DEVELOPMENT OF A PATH-BASED TRIP ASSIGNMENT MODEL UNDER TOLL IMPOSITION

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Abstract: Travel time and travel tolls are the most important criteria in traveler's route choice. In this paper, path-based trip assignment model is formulated. Also actual travel tolls can be set according to path travel distance. Gradient Projection (GP) algorithm is adopted to obtain path-based solutions and for computational efficiency, MPS(Minimal Path Search) algorithm is used to find the shortest path. The model improves accuracy of trip assignment by reflecting realistic travel tolls and is proved to converge much faster than the existing assignment models. Given the path solutions reflecting travel tolls, objective function can be specified from the relationship between travel toll and travel distance and its measure of effectiveness can be easily measured. Therefore this model cannot only describe the situation more realistically but also overcome the limited analysis of effectiveness, which makes this model applicable more widely

Keyword: Path-Based Trip Assignment Model, Gradient Projection Algorithm, K-Shortest Path Algorithm, Travel Toll

1. INTRODUCTION

Traditional trip assignment model can be categorized as Link-Based Assignment (LBA) model in that it is mathematically consisted of traffic volume and delay function of links. Since its solution is proven to be stable and unique, the LBA models have been used widely.

On the contrary, Path-Based Assignment (PBA) model, which has lately attracted considerable attention, is based on traffic volume of paths with a certain O-D pair. Under the necessity of assigning traffic volume to paths between O-D pairs, it is expected that PBA model can achieve efficient traffic operation.[1][2]

Trip assignment model, which forecasts traffic volume of links or paths with given O-D trips, describes route choice behavior of travelers at the same time. Because the most important criteria in selecting routes are travel time and cost, trip assignment model must be developed to reflect these criteria.

Furthermore, since the way of imposing toll varies with traveling path, total trip length must be known in order to reflect realistic traveling cost. However, LBA model can find only the traffic volume of links but not the total trip length.
The purpose of this paper is to establish the path-based trip assignment algorithm and consequently to develop path-based assignment model that can reflect realistic travel tolls.

2. BACKGROUND

As general ways of solving trip assignment problem, techniques such as Frank-Wolfe(F-W) algorithm or convex combination algorithm have been widely applied. F-W algorithm determines the direction and the maximum shifting size in the area of possible solution. It achieves the minimization of the objective function using link-based mathematical planning. Using this solving process, traffic volume is transferred from links to links and traffic volume of links will be found as a final result.
On the other hand, in PBA model, determinant variable is traffic volume of paths. Therefore, every path must be searched and memorized in each iteration. With insufficient memory and inefficient central process of old computers, it was difficult to apply PBA model to transportation engineering field.
However, recently advanced computer system and optimizing techniques have made it possible to assign traffic volume with PBA model in real time. With the technical support, PBA model is attracting attention in that traffic volume of paths is essential in route guidance, traffic control, and real-time traffic operation. In addition, the result of PBA can be applied to estimation of O-D tables, assessment of environmental impacts, validation of models, and establishment of logistic system.
The very recent research about PBA model reached the level of assigning traffic volume in large network. One of the most remarkable PBA model is Jayakrishnan's which uses Gradient Projection (GP) [3] algorithm. Jayakrishnan's research results show that PBA model using GP algorithm is more efficient than LBA model using F-W algorithm, in various size of networks. [4][5]
In trip assignment, finding possible paths is the one of the most important process. While minimum path algorithms, which find only one path, have been generally used, it has limitation in that it is deterministic model and can hardly be used as a Measure of Effectiveness (MOE).
In path-based trip assignment, it is necessary to list and compare all possible paths, so the number of possible paths tends to increase very rapidly with increasing network size. Yet the actual traveler will choose a route among a few possible ones in practical condition. Thus, actual number of meaningful paths doesn't change rapidly with the network size.
Recently, considering these aspects, K-shortest paths algorithms are used to find several possible paths.[6] K-shortest paths algorithms, which search K paths with given O-D pairs, are applied to route choice problem in trip assignment model, minimum path problem with multi-criteria, Pareto's optimal path problem, and other trip assignment models.
Minimal Path Search (MPS) algorithm, used in this paper, introduces the concept of reduced cost to improve the efficiency of Yen's generalization algorithm. With suitable data structure, the performance of this algorithm is enhanced. One of the efficient data structure is 'sorted forward star form.' Every stage of MPS algorithm is the same as that of Yen's generalization algorithm, but new possible paths can be found more simply.

3. GRADIENT PROJECTION ALGORITHM

Gradient Projection (GP) algorithm is one of the optimizing techniques using direction searching method and is applied to solve optimal path assigning problem. In each iteration this algorithm is performed on the basis of the minimum path for each O/D pair and its minimal value of first-order derivative costs, and the variation of traffic volume is calculated based on the second-order derivative. Of course, when the variation is so large that the traffic flow becomes negative, it will be projected into zero. This algorithm solves the optimal path assigning problem by the general methods such as Steepest Descent Method or Newton's Method with additional restrictions.

In minimizing problem of any function f, assuming that it is second-order differentiable function of n-dimensional vector $x=(x_1,\cdots,x_n)$, its gradient and Hessian matrix are as follows.

$$\nabla f(x)=\begin{bmatrix}\dfrac{\partial f(x)}{\partial x_1}\\ \vdots\\ \dfrac{\partial f(x)}{\partial x_n}\end{bmatrix},\nabla^2 f(x)=\begin{bmatrix}\dfrac{\partial^2 f(x)}{(\partial x_1)^2}\cdots\dfrac{\partial^2 f(x)}{\partial x_1\partial x_n}\\ \vdots \qquad\qquad \vdots\\ \dfrac{\partial^2 f(x)}{\partial x_n\partial x_1}\cdots\dfrac{\partial^2 f(x)}{(\partial x_n)^2}\end{bmatrix}$$

where, it is assumed that $\nabla^2 f(x)$ is positive semidefinite for any x.

The method to find the minimum value of f from the initial value of x^0 with no constraint is proposed, that is,

$$x^{k+1}=x^k-\alpha^k\nabla f(x^k),\qquad k=0,1,\cdots,n \qquad (1)$$

where, α^k is positive step-size determined by a certain rule. In this iteration the change of the value, except when the gradient equals to 0 and the solution is optimal, follows the direction of the negative gradient, that is, the direction of the value of function decreases.

Generally, the step-size α^k is determined by

$$f\left[x^k-\alpha^k\nabla f(x^k)\right]=\min_{\alpha>0} f\left[x^k-\alpha^k\nabla f(x^k)\right] \qquad (2)$$

In another way, α^k can be specified as a particular constant, that is,

$$\alpha^k\equiv\bar{\alpha}\qquad \textit{for all } k \qquad (3)$$

Under non-negative condition $x_i\geq 0$ the minimization function f can be formulated as follows.

minimize $f(x)$
subject to $x\geq 0$

GP algorithm is an optimizing technique under this constraint, which is adjusted form of Steepest Descent Method. Thus, as Eq.(4) shows, iteration procedure including projection would be performed.

$$x^{k+1} = \left[x^k - \alpha^k \nabla f(x^k)\right]^+ , \qquad k = 0,1,\cdots,n \qquad (4)$$

With respect to any vector z, [z]+, projection of z can be written as follows.

$$[z]^+ = \begin{bmatrix} \max\{0, z_1\} \\ \max\{0, z_2\} \\ \vdots \\ \max\{0, z_n\} \end{bmatrix} \qquad (5)$$

Convergence speed of GP algorithm can be enhanced by multiplying any proper positive definite scaling matrix B^k.Then, Eq.(6) can be formulated.

$$x^{k+1} = \left[x^k - \alpha^k B^k \nabla f(x^k)\right]^+ , \qquad k = 0,1,\cdots,n \qquad (6)$$

Assuming the existence of inverse function of $\nabla^2 f$ the fastest converging way is to denote B^k as follows.

$$B^k = \left[\nabla^2 f(x^2)\right]^{-1} \qquad (7)$$

With this B^k and constant α^k, the algorithm converges very fast near the lowest point. Yet this result is hardly obtained in that the inverse function of $\nabla^2 f$ must exist and be known. Then, as an approximation of optimal B^k,the diagonal entries of inversed Hessian can be used and that is,

$$B^k = \begin{bmatrix} \left[\dfrac{\partial^2 f(x^k)}{(\partial x_1)^2}\right]^{-1} & 0 & \cdots & 0 \\ 0 & \left[\dfrac{\partial^2 f(x^k)}{(\partial x_2)^2}\right]^{-1} & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & \left[\dfrac{\partial^2 f(x^k)}{(\partial x_n)^2}\right]^{-1} \end{bmatrix}$$

Where, B^k is diagonal and positive definite scaling matrix.
With B^k as an approximation of inversed Hessian, iterations can be presented as follows.

$$x_i^{k+1} = [x_i^{k_i} - \alpha^k \left[\frac{\partial^2 f(x^k)}{(\partial x_i)^2}\right]^{-1} \frac{\partial f(x^k)}{\partial x_i}]^+, \quad i = 1,\cdots,n \qquad (8)$$

It can be rewritten that

$$x_i^{k+1} = \max\{0, x_i^{k_i} - \alpha^k \left[\frac{\partial^2 f(x^k)}{(\partial x_i)^2}\right]^{-1} \frac{\partial f(x^k)}{\partial x_i}\}, \quad i = 1, \cdots, n \quad (9)$$

In iteration process step-size α^k can be determined in various way. One of them is to put it as a specific constant and in this case determination of that constant value is a critical issue. Generally, according to literatures about nonlinear problems, it is recommended to use 1 as a constant value of α^k.

GP algorithm converges slowly near the optimal value, but generally converges fast to the optimal value in early stages. In practical assigning problem, to approach the admittable range of value in the small number of iteration is much more important than to get an accurate optimal value. Therefore, it can be said that this algorithm yields quite satisfying result.

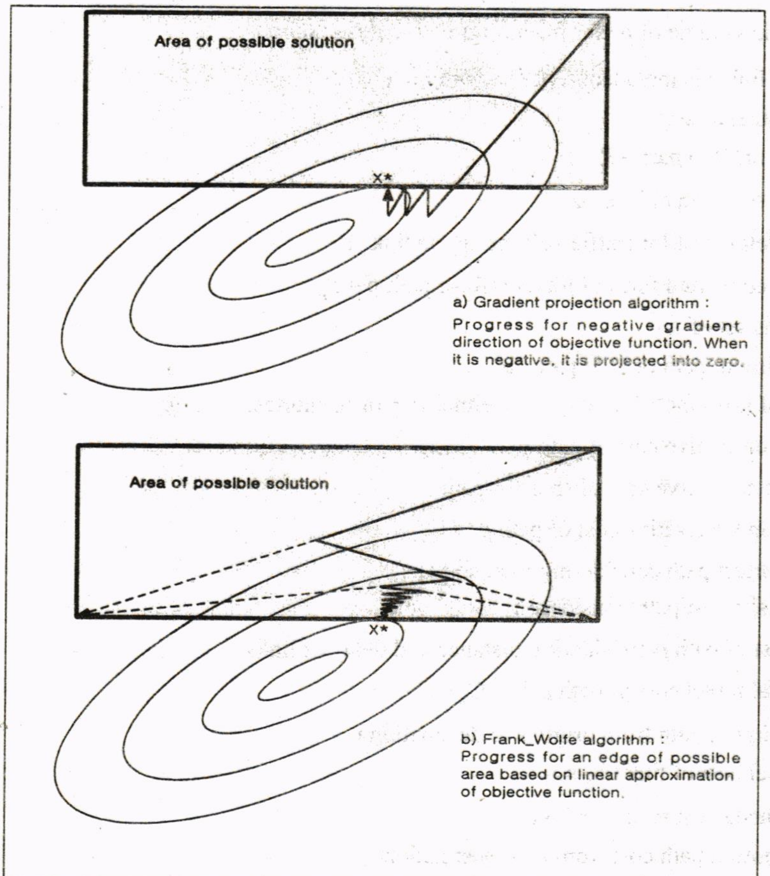

Figure 1. GP algorithm and F-W algorithm

Furthermore, this algorithm is superior to the F-W algorithm. <Fig. 1> shows the difference between GP and F-W algorithm. While iterations of F-W algorithm zigzag, GP algorithm finds the optimal solution more efficiently.

4. DEVELOPMENT OF TRIP ASSIGNMENT MODEL REFLECTING TRAVEL TOLL

4.1 NOTATION

Notation in this model is as follows.

i = origin
j = destination
a = link
ω = O/D pair in process = ij
p = path in process = ijr
V_a = traffic volume of link a = $\sum_i \sum_j \sum_r P_{ijr} \cdot \delta_{ijr}^a$
P_{ijr} = traffic volume of path r from origin i to destination j
δ_{ijr}^a = 1, if link a is included in path r from i to j
0, otherwise
P_ω = trips on O/D pair = P_{ij}
S_a =delay function of link a
$S_a(V_a)$ =delay cost for traffic volume V_a on link a
T_p = time converted form of travel toll for path p = T_{ijr}
X_ω = paths set of ω
$\bar{p}_\omega$ = minimum path of O/D pair ω
L_p = set of links included only once either in p or in shortest path $\bar{p}_\omega$
d_p = first derivative cost of path p
$d_{\bar{p}_\omega}$ = first derivative cost of shortest path
H_p = second derivative cost of path p
$p_{\min}$ = shortest path considering total cost
p_k = k-th shortest path considering LC_p only
LC_p = cost of path p considering distance and delay of links
TC_p = total travel cost of path p = $LC_p + T_p$
p_{st}^* = minimum path from origin s to destination t
c_{mn} = travel cost of link (m, n)
$\overline{c_{mn}}$ = reduced cost = $\pi_n - \pi_m + c_{mn}$
π_m = minimum path cost from m to destination
$A(v)$ = set of links included in paths from any node v to destination t
$T_{t,}^*$ = set of minimum paths for all O/D pairs
ε = convergence criteria

4.2 DEFINITION OF OBJECTIVE FUNCTION

In order to develop assignment model based on user equilibrium (UE) and travel cost, firstly, the objective function of user equilibrium is defined, that is,

$$Min. \sum_a \int_0^{V_a} S_a(x)dx \qquad (10)$$

In existing trip assignment model, travel cost was calculated by link cost of above function plus travel toll imposed on travelers where two terms must be consistent in unit. That is to say that travel toll must be converted into time T_a to calculate the objective function in terms of time. T_a means the travel time converted from the travel toll of link a which is independent of traffic volume V_a.

Moreover, it is assumed that every user has the same value of time in order to uniformly include travel toll in cost function. The dual criteria assigning problem can be avoided by this assumption, therefore, it is applied to this study.

Consequently, the objective function of user equilibrium reflecting travel toll can be presented as follows.

$$Min. \sum_a \int_0^{V_a} (S_a(x)dx + T_a) = \sum_a \int_0^{V_a} S_a(x)dx + \sum_a \int_0^{V_a} T_a dx$$
$$= \sum_a \int_0^{V_a} S_a(x)dx + \sum_a \int_0^{V_a} T_a V_a dx \qquad (11)$$

The latter term of Eq. (11), which is based on links can be substituted with paths based term. By doing that, various toll system based on paths can be applied to trip assignment model. Furthermore, travel toll can be calculated without additional steps in PBA model.

In other words, by substituting travel toll of the objective function with traffic volume of paths and time converted form of travel toll, it becomes possible to reflect more practical travel toll in assignment model. Similarly, by assuming that time converted form of travel toll T_a is independent of path volume P_{ijr} and the value of time is fixed, Eq. (11) can be rewritten as follows.

$$Min. \sum_a \int_0^{V_a} S_a(x)dx + \sum_i \sum_j \sum_r T_{ijr} P_{ijr} \qquad (12)$$

Constraints of the objective function (12) are conservation of traffic volume and non-negativity alike general UE models. Based on the discussion above, the objective function and constraints can be summarized, that is,

objective function $Min. \sum_{link} \int_0^{V_a} S_a(x)dx + \sum_i \sum_j \sum_r T_{ijr} P_{ijr}$

constraints $\sum_r P_{ijr} = P_{ij} \qquad \forall i,j$

$$P_{ijr} \geq 0 \qquad \forall i,j,r$$

$$V_a = \sum_i \sum_j \sum_r P_{ijr} \delta_{ijr}$$

It can be said that the first order condition of this objective function consists with the equilibrium condition of general UE model, so this objective function satisfies the equilibrium condition.
The solution of PBA model is unique in terms of link volume, but it is not unique in terms of path volume even if it is optimal. The objective function in this model is also convex to link volume, so it satisfies the uniqueness of link volume.

4.3 PATH-BASED ASSIGNMENT MODEL REFLECTING TRAVEL TOLL

Generally, the objective function should be second order differentiable in order to apply the GP algorithm. Since the objective function here is formulated with integral of cost function, the cost function should be first order differentiable.
The procedure of GP algorithm is as follows.

$$x_p^{k+1} = \max\{0, x_p^k - \alpha^k H_p^{-1}(d_p - d_{\overline{p_\omega}})\},$$
$$for\ all\ \omega \in W, p \in P_\omega, p \neq \overline{p_\omega} \qquad (13)$$

Where d_p and $d_{\overline{p_\omega}}$ means the first derivative cost of path p and shortest path $\overline{p_\omega}$ respectively, that is,

$$d_p = \sum_{\substack{all\ links \\ a\ on\ Path\ p}} D'_a = \{ \sum_{\substack{all\ links \\ a\ on\ Path\ p}} S_a(V_a)\} + T_p$$
$$d_{\overline{p_\omega}} = \sum_{\substack{all\ links \\ a\ on\ Path\ \overline{p_\omega}}} D'_a = \{ \sum_{\substack{all\ links \\ a\ on\ Path\ \overline{p_\omega}}} S_a(V_a)\} + T_{\overline{p_\omega}} \qquad (14)$$

H_p is second derivative cost and can be presented as

$$H_p = \sum_{a \in L_p} D''_a(V_a^k) = \sum_{a \in L_p} S'_a(V_a) \qquad (15)$$

Iterations of Eq. (15) terminate when the variation of the objective function reduced to a certain range of error. In this study, it is assumed that variation rate of the objective function is less than 0.1%.
One of the critical issues in GP algorithm is determination of the step-size α^k. As mentioned before, α^k is assumed to be a constant. Since, by doing that, calculation can be less complex without decreasing the convergence speed, many of previous research adopted constant as α^k.
As a result of trial and error, it was decided to use 1 as α^k
In iteration process, in order to transfer traffic volume to shortest path, only paths that have practical traffic volume on them would be considered. In other words, paths that have no traffic volume initially or lose their traffic volume during iterations ($x_p^{k+1} = 0$) will be excluded from next iteration. Then, the number of paths in process decreases, and the calculation becomes less complex.
Minimal path search, the sub program, is based on the path based shortest path algorithm to keep the consistency. Previous shortest path algorithms generally search the minimum cost

path based on link costs, which describe the distance and delay of links. However, travel toll is imposed not on the simple sum of link distances, but on the traveling path of a user, so previous shortest path algorithms are not appropriate for PBA model.

In addition, if general shortest path algorithm is applied to PBA model, all the possible paths from origin to destination should be searched, and, after summing up expressway tolls with path costs for each path, a path which has the smallest travel cost should be chosen. But, in this case, the number of possible paths increases very rapidly with increasing network size, so it is impossible to use it practically.

Consequently, this paper proposes a new shortest path algorithm. By using MPS (Minimal Path Search) algorithm among the K-shortest paths algorithms, the number of paths in consideration was minimized, and travel tolls were considered in the model. Furthermore, the efficiency was improved.

According to the discussion above, the shortest path algorithm in this paper is as follows. First, using general shortest path algorithm in each iteration, find the initial shortest path considering only the distance and delay of links, and then, adding the travel toll of that path, calculate the total cost of initial shortest path. Next, for each of other paths (from second to k-shortest path), calculate path cost considering the distance and delay of links using MPS algorithm, and if it is larger than the path cost of initial shortest path, then stop. For paths chosen in this way, calculate the total travel cost by adding travel toll to path cost. Then, choose a path with the smallest total cost for that iteration. In this procedure, since paths whose path costs are larger than the total cost of initial shortest path are eliminated, the shortest path reflecting travel toll can be found efficiently.

As stated above, the shortest path algorithm in this paper is an improved one in that it reduced the number of paths to be considered, and K value does not need to be chosen arbitrary.

The algorithms of PBA model considering travel toll can be arranged as follows.

1. Find the minimum paths $\overline{p_\omega}^1$ considering travel toll for each O/D pair, ω.
   → (subroutine min path)
   Include that path into path set $X_1, X_2, \cdots, X_\omega$
2. Assign initial volumes of each O/D pair to minimum paths.
3. Calculate cost of each path using cost function.
4. Calibrate objective function 1.
5. Find the min path $\overline{p_\omega}^{k+1}$ considering travel toll for each O/D pair, ω
   → (subroutine min path)
6. For each path p in X_ω, find x_p^{k+1} according to the following equation.
   $$x_p^{k+1} = \max\{0, x_p^k - \alpha H^{-1}(d_p - d_{\overline{p_\omega}})\}$$
   $$for\ all\ \omega \in W, p \in P_\omega, p \neq \overline{p_w}$$
7. Assign x_p^{k+1} as traffic volume of each path p.
   Assign $P_\omega - \sum x_p^{k+1}$ to the min path of each O/D pair, ω.
8. Include the minimum path into path set X_ω.
9. If, for any path in X_ω, traffic volume of that path equals zero (x_p^{k+1}=0), exclude it from X_ω.

Repeat 5 ~ 9 for each O/D pair.

10. Calculate path cost using cost function.
11. Calibrate objective function k+1.

12. If (objective function k - objective function k+1)/(objective function k+1)≤ ε , stop.
Otherwise, go to 5.

■ Subroutine min path(Min. path algorithm reflecting expressway toll)

1. By using min path algorithm, find min path p_1 considering LC only. Then calculate LC_1 .
2. $p_{\min} \leftarrow p_1$.
3. If $T_1 \neq 0$, then find TC_1,
   and set $TC_{\min} = TC_1$.
   If T_1=0, then $P_{\min} = p_1$.
   and go to 10.
4. i← 1
5. Find p_{i+1} by using MPS algorithm, and calculate LC_{i+1} .
   → (subroutine MPS)
6. If $LC_{i+1} \geq TC_1$, then go to 10.
7. Otherwise, find TC_{i+1} .
8. If $TC_{i+1} < TC_{\min}$, then $p_{\min} \leftarrow p_{i+1}$ and $TC_{\min} \leftarrow TC_{i+1}$.
9. Otherwise, i ← i+1 and go to 5.
10. Output $P_{\min}$ and stop.

■ Subroutine MPS(Algorithm which find K-shortest path)

1. Find the minimum path T_t^* of an O/D pair.
2. Calculate $\overline{c_{\min}}$ for all links $(m,n) \in A$.
3. Arrange all links in link cost $\overline{c_{\min}}$ order.
4. p_1 ← minimum path from origin s to destination t
   k ← 1
   X ← { p_k }
   T_k ← { p_k }
5. If k ≥ K or X= ∅ , then go to 9.
6. X← X-{ p_k }
   v_k ← node except t in p_k
7. For each node $x \in p^k_{v_k t}$,
   if $A(v) - A_{T_k}(v) \neq \varnothing$, then
   (v,x) ← the first link in $A(v) - A_{T_k}(v)$ (smallest $\overline{c_{\min}}$)
   $q \leftarrow p^k_{sv} \circ \{v,(v,x),x\} \circ p^*_{xt}$
   $X \leftarrow X \cup \{q\}$
   $q_{vt} \leftarrow \{v,(v,x),x\} \circ p^*_{xt}$
   $T_k \leftarrow T_k \cup \{q_{vt}\}$
8. k← k+1
   p_k ← minimum path in X
   output k-th min path p_k and go to 5.
9. Stop.

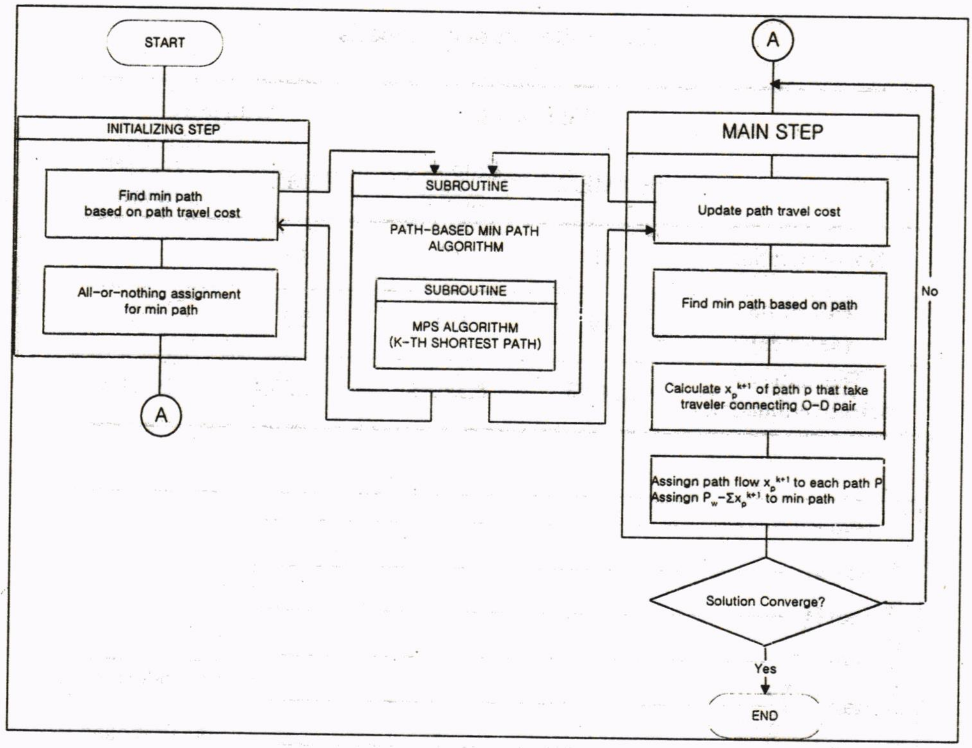

Figure 2. Algorithm of PBA model reflecting travel tolls

4.4 EVALUATIONS AND VALIDATION OF THE MODEL

For the evaluation and validation of the model proposed in this study, the results were compared with the ones of EMME/2, which was LBA model using F-W algorithm.
PENTIUM-133 MHz(32MB RAM) was used in analysis, and the value of objective function, the number of iteration, and convergence speed of two models were measured.
Two models were performed on Sioux Falls Network. For the easy check of the relationship between travel toll and travel distance, O/D and some link attributes were changed from original data.
In the case of with/without tolls, an error of assigned traffic volume between two models was less than 5 percents. (See appendix)
Table 1 shows the results of two models. The values of the objective functions of two models are almost equal to each other with/without tolls and it stands for that the proposed model converges on the optimal solution.
The iterative number of the proposed model is about a half of that of EMME/2, and it is caused by the fact that GP algorithm used in the former converges rapidly than F-W algorithm in the latter. CPU time also shows that the former is better in convergence speed.
Figure 3 shows the converging process of the objective functions, and it points out that the proposed model has an advantage in converging rapidly in its early stages. It means that if the number of iterations were restricted, the proposed model would yield a better result.

Table 1. Results of two models

	Without tolls		With tolls	
	EMME/2	Proposed model	EMME/2	Proposed model
No. of iterations	32	14	15	8
CPU time (seconds)	0.9	0.7	0.5	0.3
The value of objective function	8,342.8	8,340.3	11,223.4	11,221.5

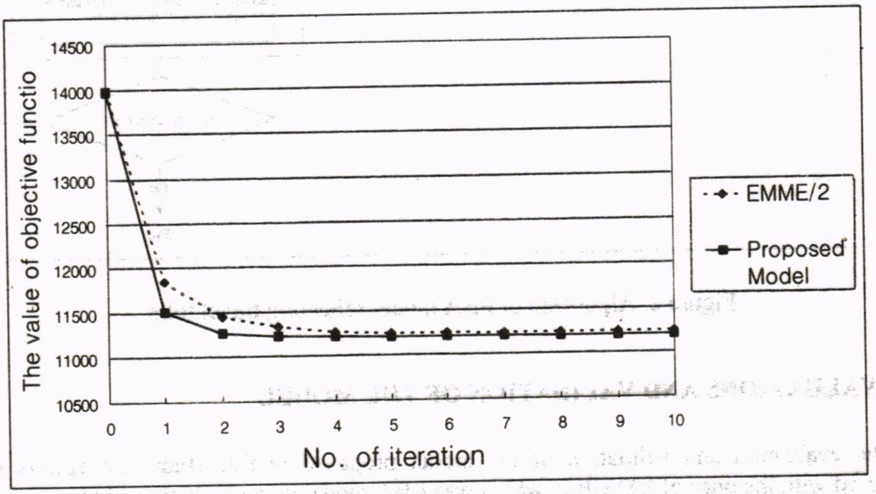

Figure 3. Comparison of the value of objective functions by No. of iterations

5. CONCLUSION

In this study, the path-based assignment model that can reflect travel tolls is developed to solve the optimal path assigning problem.

This model has two stages: the first is an assigning process using Gradient Projection(GP) algorithm, which converges faster than Frank-Wolfe algorithm mostly used in general assigning models. The second is to search the shortest path as a substructure of trip assignment model. In this study, MPS(Minimal Path Search) algorithm which is one of the K-shortest path algorithms is adopted to overcome the computational inefficiency of path-based model searching all possible paths.

The assigning results reflecting tolls between each origin and destination can be widely used not only to analyze the attributes and effects of travel behavior according to the level of tolls, but to make a political plan for transportation operation and management.

The model is applied to the Sioux Falls Network, and the assigning results are almost equal to the ones of EMME/2. Furthermore, it is better in the aspect of convergence speed and the number of iterations.

The limits of this study and the research to be continued are as follows.

To make better use of this model, a method which can be applied to a public transportation and a network as large as nationwide one has to be established.

An analysis on the problems due to non-uniqueness of solutions inherent in the PBA model is required to enhance the rationality of this model. If the volume change of each path, which is caused by the change of initial condition, would be investigated, more practical solution out of plural ones could be deduced.

For the application to the area of Intelligent Transportation System(ITS), a study which can combine this model with the dynamic assignment model should be continued.

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REFERENCES

[1] Kaysi, I., and M. Ben-Akiva. An Integrated Approach to Vehicle Routing and Congestion Prediction for Real-Time Driver Guidance. Transportation Research Record 1408, 1993, pp. 66-74.

[2] Mahmassani, H. S., and S. Peeta. Network Performance under System Optimal and User Equilibrium Dynamic Assignment. Transportation Research Record 1408, 1993, pp. 83-93.

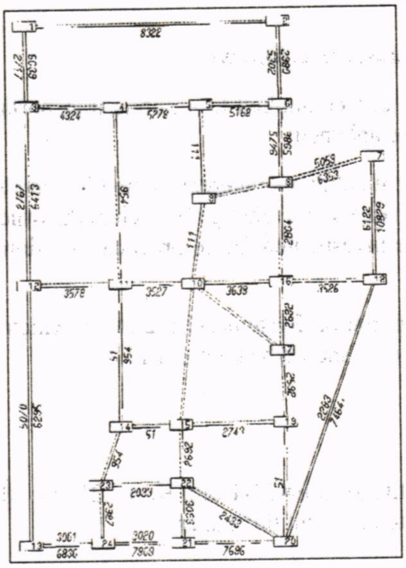
[3] Bertsekas, D. P., and R. G. Gallager. Data Networks. Englewood Cliffs, N.J.: Prentice-Hall, 1987.

[4] Jayakrishnan, R., Wei K. Tsai, Joseph N. Prashker, and Subodh Rajadhyaksha. Faster path-based algorithm for traffic assignment. Transportation Research Record 1443, 1994, pp. 75-83.

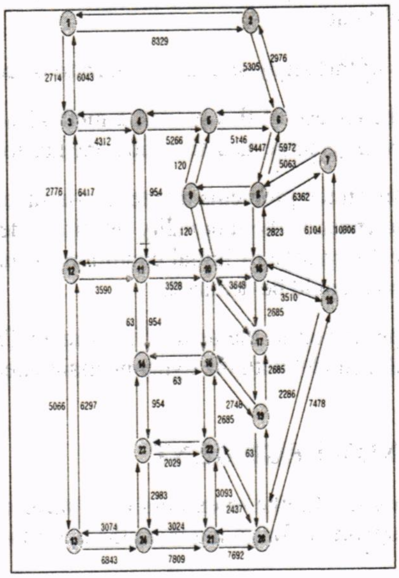
[5] Sun, Carlos, R. Jayakrishnan, and Wei K. Tsai. Computational study of a path-based algorithm and its variants for static traffic assignment. Transportation Research Record 1537, 1996, pp. 106-115.

[6] Cherkassky, B. V., A. V. Goldberg, and T. Radzik. Shortest paths algorithm: Theory and experimental evaluation. Mathematical Programming 73, 1996, pp. 129-196.

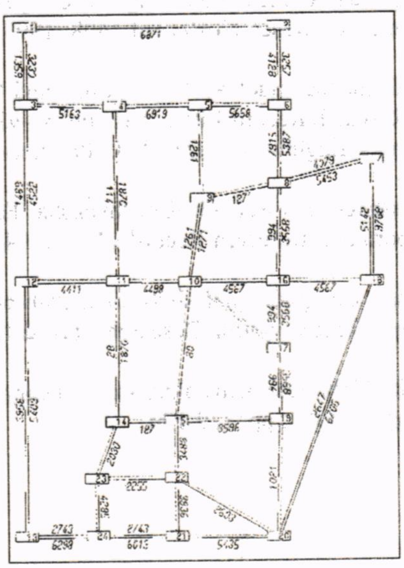
APPENDIX



<EMME/2 result without tolls>



<Proposed model result without tolls>



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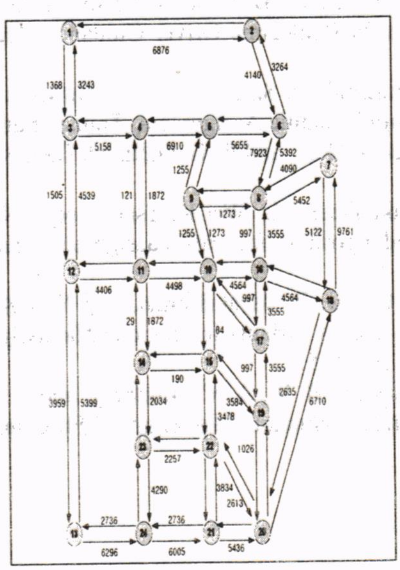

<Proposed model result with tolls>