COMPARATIVE-STATIC ANALYSIS OF CONGESTION ROAD TOLL FOR HETEROGENEOUS TRAVEL DEMAND

Chih-Peng CHU
Department of Economics
National Dong Hwa University
Hualien, Taiwan, R.O.C
Email: chpchu@mail.ndhu.edu.tw
Fax: +886-3-8662535

Abstract: Currently, McDonald (1995) concluded that the second-best optimum toll on the tollway equals the first-best congestion pricing plus an adjustment term. However, McDonald, like most researchers, adopted the assumption of homogeneity of demand characteristics to simplify their models. In our study, we assume the demand is heterogeneous. Because the demand of users is not homogeneous in reality, the characteristics of heterogeneous users should influence the results of congestion pricing policy making. This paper develops the congestion-pricing model of two routes running parallel, one highway and one local road, with n types of elastic-demands. This paper conducts the closed form for first-best and second-best tolls in the case of heterogeneous travel demand. We apply comparative-static analysis for these tolls to understand the impacts of parameter changes on these tolls. A numerical simulation is presented at the end to illustrate our theoretical analysis.

Key Words: congestion road pricing, heterogeneous travel demand, pricing policy.

1. INTRODUCTION

After Pigou opened the "door" of congestion road pricing in 1920, this concept in economics plays as an important method on travel demand management. Accompanied by the growth of economic activities and rise in household income, the demand for travel has been rapidly increased, especially the usage of private transportation modes. With the limitation of transportation supply, most of metropolitans face terrible traffic congestion problem. With improvement in electronic technologies, the road congestion pricing seems to be feasible now. However, from the failure experience of Hong Kong's electronic road pricing, the center of discussions on urban road congestion has been shifting from technological considerations toward political acceptability, the methods of introducing road pricing schemes and various alternatives to road pricing in traffic demand regulations.

As far as the researcher understands, solving road congestion problem in reality is a complicated dynamic process. There always is a dilemma faced by the analyst to choose between a dynamic model and a simple static model. In general, the dynamic model could generate the result that is closer to the real word situation. But the analytical solutions are often difficult to obtain (such as Newell (1988)). The static model is a simpler representation of reality but it would provide analytical solutions and derivation of general economics principles to the problem studied (such as Verhoef, 1999). This research concentrates on the static model approach for revealing more economic insights for the congestion toll policies.

On the other hand, the theory of road pricing has been switched its focus from first-best pricing (marginal cost pricing) to second best pricing and extended its analytical abilities to much realistic situations. The first model that introduced second-best toll is developed by Levy-Lambert (1968) and Marchand (1968), and followed by more discussions from Crew and Kleindorefer (1986) and Shermand (1989). For a typical second-best model is a simple urban highway system consisting of just two congestion routes, only one route is charged by congestion optimal toll. The basic theoretical result is the second-best toll on the tollway should be less than the first-best toll if two routes are substitutive, and the second-best toll should be higher than first-best toll if two routes are complements. McDonald (1995) utilized this approach and concluded that the second-best optimum toll equals the marginal congestion cost on the tollway plus an adjustment term. The sign of this adjustment term depends on the sign of cross-partial effect of two routes. If two routes are complements, the adjustment term is positive. If two routes are substitutive, the adjustment term is negative. Latter, in his 1999 paper, McDonald focused on the substitution case and presented the second-best toll in a modified format. However, McDonald, like most researchers, adopted the assumption of homogeneity of demand characteristics to simplify their models.

Even the heterogeneous users sometimes are indistinguishable by observation and the congestion charges always be anonymous, the heterogeneity of demand will still have a profound impact on the results of congestion pricing policy. For example, there are two groups of users A and B using a single road. Suppose that current travel time on the road equals to total number of car on the road (travel time is a function of travel flow of group A and B, that us assume the travel time is equal to the sum of both flows). For example, there are 10 cars from group A and 10 cars from group B on the road now. Suppose the value of time for group A is \$0.1 dollar per minute and for group B is \$0.2 dollar. The total cost for this system is 10*20*0.1+10*20*0.2=\$60. Consider that next entry to the road is from group A, then the total cost become 11*21*0.1+10*21*0.2=\$65.1. Consider that next entry is from group B, then the total costs become 10*21 * 0.1+11*21 * 0.2= \$67.2. Therefore, we know the marginal cost caused by additional entry depends on groups of users. As Daganzo (1995) mentioned that the travel is not equally important to everyone, either the value of time. Therefore, the assumption of homogeneity of travelers is not proper. Verhoef et. al. (1995) analyzed the general environment optimal common fees (referred as second-best toll in their definition) for different groups of users under two situations. In first situation, they calculate the environmental toll for an uncongested network and the interdependency among different groups are zero. They also assume that the average private cost is constant. In second situation, they considered the demand interdependencies but focused on tolled and non-tolled groups. Their approaches are not the same as we will discuss in this paper. This paper concentrates on road congestion pricing, road charges for environmental externality are not explicitly considered.

This paper presents a theoretical extension thę above model for the case of heterogeneous travel demand in general cases. This article considers a congestion-pricing model with n types of elastic-demands on a highway system and an alternative local road under different pricing policies. We will discuss the model in section 2. Section 3 will derive the analytical results of toll and in Section 4 some special cases will be presented. Discussions and conclusions present in last section, Section 5.

2. BASIC OF MODEL STRUCTURE

In beginning of this section we will give an informal description of the structure of the models, and the basic assumptions. These assumptions for this research are to keep the analytical process neat and results would have significant insights. Later, it will be, word by word, translated into a set of equations defining the models under different toll policies.

The network structure

Since our model will be static, we utilize a simple but good enough model to conduct an economic analysis of travelers' route choice behavior. This network structure approach has been used by other researchers, e.g. Arnott et. al. (1990) and McDonald et. al. (1995, 1999). In the network, there is only a single origin-destination pair connected by two roads: local road and highway. There is no other choice for travelers, in other words, local road and highway are substitution in nature. The scope of this research is to focus on short run, therefore, the capacities of these two roads are assumed fixed.

Government policies

The toll policies for government are: first-best scheme and second-best scheme. For the first-best scheme, government will be able to charge toll on these two roads. For the second-best scheme, government will only set toll charge devices on highway.

Characteristics of road users

This model considers n types of rational users, and users can only choose which road to use. We also assume that all trips in the system are made by automobile. That means that there is no decision on mode selection. We assume that utility is quasi-linear for every user, such that the income effect caused by the toll could be ignored. On the other hand, the marginal utility function of the user is also regarded as demand function for the trip.

Based on these assumptions, more detailed descriptions on the key parts of this model: demand function, cost function, and welfare function will be given hereafter.

2.2 Demand Function

Suppose that there are n types of travelers in this network. The trip demand functions for n types of users are known as:

$$v_i = Q_i - b_i P_i, i=1,2,..,n \text{--}(1)$$

v_i, travel flow, is the number of trip made by type *i* user, P_i is the price that type *i* user faced, Q_i is the total potential trips of type *i* users, that is the total number of travel flow when price is zero. b_i is the slope of demand function for type *i* user. The inverse demand function for type *i* users is:

$$P_i = Q_i/b_i - v_i/b_i = A_i - \beta_i v_i \text{--}(2)$$

where $A_i = Q_i/b_i$, $\beta_i = 1/b_i$, i=1,2,…,n. From equation 2, we know that P_i is a function of v_i

only, other user groups will not have impacts on single group's price, that is $\frac{\partial P_i}{\partial v_i} = -\frac{1}{b_i}$ and $\frac{\partial P_j}{\partial v_i} = 0$, for $i \neq j$. However, the interactive impacts will affect on the process of analysis when these groups using the same route. At this moment, we can calculate the gross benefit seperately. The gross benefit functions (gross consumer surplus) are the integration of inverse demand function from zero to exact trip flow, that is

$$\text{the gross benefit for type } i \text{ user} = B_i = \int_0^{v_i} P_i(v_i) dv_i \text{, } i=1,2,\ldots,n.$$

The total gross benefit (gross consumer surplus) for all the travelers on this system is the sum of above equations:

$$\text{Total benefit } (TB) = \sum_{i=1}^{n} B_i = \sum_{i=1}^{n} \int_0^{v_i} P_i(v_i) dv_i \text{ --(3)}$$

2.3 Cost Function

The cost function for type i users on road k is a function of the sum of all types of users travel on that road.

Cost functions: $AC_i^k = AC_i^k(v_1^k, v_2^k, \ldots, v_n^k)$, i=1, 2, …, n; k=L, H--------------------------(4)

Where v_i^k, travel flow of type i users on road k. L stands for local road and H stands for highway. For the type i users, their total costs for individual group are:

$$TC_i = AC_i^L(v_1^L, v_2^L, \ldots, v_n^L) \times v_i^L + AC_i^H(v_1^H, v_2^H, \ldots, v_n^H) \times v_i^H \text{ -----------------(6)}$$

The total (travel) cost on this network system is:

$$TC = \sum_{i=1}^{n} (AC_i^L(v_1^L, v_2^L, \ldots, v_n^L) \times v_i^L + AC_i^H(v_1^H, v_2^H, \ldots, v_n^H) \times v_i^H) \text{ ------------------(7)}$$

2.4 Welfare Definition

The objective value, w, in equation (8) is the difference between gross traveler's surplus and the total cost of network.

We assume individual group welfare is additive, then the total welfare is:

$$w = B - TC = \sum_{i=1}^{n} [\int_0^{v_i} P_i(v_i) dv_i - (AC_i^L \times v_i^L + AC_i^H \times v_i^H)] \text{ -------------(8)}$$

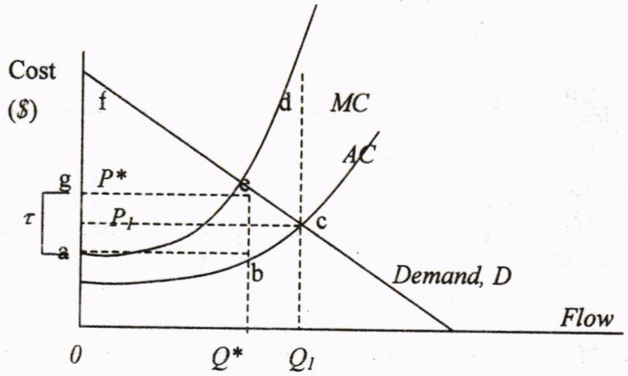

Figure 1. Congestion pricing

3. THE TOLL POLICY MODELS AND THEORETICAL ANALYSES

In this section we design models for two different toll schemes: first-best (FB) toll scheme and second-best (SB) scheme. For each model, we first present the mathematical format of the model, then the toll is presented after solving the model.

3.1 First Best Scheme

First scheme assumes that government can impose the toll on every road as long as it has traffic congestion. This scheme allows government to charge the toll on highway as well as local road. The objective of this scheme is to find a set of traffic flows that will maximize the welfare function as we defined above. The set of optimal traffic flows is the equilibrium solution of the following optimal model with constraints:

$$\text{Max } \ w = B - TC = \sum_{i=1}^{n}\left[\int_{0}^{v_i} P_i(v_i)dv_i - (AC_i^L \times v_i^L + AC_i^H \times v_i^H)\right] \text{----FB Scheme}$$

St :

$$\left.\begin{array}{l} v_i = v_i^L + v_i^H, i=1,2,\ldots,n \\ v_i \geq 0,\ v_i^L \geq 0,\ v_i^H \geq 0 \end{array}\right\} \text{----FB Scheme constraints}$$

The constraints in this scheme are the flow conservation requirement of travel flows and the nonnegative travel flows requirement.

The Lagrangian equation is as follow:

$$L = \sum_{i=1}^{n}\int_{0}^{v_i} P_i(v_i)dv_i - \left(\sum_{i=1}^{n} AC_i^L \times v_i^L\right) - \left(\sum_{i=1}^{n} AC_i^H \times v_i^H\right) - \sum_i \mu_i(v_i - v_i^L - v_i^H) \text{------(9)}$$

The first order conditions is derived by differentiating with L respect to (v_i, v_i^L, v_i^H, and μ_i)

$$\frac{\partial L}{\partial v_j^L} = -AC_j^L - \sum_{i=1}^{n}(\frac{\partial AC_i^L}{\partial v_j^L} \times v_i^L) + \mu_j = 0 \text{ ------(10)}$$

$$\frac{\partial L}{\partial v_j^H} = -AC_j^H - \sum_{i=1}^{n}(\frac{\partial AC_i^H}{\partial v_j^H} \times v_i^H) + \mu_j = 0 \text{ ------(11)}$$

$$\frac{\partial L}{\partial v_j} = P_j(v_j) - \mu_j = 0 \text{ ------(12)}$$

$$\frac{\partial L}{\partial \mu_j} = v_j - v_j^L - v_j^H = 0 \text{ ------(13)}$$

If $P_i(v_i)$ is concave function and the AC_i^j is convex function, then any solution satisfying the first order conditions is an optimal solution for this welfare maximization problem. For most cases in economics, the demand function satisfies the concavity property. In the mean time, most of the transportation cost functions is convex. Therefore, if there is a set of travel flow that satisfies equations (10) to (13), then it will maximize the objective function. To maximize the welfare, when the traffic flows reach the equilibrium, government could charge the group j users the difference between willingness to pay and the (average) travel cost. Take group j as one example, the toll for group j on the highway is shown as follow:

$$\tau_j^H = P_j(v_j) - AC_j^H$$

From (11) and (12), we have

$$\tau_j^H = \mu_j - AC_j^H$$

$$= AC_j^H + \sum_{i=1}^{n}(\frac{\partial AC_i^H}{\partial v_j^H} \times v_i^H) - AC_j^H = \sum_{i=1}^{n}(\frac{\partial AC_i^H}{\partial v_j^H} \times v_i^H) \text{ ------(14)}$$

This first-best optimal toll for group j is sum of a weight multiplies the v_i^H in optimal condition for all i groups. The weight is that differentiation of each group cost function with respect to the travel flow of the j group users. In other words, the weight is the slope of the average cost (AC) curve. Since the different groups of users may have different slope of the average cost, the tolls should be different among different groups.

For the toll on local road, we follow the similar process and have the followings:

$$\tau_j^L = \sum_{i=1}^{n}(\frac{\partial AC_i^L}{\partial v_j^L} \times v_i^L) \text{ ------(15)}$$

In real world, if the vehicles (users) are easily to be distinguished and their interactive-effect on costs are different, then government should charge each group of vehicles (users) different tolls. This is the case for Singapore government road pricing policy. They charge automobile and motorcycle users different tolls. However, in some case that vehicles (users) are not distinguishable, the common fee approach derived by Vehief et. al. would be a solution to this scheme.(Vehief, 1995).

3.2 Second-best Scheme

This scheme considers the situation that government set toll collection booths on highway only. We also assume the objective of this scheme is to maximize the welfare of whole network as in the first-best scheme. The mathematical form of this scheme is that the first-best scheme model plus n "extra" constraints ($P_i = AC_i^L$) . These constraints show the local road users (for all types) only pay the average travel cost.

$$\text{Max}\ \ w = B - TC = \sum_{i=1,2}[\int_0^{v_i} P_i(v_i)dv_i - (AC_i^L \times v_i^L + AC_i^H \times v_i^H)] \text{---------- SB Scheme}$$

St :

$$\left.\begin{array}{l} P_i = AC_i^L, i=1,2,\ldots,n \\ v_i = v_i^L + v_i^H, i=1,2,\ldots,n \\ v_i \geq 0,\ v_i^L \geq 0,\ v_i^H \geq 0 \end{array}\right\} \text{----SB Scheme Constrains}$$

The Lagrangian shown as follow:

$$L = \sum_{i=1}^{n}[\int_0^{V_i} P_i(v_i)dV_i - AC_i^L \times v_i^L - AC_i^H \times v_i^H] - \sum_{i=1}^{n}[\lambda_i(P_i(v_i) - AC_i^L)] - \sum_{i=1}^{n}\mu_i(v_i - v_i^L - v_i^H) \text{---(16)}$$

The first order conditions are derived by differentiating L with respect to (v_i, v_i^L, v_i^H, λ_i and μ_i) .

For particular group j, we have

$$\frac{\partial L}{\partial v_j^L} = -AC_j^L - \sum_{i=1}^{n}\frac{\partial AC_i^L}{\partial v_j^L} \times v_i^L + \sum_{i=1}^{n}[\lambda_i \frac{\partial AC_i^L}{\partial v_j^L}] + \mu_j = 0 \text{---(17)}$$

$$\frac{\partial L}{\partial v_j^H} = -AC_j^H - \sum_{i=1}^{n}(\frac{\partial AC_i^H}{\partial v_j^H} \times v_i^H) + \mu_j = 0 \text{---(18)}$$

$$\frac{\partial L}{\partial v_j} = P_j(v_j) - \lambda_j \frac{\partial P_j(v_j)}{\partial v_j} - \mu_j = 0 \quad \text{------(19)}$$

$$\frac{\partial L}{\partial \lambda_j} = P_j(v_j) - AC_j^{\,L} = 0 \quad \text{------(20)}$$

$$\frac{\partial L}{\partial \mu_j} = v_j - v_j^L - v_j^H = 0 \quad \text{------(21)}$$

Since there is no charge for local road, the $\tau_j^L = 0$ for all j, and the toll for highway will be:

$$\tau_j^H = P_j(v_j) - AC_j^H$$

From (18)

$$\tau_j^H = \lambda_j \frac{\partial P_j(v_j)}{\partial V_j} + \mu_j - AC_j^{\,H}$$

From (17)

$$\tau_j^H = \sum_{i=1}^{n} \frac{\partial AC_i^L}{\partial v_j^L} v_i^H + \lambda_j \frac{\partial P_j(v_j)}{\partial v_j} \quad \text{------(22)}$$

A comparison of the Equation (22) with (14) shows that the second-best toll is the first-best toll on highway plus an extra term, which depends on the slope of demand functions.

4. DISCUSSIONS ON SPECIAL CASES

4.1 Heterogeneous demand with the same cost-effect

For the case of cost functions with same cost-effect of flows, that is $\frac{\partial AC_i^H}{\partial v_j^H} = \frac{\partial AC_i^H}{\partial v_i^H}$. The same cost-effect means that other groups of user will have the same effect on the cost of group *j* as the user in its own group. With this property, we rewrite Equation (14) as:

$$\tau_j^H = \sum_{i=1}^{n} (AC_i^{H'} \times V_i^H) = \tau_{FB}^H \quad \text{------(23)}$$

In this case, government should charge an identical toll to all the groups even their demands and cost functions are different.

For the toll on local road is easy to obtain by following the similar process.

$$\tau_j^L = \sum_{i=1}^{n} (AC_i^{L'} \times V_i^L) = \tau_{FB}^L \quad \text{------(24)}$$

For the case of second-best toll with same cost-effect, we have the followings:
From (19) and (20)

$$P_j\ (v_j) - \lambda_j \frac{\partial P_j\ (v_j)}{\partial v_j} = \mu_j \text{ and } P_j\ (v_j) = AC_j^L$$

Therefore, $AC_j^L - \lambda_j \dfrac{\partial P_j\ (v_j)}{\partial v_j} = \mu_j$

From (17), we also have

$$\mu_j = AC_j{}^L + \sum_{i=1}^{n} \left(\frac{\partial AC_i^L}{\partial v_j^L} \times v_i^L\right) + \sum_{i=1}^{n} \left(\lambda_i \times \frac{\partial AC_i^L}{\partial v_j^L}\right)$$

$$\text{so, } -\lambda_j \frac{\partial P_j\ (v_j)}{\partial v_j} = \sum_{i=1}^{n} \left(\frac{\partial AC_i^L}{\partial v_j^L} \times v_i^L\right) + \sum_{i=1}^{n} \left(\lambda_i \times \frac{\partial AC_i^L}{\partial v_j^L}\right) \text{------------------------------(25)}$$

then $\lambda_j \dfrac{\partial P_j\ (v_j)}{\partial v_j}$ will be independent of j. That means $\lambda_j \dfrac{\partial P_j\ (v_j)}{\partial v_j}$ will be the same for all j. In that case, we conclude $\tau_j^H = \tau_{SB}^H$ for all i and we can rewrite it as

$$\tau_{SB}^H = \sum_{i=1}^{n} AC_i^{H'} \times v_i^H + \lambda_j \frac{\partial P_j\ (v_j)}{\partial v_j} \text{--(26)}$$

Notice these congestion tolls are not determined until these traffic flows reach the equilibrium situation. A simulation of specific case study will demonstrate some insight information for these impacts of toll policies. Following is the description of the case.

Assume there are only two different groups of uses, *1* and *2*, with cost functions as BPR (Bureau of public Roads) function forms. For groups *1* and *2*, the demand functions are

$$v_i = Q_i - b_i P_i,\ i=1,2$$

v_i : Aggregated traffic volume of group i,

P_i: The trip price for group i,

Q_i: The potential demand of group i,

We set Q_1=6000 and Q_2=5000, for example. Q_i is also the total travel volume if the trip price equals zero. We also assume that b_1=2 and b_2=3.

The cost functions are in the BPR (Bureau of Public Roads) form.

$$AC_i^j = vot_i \times t_0^j[1+\alpha^j(\sum_i v_i^j / k^j)^{\beta^j}], i=1、2; j=L、H$$

vot_1=1 and vot_2=3 are the values of time for group 1 and 2. t_0^H=30 and t_0^L=60 are the free flow time on highway and local road, respectively. k^H=2500 and k^L=2000 are the capacities of highway and local road. $\alpha^H = \alpha^L$=0.15 and $\beta^H = \beta^L$=4 are the default setting parameters for BPR cost functions. Table1 shows the simulation results.

Table 1. Simulation Results for Same Cost-effect Case

	v_1^L	v_1^H	v_2^L	v_2^H	AC_1^L	AC_1^H	AC_2^L	AC_2^H
No Toll	3590	2106	0	3619	153.46	153.46	460.39	460.39
Second-Best	4262	1247	0	3677	245.55	97.73	736.65	293.19
First-Best	3550	1437	0	3026	149.31	75.72	447.92	227.15
	P_1	P_2	τ^L	τ^H				
No Toll	153.46	460.39	0.00	0.00				
Second-Best	245.55	441.01	0.00	147.82				
First-Best	506.55	657.97	357.24	430.82				

Table 1 shows the results of simulation for the case of same cost-effect case. Group 1 users travel on both local road and highway with cost \$153.46 at no toll scheme, Group 2,on the other hand, have another cost \$460.39 on both routes. The average costs for different groups of users on local road are different because of their values of time. In Second-Best Scheme, since there is no toll on local road, the group 1 users their average cost equal to their price, \$245.55. Group 2 users on local road face a willingness-to-pay (price) \$441.01 higher than the cost to them, \$736.65. Therefore, there is no group 2 flow on local road. Government will charge a toll, \$147.82, on the highway users regardless the travel type. For group 1 users on highway, their average cost, \$97.73, plus this toll equals their willingness-to-pay (price) and also equals to the cost of using local road. For group 2 user on highway, their average cost plus this toll equals their willingness-to-pay and smaller than the cost of using local road. In the scheme of First-Best, government charge toll for using highway higher than that of local road. The results also show that for particular path government should charge both groups of users same toll which will make their "total cost" equals their willingness-to-pay.

4.2 Heterogeneous demand with common cost functions

In this section, we simplify our model by assuming the cost functions for each group of users are the same. In other words, $AC_i^k = AC^k$, $i=1,2,...,n.,k=L,H$. Therefore, we rewrite the toll for First-best scheme will be:

From (14)

$$\tau_j^H = P_j(V_j) - AC^H$$

$$= AC^{H'} \times (\sum_{i=1}^{n} v_i^H) = AC^{H'} \times v^H = \tau_{FB}^H \text{ ----------------(27)}$$

From (15) $\tau_j^L = P_j(v_j) - AC^L$

$$= AC^{L'} \times (\sum_{i=1}^{n} v_i^L) = AC^{L'} \times v^L = \tau_{FB}^L \text{ ------------------(28)}$$

For Second-Best scheme:

From (25), we have

$$\lambda_j P_j^{'}(v_j) = \lambda_i P_i^{'}(v_i)$$

$$\lambda_i = \frac{\lambda_j P_j^{'}(V_j)}{P_i^{'}(V_i)}$$

$$\sum_{i=1}^{n} \lambda_i = \lambda_j P_j^{'}(V_j) * \sum_{i=1}^{n} (\frac{1}{P_i^{'}(V_i)})$$

Also we have

$$(\sum_{i=1}^{n} \lambda_i) \cdot AC^{L'} = AC^L + AC^{L'} - AC^H - AC^{H'}$$

$$= MC^L - MC^H$$

$$\sum_{i=1}^{n} \lambda_i = \frac{MC^L - MC^H}{AC^{L'}}$$

$$\lambda_j P_j^{'}(v_j) = \frac{MC^L - MC^H}{\sum_{i=1}^{n} (\frac{1}{P_i^{'}(v_i)}) \cdot AC^{L'}}$$

$$\tau_j^H = AC^{H'} \times (\sum_{i=1}^{n} v_i^H) + \frac{MC^L - MC^H}{\sum (\frac{1}{P_i'(v_i)}) \cdot AC^{L'}}$$

$$\tau_j^H = AC^{H'} \times v^H + \frac{MC^L - MC^H}{\sum (\frac{1}{P_i'(v_i)}) \cdot AC^{L'}} = \tau_{SB}^H \text{ ------------------(29)}$$

Following is the result of the simulation for the case that we analyze above. The difference is the values of time for two group users are equal, assuming it is 1. Table 2 shows the results of another simulation.

Table 2. Simulation Results for Common Cost Case

	v_1^L	v_1^H	v_2^L	v_2^H	AC_1^L	AC_1^H	AC_2^L	AC_2^H
No Toll	2578	3038	1337	3087	192.11	192.11	192.11	192.11
Second-Best	3001	2531	1190	3110	233.56	146.66	233.56	146.66
First-Best	2692	2384	766	2849	140.38	116.38	140.38	116.38
	P_1	P_2	τ^L	τ^H				
No Toll	192.11	192.11	0	0				
Second-Best	233.56	233.56	0	86.90				
First-Best	461.92	461.92	321.53	345.53				

Table 2 shows the results of simulation for the case of common cost case. In this case, Group *1* and *2* have different price elasticity but same value of time. Group *1* same as Group *2* users travel on both local road and highway with cost \$192.11 at no toll scheme. In Second-Best Scheme, since there is no toll on local road, the group 1 and 2 both users have same average cost, \$245.55, which equals to their price. Government will charge a toll \$86.90 on the highway users regardless the travel type. For group *1* and *2* users on highway, their average cost plus this toll equals their willingness-to-pay (price) and also equals to the cost of using local road. In the scheme of First-Best, government charge toll for using highway higher than that of local road. The results also show that for particular path government should charge both groups of users same toll which will make their "total cost" equals their willingness-to-pay.

4.3 Heterogeneous demand with linear cost functions

In this case, the cost functions will be even simplified to be a linear relationship with travel flow. Assume the $AC^L(v_1^L, v_2^L, ..., v_n^L) = \alpha^L \times (v_1^L + v_2^L + ... + v_n^L)$

And $AC^H(v_1^H, v_2^H, ..., v_n^H) = \alpha^H \times (v_1^H + v_2^H + ... + v_n^H)$

Therefore, we have $\frac{\partial AC^L}{\partial v_j^L} = \alpha^L$ and $\frac{\partial AC^H}{\partial v_j^H} = \alpha^H$

In this case, the first-best tolls will be

$$\tau_j^L = AC^{L'}(\sum_{i=1}^{n'} v_i^L) = AC^{L'} \times v^L = \alpha^L \times v^l = AC^L = \tau_{FB}^L \text{ ------------------(30)}$$

$$\tau_j^H = AC^H(\sum_{i=1}^{n} v_i^H) = AC^{H'} \times v^H = \alpha^H \times v^H = AC^H = \tau_{FB}^H \text{ ----------------(31)}$$

For Second-Best toll,

$$\tau_j^H = \alpha^H \times v^H + \frac{\alpha^L(v^L+1) - \alpha^H(v^H+1)}{\alpha^L \times \sum(\frac{1}{P_i'(v_i)})\cdot}$$

$$\tau_j^H = AC^H + \frac{(v^L+1) - \frac{\alpha^H}{\alpha^L}(v^H+1)}{\sum(\frac{1}{P_i'(v_i)})\cdot} = \tau_{SB}^H \text{ ------------------(32)}$$

For that case of linear cost functions, the first-best toll, Equation (31), is exactly the average cost for using the road and is independence of the type of travel flow. The second-best toll also is type independence and has the format of Equation (32). The second term of this toll is an adjustment of first-best toll. This term can decomposed into two parts: first part is the marginal cost difference between local road and highway which turns out to be a nonnegative term; the second part is the summation of the slops of inverse demand functions which turns out to be negative value. Therefore, the adjustment term of second-best toll is non-positive.

5. CONCLUSIONS

This study shows that an equilibrium flow pattern that satisfies all the optimal necessary conditions can be obtained in both first-best and second-best pricing. For both of these pricing schemes, we demonstrate the differences between price and cost as the road toll to maximize the social welfare.

The results of this paper are clear. For heterogeneous elastic users, the toll is still the same to all the users, both in first-best scheme and second-best scheme as long as the symmetric cost effect holds. In the case of first-best scheme, the toll is the sum of differentiations of cost function of each group users multiple the travel-flows. That is, the first-best optimal toll turns out to be a weighted average of the differentiated cost function with the optimum group sizes. The first-best road pricing assumed a perfect charging system that would be difficult to implement it practically. This paper also develops another practical toll scheme, the second-best pricing. It turns out the second-best toll could be interpreted as the first-best toll plus an adjustment term (shadow price of travel flows multiples the slope of inverse demand function). Liu and McDonald (1999) presented similar result in their paper in the case of single group of users. Our paper develops the general form of second-best toll.

This paper provides analytical results for evaluating the first-best toll and second-best toll for a simple network. Future research could work on the simulation process with more accurate cost and demand functions. Our simulation results conform with previous research results: the welfare of first-best is higher than that of second-best one, and the total travel flow of first-best, on the other hand, is smaller than that of second-best one.

REFERENCES

Thomson J. M., "Reflections on The Economics of Traffic Congestion", **Journal of Transport Economics and Policy, Vol. 32**, Part 1,1997, pp93-112.

Arnott, R. and K. Marvin, "When Are Anonymous Congestion Charges Consistent with Marginal Cost Pricing", **Journal of Public Economics, No. 67**,1998, pp45-64。

Morrison, S. A., "A Survey of Road Pricing", **Transportation Research-Part A, Vol. 20, No. 2**, 1986, pp87-97.

Verhoef, E. T., N. Peter and R. Piet, "Second-best Regulation of Road Transport Externalities", **Journal of Transport Economics and Policy,** 1995, pp147-167.

McDonald, J. F., " Urban Highway Congestion—An Analysis of Second-Best Toll", **Transportation, Vol. 22**, 1995, pp353-369。

Yang, H. & W. H. K. Lam, "Optimal Road Tolls Under Conditions of Queueing and Congestion" ,**Transportation Research-Part A, Vol. 30, No. 5**, 1996, pp319-332.

Yang, H. and M. G. H. Bell, " Traffic Restraint, Road Pricing and Network Equilibrium" ,**Transportation Research Part B, Vol.31, No. 4**, 1997, pp303-314.

Liu, L. N. and J. F. McDonald, "Economic Efficiency of Second-best Congestion Pricing Schemes in Urban Highway Systems", **Transportation Research-Part B, Vol. 33**, 1999, pp157-188。