Designing Freight Transport Network Using Cooperative Particle Swarm Optimisation in Supply Chain–Transport Supernetworks

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Abstract: This paper proposes a cooperative modified probability-based discrete binary particle swarm optimisation (C-MPBPSO) for designing freight transport network, which is formulate as a mathematical programme with equilibrium constraints. The lower level applies a supply chain-transport supernetwork equilibrium, which integrates supply chain networks (SCNs) with a transport network. The upper level discretely optimises the set of road network improvement actions such that the SCNs' efficiency is maximised. The C-MPBPSO algorithm is applied to approximately solve the upper level. In order to confirm the superiority of C-MPBPSO, the appropriate parameters for C-MPBPSO are tested, and then its performance is compared with those of several conventional single-swarm-based discrete binary PSO algorithms. The results with a hypothesised small network exhibits that C-MPBPSO can offer better performance than those conventional PSO algorithms, considering computation times and quality of solutions.

Keywords: Particle Swarm Optimisation, Cooperative Approach, Freight Transport Network Design, Supply Chain

1. INTRODUCTION

Supply chain management (SCM) is widely recognized by companies as a key factor for remaining competitive, since intense international sales competition has forced them to offer low-cost and high-quality products. Although the term of SCM is used in many ways, its fundamental objective is to develop effective networks among companies, that is, to create efficient supply chain networks (SCNs). In order to achieve the ultimate goal of SCM, the collaboration among the different economic entities in a whole SCN is certainly required, especially on product distribution and freight transport. SCN is a recent term used to represent the linkage among the economic entities (e.g., manufacturers, wholesalers, retailers and consumers) and the resulting of their behavioural interactions. Nagurney *et al*. (2002) initiated an equilibrium model for a multi-tiered competitive SCN (i.e., supply chain network equilibrium (SCNE) model), which can provide several notable outputs for the SCN. The comprehension of the entities' behaviour and their interaction using the SCNE models, allows administrators and planners to understand the generation mechanism of product movement and to explore the effects of transport- and logistics-related measures.

Recently, there has been a lot of significant effort to expand the SCNE model (see the details in Yamada *et al.*, 2011), including Nagurney (2006) showing that the SCNE models can be reformulated and solved as a transport network equilibrium problem. However, even this has not integrated both transport and supply chain networks into a model. Hence, the effects of any proposed freight transport measure cannot be assessed over the entire SCNs. As freight transport deals primarily with the distribution of goods in the entire SCNs, Yamada *et al.* (2011) remarkably proposed a supply chain–transport supernetwork equilibrium (SC–T–SNE) model, which encompasses the behaviours of freight carriers and transport network users within the SCNE model. The model enables to investigate the effects of traffic conditions in a transport network (TN) on the behaviour of each entity on the SCNs and vice versa.

The adoption of accurate methods in freight transport planning offers promising opportunities for urban society and SCN actors to become more efficient and sustainable. This paper deals therefore, with the problem of investment planning in developing a freight TN, which can facilitate the evaluation of the decision-making in the freight TN as well as the selection of suitable actions in it to improve the efficiency of SCNs. The actions involve improving existing roads or establishing new roads. The most feasible set of road projects is selected for the efficient design of the freight TN. The research utilises a model based on MPEC (mathematical programmes with equilibrium constraints), where the SC-T-SNE is used as a constraint. This can be formulated using variational inequalities (VIs) in the lower level. If the SC-T-SNE is incorporated within the lower level, then the upper level approximately optimises the combination of the actions using metaheuristics-based solution procedures. The advantage of adopting metaheuristic techniques is that such techniques can handle combinatorial optimisation problems in relatively shorter computational times. In addition, some conventional solution methods, such as the branch-and-bound or branch-and-cut method are not available for solving the MPEC due to the parameterised VI constraints. Hence, there have been several researches incorporating metaheuristics techniques into the MPEC (e.g., Yamada *et al.*, 2009; Meng *et al.*, 2009).

In this research, a cooperative modified probability-based discrete binary particle swarm optimisation (C-MPBPSO) is developed and applied as a solution technique in the upper level. C-MPBPSO is a variant of particle swarm optimisation (PSO), which is an evolutionary computation technique inspired by flocking behaviour of birds. Eberhart and Kennedy (1995) originally designed PSO algorithms as an optimisation technique for use in real-number spaces. Furthermore, they extended the continuous PSO to deal with discrete optimisation problems, which is also known as the discrete binary PSO (DBPSO) (Kennedy and Eberhart, 1997). Shen *et al.* (2004) then presented the modified binary PSO (MBPSO) algorithm incorporating a strategy to update the positions of particles within DBPSO. Recently, Menhas *et al.* (2012a) applied the probability-based discrete binary PSO (PBPSO) algorithm (see also Menhas *et al.* (2011); Wang *et al.* (2008) for the PBPSO) to a single objective combinatorial optimisation problem. They indicate that PBPSO can offer a better performance than DBPSO and MBPSO in terms of quality and stability of solutions. As PBPSO tends to have relatively slow convergence, Zukhruf *et al.* (2012) proposed the modified PBPSO (MPBPSO) algorithm, where an updating strategy for the change in position is embedded in the existing PBPSO algorithm. Preliminary testing on a small-sized hypothetical supply chain–transport supernetwork reveals that when the search space is relatively small, MPBPSO can deliver a better performance than PBPSO with its faster and stable searching ability.

PSO and its variants have been acknowledged as powerful evolutionary computation techniques. However, in general, their performance is degraded with the increase in the number of dimensions. The past researches showed that incorporating cooperative behaviour within PSO significantly improves its performance (e.g., Van den Bergh and Engelbrecht, 2004; El-Abd, 2008; and Menhas *et al.*, 2012b). The basic scheme of the cooperative PSO involves multiple swarms for exchanging the information among them in order to explore the search space more efficiently and achieve better solutions. Hence, this paper incorporates the cooperative behaviour within MPBPSO and develops a C-MPBPSO algorithm. To confirm the superiority of C-MPBPSO, namely, MPBPSO with multiple cooperative swarms, its performance will be compared with those of several single-swarm-based PSO algorithms.

The rest of the paper is organised as follows. In the following section, the overall modelling framework is described, and the formulation of the model is given. In the third section, the performance of C-MPBPSO to be incorporated within the upper level is investigated on a hypothesised small test network. The model is then tested and applied in the fourth section to a hypothesised but relatively larger-sized test network, and explores the implications of how to expand the freight TN. Finally, in the fifth section, the methodologies, results, and analyses in the paper are summarised.

2. MPEC-based Model

2.1. Overall Framework and Upper Level

The model consists of two levels, where the lower level employs SC-T-SNE, estimating the quantities of products transacted (i.e., those transported or distributed) between SCN entities, the prices of the products, and traffic conditions on the TN. In the upper level the combination of TN improvement actions is optimised, and the solutions derived in both levels influence each other. The model can be formulated as follows, where the term �$\langle \bullet, \bullet \rangle$� represents the inner product in N-dimensional Euclidean space, and equilibrium solutions are denoted by "*".

$$\max_{u} \quad P\left(u, Z^{*}\right) \tag{1}$$

$$\text{subject to } \left(u, Z\right) \in K \tag{2}$$

$$\left\langle G\left(u, Z^{*}\right), Z - Z^{*} \right\rangle \geq 0 \tag{3}$$

where,

u: vector of the set of TN improvement actions,

$\boldsymbol{Z}$: vector of state variables on a supernetwork,

K: non-empty feasible space.

Maximising objective function (1) involves the upper level, a combinatorial optimisation problem with 0-1 variables. Constraint (2) corresponds to each variable condition, constraint (3) to SC-T-SNE (specifically, VI (32)), and both makes the lower level. Here, two sets of links ($A = A_1 \cup A_2$) are defined, where A_2 is only relevant to the design variables. When link a is newly built or renovated, u_a is equal to 1; otherwise u_a is 0 (i.e., $u_a \in \{0,1\}$ and $u \in \{u_a | a \in A_2\}$).

2.2. SC-T-SNE at Lower Level

The lower level has the same mathematical formulation as indicated in Yamada *et al.* (2011). Therefore, this paper outlines the SC-TSNE model as below (see the details in Yamada *et al.*, 2011).

SCNs for various different products are involved in a TN of D (V, A) with the set of nodes V and that of links A. Y kinds of four-tier SCNs lie on the TN, each providing product y (y=1,...,Y). The SCN for product y consists of I^y manufacturers, with a typical manufacturer denoted by i^y; J^y wholesalers, with a typical wholesaler denoted by j^y; K^y retailers, with a typical retailer denoted by k^y; H^y freight carriers, with a typical freight carrier denoted by h^y, and consumers associated with L demand markets, with a typical demand market denoted by l. The links in the SCN represent those for transport and transaction.

The manufacturers, wholesalers, retailers and demand markets in Y kinds of SCNs exist on the nodes in the TN. It is not possible for more than one decision-maker to deal with the same kind of product at a single node in the TN. Each node on the TN is capable of generating and attracting freight trips, because the products are transacted and distributed among its decision-makers. Passenger car trips can also be generated from and attracted at any of its nodes. The set of origins for all the trips is represented as $R \subseteq V$, and that of the destinations as $S \subseteq V$.

2.2.1 The Behaviour of Manufacturers and their Optimality Conditions.

Manufacturer i^y on the SCN for product y is involved in its production, which can then be purchased by wholesaler j^y. Let $E^{1y}\left(=E_{i^y j^y}\right)$ be the set of paths for transporting product y between manufacturer i^y and wholesaler j^y on the transport network, and dim $p^{1y} = e^{1y}$ is given to path $p^{1y}\left(=p_{i^y j^y}\right) \in E^{1y}$ between OD pair$\left(i^y, j^y\right)$ $\left(i^y \in R, j^y \in S\right)$. The behaviour of manufacturer i^y dealing with product y is formulated below as a profit maximisation problem.

$$\begin{aligned}\underset{q_{i^y}}{\text{Max}} \quad & \sum_{j^y=1}^{J^y} \rho_{i^y j^y}^{1*} \sum_{h^y=1}^{H^y} \sum_{p^{1y} \in E^{1y}} q_{h^y i^y j^y}^{p^{1y}} - f_{i^y}\left(Q^{1y}\right) - g_{i^y}\left(Q^{1y}\right) \\ & - \sum_{j^y=1}^{J^y} c_{i^y j^y}\left(Q^{1y}\right) - \sum_{h^y=1}^{H^y} \sum_{j^y=1}^{J^y} \rho_{h^y i^y j^y}^{1*} \sum_{p^{1y} \in E^{1y}} q_{h^y i^y j^y}^{p^{1y}}\end{aligned} \tag{4}$$

$$\text{subject to} \quad q_{h^y i^y j^y}^{p^{1y}} \geq 0 \quad \forall h^y, j^y, p^{1y} \tag{5}$$

where,

$\rho_{i^y j^y}^{1}$: price charged for product y by manufacturer i^y to wholesaler j^y,

$q_{h^y i^y j^y}^{p^{1y}}$: amount of product y transacted/transported from manufacturer i^y to wholesaler j^y by freight carrier h^y using path p^{1y},

q_{i^y} : $H^y J^y e^{1y}$-dimensional vector with component $h^y j^y p^{1y}$ denoted by $q_{h^y i^y j^y}^{p^{1y}}$ representing production output of product y by manufacturer i^y

Q^{1y} : $H^y I^y J^y e^{1y}$-dimensional vector with component $h^y i^y j^y p^{1y}$ denoted by $q_{h^y i^y j^y}^{p^{1y}}$ representing shipments of product y between manufacturers and wholesalers,

$f_{i^y}\left(Q^{1y}\right)$: production cost to manufacturer i^y for product y,

$g_{i^y}\left(Q^{1y}\right)$: facility cost to manufacturer i^y,

$c_{i^y j^y}\left(Q^{1y}\right)$: transaction cost for product y incurred between manufacturer i^y and wholesaler j^y (excluding transport cost incurred between i^y and j^y),

$\rho_{h^y i^y j^y}^{1}$: carriage charged by freight carrier h^y for transporting product y between manufacturer i^y and wholesaler j^y.

Let Q^1 be an S^1-dimensional vector with components: Q^{11}, …, Q^{1Y}, where $S^1 = \sum_{y=1}^{Y}\left(H^y I^y J^y e^{1y}\right)$. Assuming that the production cost functions, facility cost functions and transaction cost functions for each manufacturer are continuously differentiable and convex as well as that the manufacturers compete in a noncooperative fashion, the optimality conditions for all manufacturers for all kinds of products can simultaneously be expressed as the following VI: determine $Q^{1*} \in R_{+}^{S^1}$, which satisfies:

$$\sum_{y=1}^{Y}\sum_{h^y=1}^{H^y}\sum_{i^y=1}^{I^y}\sum_{j^y=1}^{J^y}\sum_{p^{1y}\in E^{1y}}\left[\frac{\partial f_{i^y}\left(Q^{1y*}\right)}{\partial q_{h^y i^y j^y}^{p^{1y}}}+\frac{\partial g_{i^y}\left(Q^{1y*}\right)}{\partial q_{h^y i^y j^y}^{p^{1y}}}+\frac{\partial c_{i^y j^y}\left(Q^{1y*}\right)}{\partial q_{h^y i^y j^y}^{p^{1y}}}\right. \\ \left.+\rho_{h^y i^y j^y}^{1*}-\rho_{i^y j^y}^{1*}\right]\times\left[q_{h^y i^y j^y}^{p^{1y}}-q_{h^y i^y j^y}^{p^{1y}*}\right]\geq 0 \quad \forall Q^1\in R_{+}^{S^1} \tag{6}$$

Although the prices charged are not considered variables (as well as those in the subsequent derivation of VIs (10), (14) and (26)), they can be treated as endogenous variables in the complete equilibrium model (i.e., VI (32)) (see Nagurney, Dong, and Zhang 2002; Hammond and Beullens 2007; and Yamada *et al*. 2011).

2.2.2 The Behaviour of Wholesalers and their Optimality Conditions.

Wholesalers are involved in transactions with both manufacturers and retailers. Let $E^{2y}\left(=E_{j^y k^y}\right)$ denote the set of paths for transporting product y between wholesaler j^y and retailers k^y on the transport network, and $p^{2y}\left(=p_{j^y k^y}\right)\in E^{2y}$ (dim $p^{2y}=e^{2y}$) be the path travelled between OD pair $\left(j^y,k^y\right)$ $\left(j^y\in R, k^y\in S\right)$ in it. The behaviour of wholesaler j^y dealing with product y is formulated with the following criterion of profit maximisation.

$$\max_{q_{i^y},q_{j^y}} \sum_{k^y=1}^{K^y}\rho_{j^y k^y}^{2*}\sum_{h^y=1}^{H^y}\sum_{p^{2y}\in E^{2y}} q_{h^y j^y k^y}^{p^{2y}}-c_{j^y}\left(Q^{1y}\right)-g_{j^y}\left(Q^{1y}\right)-\sum_{k^y=1}^{K^y}c_{j^y k^y}\left(Q^{2y}\right) \\ -\sum_{h^y=1}^{H^y}\sum_{k^y=1}^{K^y}\rho_{h^y j^y k^y}^{2*}\sum_{p^{2y}\in E^{2y}} q_{h^y j^y k^y}^{p^{2y}}-\sum_{i^y=1}^{I^y}\rho_{i^y j^y}^{1*}\sum_{h^y=1}^{H^y}\sum_{p^{1y}\in E^{1y}} q_{h^y i^y j^y}^{p^{1y}} \tag{7}$$

$$\text{subject to} \quad \sum_{h^y=1}^{H^y}\sum_{k^y=1}^{K^y}\sum_{p^{2y}\in E^{2y}} q_{h^y j^y k^y}^{p^{2y}} \leq \sum_{h^y=1}^{H^y}\sum_{i^y=1}^{I^y}\sum_{p^{1y}\in E^{1y}} q_{h^y i^y j^y}^{p^{1y}} \tag{8}$$

$$q_{h^y i^y j^y}^{p^{1y}}\geq 0 \ \forall h^y, i^y, p^{1y}, \quad q_{h^y j^y k^y}^{p^{2y}}\geq 0 \ \forall h^y, k^y, p^{2y} \tag{9}$$

where,

$\rho_{j^y k^y}^{2}$: sales price charged for product y by wholesaler j^y to retailers k^y,

$q_{h^y j^y k^y}^{p^{2y}}$: amount of product y transacted/transported from wholesaler j^y to retailer k^y by freight carrier h^y using path p^{2y},

q_{j^y} : $H^y K^y e^{2y}$-dimensional vector with component $h^y k^y p^{2y}$ denoted by $q_{h^y j^y k^y}^{p^{2y}}$ representing shipments of product y by wholesaler j^y,

Q^{2y} : $H^y J^y K^y e^{2y}$-dimensional vector with component $h^y j^y k^y p^{2y}$ denoted by $q_{h^y j^y k^y}^{p^{2y}}$ representing shipments of product y between wholesalers and retailers,

$c_{j^y}\left(Q^{1y}\right)$: handling/inventory costs to wholesaler j^y,

$g_{j^y}\left(Q^{1y}\right)$: facility cost to wholesaler j^y,

$c_{j^y k^y}\left(Q^{2y}\right)$: transaction cost for product y incurred between wholesaler j^y and retailer k^y (excluding transport cost incurred between j^y and k^y),

$\rho^{2}_{h^y j^y k^y}$: carriage charged by freight carrier h^y for transporting product y between wholesaler j^y and retailer k^y.

Constraint (8) simply expresses the fact that retailers cannot purchase more of the product from a wholesaler than is available in stock.

Let Q^2 be an S^2-dimensional vector with components: Q^{21}, …, Q^{2Y}, where $S^2\left(=\sum_{y=1}^{Y}\left(H^y J^y K^y e^{2y}\right)\right)$. If handling/inventory cost functions, facility cost functions and transaction cost functions are continuously differentiable and convex, then the optimality conditions for all wholesalers for all kinds of products simultaneously coincide with the solution of the following VI: determine $\left(Q^{1*},Q^{2*},\gamma^*\right)\in R_+^{S^1+S^2+S^6}$ which satisfies:

$$\sum_{y=1}^{Y}\sum_{h^y=1}^{H^y}\sum_{i^y=1}^{I^y}\sum_{j^y=1}^{J^y}\sum_{p^{1y}\in E^{1y}}\left[\frac{\partial c_{j^y}\left(Q^{1y*}\right)}{\partial q^{p^{1y}}_{h^y i^y j^y}}+\frac{\partial g_{j^y}\left(Q^{1y*}\right)}{\partial q^{p^{1y}}_{h^y i^y j^y}}+\rho^{1*}_{i^y j^y}-\gamma^*_{j^y}\right]\times\left[q^{p^{1y}}_{h^y i^y j^y}-q^{p^{1y}*}_{h^y i^y j^y}\right] \tag{10}$$
$$+\sum_{y=1}^{Y}\sum_{j^y}^{J^y}\left[\sum_{h^y=1}^{H^y}\sum_{i^y=1}^{I^y}\sum_{p^{1y}\in E^{1y}}q^{p^{1y}*}_{h^y i^y j^y}-\sum_{h^y=1}^{H^y}\sum_{k^y=1}^{K^y}\sum_{p^{2y}\in E^{2y}}q^{p^{2y}*}_{h^y j^y k^y}\right]\times\left[\gamma_{j^y}-\gamma^*_{j^y}\right]\geq 0\quad\forall\left(Q^1,Q^2,\gamma\right)\in R_+^{S^1+S^2+S^6}$$

Here, the term γ_{j^y} is the Lagrange multiplier associated with constraint (8), and γ is an S^6-dimensional vector with component y denoted by γ_{j^y}, where $S^6=\sum_{y=1}^{Y}J^y$.

2.2.3 The Behaviour of Retailers and their Optimality Conditions

The retailers, in turn, are involved in transactions with the wholesalers, since they wish to obtain the products for their retail outlets, also with the consumers who are the ultimate purchasers of the products. Given path $p^{3y}\left(=p_{k^y l}\right)\in E^{3y}$ (dim $p^{3y}=e^{3y}$) used between OD pair of $\left(k^y,l\right)$ $\left(k^y\in R, l\in S\right)$ on the transport network, where $E^{3y}\left(=E_{k^y l}\right)$ is the set of paths between retailer k^y and demand market l, the behaviour of retailer k^y who deals with product y and seeks a maximum profit can be formulated as follows:

$$\max_{q_{j^y},q_{k^y}}\ \sum_{l=1}^{L}\rho^{3*}_{k^y l}\sum_{h^y=1}^{H^y}\sum_{p^{3y}\in E^{3y}}q^{p^{3y}}_{h^y k^y l}-c_{k^y}\left(Q^{2y}\right)-g_{k^y}\left(Q^{2y}\right)-\sum_{l=1}^{L}c_{k^y l}\left(Q^{3y}\right)$$
$$-\sum_{h^y=1}^{H^y}\sum_{l=1}^{L}\rho^{3*}_{h^y k^y l}\sum_{p^{3y}\in E^{3y}}q^{p^{3y}}_{h^y k^y l}-\sum_{j^y=1}^{J^y}\rho^{2*}_{j^y k^y}\sum_{h^y=1}^{H^y}\sum_{p^{2y}\in E^{2y}}q^{p^{2y}}_{h^y j^y k^y} \tag{11}$$

$$\text{subject to}\quad\sum_{h^y=1}^{H^y}\sum_{l=1}^{L}\sum_{p^{3y}\in E^{3y}}q^{p^{3y}}_{h^y k^y l}\leq\sum_{h^y=1}^{H^y}\sum_{j^y=1}^{J^y}\sum_{p^{2y}\in E^{2y}}q^{p^{2y}}_{h^y j^y k^y} \tag{12}$$

$$q^{p^{2y}}_{h^y j^y k^y}\geq 0\ \forall h^y, j^y, p^{2y},\quad q^{p^{3y}}_{h^y k^y l}\geq 0\ \forall h^y, l, p^{3y} \tag{13}$$

where,

$\rho^{3}_{k^y l}$: sales price charged for product y by retailer k^y to demand market l,

$q^{p^{3y}}_{h^y k^y l}$: amount of product y transacted/transported from retailer k^y to demand market l by freight carrier h^y using path p^{3y},

q_{k^y} : $H^y L e^{3y}$-dimensional vector with component $k^y l p^{3y}$ denoted by $q^{p^{3y}}_{h^y k^y l}$ representing shipments of product y by retailer k^y,

Q^{3y} : $H^yK^yLe^{3y}$-dimensional vector with component $h^yk^ylp^{3y}$ denoted by $q_{h^yk^yl}^{p^{3y}}$ representing shipments of product y between retailers and demand markets,

$c_{k^y}\left(Q^{2y}\right)$: handling/inventory costs to retailer k^y,

$g_{k^y}\left(Q^{2y}\right)$: facility cost to retailer k^y,

$c_{k^yl}\left(Q^{3y}\right)$: transaction cost for product y incurred between retailer k^y and demand market
l (excluding transport cost incurred between k^y and l),

$\rho_{h^yk^yl}^{3}$: carriage charged by freight carrier h^y for transporting product y between retailer k^y and demand market l.

Here, Q^3 denotes an S^3-dimensional vector with components: Q^{31}, …, Q^{3Y}, where $S^3\left(=\sum_{y=1}^{Y}\left(H^yK^yLe^{3y}\right)\right)$, and assuming that handling/inventory cost functions, facility cost functions and transaction cost functions are continuously differentiable and convex, the optimality conditions for all retailers for all kinds of products can simultaneously be formulated as the following VI: determine $\left(Q^{2*},Q^{3*},\delta^*\right)\in R_+^{S^2+S^3+S^7}$ satisfying:

$$\begin{aligned}&\sum_{y=1}^{Y}\sum_{h^y=1}^{H^y}\sum_{j^y=1}^{J^y}\sum_{k^y=1}^{K^y}\sum_{p^{2y}\in E^{2y}}\left[\frac{\partial c_{k^y}\left(Q^{2y*}\right)}{\partial q_{h^yj^yk^y}^{p^{2y}}}+\frac{\partial g_{k^y}\left(Q^{2y*}\right)}{\partial q_{h^yj^yk^y}^{p^{2y}}}+\rho_{j^yk^y}^{2*}-\delta_{k^y}^*\right]\times\left[q_{h^yj^yk^y}^{p^{2y}}-q_{h^yj^yk^y}^{p^{2y}*}\right]\\&+\sum_{y=1}^{Y}\sum_{h^y=1}^{H^y}\sum_{k^y=1}^{K^y}\sum_{l=1}^{L}\sum_{p^{3y}\in E^{3y}}\left[-\rho_{k^yl}^{3*}+\frac{\partial c_{k^yl}\left(Q^{3y*}\right)}{\partial q_{h^yk^yl}^{p^{3y}}}+\rho_{h^yk^yl}^{3*}+\delta_{k^y}^*\right]\times\left[q_{h^yk^yl}^{p^{3y}}-q_{h^yk^yl}^{p^{3y}*}\right]\\&+\sum_{y=1}^{Y}\sum_{k^y}^{K^y}\left[\sum_{h^y=1}^{H^y}\sum_{j^y=1}^{J^y}\sum_{p^{2y}\in E^{2y}}q_{h^yj^yk^y}^{p^{2y}*}-\sum_{h^y=1}^{H^y}\sum_{l=1}^{L}\sum_{p^{3y}\in E^{3y}}q_{h^yk^yl}^{p^{3y}*}\right]\times\left[\delta_{k^y}-\delta_{k^y}^*\right]\geq 0\ \forall\left(Q^2,Q^3,\delta\right)\in R_+^{S^2+S^3+S^7}\end{aligned}\tag{14}$$

Here, the term δ_{k^y} is the Lagrange multiplier associated with constraint (12), and δ is an S^7-dimensional vector with component y denoted by δ_{k^y}, where $S^7=\sum_{y=1}^{Y}K^y$.

2.2.4 The Consumers in Demand Markets and the Equilibrium Conditions.

The consumers take into account the prices charged for the products by the retailers in making their consumption decisions. The demand function is assumed to be continuous, and the following complementarity conditions hold for demand market l.

$$\rho_{k^yl}^{3*}\begin{cases}=\rho_l^{4y*} & \text{if } q_{h^yk^yl}^{p^{3y}*}>0\\ \geq\rho_l^{4y*} & \text{if } q_{h^yk^yl}^{p^{3y}*}=0\end{cases}\tag{15}$$

$$d_l^y\left(\rho^{4y*}\right)\begin{cases}=\sum_{h^y=1}^{H^y}\sum_{k^y=1}^{K^y}\sum_{p^{3y}\in E^{3y}}q_{h^yk^yl}^{p^{3y}*} & \text{if } \rho_l^{4y*}>0\\ \leq\sum_{h^y=1}^{H^y}\sum_{k^y=1}^{K^y}\sum_{p^{3y}\in E^{3y}}q_{h^yk^yl}^{p^{3y}*} & \text{if } \rho_l^{4y*}=0\end{cases}\tag{16}$$

where,

ρ_l^{4y} : market price of product y at demand market l,

ρ^{4y} : L-dimensional vector for product y with component l denoted by ρ_l^{4y},

$d_l^y\left(\rho^{4y}\right)$: demand function of product y at demand market l.

Let ρ^4 be an LY-dimensional vector for product y with component ly denoted by ρ_l^{4y}. In equilibrium, conditions (15) and (16) will have to hold for all demand markets for all kinds of products, and these, in turn, can also be expressed as a VI, and given by: determine $\left(Q^{3*},\rho^{4*}\right)\in R_+^{S^3+LY}$ such that:

$$\sum_{y=1}^{Y}\sum_{h^y=1}^{H^y}\sum_{k^y=1}^{K^y}\sum_{l=1}^{L}\sum_{p^{3y}\in E^{3y}}\left[\rho_{k^yl}^{3*}-\rho_l^{4y*}\right]\times\left[q_{h^yk^yl}^{p^{3y}}-q_{h^yk^yl}^{p^{3y}*}\right]$$
$$+\sum_{y=1}^{Y}\sum_{l=1}^{L}\left[\sum_{h^y=1}^{H^y}\sum_{k^y=1}^{K^y}\sum_{p^{3y}\in E^{3y}}q_{h^yk^yl}^{p^{3y}*}-d_l^y\left(\rho^{4y*}\right)\right]\times\left[\rho_l^{4y}-\rho_l^{4y*}\right]\geq 0\quad\forall\left(Q^3,\rho^4\right)\in R_+^{S^3+LY} \tag{17}$$

2.2.5 The Behaviour of Freight Carriers and their Optimality Conditions.

The freight carriers are not only decision-makers in the SCNs but transport network users. A road network is assumed as transport network with two kinds of user groups: freight vehicles operated by the freight carriers on the SCNs and other vehicles (i.e., ‘‘passenger car traffic’’ or ‘‘passenger car’’ hereafter). The origin node for passenger car trips is expressed with $r\in R$, and the attraction node with $s\in S$. The path $p_{rs}\in E_{rs}\left(u\right)$ between OD pair (r, s), where E_{rs} is the set of paths between r and s, is given as $\dim p_{rs}=e^5$.

The freight carriers are also profit-maximisers, and the optimisation problem for freight carrier h^y is given as below:

$$\begin{aligned}\underset{q_{i^y},q_{j^y},q_{k^y}}{\text{Max}}\quad&\sum_{i^y=1}^{I^y}\sum_{j^y=1}^{J^y}\rho_{h^yi^yj^y}^{1*}\sum_{p^{1y}\in E^{1y}}q_{h^yi^yj^y}^{p^{1y}}+\sum_{j^y=1}^{J^y}\sum_{k^y=1}^{K^y}\rho_{h^yj^yk^y}^{2*}\sum_{p^{2y}\in E^{2y}}q_{h^yj^yk^y}^{p^{2y}}\\&+\sum_{k^y=1}^{K^y}\sum_{l=1}^{L}\rho_{h^yk^yl}^{3*}\sum_{p^{3y}\in E^{3y}}q_{h^yk^yl}^{p^{3y}}-g_{h^y}\left(Q^{1y},Q^{2y},Q^{3y}\right)\\&-\sum_{i^y=1}^{I^y}\sum_{j^y=1}^{J^y}\sum_{p^{1y}\in E^{1y}}q_{h^yi^yj^y}^{p^{1y}}C_{h^yi^yj^y}^{p^{1y}}\left(Q^1,Q^2,Q^3,X\right)\\&-\sum_{j^y=1}^{J^y}\sum_{k^y=1}^{K^y}\sum_{p^{2y}\in E^{2y}}q_{h^yj^yk^y}^{p^{2y}}C_{h^yj^yk^y}^{p^{2y}}\left(Q^1,Q^2,Q^3,X\right)-\sum_{k^y=1}^{K^y}\sum_{l=1}^{L}\sum_{p^{3y}\in E^{3y}}q_{h^yk^yl}^{p^{3y}}C_{h^yk^yl}^{p^{3y}}\left(Q^1,Q^2,Q^3,X\right)\end{aligned}\tag{18}$$

$$\text{subject to}\quad q_{h^yi^yj^y}^{p^{1y}}\geq 0\ \forall i^y,j^y,p^{1y},\quad q_{h^yj^yk^y}^{p^{2y}}\geq 0\ \forall j^y,k^y,p^{2y}$$
$$q_{h^yk^yl}^{p^{3y}}\geq 0\ \ \forall k^y,l,p^{3y}\tag{19}$$

where,

$g_{h^y}\left(\bullet\right)$: facility cost to freight carrier h^y,

$C_{h^yi^yj^y}^{p^{1y}}\left(\bullet\right),C_{h^yj^yk^y}^{p^{2y}}\left(\bullet\right),C_{h^yk^yl}^{p^{3y}}\left(\bullet\right)$: unit operation cost (per transport volume) of freight carrier h^y for transporting product y using path p^{1y}, p^{2y}, and p^{3y}, respectively,

e^r : number of origin nodes for passenger cars,

e^s : number of destination nodes for passenger cars,

X : $e^re^se^5$-dimensional vector with component rsp_{rs} denoted by $x_{rs}^{p_{rs}}$,

$x_{rs}^{p_{rs}}$: traffic volume of passenger cars travelling between r and s using path p_{rs}.

The unit operation cost can be calculated as follows:

$$C_{h^y i^y j^y}^{p^{1y}}\left(Q^1,Q^2,Q^3,X\right)=\frac{\eta t_{i^y j^y}^{p^{1y}}\left(Q^1,Q^2,Q^3,X\right)}{\iota\kappa} \tag{20}$$

$$C_{h^y j^y k^y}^{p^{2y}}\left(Q^1,Q^2,Q^3,X\right)=\frac{\eta t_{j^y k^y}^{p^{2y}}\left(Q^1,Q^2,Q^3,X\right)}{\iota\kappa} \tag{21}$$

$$C_{h^y k^y l}^{p^{3y}}\left(Q^1,Q^2,Q^3,X\right)=\frac{\eta t_{k^y l}^{p^{3y}}\left(Q^1,Q^2,Q^3,X\right)}{\iota\kappa} \tag{22}$$

where,

η : operation cost for a freight per unit time,

$t_{i^y j^y}^{p^{1y}}(\bullet)$, $t_{j^y k^y}^{p^{2y}}(\bullet)$, $t_{k^y l}^{p^{3y}}(\bullet)$: travel time on path p^{1y}, p^{2y}, p^{3y}, respectively,

ι : capacity of a freight vehicle,

κ : average loading factor of a freight vehicle.

The path travel times can also be derived as below:

$$t_{i^y j^y}^{p^{1y}}\left(Q^1,Q^2,Q^3,X\right)=\sum_{a\in A} t_a\left(x_a\right)\delta_{a,p^{1y}}^{i^y j^y} \tag{23}$$

$$t_{j^y k^y}^{p^{2y}}\left(Q^1,Q^2,Q^3,X\right)=\sum_{a\in A} t_a\left(x_a\right)\delta_{a,p^{2y}}^{j^y k^y} \tag{24}$$

$$t_{k^y l}^{p^{3y}}\left(Q^1,Q^2,Q^3,X\right)=\sum_{a\in A} t_a\left(x_a\right)\delta_{a,p^{3y}}^{k^y l} \tag{25}$$

where,

$\delta_{a,p^{1y}}^{i^y j^y}$: binary value of 1 if link a is contained in path p^{1y} between i^y and j^y; 0 if it is otherwise,

$\delta_{a,p^{2y}}^{j^y k^y}$: binary value of 1 if link a is contained in path p^{2y} between j^y and k^y; 0 if it is otherwise,

$\delta_{a,p^{3y}}^{k^y l}$: binary value of 1 if link a is contained in path p^{3y} between k^y and l; 0 if it is otherwise,

$t_a(x_a)$: travel time on link a,

x_a : traffic volume on link a (see equation (30)).

Provided that the facility cost functions are continuously differentiable and convex, and the operation cost functions are continuously differentiable, non-decreasing and convex; the optimality conditions for all freight carriers for all kinds of products can simultaneously be formulated as the following VI: determine $\left(Q^{1*},Q^{2*},Q^{3*}\right)\in R_+^{S^1+S^2+S^3}$ satisfying:

$$\begin{aligned}\sum_{y=1}^{Y}\sum_{h^y=1}^{H^y}\sum_{i^y=1}^{I^y}\sum_{j^y=1}^{J^y}\sum_{p^{1y}\in E^{1y}}\Bigg[&\frac{\partial g_{h^y}\left(Q^{1y*},Q^{2y*},Q^{3y*}\right)}{\partial q_{h^y i^y j^y}^{p^{1y}}}+C_{h^y i^y j^y}^{p^{1y}}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)\\
&+\sum_{i^y=1}^{I^y}\sum_{j^y=1}^{J^y}\sum_{p^{1y}\in E^{1y}} q_{h^y i^y j^y}^{p^{1y}*}\frac{\partial C_{h^y i^y j^y}^{p^{1y}}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)}{\partial q_{h^y i^y j^y}^{p^{1y}}}+\sum_{j^y=1}^{J^y}\sum_{k^y=1}^{K^y}\sum_{p^{2y}\in E^{2y}} q_{h^y j^y k^y}^{p^{2y}*}\frac{\partial C_{h^y j^y k^y}^{p^{2y}}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)}{\partial q_{h^y i^y j^y}^{p^{1y}}}\\
&+\sum_{k^y=1}^{K^y}\sum_{l=1}^{L}\sum_{p^{3y}\in E^{3y}} q_{h^y k^y l}^{p^{3y}*}\frac{\partial C_{h^y k^y l}^{p^{3y}}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)}{\partial q_{h^y i^y j^y}^{p^{1y}}}-\rho_{h^y i^y j^y}^{1*}\Bigg]\times\left[q_{h^y i^y j^y}^{p^{1y}}-q_{h^y i^y j^y}^{p^{1y}*}\right]\end{aligned} \tag{26}$$

$$
\begin{aligned}
&+\sum_{y=1}^{Y}\sum_{h^y=1}^{H^y}\sum_{j^y=1}^{J^y}\sum_{k^y=1}^{K^y}\sum_{p^{2y}\in E^{2y}}\left[\frac{\partial g_{h^y}\left(Q^{1y*},Q^{2y*},Q^{3y*}\right)}{\partial q^{p^{2y}}_{h^y j^y k^y}}+C^{p^{2y}}_{h^y j^y k^y}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)\right.\\
&+\sum_{i^y=1}^{I^y}\sum_{j^y=1}^{J^y}\sum_{p^{1y}\in E^{1y}} q^{p^{1y}*}_{h^y i^y j^y}\frac{\partial C^{p^{1y}}_{h^y i^y j^y}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)}{\partial q^{p^{2y}}_{h^y j^y k^y}}+\sum_{j^y=1}^{J^y}\sum_{k^y=1}^{K^y}\sum_{p^{2y}\in E^{2y}} q^{p^{2y}*}_{h^y j^y k^y}\frac{\partial C^{p^{2y}}_{h^y j^y k^y}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)}{\partial q^{p^{2y}}_{h^y j^y k^y}}\\
&\left.+\sum_{k^y=1}^{K^y}\sum_{l=1}^{L}\sum_{p^{3y}\in E^{3y}} q^{p^{3y}*}_{h^y k^y l}\frac{\partial C^{p^{3y}}_{h^y k^y l}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)}{\partial q^{p^{2y}}_{h^y j^y k^y}}-\rho^{2*}_{h^y j^y k^y}\right]\times\left[q^{p^{2y}}_{h^y j^y k^y}-q^{p^{2y}*}_{h^y j^y k^y}\right]\\
&+\sum_{y=1}^{Y}\sum_{h^y=1}^{H^y}\sum_{k^y=1}^{K^y}\sum_{l=1}^{L}\sum_{p^{3y}\in E^{3y}}\left[\frac{\partial g_{h^y}\left(Q^{1y*},Q^{2y*},Q^{3y*}\right)}{\partial q^{p^{3y}}_{h^y k^y l}}+C^{p^{3y}}_{h^y k^y l}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)\right.\\
&+\sum_{i^y=1}^{I^y}\sum_{j^y=1}^{J^y}\sum_{p^{1y}\in E^{1y}} q^{p^{1y}*}_{h^y i^y j^y}\frac{\partial C^{p^{1y}}_{h^y i^y j^y}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)}{\partial q^{p^{3y}}_{h^y k^y l}}+\sum_{j^y=1}^{J^y}\sum_{k^y=1}^{K^y}\sum_{p^{2y}\in E^{2y}} q^{p^{2y}*}_{h^y j^y k^y}\frac{\partial C^{p^{2y}}_{h^y j^y k^y}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)}{\partial q^{p^{3y}}_{h^y k^y l}}\\
&\left.+\sum_{k^y=1}^{K^y}\sum_{l=1}^{L}\sum_{p^{3y}\in E^{3y}} q^{p^{3y}*}_{h^y k^y l}\frac{\partial C^{p^{3y}}_{h^y k^y l}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)}{\partial q^{p^{3y}}_{h^y k^y l}}-\rho^{3*}_{h^y k^y l}\right]\times\left[q^{p^{3y}}_{h^y k^y l}-q^{p^{3y}*}_{h^y k^y l}\right]\geq 0\ \forall\left(Q^{1*},Q^{2*},Q^{3*}\right)\in R_{+}^{S^1+S^2+S^3}
\end{aligned}
$$

2.2.6 The Passenger Car Traffic on Road Network and the Equilibrium Conditions.
The behaviour of passenger cars is assumed to follow the user equilibrium traffic conditions with variable demand. In this case, the behaviour of passenger cars in the road network is formulated as below:

$$
\begin{cases}
t^{p_{rs}}_{rs}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)=c^*_{rs} & \text{if } x^{p_{rs}*}_{rs}>0\\
t^{p_{rs}}_{rs}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)\geq c^*_{rs} & \text{if } x^{p_{rs}*}_{rs}=0
\end{cases}
\tag{27}
$$

$$
d_{rs}\left(c^*_{rs}\right)
\begin{cases}
=\sum_{p_{rs}\in E_{rs}} x^{p_{rs}*}_{rs} & \text{if } c^*_{rs}>0\\
\leq \sum_{p_{rs}\in E_{rs}} x^{p_{rs}*}_{rs} & \text{if } c^*_{rs}=0
\end{cases}
\tag{28}
$$

where,

$$
t^{p_{rs}}_{rs}\left(Q^1,Q^2,Q^3,X\right)=\sum_{a\in A} t_a\left(x_a\right)\delta^{rs}_{a,p}
\tag{29}
$$

$$
x_a=\sum_{p_{rs}\in E_{rs}}\delta^{rs}_{a,p}x^{p_{rs}}_{rs}+v\left(\sum_{p^{1y}\in E^{1y}}\delta^{i^y j^y}_{a,p^{1y}}\frac{q^{p^{1y}}_{h^y i^y j^y}}{\iota\kappa}+\sum_{p^{2y}\in E^{2y}}\delta^{j^y k^y}_{a,p^{2y}}\frac{q^{p^{2y}}_{h^y j^y k^y}}{\iota\kappa}+\sum_{p^{3y}\in E^{3y}}\delta^{k^y l}_{a,p^{3y}}\frac{q^{p^{3y}}_{h^y k^y l}}{\iota\kappa}\right)
\tag{30}
$$

where,

$t^{p_{rs}}_{rs}(\bullet)$: travel time on path p_{rs},

$\delta^{rs}_{a,p_{rs}}$: binary value of 1 if link a is contained in path p_{rs} between r and s; 0 if it is otherwise,

c_{rs} : travel cost incurred between r and s,

$d_{rs}(\bullet)$: traffic demand function between r and s.

v : passenger car equivalent.

Conditions (27) represent the equilibrium conditions known as Wardrop's first principle (Wardrop, 1952). Conditions (28) show the requirements to be fulfilled for OD traffic demand with price formulations (Nagurney, 1999). The model handles products and passenger as multiclass users with variable demand (e.g., Boyce and Bar-Gera, 2004). Conditions (27) and (28) must hold for all OD pairs in equilibrium. Hence, these conditions are equivalent to $\left(X^*, c_{rs}^*\right) \in R_+^{e^r e^s e^5 + e^r e^s}$ which satisfies:

$$\sum_{r\in R}\sum_{s\in S}\sum_{p_{rs}\in E_{rs}}\left[t_{rs}^{p_{rs}}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)-c_{rs}^*\right]\times\left[x_{rs}^{p_{rs}}-x_{rs}^{p_{rs}*}\right]$$
$$+\sum_{r\in R}\sum_{s\in S}\left[\sum_{p_{rs}\in E_{rs}}x_{rs}^{p_{rs}*}-d_{rs}\left(c_{rs}^*\right)\right]\times\left[c_{rs}-c_{rs}^*\right]\geq 0\;\forall\left(X,c_{rs}\right)\in R_+^{e^r e^s e^5+e^r e^s} \tag{31}$$

2.2.7 The Equilibrium Conditions of the Supply Chain–Transport Supernetwork.

The equilibrium state of the supply chain-transport supernetwork can be characterised as one where the optimality conditions (6), (10), (14) and (26) and the equilibrium conditions (17) and (31) hold simultaneously, such that no decision-maker has any incentive to alter his decisions. The equilibrium conditions governing the supply chain-transport supernetwork model are equivalent to the solution to the VI given by: determine $Q^{1*}, Q^{2*}, Q^{3*}, \gamma^*, \delta^*, \rho^{4*}, X^*, c_{rs}^* \in R_+^{S^1+S^2+S^3+S^6+S^7+LY+e^r e^s e^5+e^r e^s}$

$$\sum_{y=1}^{Y}\sum_{h^y=1}^{H^y}\sum_{i^y=1}^{I^y}\sum_{j^y=1}^{J^y}\sum_{p^{1y}\in E^{1y}}\left[\frac{\partial f_{i^y}\left(Q^{1y*}\right)}{\partial q_{h^y i^y j^y}^{p^{1y}}}+\frac{\partial g_{i^y}\left(Q^{1y*}\right)}{\partial q_{h^y i^y j^y}^{p^{1y}}}+\frac{\partial c_{i^y j^y}\left(Q^{1y*}\right)}{\partial q_{h^y i^y j^y}^{p^{1y}}}+\frac{\partial c_{j^y}\left(Q^{1y*}\right)}{\partial q_{h^y i^y j^y}^{p^{1y}}}+\frac{\partial g_{j^y}\left(Q^{1y*}\right)}{\partial q_{h^y i^y j^y}^{p^{1y}}}\right.$$
$$+\frac{\partial g_{h^y}\left(Q^{1y*},Q^{2y*},Q^{3y*}\right)}{\partial q_{h^y i^y j^y}^{p^{1y}}}+C_{h^y i^y j^y}^{p^{1y}}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)+\sum_{i^y=1}^{I^y}\sum_{j^y=1}^{J^y}\sum_{p^{1y}\in E^{1y}}q_{h^y i^y j^y}^{p^{1y}*}\frac{\partial C_{h^y i^y j^y}^{p^{1y}}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)}{\partial q_{h^y i^y j^y}^{p^{1y}}}$$
$$\left.+\sum_{k^y=1}^{K^y}\sum_{l=1}^{L}\sum_{p^{3y}\in E^{3y}}q_{h^y k^y l}^{p^{3y}*}\frac{\partial C_{h^y k^y l}^{p^{3y}}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)}{\partial q_{h^y i^y j^y}^{p^{1y}}}-\gamma_{j^y}^*\right]\times\left[q_{h^y i^y j^y}^{p^{1y}}-q_{h^y i^y j^y}^{p^{1y}*}\right]$$
$$+\sum_{y=1}^{Y}\sum_{h^y=1}^{H^y}\sum_{j^y=1}^{J^y}\sum_{k^y=1}^{K^y}\sum_{p^{2y}\in E^{2y}}\left[\frac{\partial c_{k^y}\left(Q^{2y*}\right)}{\partial q_{h^y j^y k^y}^{p^{2y}}}+\frac{\partial g_{k^y}\left(Q^{2y*}\right)}{\partial q_{h^y j^y k^y}^{p^{2y}}}+\frac{\partial c_{j^y k^y}\left(Q^{2y*}\right)}{\partial q_{h^y j^y k^y}^{p^{2y}}}+\frac{\partial g_{h^y}\left(Q^{1y*},Q^{2y*},Q^{3y*}\right)}{\partial q_{h^y j^y k^y}^{p^{2y}}}\right.$$
$$+C_{h^y j^y k^y}^{p^{2y}}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)+\sum_{j^y=1}^{J^y}\sum_{k^y=1}^{K^y}\sum_{p^{2y}\in E^{2y}}q_{h^y j^y k^y}^{p^{2y}*}\frac{\partial C_{h^y j^y k^y}^{p^{2y}}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)}{\partial q_{h^y j^y k^y}^{p^{2y}}}$$
$$+\sum_{i^y=1}^{I^y}\sum_{j^y=1}^{J^y}\sum_{p^{1y}\in E^{1y}}q_{h^y i^y j^y}^{p^{1y}*}\frac{\partial C_{h^y i^y j^y}^{p^{1y}}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)}{\partial q_{h^y j^y k^y}^{p^{2y}}}$$
$$\left.+\sum_{k^y=1}^{K^y}\sum_{l=1}^{L}\sum_{p^{3y}\in E^{3y}}q_{h^y k^y l}^{p^{3y}*}\frac{\partial C_{h^y k^y l}^{p^{3y}}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)}{\partial q_{h^y j^y k^y}^{p^{2y}}}+\gamma_{j^y}^*-\delta_{k^y}^*\right]\times\left[q_{h^y j^y k^y}^{p^{2y}}-q_{h^y j^y k^y}^{p^{2y}*}\right] \tag{32}$$
$$+\sum_{y=1}^{Y}\sum_{h^y=1}^{H^y}\sum_{k^y=1}^{K^y}\sum_{l=1}^{L}\sum_{p^{3y}\in E^{3y}}\left[\frac{\partial c_{k^y l}\left(Q^{3y*}\right)}{\partial q_{h^y k^y l}^{p^{3y}}}+\frac{\partial g_{h^y}\left(Q^{1y*},Q^{2y*},Q^{3y*}\right)}{\partial q_{h^y k^y l}^{p^{3y}}}+C_{h^y k^y l}^{p^{3y}}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)\right.$$
$$+\sum_{k^y=1}^{K^y}\sum_{l=1}^{L}\sum_{p^{3y}\in E^{3y}}q_{h^y k^y l}^{p^{3y}*}\frac{\partial C_{h^y k^y l}^{p^{3y}}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)}{\partial q_{h^y k^y l}^{p^{3y}}}+\sum_{i^y=1}^{I^y}\sum_{j^y=1}^{J^y}\sum_{p^{1y}\in E^{1y}}q_{h^y i^y j^y}^{p^{1y}*}\frac{\partial C_{h^y i^y j^y}^{p^{1y}}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)}{\partial q_{h^y k^y l}^{p^{3y}}}$$

$$+\sum_{j^y=1}^{J^y}\sum_{k^y=1}^{K^y}\sum_{p^{2y}\in E^{2y}} q_{h^y j^y k^y}^{p^{2y}*}\frac{\partial C_{h^y j^y k^y}^{p^{2y}}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)}{\partial q_{h^y k^y l}^{p^{3y}}}+\delta_{k^y}^*-\rho_l^{4y*}\Big]\times\left[q_{h^y k^y l}^{p^{3y}}-q_{h^y k^y l}^{p^{3y}*}\right]$$

$$+\sum_{y=1}^{Y}\sum_{j^y=1}^{J^y}\left[\sum_{h^y=1}^{H^y}\left(\sum_{i^y=1}^{I^y}\sum_{p^{1y}\in E^{1y}} q_{h^y i^y j^y}^{p^{1y}*}-\sum_{k^y=1}^{K^y}\sum_{p^{2y}\in E^{2y}} q_{h^y j^y k^y}^{p^{2y}*}\right)\right]\times\left[\gamma_{j^y}-\gamma_{j^y}^*\right]$$

$$+\sum_{y=1}^{Y}\sum_{k^y=1}^{K^y}\left[\sum_{h^y=1}^{H^y}\left(\sum_{j^y=1}^{J^y}\sum_{p^{2y}\in E^{2y}} q_{h^y j^y k^y}^{p^{2y}*}-\sum_{l=1}^{L}\sum_{p^{3y}\in E^{3y}} q_{h^y k^y l}^{p^{3y}*}\right)\right]\times\left[\delta_{k^y}-\delta_{k^y}^*\right]$$

$$+\sum_{y=1}^{Y}\sum_{l=1}^{L}\left[\sum_{h^y=1}^{H^y}\sum_{k^y=1}^{K^y}\sum_{p^{3y}\in E^{3y}} q_{h^y k^y l}^{p^{3y}*}-d_l^y\left(\rho^{4y*}\right)\right]\times\left[\rho_l^{4y}-\rho_l^{4y*}\right]$$

$$+\sum_{r\in R}\sum_{s\in S}\sum_{p_{rs}\in E_{rs}}\left[\zeta t_{rs}^{p_{rs}}\left(Q^{1*},Q^{2*},Q^{3*},X^*\right)-c_{rs}^*\right]\times\left[x_{rs}^{p_{rs}}-x_{rs}^{p_{rs}*}\right]$$

$$+\sum_{r\in R}\sum_{s\in S}\left[\sum_{p_{rs}\in E_{rs}} x_{rs}^{p_{rs}*}-d_{rs}\left(c_{rs}^*\right)\right]\times\left[c_{rs}-c_{rs}^*\right]\geq 0 \quad \forall\left(Q^1,Q^2,Q^3,\gamma,\delta,\rho^4,X,c_{rs}\right)\in R_+^{S^1+S^2+S^3+S^6+S^7+LY+e^r e^s e^5+e^r e^s}$$

The proof to the existence and uniqueness of the solution of VI (32), as well as its solution procedures, follow those demonstrated in Yamada *et al.* (2011).

3. COOPERATIVE MODIFIED PROBABILITY-BASED DISCRETE BINARY PARTICLE SWARM OPTIMISATION

3.1 Outlines

The efficiency of SCNs is assessed using total surplus being calculated as the sum of producer surplus and consumer surplus. The producer surplus is estimated as the sum of the profits for all manufacturers, all wholesalers, all retailers and all freight carriers, which can be computed with the solutions obtained by solving SC-T-SNE. Therefore, the objective function of the upper level is to maximise the Benefit Cost Ratio (BCR), namely the ratio of the increased total surplus with the actions implemented as compared to without them to the investment/operational cost required for implementing them. The objective function can be represented as follows:

$$P\left(u,Z^*\right)=\left(U\left(u,Z^*\right)-U_0\left(Z^*\right)\right)\Big/\sum\nolimits_{a\in A_2}\alpha_a u_a \tag{33}$$

where,

$U(\bullet)$: total surplus obtained in SCNs with actions implemented,

$U_0(\bullet)$: total surplus obtained in SCNs without any action implemented,

α_a : investment/operation costs for link a.

In this paper, it is assumed that there is no restriction to the investment budget, even though this could be possible by involving additional procedures (e.g., penalty method) within the solution procedures for the upper level. BCR values are commonly estimated over the standard planning horizon in which the number of years of the project is considered. The analyses shown in this paper are only preliminary, and hence the project life and social discount rate are not taken into account. A particle of PSO, which can be considered as a candidate solution to an optimisation problem, flies through the problem space looking for the optimal solution. Each particle has a position and a fitness value. The particles adjust their position according to their most recent velocity vector. The velocity vector is determined on the basis of their own and their neighbouring-particles' experience. Specifically, the velocity includes the best position achieved so far by the particle itself (*pbest*) and that visited by any particle in the population (*gbest*). In DBPSO, a particle moves in a search space bounded to zero or one on each dimension. Therefore, the velocity in the binary version represents the pseudo probability of bits taking value of 1. PBPSO, and MPBPSO as well, are variants of the DBPSO, which replace the sigmoid function applied to the DBPSO with a bounded linear transformation function. It is used to estimate the actual probability for bits taking value of 1 or 0.

A set of TN improvement actions, u, is regarded as a particle of PSO, where its dimension represents the total number of possible actions to be implemented. The length of the particle is assumed to be 16 (i.e., D=16 in the algorithm shown below), which is the same as that used by Yamada *et al*. (2009). Every position of a particle, namely action implementation indicator u_a, is formed in such a way that it takes a binary value of 1 if the corresponding action is implemented and 0 if it is otherwise. The value of objective function is calculated for each particle, and its fitness is evaluated. The swarm consists of a specific number of sets of actions.

MPBPSO is an extended version of PBPSO, where an updating scheme for changing the position of particles is added to PBPSO (Zukhruf *et al.*, 2012), which still can be improved. Accelerating convergence speed and avoiding local optima are the two most important and appealing goals in PSO research. The past research showed that incorporating cooperative behaviour within PSO significantly improves its performance. Therefore, this paper incorporates a cooperative behaviour within the MPBPSO algorithm (i.e., C-MPBPSO). The cooperative multiple swarms utilise the cooperative search to optimise different components of solution vector in which the original n-dimensional problem is divided into low-dimensional subcomponents. Let $D_d^1, D_d^2, \ldots, D_d^{sw}, \ldots, D_d^{SW}$ be the dimension of each of sw subcomponents, then

$$D_d = \sum_{sw=1}^{SW} D_d^{sw} \qquad (34)$$

where D_d^{sw} is the dimension of the original problem. C-MPBPSO employs SW subswarms, $swarm^1$, $swarm^2$, …, $swarm^{SW}$, with their sizes of pop^1, pop^2, …, pop^{SW}, respectively. In this paper, each swarm employs the same number of particles (i.e., *pop*), which is applied by Menhas *et al.* (2012b) and El-Abd (2008). Subcomponent sw of the problem is explored by $swarm^{sw}$. Then, the overall solution vector combines the sub solutions that are determined by the subswarms.

The way to update the particles within each sub-swarm is identical to that applied in the standard MPBPSO. However, since the particles in a subswarm can only represent a subspace of smaller dimension than the original search space, the particles cannot directly be evaluated with the objective function due to the missing components. Hence, the sharing mechanism among subswarms plays an important role in C-MPBPSO.

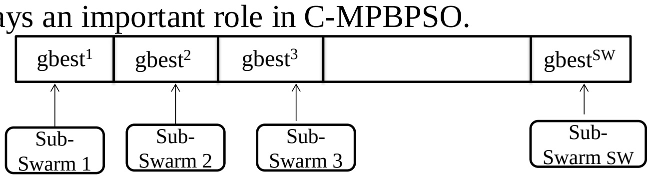


Figure 1. Context Vector

Van den Bergh and Engelbrecht (2004) introduced a shared buffer vector, which is also called a context vector, for evaluating the particles' fitness. The context vector provides the missing information required for the fitness evaluation. This vector can also be considered as indirect cooperation among the subswarms. More specifically, let *CV* be the *n*-dimensional context vector where each subswarm deposits its contribution. If $cv^{sw} = \left[cv_1^{sw}, cv_2^{sw}, ..., cv_{D_d^{sw}}^{sw} \right]$ is a contributed D_d^{sw}-dimensional vector by *sw*-th subswarm of *swarm*^{sw}, the context vector is defined as:

$$CV = \left[cv_1^1, ..., cv_{D_d^1}^1, cv_1^2, ..., cv_{D_d^2}^2, ..., cv_1^{sw}, ..., cv_{D_d^{sw}}^{sw}, ..., cv_1^{SW}, ..., cv_{D_d^{SW}}^{SW} \right] \quad (35)$$

The *m*th particle of *sw*-th subswarm, denoted as $u_m^{sw} = \left[u_{m1}^{sw}, u_{m2}^{sw}, \ldots, u_{mD_d^{sw}}^{sw} \right]$ is evaluated using the context vector to complement the missing components. This is achieved by substituting the buffer components that corresponds to the contribution of the *sw*th swarm, with the components of u_m^{sw} retaining the rest of the context unchanged. Hence, the objective value assigned to u_m^{sw} is described as follows:

$$fit\left(u_m^{sw}\right) = fit\left(CV_m^{sw}\right) \quad (36)$$

where,

$$CV_{sm}^{sw} = \left[cv_1^1, ..., cv_{D_d^1}^1, ..., u_{m1}^{sw}, ..., u_{mD_d^{sw}}^{sw}, ..., cv_1^{SW}, ..., cv_{D_d^{SW}}^{SW} \right] \quad (37)$$

with $sw = 1,2, \ldots, SW$ and $m = 1,2, \ldots, pop$.

A reasonable choice for the contributed information of each subswarm is its global best position for *sw*-th subswarm, i.e,

$$cv^{sw} = gbest^{sw} = \left[gbest_1^{sw}, gbest_2^{sw}, ..., gbest_{D_d^{sw}}^{sw} \right] \quad (38)$$

This produces a context, which contains the global best positions of all subswarms, namely,

$$CV = \left[gbest_1^1, ..., gbest_{D_d^1}^1, ..., gbest_1^{sw}, ..., gbest_{D_d^{sw}}^{sw}, ..., gbest_1^{SW}, ..., gbest_{D_d^{SW}}^{SW} \right] \quad (39)$$

3.2 C-MPBPSO Algorithm

Here, the number of particles per swarm is denoted by *pop* and each particle by *m* in subswarm *sw*, where $m = 1, 2, . . . , pop$, $sw = 1, 2, . . . , SW$ and $ß = 1, 2, \ldots, D_d^{sw}$. Let $u_m^{nsw} = \left[u_{m1}^{nsw}, u_{m2}^{nsw}, ..., u_{m\beta}^{nsw}, ..., u_{mD_d^{sw}}^{nsw} \right]$ denote position vector of particle *m* in subswarm *sw* at iteration *n*, where D_d^{sw} be the dimension of each of *sw* subcomponents and $u_m^{nsw} \in \{0,1\}$. Moreover, $w_m^{nsw} = \left[w_{m1}^{nsw}, w_{m2}^{nsw}, ..., w_{m\beta}^{nsw}, ..., w_{mD_d^{sw}}^{nsw} \right]$ denotes decimal position of particle *m* in subswarm *sw* at iteration *n*. Let personal best position for particle *m* in subswarm *sw* be $pbest_m^{nsw} = \left[pbest_{m1}^{nsw}, pbest_{m2}^{nsw}, ..., pbest_{m\beta}^{nsw}, ..., pbest_{mD_d^{sw}}^{nsw} \right]$ and $pbest_m^{nsw} \in \{0,1\}$. In addition, $gbest^{nsw}$ denotes global best position for sub-swarm *sw*, where $gbest^{nsw} = \left[gbest_1^{nsw}, gbest_2^{nsw}, ..., gbest_\beta^{nsw} ..., gbest_{D_d^{sw}}^{nsw} \right]$ and $gbest^{nsw} \in \{0,1\}$. $z_m^{nsw} = \left[z_{m1}^{nsw}, z_{m2}^{nsw}, ..., z_{m\beta}^{nsw}, ..., z_{mD_d^{sw}}^{nsw} \right]$ is the velocity vector associated with *m*-th particle in subswarm *sw*. Let $prob_{max}$ and $prob_{min}$ ($prob_{min} = -\, prob_{max}$) be the predefined upper and lower bounds of linear pseudo probability. The maximum and minimum velocity levels, vel_{max} ($vel_{max} = prob_{max}$) and vel_{min} ($vel_{min} = -\, vel_{max}$), are defined to bound the velocity of particles. The cooperative MPBPSO algorithm is given as follows:

Step 1. Initialisation (n=0)

(i) Set parameter values of ω, θ_1, θ_2, $prob_{max}$, $prob_{min}$, vel_{max}, and vel_{min} where $prob_{min} = -prob_{max}$, $vel_{max} = prob_{max}$ and $vel_{min} = -vel_{max}$.

(ii) Initialise velocity vector z_m^{0sw} and initial pseudo probability vector w_m^{0sw} for m = 1, 2, . . . , pop, sw = 1, 2, . . . , SW and ß=1, 2, …, D_d^{sw} as:

$$z_m^{0sw} = \left[z_{m1}^{0sw}, z_{m2}^{0sw}, ..., z_{m\beta}^{0sw}, ..., z_{mD_d^{sw}}^{0sw} \right] = [0.000, 0.000, ..., 0.000] \quad (40)$$

$$w_m^{0sw} = \left[w_{m1}^{0sw}, w_{m2}^{0sw}, ..., w_{m\beta}^{0sw}, ..., w_{mD_d^{sw}}^{0sw} \right] = [0.000, 0.000, ..., 0.000] \quad (41)$$

$$pbest_m^{0sw} = \left[pbest_{m1}^{0sw}, pbest_{m2}^{0sw}, ..., pbest_{m\beta}^{0sw}, ..., pbest_{mD_d^{sw}}^{0sw} \right] = [0, 0, ..., 0] \quad (42)$$

$$gbest^{nsw} = \left[gbest_1^{nsw}, gbest_2^{nsw}, ..., gbest_\beta^{nsw}, ..., gbest_{D_d^{sw}}^{nsw} \right] = [0, 0, ..., 0] \quad (43)$$

(iii) Generate pop particles of candidate solution vectors $\left\{ u_m^{1sw} = \left[u_{m1}^{1sw}, u_{m2}^{1sw}, ..., u_{m\beta}^{1sw}, ..., u_{mD_d^{sw}}^{1sw} \right] \right\}$ using the following relationships:

$$prob_{m\beta} = \left(w_{m\beta}^{0sw} - prob_{\min} \right) / \left(prob_{\max} - prob_{\min} \right) \quad (44)$$

$$u_{m\beta}^1 = \begin{cases} 1 & \text{if } rand \le prob_{m\beta} \; \left(0 \le prob_{m\beta} \le 1 \right) \\ 0 & \text{else} \end{cases} \quad (45)$$

(iv) Set $z_m^{0sw} = z_m^{1sw}$ and $w_m^{0sw} = w_m^{1sw}$.

Step 2. Initialise context vector CV_m^{nsw} using the global best position of each subswarm.

Step 3. Evaluate fitness value of each particle, i.e., $fit\left(u_m^{nsw}\right) \; \left(= fit\left(CV_m^{nsw}\right)\right)$.

Step 4. Update historical best position $pbest_m^{nsw}$ and the global best position $gbest^{nsw}$ based on their current fitness.

Step 5. Calculate the velocity and the pseudo probability of each particle using Equations (46) through (49).

$$z_{m\beta}'^{nsw} = \omega z_{m\beta}^{nsw} + \theta_1 rand_1 \left(pbest_{m\beta}^{nsw} - u_{m\beta}^{nsw} \right) + \theta_2 rand_2 \left(gbest_{m\beta}^{nsw} - u_{m\beta}^{nsw} \right) \quad (46)$$

$$z_{m\beta}^{nsw} = \begin{cases} vel_{\min} & \text{if } z_{m\beta}'^{nsw} \le vel_{\min} \\ z_{m\beta}'^{nsw} & \text{if } vel_{\min} < z_{m\beta}'^{nsw} < vel_{\max} \\ vel_{\max} & \text{if } vel_{\max} \le z_{m\beta}'^{nsw} \end{cases} \quad (47)$$

$$w_{m\beta}'^{nsw} = w_{m\beta}^{nsw} + z_{m\beta}^{nsw} \quad (48)$$

$$w_{m\beta}^{nsw} = \begin{cases} prob_{\min} & \text{if } w_{m\beta}'^{nsw} \le prob_{\min} \\ w_{m\beta}'^{nsw} & \text{if } prob_{\min} < w_{m\beta}'^{nsw} < prob_{\max} \\ prob_{\max} & \text{if } prob_{\max} \le w_{m\beta}'^{nsw} \end{cases} \quad (49)$$

Step 6. Generate $u_m'^{nsw}$ using Equations (44) and (45),

Step 7. Copy $u_m'^{nsw}$ in the proper position of CV_m^{nsw}, then evaluate $fit\left(u_m'^{nsw}\right)$

Step 8. If $n \ge 1$, check the fitness of the particles. If $fit\left(u_m'^{nsw}\right) - fit\left(u_m^{nsw}\right) \le 0$, update the position of particle $u_m'^{nsw}$ using the following rules:

Repeat D_d^{sw} times

$$u'^{nsw}_{m\beta} = \begin{cases} gbest^{nsw}_{\beta} & \text{if } 0 \le rand < \tau \\ pbest^{nsw}_{m\beta} & \text{if } \tau \le rand < (2\tau+1)/3 \\ irand & \text{if } (2\tau+1)/3 \le rand < (\tau+2)/3 \\ u'^{nsw}_{m\beta} & \text{if } (\tau+2)/3 \le rand \le 1 \end{cases} \quad (50)$$

where $\tau = \tau_1 + \tau_2 \times rand$

Step 9. Update CV^{nsw}_{m} and evaluate $fit\left(u'^{nsw}_{m}\right)$. Then, update u^{nsw}_{m} as follows

$$u^{nsw}_{m} = \begin{cases} u^{nsw}_{m} & \text{if } fit\left(u'^{nsw}_{m}\right) < fit\left(u^{nsw}_{m}\right) \\ u'^{nsw}_{m} & \text{if } fit\left(u'^{nsw}_{m}\right) \ge fit\left(u^{nsw}_{m}\right) \end{cases} \quad (51)$$

Step 10. Compare the current position of each particle with its historical best position based on the fitness value, and renew the global best.

Step 11. Change $n = n+1$. If the algorithm has reached a stopping criterion, then stop the iterations. Otherwise, return to step 7.

3.3 Parameter Settings and Performance Tests

The values of parameters of the PSO algorithms significantly affect their performance. The existing cooperative PSO-related research noted that the decomposition strategy significantly influences its performance. This strategy involves the determination of the type of information exchanged, period of exchange, and sharing topology. Several decomposition strategies have been introduced for the cooperative PSO. The comprehensive survey of these strategies is given by El-Abd (2008). Synchronous and asynchronous communications are basically utilised as strategies in the cooperative PSO. In the synchronous communication, the subswarms exchange the information in every predetermined number of iterations. On the other hand, the asynchronous one includes a certain condition (e.g., the fitness value does not improve for a specified number of iterations) as an indicator for exchanging information. The best experience of cooperating swarms, referred to as "*gbest*", is mostly used as the type of exchangeable information, where the exchange is updated in every predetermined number of iterations (El-Abd, 2008).

In the cooperative PSO, the number of subswarms and that of particle form the swarm size. El-Abd (2008) found that increase in the number of subswarms, which in turn, means decrease in the number of particles, should be followed by accelerating the synchronisation period. Otherwise, the swarms will separate, being difficult to escape from local optimal. He also suggested that the number of subswarms up to five with a shorter synchronisation period. Therefore, the succeeding parameters tests are to be conducted to find the proper configuration of swarm size.

The model is initially tested on a hypothesised small network (see Figure 2) to investigate the proper swarm configuration of C-MPBPSO. The similar network is conducted by previous research for investigating the parameter values of the standard MPBPSO (Zukhruf *et al.*, 2012). Therefore, the related parameter values are determined as those used by Zukhruf *et al.* (2012) except for the number of swarms and particles. The supernetwork shown in Figure 2 is composed of 32 links and 9 nodes where one manufacturer, two wholesalers, two retailers, two demand markets, two passenger car ODs exist. The parameter values and functional forms applied to the lower level are the same as those used by Yamada *et al.* (2011). There are 16 alternative actions, as shown in Table 1, including the road capacity improvement and the new road establishment. The parameter of the cooperative

MPBPSO is tested for a total of 10 runs with different seeds of random values, because the algorithms incorporate randomised processes that influence their computational results.

The performance comparisons are conducted for the best, average, and worst solutions and the computational time. The computational times are estimated with a PC of Intel Core i7 2.2 GHz CPU and 8.0 GB RAM.

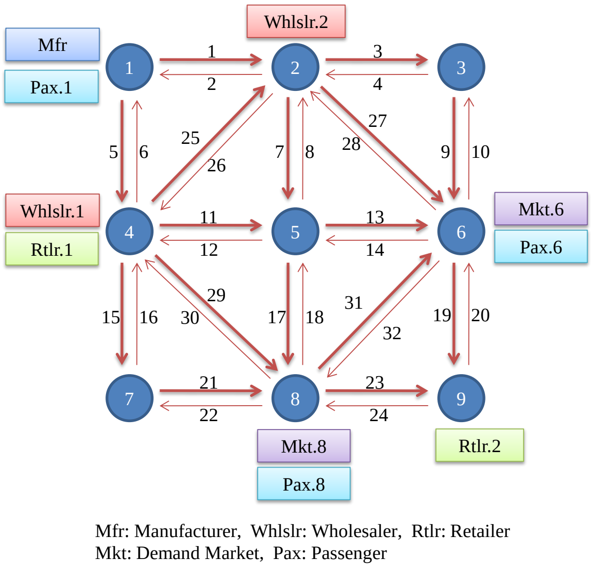


Figure 2. Test Network

Table 1. Listing of Actions for Test Network

No	Type of action	Location	Investment/operation cost
1	Road Widening	1, 2	1800
2	Road Widening	3, 4	500
3	Road Widening	5, 6	1800
4	Road Widening	7, 8	200
5	Road Widening	9, 10	500
6	Road Widening	11, 12	200
7	Road Widening	13, 14	600
8	Road Widening	15, 16	500
9	Road Widening	17, 18	600
10	Road Widening	19, 20	500
11	Road Widening	21, 22	500
12	Road Widening	23, 24	500
13	New Way	25, 26	1800
14	New Way	27, 28	1800
15	New Way	29, 30	1800
16	New Way	31, 32	1800

The proper number of subswarms is investigated first. In the case of comparing the performance of C-MPBPSO with different number of swarms, the maximum possible number of particles is kept fixed (Number of particles per subswarm x Number of subswarms x Number of iterations = 4320). The increasing number of subswarms will decrease the number of iterations, since the tests are conducted with three different numbers of swarms (i.e., 2, 4, and 8) in which the number of particles per subswarm is fixed at 30. The results (Table 2) show that the larger number of subswarms potentially converges into premature solutions. This indicates that increased number of subswarms will enhance the communication needs. In addition, if the number of swarms is increased, the computation time required will be decreased. This result represents that the smaller dimension of subspace is likely to significantly improve the convergence speed, even though to increase the risk of premature convergence.

Table 2. Tests for number of subswarms in C-MPBPSO

No. of subswarms	2	4	8
No. of iterations	36	18	9
Max. No. of Solutions	4320	4320	4320
Best	1.024	1.024	1.024
Average	0.957	1.024	0.798
Worst	0.944	1.024	0.701
Ave. CPU times (sec)	4780	556	162

In order to investigate the proper swarm size, setting the maximum possible number of solutions (i.e., *iter* x *pop* x SW x 2 (=possible number of particles to be searched in Steps 9 & 10) to around 4.500, which is applied by the similar MPEC problem (e.g., Yamada *et al.*, 2009; Zukhruf *et al.*, 2012), the test is undertaken with the same parameter values as those applied in Zukhruf *et al.* (2012) except for the number of particles and iterations. The tests are conducted with four subswarms and five different population sizes (i.e., five different numbers of particles). Table 3 shows that increased number of particles up to a certain limit potentially improve the C-MPBPSO performance. Smaller number of particles is likely to be trapped into local optimal. The best parameter values are found to be 4 for *SW*, and 28 x 20 for *iter* x *pop;* with the values of the objective function being 1.024 for the best, average, and worst solutions, containing action numbers 1 and 4.

Table 3. Tests for number of particles in C-MPBPSO

No. of Particles per Subswarm	5	10	20	30	40
No. of Subswarms	4	4	4	4	4
No. of Iterations	112	56	28	18	14
Max. No. of Solutions	4480	4480	4480	4320	4480
Best	1.024	1.024	1.024	1.024	1.024
Average	1.016	1.024	1.024	1.024	1.024
Worst	0.982	1.024	1.024	1.024	1.024
Ave. CPU times (sec)	1448	537	535	556	1832

3.4 Comparison with PSOs

The performance comparison with the original MPBPSO is conducted to prove the superiority of C-MPBPSO. The performances of DBPSO and PBPSO are also compared for a thorough examination of C-MPBPSO performance. The best parameter values of the DBPSO algorithms were calculated for a total of ten runs and estimated as 30 x 150 for *pop* x *iter*, 1.0 for ω , 2 for both θ_1 and θ_2 ,, and 20 for vel_{max}. Those of PBPSO and MPBPSO were taken from the previous numerical test (e.g., Zukhruf *et al.*, 2012). The best parameter values are found to be 25 x 90 for *pop* x *iter*, 0.8 for ω , 2 for both θ_1 and θ_2, 50 for vel_{max}, 0.4 for τ_1, and 0.5 for τ_2 in MPBPSO; and 45 x 100 for *pop* x *iter*, 0.8 for ω , 2 for both θ_1 and θ_2, and 50 for vel_{max} in PBPSO, respectively.

Table 4 compares the results of four types of discrete binary PSO algorithms for the values of objective function in the best, average, and worst solutions, as well as for the average computational time taken to complete the search. The results show that C-MPBPSO has better stability and faster searching ability than DBPSO.

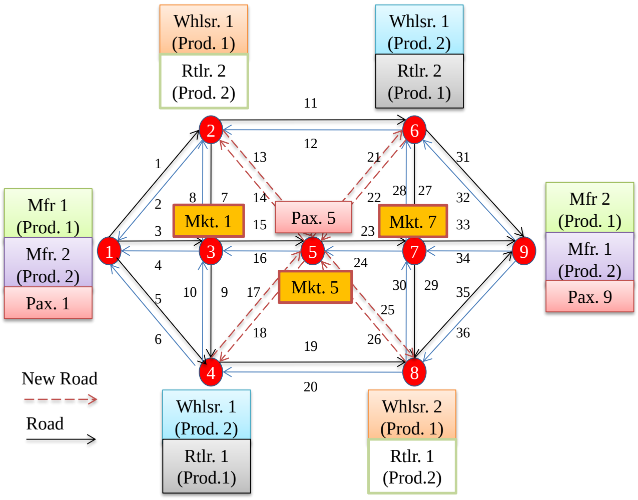
Table 4. Performance Comparison among PSOs (ten runs)

	DBPSO	PBPSO	MPBPSO	C-MPBPSO
Best	1.024	1.024	1.024	1.024
Average	1.008	1.024	1.024	1.024
Worst	0.227	1.024	1.024	1.024
Ave. CPU times (sec)	3763	1446	1236	535

It can also be seen from Table 4 that C-MPBPSO, MPBPSO and PBPSO are capable of providing the same quality and stability of solutions. However, C-MPBPSO requires shorter times to compute than the others. This can be explained by the fact that C-MPBPSO requires a lower number of particles, wherein the accumulated numbers of particles evaluated for C-MPBPSO, MPBPSO and PBPSO are 372, 482 and 531, respectively. This fact also indicates that the decomposition strategy of cooperative approach significantly accelerates the rate of convergence.

4. APPLICATION RESULTS

The MPEC model is then applied to a relatively larger-sized supernetwork to investigate a possible development strategy on a transport network for enhancing the efficiency of SCNs. C-MPBPSO is applied as the solution techniques for the upper level. The supernetwork as illustrated in Figure 3 is composed of 36 links and 9 nodes with two manufacturers, two wholesalers, two retailers, three demand markets, three passenger car ODs. The parameter values and functional forms applied to the lower level are the same as those used by Yamada *et al.* (2011). Here, 16 transport-related projects are considered, including the roads capacity improvement and the new road establishment (Table 5).



Mfr: Manufacturer, Whslr: Wholesaler, Rtl: Retailer,
Mkt: Demand Market, Pax: Passenger Car

Figure 3. Larger-sized Test Network

Table 5. Listing of Actions for Larger-sized Test Network

No	Type of action	Location	Investment/operation cost
1	Road Widening	1,2	875
2	Road Widening	3,4	625
3	Road Widening	5,6	875
4	Road Widening	9,10	625
5	Road Widening	11,12	1250
6	Road Widening	15,16	625
7	Road Widening	19,20	1250
8	Road Widening	23,24	625
9	Road Widening	29,30	625
10	Road Widening	31,32	875
11	Road Widening	33,34	625
12	Road Widening	35,36	875
13	New Way	13,14	1750
14	New Way	17,18	1750
15	New Way	21,22	1750
16	New Way	25,26	1750

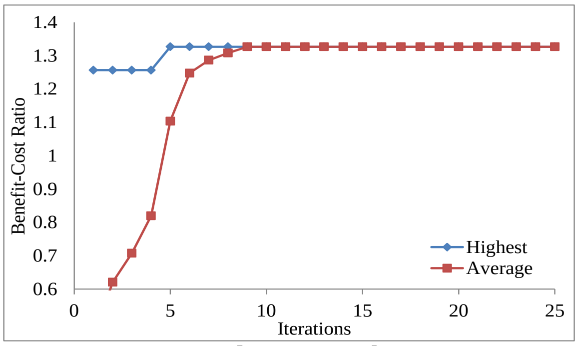


Figure 4. BCR values in each iteration

Table 6. Amount of products and surpluses

	Without actions	With actions implemented
Amount of products	236	248
Producer Surplus	30,806	32,546
Consumer Surplus	6,523	6,937
Total Surplus	37,329	39,483

A combination of two actions is found to be the best solution with a BCR value of 1.33. It includes action number 2 and 4 for road widening. This result corresponds with the findings of Yamada *et al.* (2009), where the cost effectiveness is higher in road widening than new construction. The best BCR value appears in the 5th iteration (Figure 4). Figure 4 also shows that C-MPBPSO reaches convergence in the 9th iteration. Table 6 demonstrates the changes in surpluses and total amount of products transacted/distributed as a result of the improvement in the transport network. The increase in total surplus means the enhancement of the efficiency of SCNs. The results obtained from the model are plausible, since links 3 and 4 are found to be the congested links in case without any actions implemented. The capacity expansion of these congested links would improve the efficiency of SCNs.

5. CONCLUSIONS

This paper presented a new cooperative discrete binary PSO algorithm, namely, C-MPBPSO, for solving the MPEC-based freight transport network design problem. The problem evaluates the decision-making in a freight transport network and selects suitable actions in it to improve the efficiency of SCNs. The lower level describes supply chain-transport supernetwork equilibrium (SC-T-SNE) integrating SCNs with a transport network; whilst the upper level determines the best combination of the actions for road network improvement, where C-MPBPSO is applied as its solution procedures. In order to investigate the proper parameter values of C-MPBPSO, the several numerical tests were conducted. After the proper values were determined, the performance of the C-MPBPSO and the existing discrete binary PSO algorithm were compared. The testing results with a hypothesised small network exhibited that the C-MPBPSO can offer better performance than the conventional discrete binary PSO algorithms, considering its computation times and quality of solutions. The model was then applied to a relatively larger-sized supernetwork. Results indicate that road-widening actions could enhance the efficiency of SCNs. For future research, the model would be extended for considering reverse logistics, and applied to the actual large transportation network.

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