right] \leq 0 \quad (11)$$

$$x_{do}^{kn}, y_o^t, z_o, q_d^{kn} \in \{0,1\} \quad \forall o \in O, \forall d \in D, \forall t \in T, \forall k \in K, \forall n \in N_d^k. \quad (12)$$

The Objective Function (1) maximizes profit by subtracting the total costs ( $C$ ) from the total revenue ( $R$ ). Equations (2) and (3) are the total revenue and the total costs from three service types respectively. Constraints (4) ensure that all mandatory shipment demands are serviced by one service type while Constraints (5) allow non-mandatory shipment demands to be serviced or rejected depending on their impacts to the total profit. Constraints (6) and (7) restrict the volume and weight of shipments placed in containers using the containers' effective volume and weight capacities, respectively. Constraints (8) ensure containers are booked in order from the 1<sup>st</sup> container to the next. Constraints (9) specify that the first leg of the transshipment services ( $y_o^t$ ) must be booked on a container going to a transshipment port ( $x_{do}^{kn}$ ). Constraints (10) limit the number of shipments diverted to partner service in each destination. This is done to avoid excessive diversion to partner services. Inequality (11) ensures that the overall damage probability does not exceed the maximum allowable level. We do not include the risk from using partner's service in Inequality (11) because the forwarder does not have direct control over the partner's service and the overall risk from using the partner's service is controlled indirectly through Constraints (10), which limit the number of shipments being directed to the partner's service. All variables are binary.

While Formulation (1)-(12) provides the overall view of the problem and explain the relationships between different components, it suffers from the scale and complexity of the problem, which leads to intractability when solving using brute-force method. To solve the ISRCB model, we propose two solution algorithms: the Two-Phase Method (2PM) and the Incremental Solution Method (ISM). We describe these algorithms next.

#### 4. THE TWO-PHASE METHOD (2PM)

The 2PM is a heuristic that solves Formulation (1)-(12) by decomposing the solution process into two phases. In Phase I, the 2PM solves a modified problem to establish *the number of containers* of different types that have to be booked to each destination port  $d$ . This ignores the discreteness of the individual containers. Consequently, the resulting assignment of shipments to containers in Phase I may violate the no-split assumption (Assumption 1 of the problem). To correct this issue, the 2PM enters Phase II, where the container booking decision (i.e., the numbers of booked containers) is fixed (from Phase I). With the container booking decision fixed, the capacities and costs of the direct service on different routes are now fixed and known. The 2PM then solves a shipment routing model with the known container capacity, reinstating the discreteness of the individual containers into the model to

ensure feasible flow of shipments through the network. We describe the details of the 2PM next.

### Phase I

We define a new integer variable  $q_d^k$  as the number of type- $k$  containers going to destination port  $d$  that is booked for service. Notice that  $q_d^k = \sum_{n \in N_d^k} q_d^{kn}$ . Consequently,  $x_{do}^k$  equals to 1 if demand  $o$  is shipped using a direct service going to destination port  $d$  using type- $k$  container, 0 otherwise. By using the assignment variable  $x_{do}^k$  instead of the original  $x_{do}^{kn}$ , the model in Phase I loses the discreteness of the individual containers. With these transformations, Formulation (1)-(12) now reads:

$$\text{[Phase I Model]} \quad \text{Maximize } z = R - C \quad (13)$$

Subject to

$$R = \sum_{o \in O} \left[ \sum_{d \in D} \sum_{k \in K} \sum_{n \in N_d^k} r_o^x \delta_{do} x_{do}^{kn} + \sum_{t \in T} r_o^y y_o^t + r_o^z z_o \right] \quad (14)$$

$$C = \sum_{d \in D} \sum_{k \in K} c_d^k q_d^k + \sum_{o \in O} \sum_{t \in T} c_o^t y_o^t + \sum_{o \in O} c_o z_o \quad (15)$$

$$\sum_{d \in D} \sum_{k \in K} \sum_{n \in N_d^k} \delta_{do} x_{do}^k + \sum_{t \in T} y_o^t + c_o z_o = 1 \quad \forall o \in \hat{O} \quad (16)$$

$$\sum_{d \in D} \sum_{k \in K} \sum_{n \in N_d^k} \delta_{do} x_{do}^k + \sum_{t \in T} y_o^t + c_o z_o \leq 1 \quad \forall o \in O \setminus \hat{O} \quad (17)$$

$$\sum_{o \in O} v_o x_{do}^k - V^k q_d^k \leq 0 \quad \forall d \in D, \forall k \in K \quad (18)$$

$$\sum_{o \in O} w_o x_{do}^k - W^k q_d^k \leq 0 \quad \forall d \in D, \forall k \in K \quad (19)$$

$$\sum_{k \in K} \sum_{n \in N_d^k} x_{to}^k - y_o^t = 0 \quad \forall o \in O, \forall t \in T \quad (20)$$

$$\sum_{o \in O} \delta_{do} v_o z_o \leq \mu_d \quad \forall d \in D \quad (21)$$

$$\sum_{o \in O} \left[ (\beta^x - \beta^t) \sum_{d \in D} \sum_{k \in K} \delta_{do} x_{do}^k + (\beta^y - \beta^t) \sum_{t \in T} y_o^t \right] \leq 0 \quad (22)$$

$$x_{do}^k, y_o^t, z_o \in \{0,1\} \quad \forall o \in O, \forall d \in D, \forall t \in T, \forall k \in K \quad (23)$$

$$q_d^k \in \mathbb{N} \quad \forall d \in D, \forall k \in K. \quad (24)$$

Constraints (8) from the original formulation are not needed in the Phase I model because the discreteness of the individual containers is removed.

We solve Formulation (13)-(24) using the branch-and-bound algorithm to obtain the optimal set of  $q_d^k$  for each container type  $k$  and destination port  $d$ .

### Phase II

In Phase II, we use the numbers of containers  $q_d^k$  for each container type  $k$  and destination

port  $d$  to establish the costs and capacities of the direct services. We set  $|N_d^k| = q_d^k$  and use the parameter  $\tilde{q}_d^{kn}$  to denote the availability of the  $n^{\text{th}}$  type- $k$  container going to destination port  $d$ . We then solve the shipment routing model described by Formulation (25)-(35) to obtain the optimal routing given the capacity from Phase I.

**[Phase II Model]** Maximize  $z = R - C$  (25)

Subject to

$$R = \sum_{o \in O} \left[ \sum_{d \in D} \sum_{k \in K} \sum_{n \in N_d^k} r_o^x \delta_{do} x_{do}^{kn} + \sum_{t \in T} r_o^y y_o^t + r_o^z z_o \right] \tag{26}$$

$$C = \sum_{o \in O} \sum_{t \in T} c_o^t y_o^t + \sum_{o \in O} c_o z_o \tag{27}$$

$$\sum_{d \in D} \sum_{k \in K} \sum_{n \in N_d^k} \delta_{do} x_{do}^{kn} + \sum_{t \in T} y_o^t + c_o z_o = 1 \quad \forall o \in \hat{O} \tag{28}$$

$$\sum_{d \in D} \sum_{k \in K} \sum_{n \in N_d^k} \delta_{do} x_{do}^{kn} + \sum_{t \in T} y_o^t + c_o z_o \leq 1 \quad \forall o \in O \setminus \hat{O} \tag{29}$$

$$\sum_{o \in O} v_o x_{do}^{kn} \leq V^k \tilde{q}_d^{kn} \quad \forall d \in D, \forall k \in K, \forall n \in N_d^k \tag{30}$$

$$\sum_{o \in O} w_o x_{do}^{kn} \leq W^k \tilde{q}_d^{kn} \quad \forall d \in D, \forall k \in K, \forall n \in N_d^k \tag{31}$$

$$\sum_{k \in K} \sum_{n \in N_d^k} x_{to}^{kn} - y_o^t = 0 \quad \forall o \in O, \forall t \in T \tag{32}$$

$$\sum_{o \in O} \delta_{do} v_o z_o \leq \mu_d \quad \forall d \in D \tag{33}$$

$$\sum_{o \in O} \left[ (\beta^x - \beta^t) \sum_{d \in D} \sum_{k \in K} \sum_{n \in N_d^k} \delta_{do} x_{do}^{kn} + (\beta^y - \beta^t) \sum_{t \in T} y_o^t \right] \leq 0 \tag{34}$$

$$x_{do}^{kn}, y_o^t, z_o \in \{0,1\} \quad \forall o \in O, \forall d \in D, \forall t \in T, \forall k \in K, \forall n \in N_d^k. \tag{35}$$

## 5. THE INCREMENTAL SOLUTION METHOD (ISM)

The Incremental Solution Method (ISM) is an iterative heuristic. In each iteration, the ISM determines two categories of booking of type- $k$  containers for each destination port  $d$ : firmed booking and potential booking. For firmed bookings, the decision to book those containers as well as the assignment of shipments to those containers (called firmed assignments) are firmed and final. That is, firmed bookings and firmed shipment assignments from each iteration are considered final and will be removed from consideration in the next iteration of the ISM. For potential bookings, the decision to book those containers as well as the assignment of shipment to those containers are only approximate and may be revised in the next iteration. The shipments that have been assigned in the current iteration to potentially booked containers will be reset for new assignments in the next iteration. In each iteration of the ISM, only one container booking of each type ( $k$ ) and destination ( $d$ ) will be designated as firmed. The ISM repeated iteratively until the potential booking of every container type and destination is at most one, at which point all shipment assignments are firmed.

### The ISRCB Model for the ISM

Let a binary variable  $\hat{q}_d^k$  equals to 1 if a type- $k$  container going to destination port  $d$  is firmed in the current iteration, 0 otherwise. And let an integer variable  $q_d^k$  be the number of potential bookings of type- $k$  containers going to destination port  $d$  in the current iteration. Similarly, let  $\hat{x}_{do}^k$  be the firmed assignment and equals to 1 if shipment demand  $o$  is assigned to a firmed booking of type- $k$  container going to destination port  $d$ , 0 otherwise, while  $x_{do}^k$  be the approximate assignment in the current iteration.

In each iteration of the ISM, we only allow one firmed booking to be made for a type- $k$  container going to destination port  $d$ . Note that a potential booking cannot be made unless a firmed booking has already been made in that iteration. Furthermore, from our implementation, it is necessary to enforce a lower bound on the utilization ( $\alpha_v$  for volume utilization and  $\alpha_w$  for weight utilization) of the firmed booking because the model cannot differentiate the impacts of the firmed and approximate assignments. Lastly, because the ISM iteratively removes firmed booking from consideration in the next iteration, it has to account for the damage probability of those firmed assignments in the next iteration. Let  $\tau$  be the average damage probability already incurred from firmed assignments.

We create a modified formulation for use with the ISM as follows:

$$\text{[ISM Model]} \quad \text{Maximize } z = R - C \quad (36)$$

Subject to

$$R = \sum_{o \in O} \left[ \sum_{d \in D} \sum_{k \in K} r_o^x \delta_{do} (\hat{x}_{do}^k + x_{do}^k) + \sum_{t \in T} r_o^y y_o^t + r_o^z z_o \right] \quad (37)$$

$$C = \sum_{d \in D} \sum_{k \in K} c_d^k (\hat{q}_d^k + q_d^k) + \sum_{o \in O} \sum_{t \in T} c_o^t y_o^t + \sum_{o \in O} c_o z_o \quad (38)$$

$$\sum_{d \in D} \sum_{k \in K} \delta_{do} (\hat{x}_{do}^k + x_{do}^k) + \sum_{t \in T} y_o^t + c_o z_o = 1 \quad \forall o \in \hat{O} \quad (39)$$

$$\sum_{d \in D} \sum_{k \in K} \delta_{do} (\hat{x}_{do}^k + x_{do}^k) + \sum_{t \in T} y_o^t + c_o z_o \leq 1 \quad \forall o \in O \setminus \hat{O} \quad (40)$$

$$q_d^k - M \hat{q}_d^k \geq 0 \quad \forall d \in D, \forall k \in K \quad (41)$$

$$\sum_{o \in O} v_o \hat{x}_{do}^k - V^k \hat{q}_d^k \leq 0 \quad \forall d \in D, \forall k \in K \quad (42)$$

$$\sum_{o \in O} w_o \hat{x}_{do}^k - W^k \hat{q}_d^k \leq 0 \quad \forall d \in D, \forall k \in K \quad (43)$$

$$\sum_{o \in O} v_o \hat{x}_{do}^k - \alpha_v V^k \hat{q}_d^k \geq 0 \quad \forall d \in D, \forall k \in K \quad (44)$$

$$\sum_{o \in O} w_o \hat{x}_{do}^k - \alpha_w W^k \hat{q}_d^k \geq 0 \quad \forall d \in D, \forall k \in K \quad (45)$$

$$\sum_{o \in O} v_o x_{do}^k - V^k q_d^k \leq 0 \quad \forall d \in D, \forall k \in K \quad (46)$$

$$\sum_{o \in O} w_o x_{do}^k - W^k q_d^k \leq 0 \quad \forall d \in D, \forall k \in K \quad (47)$$

$$\sum_{k \in K} (\hat{x}_{do}^k + x_{do}^k) - y_o^t = 0 \quad \forall o \in O, \forall d \in D, \forall t \in T \quad (48)$$

$$\sum_{o \in O} \delta_{do} v_o z_o \leq \mu_d \quad \forall d \in D \quad (49)$$

$$\sum_{o \in O} \left[ (\beta^x - \beta^t) \sum_{d \in D} \sum_{k \in K} \delta_{do} (\hat{x}_{do}^k + x_{do}^k) + (\beta^y - \beta^t) \sum_{t \in T} y_o^t \right] + \tau \leq 0 \quad (50)$$

$$\hat{x}_{do}^k, x_{do}^k, y_o^t, z_o, \hat{q}_d^k \in \{0,1\} \quad \forall o \in O, \forall d \in D, \forall t \in T, \forall k \in K \quad (51)$$

$$q_d^k \in \mathbb{N} \quad \forall d \in D, \forall k \in K. \quad (52)$$

The differences from the previously described formulations are as follow. Constraints (41) ensure that no potential bookings are allowed unless a firmed booking has already been made for the container type and destination in the current iteration. Constraints (44) and (45) ensure minimum utilization of the firmly booked containers. Inequality (50) modifies the damage risk by taking into account those risks incurred from firmed assignments from the previous iterations.

## 6. COMPUTATIONAL RESULTS

### 6.1 Data

This paper studies the operational planning of a LCL ocean freight forwarder in Thailand. The company accepts export shipping orders from customers out of Laem Chabang port in Thailand and delivers those shipments to 29 destination ports in 27 countries in 5 continents. Direct service can serve 14 destinations, transshipment service can serve 25 destinations, and partner's service can serve 9 destinations. The details are shown in Table 1. For each destination, the number of customers varies ranging from a minimum of 2 to a maximum of 30 customers per destination. Shipping orders from customers are collected weekly over one year period, leading to 52 data sets. The distribution of the number of shipping orders is shown in Figure 3. The default risk probability values (see Section 3.1) are used.

Three types of containers are considered in this operation: 20-foot, 40-foot, and High-Cube 40 foot. The effective capacities for each container type differ slightly by destination. The total operation costs for one container of type  $k$  going to destination  $d$  comprises of three components: origin charge, ocean freight surcharge, and destination charge and is charged per container. The costs for transshipment and partner's service are by shipment's weight and volume depending on shipment.

From the actual data set described above, we generate additional data instances to test our model and solution algorithms. In total, 25 data instances are used.

The model is implemented using the C# Programming Language and the IBM ILOG CPLEX Version 12 on an Intel Core 2 Duo T5750 PC with 2GB of memory.

### 6.2 Performance and Effectiveness of Proposed Algorithms

We present a summary of results in this section. Both proposed solution algorithms have a number of settings that we can adjust in order to improve the performance. The results shown reflect the most effective parameter configuration in our experiments. For example, in the 2PM, we limit the solution in each phase to 180 seconds, while in the ISM, we limit the solution time in each iteration to 120 seconds. In all cases, the ISM takes at most five iterations to converge. To provide a basis for comparison, we solve the ISRCB model (Formulation (1)-(12) using Branch-and-Bound (B&B) with 3-hour time limit.

Table 1. Destination Ports Serviced by the Sample Forwarder

Destination Ports		Available Service			
Port	Country	Direct	Transshipment		Partner
			Hong Kong	Singapore	
Aarhus	Denmark		y		
Antwerp	Belgium	y		y	
Ashdod	Israel		y		y
Bratislava	Slovakia		y		
Budapest	Hungary		y		
Fos -Sur-Mer	France		y		
Genoa	Italy	y		y	
Göteborg	Sweden	y			
Gdynia	Poland		y		
Hamburg	Germany	y		y	
Helsinki	Finland			y	
Hong Kong	Hong Kong	y			y
Istanbul	Turkey	y	y		
Le Harve	France	y		y	
Leixoes	Portugal			y	
Los Angeles	USA	y			y
Mazaniilo	Mexico		y		y
Melbourne	Australia	y		y	y
Oslo	Norway			y	
Paranagua	Brasil		y		y
Prague	Czech		y		
Rotterdam	Netherlands	y		y	
Singapore	Singapore	y			y
Southampton	Great Britain	y		y	y
Sydney	Australia	y		y	y
Ushuaia	Argentina		y		
Vancouver	Canada	y		y	
Vienna	Austria			y	
Zurich	Switzerland		y		

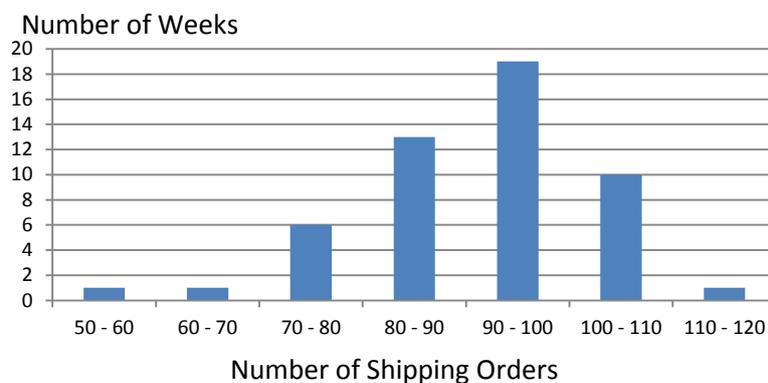


Figure 3. The Distribution of the Number of Shipping Orders

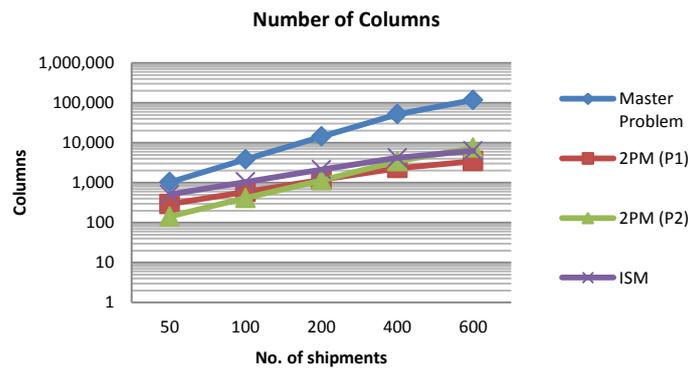


Figure 4. Growth Trend of the Number of Columns

Figure 4 shows the growth in the number of columns as the problem size, measured in term of the number of shipments, increases. Note that the vertical axis is reported in logarithmic scale. As seen in Figure 4, all methods exhibit exponential growth in the number of columns, however, the decomposition methods used in the 2PM and ISM help reduced the number of columns significantly compared to the original growth trend.

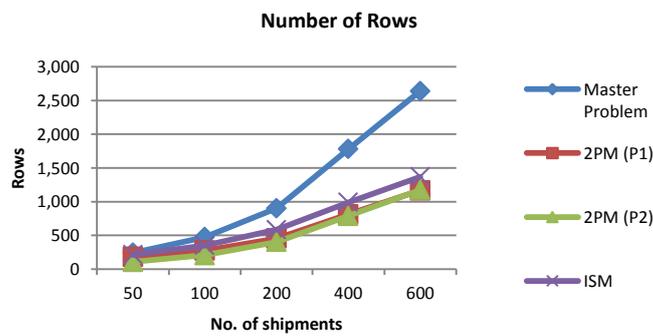


Figure 5. Growth Trend of the Number of Rows

The number of rows, in contrast, does not increase as rapidly as the problem size grows. (Figure 5.) The solution methods proposed also help decrease the number of rows.

Figure 6 shows the growth of the constraint matrix in term of the number of non-zero elements. Note that the vertical axis is in logarithmic scale. Again, similar to the number of columns shown in Figure 4, all models exhibit exponential growth but the two decomposition methods improve markedly compared to the original model.

Table 2 summarizes the objective function values and solution times for the different algorithms. Figures 7 to 9 highlight important results from the table.

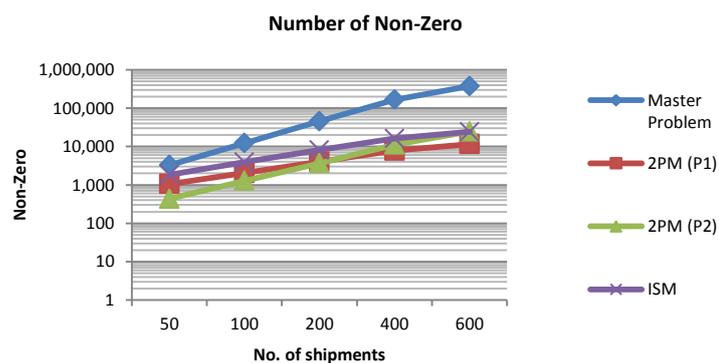


Figure 6. Growth Trend of the Number of Non-Zeros in the Constraint Matrices

Table 2. Result Summary

Data		B&B		2PM		ISM	
No. of Shipments	Data Set	Obj. Value	Time (sec)	Obj. Value	Time (sec)	Obj. Value	Time (sec)
50	1	-1,683	0.33	-1,683	0.27	-1,683	0.47
	2	-250	0.16	-250	0.11	-250	0.13
	3	197	0.81	197	0.33	197	0.87
	4	-288	0.16	-288	0.28	-288	0.13
	5	198	0.47	198	0.25	198	0.25
100	1	3,545	2.54	3,549	1.14	3,550	3.51
	2	2,958	1.89	2,959	3.76	2,960	5.62
	3	3,202	16.22	3,207	1.42	3,208	2.12
	4	3,873	4.01	3,872	1.33	3,874	1.87
	5	1,067	4.20	1,067	1.25	1,068	60.17
200	1	15,811	44.66	15,840	5.52	15,843	62.42
	2	9,323	57.36	9,322	1.51	9,324	4.65
	3	8,492	3,998.29	8,497	181.98	8,492	60.37
	4	10,641	159.23	10,648	360.05	10,664	122.37
	5	9,669	99.72	9,680	183.15	9,681	58.61
400	1	37,335	6,778.04	37,350	360.00	37,458	243.84
	2	26,353	1,634.45	26,669	360.00	26,607	79.01
	3	24,270	5,636.94	24,480	208.46	24,669	103.66
	4	21,333	2,221.95	21,730	360.00	21,753	171.69
	5	29,753	1,804.65	29,762	360.00	29,810	125.96
600	1	50,199	10,001.14	48,394	360.43	50,150	244.14
	2	55,781	10,052.92	55,916	360.15	56,226	246.17
	3	48,544	8,032.20	50,809	189.53	51,084	256.67
	4	47,004	10,003.24	47,523	355.56	47,748	241.16
	5	48,601	9,463.07	48,617	237.48	49,382	230.17

Next we demonstrate the solvability of the model as well as the effectiveness of our proposed solution algorithms. As shown in Figure 7, the solution times for B&B increases significantly as the problem size, measured in term of the number of shipments, increases. The solution times for the proposed solution algorithms, 2PM and ISM, on the other hands, are exceptionally small compared to that of B&B. Figure 8 provides the comparison between solution times of 2PM and ISM. For the largest instance with 600 customers, the 2PM takes on average less than 200 seconds and the ISM takes slightly under 6 minutes, while B&B takes on average slightly over 2.5 hours.

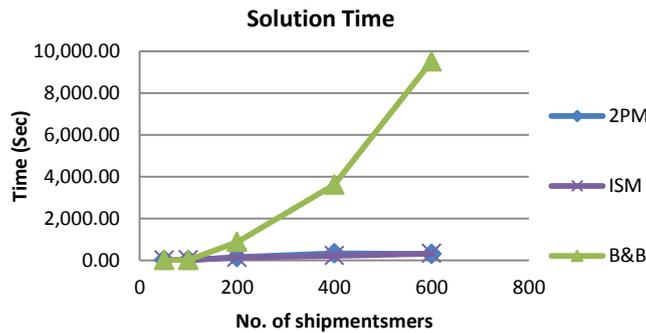


Figure 7. Solution Time

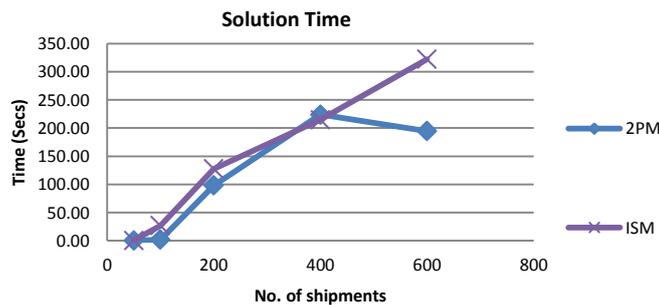


Figure 8. Solution Time Comparison between 2PM and ISM

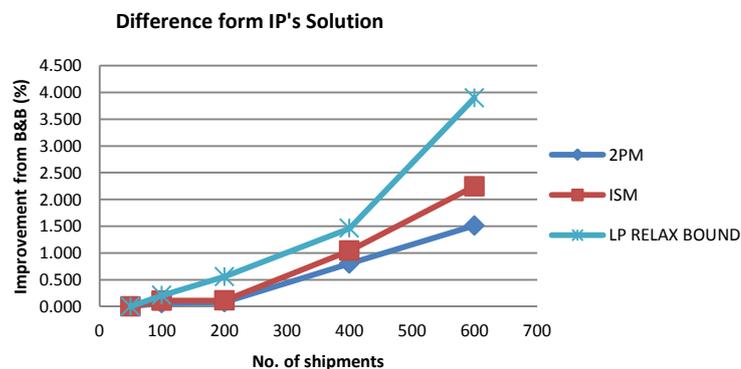


Figure 9. Solution Quality

Figure 9 illustrates the solution quality. The vertical axis measures the improvement of the objective function value over the solution obtained from solving the original ISRCB model (Formulation (1)-(12)) using branch-and-bound. The LP Relax Bound in Figure 9

shows the upperbound obtained from the LP relaxation of Formulation (1)-(12). The other two lines demonstrate the solution quality of the proposed algorithms. Both algorithms outperform B&B by a significant margin. Between the two proposed methods, the ISM gives better performance than the 2PM.

From the results shown in this section, we conclude that the two proposed algorithms perform exceptionally well in practices. Both give better solution quality compared to that of B&B in fraction of the time that B&B requires.

### 6.3 Solution Comparison

In this section we compare the solutions obtained from the model to the actual historical operation of the forwarder.

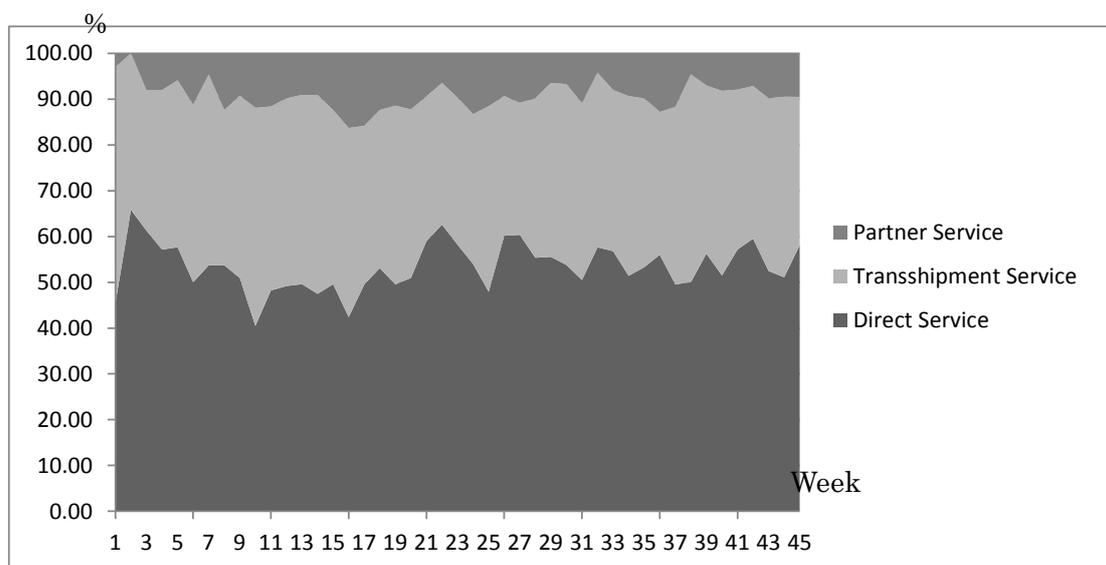


Figure 10. Actual Mix of Service Selection

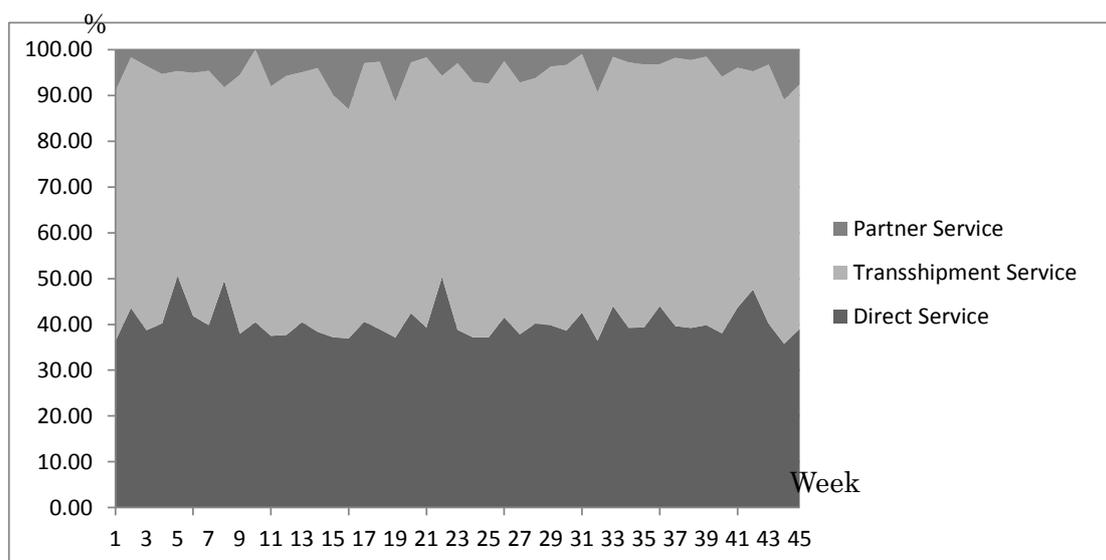


Figure 11. Model's Mix of Service Selection

In Figures 10 and 11, the horizontal axis is the week of the year and the vertical axis is the percentage mix of service type. The first observation is that the actual operation gives higher preference to the direct service (approximately 50%). The second preference is the transshipment service at approximately 40%. Then about 10% of the work is diverted to the partner service. The model, on the other hand, tends to prefer the transshipment service at approximately 55%. The direct service takes a ratio of approximately 40% and the partner service is only approximately 5%. This reflects the manual operation process where a container is often booked when sufficient loads is found without detailed analysis of profit impacts.

From Figures 10 and 11, observe also that the model's service selection is more consistent week by week, compared to the manual operation, which exhibits more variation across the week.

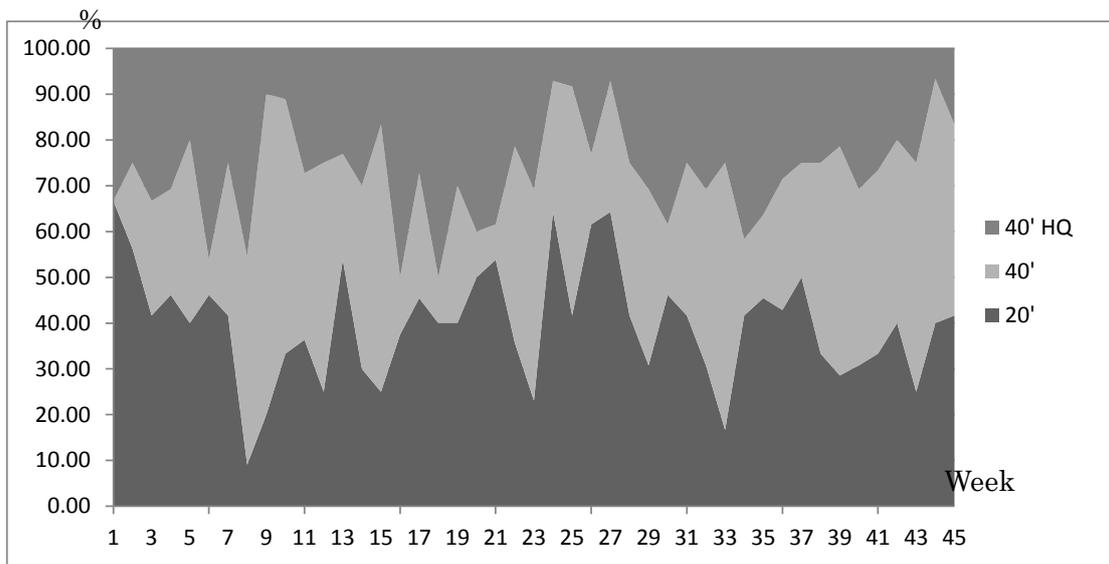


Figure 12. Actual Mix of Container Selection

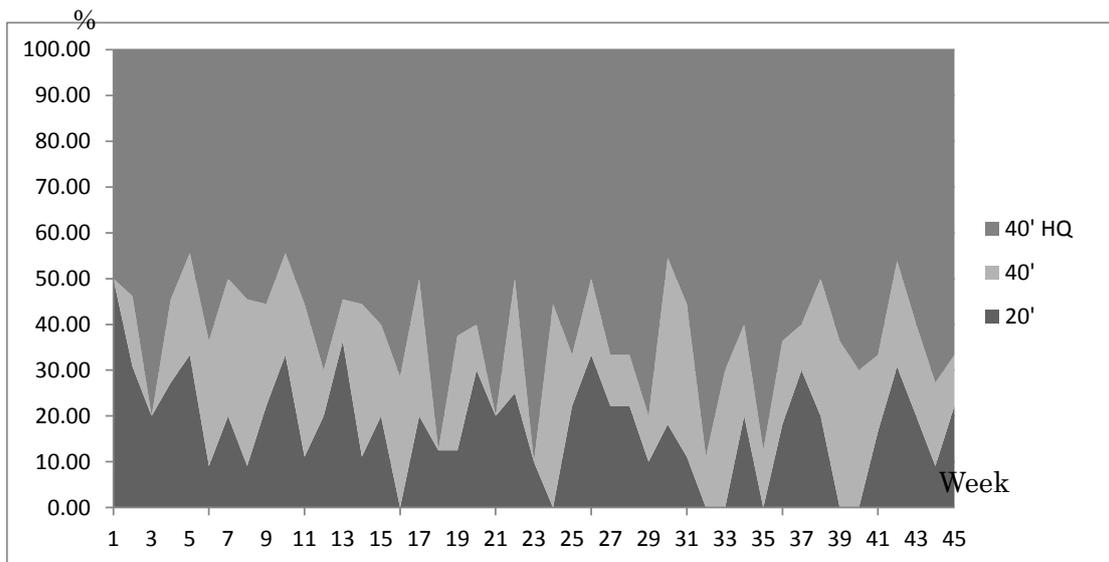


Figure 13. Model's Mix of Container Selection

From Figures 12 and 13, it is clear that the model gives strong preference to 40' High Cube container compared to the actual operation. The 40' High Cube container has cost advantage compared to the other two types of containers because of the increased capacity and hence reduced unit cost per shipment. The model takes full advantage of this cost advantage in the solution while planning the operation.

Last we compare the damage probability from the model to the actual operation. From the results shown thus far, it is noted that the model exploits the lower cost from transshipment. This leads to a higher level of damage probability incurred, compared to the actual operation, which tends to be more conservative and is not willing to exploit the damage risk, keep it on average at approximately 0.04%. It is noted that the maximum allowable risk probability is never exceeded. Thus, it is possible to match the risk probability level of the actual operation and obtain a more conservative solution.

## 7. CONCLUSION AND DISCUSSION

In this paper, the authors present the Integrated Shipment Routing and Container Booking (ISRCB) Model for ocean freight forwarder. We provide the formulation and propose two solution algorithms, namely, the Two-Phase Method (2PM) and the Incremental Solution Method (ISM). We implement the model and the solution algorithms and test them using real data from a major forwarder in Thailand with operation out of Laem Chabang port in Thailand.

From academic perspective, our proposed solution algorithms outperform the traditional branch-and-bound algorithm applied to the original ISRCB model. Both the 2PM and the ISM solves large instances of the problem far quicker (under 3 minutes vs. more than 2.5 hours) and obtains better solutions (the IP solutions are closer to the LP relaxation bound).

From practical standpoint, our proposed model and algorithms provide a tool for planning shipment routing and container booking that can aid planner in daily operation. The model takes a global view of the operation and suggests the best decision based on detailed financial analytics, which is too complicated for human to process manually. The solution from the model, however, should be viewed as suggestions only. Actual operations will encounter other constraints that are not captured in the model. Nonetheless, planners can use the model's solution as a starting point for actual planning.

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