COMMUTER BEHAVIOR FORECASTING UNDER THE OPTIMAL SINGLE- AND MULTI-STEP TOLL SCHEMES

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Abstract: This paper derives commuters' equilibrium queuing costs and schedule delay costs under the optimal single- and multi-step toll schemes. By comparing these equilibrium costs with the same costs in the present pre-tolled case, one can forecast some changes in commuter behavior from the no toll to the tolled cases. There are some valuable forecasting results for decision making presented in this paper. Firstly, the number of commuters who will or will not pay the tolls can be investigated before tolling a queuing bottleneck. Secondly, all commuters' departure time switching decisions from the no toll case to both the optimal single- or multi-step toll cases also can be investigated before tolling. Such commuter behavior forecasting results, that the previous literature has failed to provide, are valuable and helpful to policy makers if the optimal step toll scheme is considered to put into practice.

Key Words: queuing costs, schedule delay costs, bottleneck, commuter behavior, step toll

1. INTRODUCTION

Laih (1994) developed a flexible pricing mechanism including the optimal single- and multi-step tolls to relieve commuting queuing in the morning at a road bottleneck. Four years later, Singapore applied the optimal double-step toll scheme as the toll structure of Electronic Road Pricing (ERP) System in East Coast Parkway (ECP). Take the passenger car for instance, ERP rates in ECP for three time periods of AM 7:30~8:00, 8:00~9:00 and 9:00~9:30 from April 1998 until March 1999 are $1, $2 and $1, respectively. Land Transport Authority (LTA) of Singapore calls this toll structure "the shoulder pricing". In fact, one may observe that the shoulder pricing completely matches the optimal double-step toll structure. However, ERP rates for passenger cars in ECP and other expressways from April 1999 have becoming changeable and no longer match the optimal step toll structure to respond the multiple changes in commuter behavior. Commuter behavior changes are complicated and very difficult to forecast especially in urban areas because there are many commuting alternatives such as different modes and routes that commuters can take to reach their workplaces.

To forecast commuter behavior changes from the no toll to the tolled cases is a new topic in the road pricing theory. Commuter behavior forecasts are also valuable and useful in decision making because such information helps the authorities comprehensively evaluate the toll schemes in the present pre-tolled case. Unfortunately, except for two conference papers by Laih (1998, 2000), the related literatures concerning to provide detailed commuter behavior forecasts if the congestion toll is put into practice are not available. In order to fill up such a deficiency to a certain degree, this paper provides a methodological framework to forecast commuter behavior changes from the no toll case to both the optimal single- and multi-step toll cases.

This paper first derives the complete and regular values in toll structures of the optimal n-step toll schemes (where $n = 1, 2, 3, \cdots$) that are not provided in Laih's work (1994). The optimal n-step toll structure is the basis to develop the complete methodological framework for commuter behavior forecasts. Next, this paper derives the equilibrium queuing cost and schedule delay cost to each commuter before and after implementing the optimal n-step toll schemes. The above equilibrium costs act as the indispensable tools to forecast commuter behavior changes from the no toll to the tolled cases. By using the tools, commuter behavior forecasts including the number of commuters who will or will not pay the tolls, and all commuters' departure time switching decisions can be investigated.

The paper is organized as follows. A concise review of the no toll equilibrium, the optimal fine toll, and the basic optimal step toll structure for a queuing bottleneck model that have been discussed in the previous literature is given in Section 2. Methodological frameworks used to forecast commuter behavior changes from the no toll case to the optimal single- and multi-step toll cases are developed in Sections 3 and 4, respectively. Accordingly, some valuable commuter behavior forecasts under the optimal n-step toll schemes are provided. Finally, practical implications of the commuter behavior forecasts that this paper provides are addressed carefully in Section 5.

2. BACKGROUND REVIEWS

Queuing often develops in front of the entry to a road bottleneck during the morning rush hour due to a limited capacity. Queuing as the result of stochastic changes in capacity, such as car accidents, is not considered in this paper.

The basic assumptions for a queuing bottleneck model developed by Vickrey (1969) and elaborated by Laih (1994, 1998, 2000) are as follows. First, there are a fixed number of homogeneous commuters, one per car, that choose their departure times rationally based on the commuting cost minimization principle. Second, the total commuting cost for every commuter includes the queuing cost, the schedule delay cost (the costs of arriving at work earlier or later than the work start time) and the toll (if any). For simplicity, both the queuing and schedule delay costs are usually assumed to be linear. Third, commuting demands to the bottleneck are perfectly inelastic. There are two examples that can be raised to justify this assumption: 1. Queuing often occurs at the nearest highway interchange to the downtown area during the morning rush hour. Because most interchanges only have one lane, a bottleneck is formed in this situation. Almost all commuting cars in the highway choose to queue to enter the nearest interchange to the downtown. Consequently, commuting demands to the interchange can be treated as perfectly inelastic in this case. 2. Because of topographic restraints, such as tunnels and bridges, commuting roads often have a narrow segment. A bottleneck is also formed in this situation. Since alternative roads to the downtown are not existent, there are often long and persistent queues at the bottleneck during the morning rush hour. Therefore, commuting demands to such commuting roads also can be treated as perfectly inelastic. Fourth, the route segments before and after the bottleneck have a sufficient capacity so that no queuing would occur there. Fifth, any commuting time other than waiting in the queue due to the bottleneck is constant for departure time decisions. Therefore, one may consider that a commuter arrives at the bottleneck as soon as he/she departs from home, and arrives at the workplace immediately after leaving the bottleneck.

Definitions to all notations used in this paper are listed in Appendix so that one can find them conveniently and quickly. Let's first review the no toll equilibrium case. Because every commuter seeks his departure time to minimize his commuting costs, a stable equilibrium can be reached when all commuters' commuting costs are equal. Accordingly, the equilibrium condition can be expressed as $d(TC)/dt = 0$ for all t. Applying this rule, the no toll equilibrium queueing cost ($\alpha \cdot T_Q^e(t)$) to all early arrivals at work increases linearly from t_q to a maximum value at $\tilde{t}$, and then decreases linearly from $\tilde{t}$ to $t_{q'}$ to all late arrivals at work. Accordingly, the values of t_q, $\tilde{t}$ and $t_{q'}$ can be determined as $t_q = t^* - \dfrac{\gamma}{\beta+\gamma}(N/s)$, $\tilde{t} = t^* - \dfrac{\beta\gamma}{\alpha(\beta+\gamma)}(N/s)$ and $t_{q'} = t^* + \dfrac{\beta}{\beta+\gamma}(N/s)$. Since all commuters who depart during $[t_q, t_{q'}]$ have the same commuting cost in equilibrium, the equilibrium commuting cost for each commuter can be obtained as $TC^e = \alpha \cdot T_Q^e(\tilde{t}) = \dfrac{\beta\gamma}{\beta+\gamma}(N/s)$.

The optimal fine toll is defined as a series of tolls that will completely eliminate the efficiency loss of all commuters' queueing times. Consequently, the shape of the optimal fine toll scheme is triangular because of continuously changeable charges throughout the queuing period $[t_q, t_{q'}]$. Because the optimal fine toll eliminates queuing completely, all commuters' departure times from home can be considered as their arrival times at work. The maximum optimal fine toll is located at the work start time, i.e., $\tau(t^*)$. This is reasonable because commuters are willing to pay the highest toll in order to arrive at work on time without incurring any schedule delay costs.

The single- and multi-step tolls inscribed in the optimal fine toll triangle are first developed by Laih (1994) to reduce the queuing costs to a desired level. The single- and multi-step toll schemes are shaped as a rectangle, and a pyramid made up of multiple rectangles, respectively. Because the step toll with the maximum revenue inscribed in the optimal fine toll triangle is defined as the optimal step toll, the optimal single-, double- and triple-step tolls divide the maximum optimal fine toll $\tau(t^*)$ (or the equilibrium commuting cost TC^e) into two, three and four equal amounts, respectively. The effects of the optimal single-, double- and triple-step tolls on queuing reduction have been derived to be 1/2, 2/3 and 3/4, respectively, of the total queuing time that existed in the no toll equilibrium. These queuing reduction effects are obtained simply because the maximum toll revenues from the optimal single-, double- and triple-step toll schemes are 1/2, 2/3 and 3/4, respectively, of the total equilibrium queuing costs in the no toll case. The above results provided by Laih (1994) are valuable information for decision-making by the authorities, but, unfortunately, little information of commuter behavior changes from the no toll to the tolled cases can be provided in the present pre-tolled situation. These problems will be dealt with in the following Sections.

3. COMMUTER BEHAVIOR IN THE OPTIMAL SINGLE-STEP TOLL CASE

This Section derives the equilibrium queuing costs and schedule delay costs under the optimal single-step toll scheme. By comparing the equilibrium queuing costs before and after tolling the bottleneck, the differences in distribution of departure rates throughout the queuing period can be known. Meanwhile, the size of departures for all types of commuters under the optimal single-step toll scheme can be obtained. Finally, the equilibrium schedule delay costs before and after tolling the bottleneck will be compared to investigate all commuters' departure time switching decisions.

3.1 Equilibrium Queuing Costs and Departure Rates

In order to collect the maximum step toll revenue without making commuters worse off than they would be in the no toll equilibrium, an optimal single-step toll ρ (=TC^e/2), inscribed within the optimal fine toll $\triangle t_q R t_{q'}$ in Figure 1, is applied at t^+ and lifted at t^-. $\triangle t_q M t_{q'}$, on the left hand side in this Figure, illustrates the no toll equilibrium queuing cost. The slopes of $\overline{t_q M}$, $\overline{Mt_{q'}}$, $\overline{t_q R}$ and $\overline{Rt_{q'}}$ are $\dfrac{\alpha\beta}{\alpha-\beta}$, $\dfrac{-\alpha\gamma}{\alpha+\gamma}$, β and $-\gamma$, respectively. The detailed derivations of $\triangle t_q R t_{q'}$ and $\triangle t_q M t_{q'}$ can be referred to Laih's work (1994).

In Figure 1, t' and $t^{\#}$ are two important time spots that need to make some discussions. Let's discuss t' first. Because a bottleneck is fully utilized throughout the queuing period $[t_q, t_{q'}]$, the last person who will not pay the toll before t^+ arrives at his workplace just before the first person who will pay the toll at t^+. This means that both of them have almost the same schedule delay costs. Because the queuing cost to the latter is zero, and also because both have the same commuting cost in equilibrium, the former must incur a queuing cost that is equal to the amount of the toll ρ, and consequently must depart ρ/α earlier. Therefore, there are no departures during the period $[t', t^+)$. The interval between t' and t^+ in Figure 1 therefore is ρ/α.

Discussing $t^{\#}$ is similar to t'. Because the last person who will pay the toll just before t^- must have the same commuting cost as the first person who will not pay the toll when the toll is lifted on t^-, and also because the queuing cost to the former is zero, the latter must incur a queuing cost that is ρ higher than the former. This is impossible unless the latter has queued for a period of ρ/α before t^-. Therefore, we may consider that there is a mass of departing commuters wait at a temporary parking area somewhere in front of the tollgate entry to the bottleneck from $t^{\#}$ to t^-. Then they will enter the bottleneck after t^- without paying the toll. Consequently, the interval between $t^{\#}$ and t^- in Figure 1 is ρ/α.

Values of the toll and departure times appearing in Figure 1 are listed in Table 1. Because the value of t_q is assumed to be zero for simplicity, the values of t_q, $\tilde{t}$, t^+, t^-, t^* and $t_{q'}$ listed in Table 1 are $\left(t^* - \dfrac{\gamma}{\beta+\gamma}\left(\dfrac{N}{s}\right)\right)$ earlier than those appeared in Laih's work (1994).

Table 2 illustrates equilibrium results for all departure intervals under the optimal single-step toll scheme. Note that there exists a blanket departure time interval $[t', t^+)$ because nobody departs during this time period that we have mentioned before. Commuters from groups B, C and D depart during the tolled period $[t^+, t^-)$. Except for group D, which avoids paying the toll, groups B and C will pay the toll to cross the bottleneck. Groups A and E do not need to pay the toll because they depart during the no toll periods. Moreover, only groups A and B

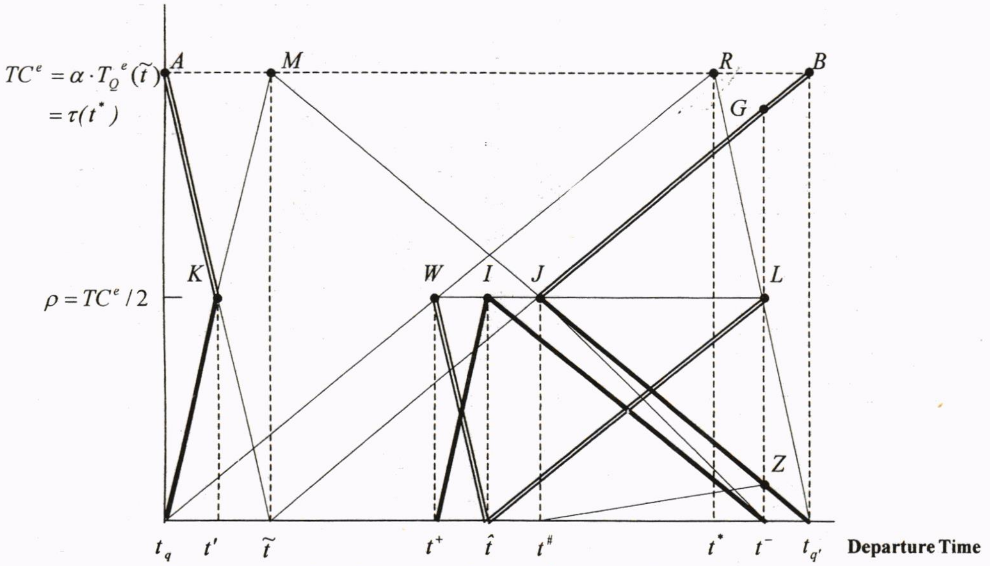

Figure 1. Equilibrium Queuing Costs & Schedule Delay Costs in the No Toll & Optimal Single-Step Toll Cases

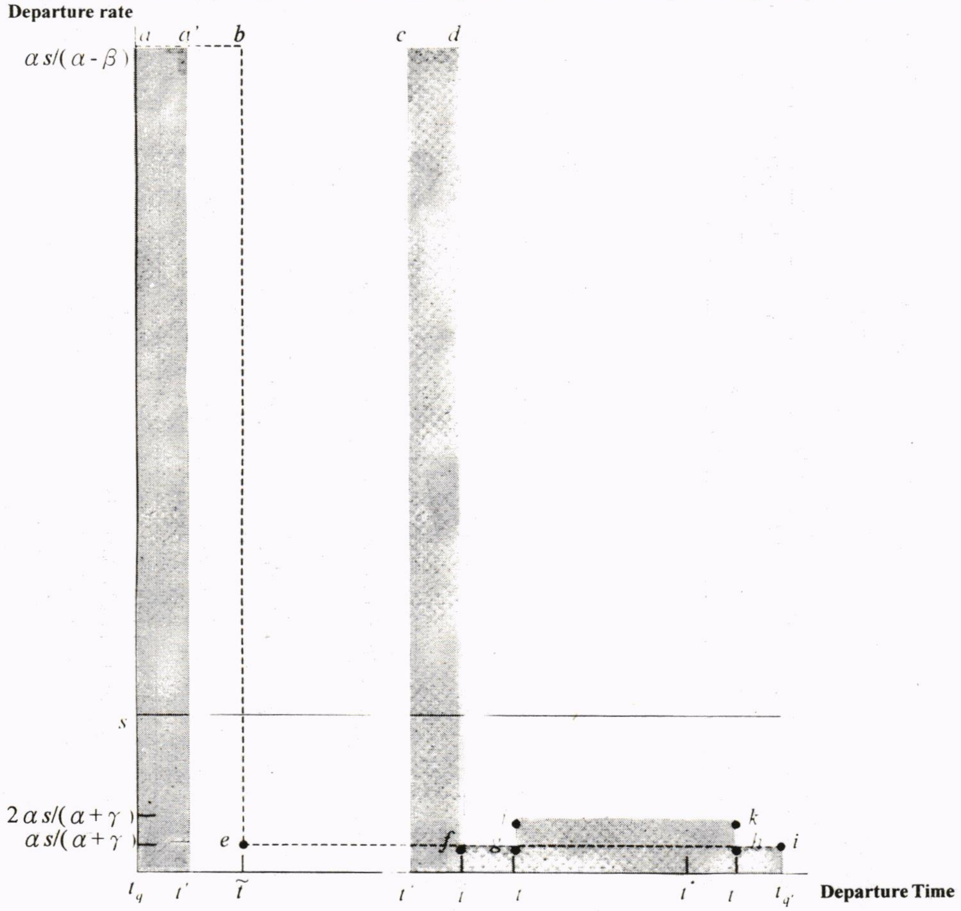

Figure 2. Equilibrium Departure Rates in the No Toll & Optimal Single-Step Toll Cases

Table 1. *Values of the Optimal Single-Step Toll & Some Important Departure Times*

$\rho\left(=\frac{TC^e}{2}\right)$	t_q	$t_{q'}$	$\tilde{t}\left(=t_q+\frac{TC^e}{\alpha\beta/(\alpha-\beta)}\right)$	$t^*\left(=\tilde{t}+\frac{TC^e}{\alpha}\right)$	$\hat{t}\left(=t^*-\frac{\rho}{\alpha}\right)$	$t^+\left(=t_q+\frac{TC^e}{2\beta}\right)$	$t'\left(=t^+-\frac{\rho}{\alpha}\right)$	$t^-\left(=t_{q'}-\frac{TC^e}{2\gamma}\right)$	$t^\#\left(=t^--\frac{\rho}{\alpha}\right)$
$\frac{\beta\gamma}{2(\beta+\gamma)}\left(\frac{N}{s}\right)$	0	$\frac{N}{s}$	$\frac{\gamma(\alpha-\beta)}{\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma}{\beta+\gamma}\left(\frac{N}{s}\right)$	$\frac{\gamma(2\alpha-\beta)}{2\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma}{2(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma(\alpha-\beta)}{2\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\beta+2\gamma}{2(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{2\alpha\gamma+\alpha\beta-\beta\gamma}{2\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$

Table 2. Equilibrium Results under the Optimal Single-Step Toll Scheme

(I) Groups	(II) Departure Intervals	(III) Types of Commuters	(IV) EQC : $\alpha\cdot T_Q^e(t)$	(V) EDR	(VI) Sizes of Departures	(VII) ESDC : $\beta\cdot T_E^e(t)$ or $\gamma\cdot T_L^e(t)$
A	$t_q\le t<t'$ (No Toll Period)	(a).Toll Free (b).Early Arrivals	$\frac{\alpha\beta\cdot t}{\alpha-\beta}$	$\frac{\alpha\cdot s}{\alpha-\beta}$	$\frac{\gamma\cdot N}{2(\beta+\gamma)}$	$\frac{-\alpha\beta\cdot t}{\alpha-\beta}+\frac{\beta\gamma}{(\beta+\gamma)}\left(\frac{N}{s}\right)$
	$t'\le t<t^+$ (No Toll Period)	None	0	0	0	0
B	$t^+\le t<\hat{t}$ (Tolled Period)	(a).Toll Payers (b).Early Arrivals	$\frac{\alpha\beta\cdot t}{\alpha-\beta}-\frac{\alpha\beta\gamma}{2(\beta+\gamma)(\alpha-\beta)}\left(\frac{N}{s}\right)$	$\frac{\alpha\cdot s}{\alpha-\beta}$	$\frac{\gamma\cdot N}{2(\beta+\gamma)}$	$\frac{-\alpha\beta\cdot t}{\alpha-\beta}+\frac{\beta\gamma(2\alpha-\beta)}{2(\beta+\gamma)(\alpha-\beta)}\left(\frac{N}{s}\right)$
C	$\hat{t}<t<t^-$ (Tolled Period)	(a).Toll Payers (b).Late Arrivals	$\frac{-\alpha\gamma\cdot t}{\alpha+\gamma}+\frac{\alpha\gamma(\beta+2\gamma)}{2(\beta+\gamma)(\alpha+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\alpha\cdot s}{\alpha+\gamma}$	$\frac{\beta\cdot N}{2(\beta+\gamma)}$	$\frac{\alpha\gamma\cdot t}{\alpha+\gamma}-\frac{\gamma^2(2\alpha-\beta)}{2(\beta+\gamma)(\alpha+\gamma)}\left(\frac{N}{s}\right)$
D	$t^\#\le t<t^-$ (Tolled Period)	(a).Toll Free (b).Late Arrivals	$\frac{-\alpha\gamma\cdot t}{\alpha+\gamma}+\frac{\alpha\gamma}{\alpha+\gamma}\left(\frac{N}{s}\right)$	$\frac{\alpha\cdot s}{\alpha+\gamma}$	$\frac{\beta\gamma\cdot N}{2(\alpha+\gamma)(\beta+\gamma)}$	$\frac{\alpha\gamma\cdot t}{\alpha+\gamma}-\frac{\gamma^2(\alpha-\beta)}{(\beta+\gamma)(\alpha+\gamma)}\left(\frac{N}{s}\right)$
E	$t^-\le t\le t_{q'}$ (No Toll Period)	(a).Toll Free (b).Late Arrivals	$\frac{-\alpha\gamma\cdot t}{\alpha+\gamma}+\frac{\alpha\gamma}{\alpha+\gamma}\left(\frac{N}{s}\right)$	$\frac{\alpha\cdot s}{\alpha+\gamma}$	$\frac{\alpha\beta\cdot N}{2(\alpha+\gamma)(\beta+\gamma)}$	$\frac{\alpha\gamma\cdot t}{\alpha+\gamma}-\frac{\gamma^2(\alpha-\beta)}{(\beta+\gamma)(\alpha+\gamma)}\left(\frac{N}{s}\right)$

will arrive at their workplaces earlier than the work start time because they depart before the departure time $\hat{t}$. These results are arranged as columns (I)~(III) in Table 2.

Because equilibrium will be achieved as long as all commuters have the same commuting costs throughout the queuing period, $TC(t) = TC(t_q)$ and $TC(t) = TC(t_{q'})$ are two equilibrium conditions for all early and late arrivals at work, respectively. Equilibrium conditions for groups A, B, C and D (or E) in Table 2 then can be expressed as equations (1), (2), (3) and (4), respectively:

$$\alpha \cdot T_Q(t) + \beta\left[t^* - \left(t + T_Q(t)\right)\right] = \beta \cdot t^*, \quad \text{for } t_q \le t \le t' \tag{1}$$

$$\alpha \cdot T_Q(t) + \beta\left[t^* - \left(t + T_Q(t)\right)\right] + \rho = \beta \cdot t^*, \quad \text{for } t^+ \le t < \hat{t} \tag{2}$$

$$\alpha \cdot T_Q(t) + \gamma\left[\left(t + T_Q(t)\right) - t^*\right] + \rho = \gamma(t_{q'} - t^*), \quad \text{for } \hat{t} < t < t^- \tag{3}$$

$$\alpha \cdot T_Q(t) + \gamma\left[\left(t + T_Q(t)\right) - t^*\right] = \gamma(t_{q'} - t^*), \quad \text{for } t^\# \le t < t^- \text{ or } t^- \le t \le t_{q'} \tag{4}$$

$\left[t^* - \left(t + T_Q(t)\right)\right]$ in equation (1) or (2) indicates the time periods for commuters arriving at work early (*i.e.*, T_E). On the other hand, $\left[\left(t + T_Q(t)\right) - t^*\right]$ in equation (3) or (4) indicates the time periods for commuters arriving at work late (*i.e.*, T_L). The values of $TC(t_q)$ and $TC(t_{q'})$ are $\beta \cdot t^*$ and $\gamma\left(t_{q'} - t^*\right)$, respectively because $t_q = 0$ and $T_Q(t_q) = T_Q(t_{q'}) = 0$.

Equilibrium queuing costs $(EQC: \alpha \cdot T_Q^{\,e}(t))$, for groups A~E, listed in column (IV) of Table 2 are obtained according to equations (1)~(4). As shown in Figure 1, the equilibrium queuing costs for groups A~E under the optimal single-step toll scheme are thick lines $\overline{t_q K}$, $\overline{t^+ I}$, $\overline{It^-}$, $\overline{JZ}$ and $\overline{Zt_{q'}}$, respectively. The slope of $\overline{t_q K}$ and $\overline{t^+ I}$ for all early arrivals is $\dfrac{\alpha\beta}{\alpha - \beta}$, which is the same as the slope of the equilibrium queuing cost ($\overline{t_q M}$) to all early arrivals in the no toll case. Note that there is no thick lines of equilibrium queuing costs through the departure period $[t', t^+)$ since no one departs during this period. Therefore, the length of the queue is reduced to zero at t^+. On the other hand, the slope of $\overline{It^-}$, $\overline{JZ}$ and $\overline{Zt_{q'}}$ for all late arrivals is $\dfrac{-\alpha\gamma}{\alpha + \gamma}$, which is the same as the slope of the equilibrium queuing cost ($\overline{Mt_{q'}}$) to all late arrivals in the no toll case. Note that the equilibrium queuing costs incurred before and after t^-, for group D in Figure 1, are $\overline{Jt^-}$ and $\overline{t^\# Z}$, respectively. The former is the decreasing equilibrium queuing costs from ρ to zero, spent at the temporary parking area to avoid entering the bottleneck during the tolled period $[t^\#, t^-)$. The latter is the increasing equilibrium queuing costs from zero to $\dfrac{\alpha \cdot \rho}{\alpha + \gamma}$ (the height of Z in Figure 1), spent to enter the bottleneck free after t^-. Consequently, the total equilibrium queuing cost for group D is $\overline{JZ}$ ($= \overline{Jt^-} + \overline{t^\# Z}$).

The equilibrium departure rates (EDR) for groups A~E in column (V) of Table 2 are obtained by using the corresponding equilibrium queuing time $(T_Q^{\,e}(t))$. Figure 2 shows the EDR distributions for both the no toll and the optimal single-step toll cases. The two cases are shown as the dotted line area and the shadowed area, respectively. Certainly, the total departure is *"N"* for each of the two cases.

In the no toll case, the equilibrium queuing costs $(\alpha \cdot T_Q^e(t))$ for all early and late arrivals in Figure 1 are $\overline{t_q M}$ and $\overline{Mt_{q'}}$, respectively, and the slopes for the former and latter are $\frac{\alpha\beta}{\alpha-\beta}$ and $\frac{-\alpha\gamma}{\alpha+\gamma}$, respectively. Accordingly, their marginal departure rates $\left(=\frac{\mathrm{d}(s \cdot T_Q^e(t))}{\mathrm{d}t}\right)$ are $\frac{\beta s}{\alpha-\beta}$ and $\frac{-\gamma s}{\alpha+\gamma}$, respectively. Since the bottleneck is fully utilized through the queuing period, EDR for the early and late arrivals in the no toll case are therefore equal to $\frac{\alpha s}{\alpha-\beta}\left(=s+\frac{\beta s}{\alpha-\beta}\right)$ and $\frac{\alpha s}{\alpha+\gamma}\left(=s-\frac{\gamma s}{\alpha+\gamma}\right)$, respectively. See the dotted line area in Figure 2.

The EDR for the optimal single-step toll case is somewhat more complicated than the no toll case. Firstly, because $\overline{t_q K}$ and $\overline{t_q M}$ in Figure 1 coincide during $[t_q, t')$, the EDR for this departure period is the same as that in the no toll case. Secondly, since there are no departures during $[t', t^+)$, the EDR is zero for this departure period. Thirdly, because the slopes of $\overline{t^+ I}$ and $\overline{It^-}$ in Figure 1 are the same as the slope of $\overline{t_q M}$ and $\overline{Mt_{q'}}$, respectively, the EDR for the two periods $[t^+, \hat{t})$ and $[\hat{t}, t^-)$ are $\frac{\alpha s}{\alpha-\beta}$ and $\frac{\alpha s}{\alpha+\gamma}$, respectively. Note that there exist group D commuters who depart during the tolled period $[t^\#, t^-)$ but decide to enter the bottleneck free after t^-. Because the departure time period for group D overlaps with the departure time period for group C, also because the marginal departure rate for group D is equal to $\frac{\alpha s}{\alpha+\gamma}$ (since the slope of $\overline{t^\# Z}$ equals $\frac{\alpha^2}{\alpha+\gamma}$), the EDR for the departure period $[t^\#, t^-)$ in Figure 2 therefore becomes $\frac{2\alpha s}{\alpha+\gamma}$. Finally, $\overline{Zt_{q'}}$ and $\overline{Mt_{q'}}$ in Figure 1 coincide during $[t^-, t_{q'}]$, so the EDR for this departure period is also $\frac{\alpha s}{\alpha+\gamma}$.

The sizes of departures listed in column (VI) of Table 2 are computed by multiplying the lengths of departure intervals and corresponding values of EDR together. The former can be obtained from column (II) by using the related values listed in Table 1, and the latter are already shown in column (V).

3.2 Equilibrium Departure Time Switching Decisions

Section 3.1 showed the distribution differences in the equilibrium departure rate before and after pricing a queuing bottleneck with the optimal single-step toll. However, these results give no idea as to investigate commuters' departure time switching decisions if the bottleneck is tolled. This Section solves these problems by comparing the equilibrium schedule delay costs for both the no toll and the optimal single-step toll cases.

Since the equilibrium queuing costs for all departure intervals under the optimal single-step toll scheme have been derived, the corresponding values of equilibrium schedule delay costs (ESDC) required to achieve the equilibrium commuting cost $TC^e \left(=\frac{\beta\gamma}{\beta+\gamma}(N/s)\right)$ can be easily obtained as shown in column (VII) of Table 2. In Figure 1, ESDC are drawn as doubled lines $\overline{\overline{AK}}$, $\overline{\overline{W\hat{t}}}$, $\overline{\overline{\hat{t}L}}$, $\overline{\overline{JG}}$ and $\overline{\overline{GB}}$ for groups A~E, respectively. The slope of $\overline{\overline{AK}}$ and $\overline{\overline{W\hat{t}}}$ for all early arrivals is $\frac{-\alpha\beta}{\alpha-\beta}$, which is the same as the slope of $\overline{A\tilde{t}}$ for all early arrivals in the no toll case. On the other hand, the slope of $\overline{\overline{\hat{t}L}}$, $\overline{\overline{JG}}$ and $\overline{\overline{GB}}$ for all late arrivals is $\frac{\alpha\gamma}{\alpha+\gamma}$, which is also the same as the slope of $\overline{\tilde{t}B}$ for all late arrivals in the no toll case.

ESDC to groups A and B is the equilibrium early cost $\beta \cdot T_E^{\ e}(t)$. On the other hand, ESDC to other groups C, D and E is the equilibrium late cost, $\gamma \cdot T_L^{\ e}(t)$. Therefore, the contents of the equilibrium commuting cost (TC^e) to groups A~E under the optimal single-step toll scheme can be shown as $\alpha \cdot T_Q^{\ e}(t)+\beta \cdot T_E^{\ e}(t)$, $\alpha \cdot T_Q^{\ e}(t)+\beta \cdot T_E^{\ e}(t)+\rho$, $\alpha \cdot T_Q^{\ e}(t)+\gamma \cdot T_L^{\ e}(t)+\rho$, $\alpha \cdot T_Q^{\ e}(t)+\gamma \cdot T_L^{\ e}(t)$ and $\alpha \cdot T_Q^{\ e}(t)+\gamma \cdot T_L^{\ e}(t)$, respectively.

Because the congestion toll derived from our model is simply the money cost to the toll payer required to save the same amount of queuing costs, the equilibrium schedule delay cost (i.e., either the early cost, $\beta \cdot T_E^{\ e}(t)$ or the late cost, $\gamma \cdot T_L^{\ e}(t)$) in the optimal step toll case must be the same as that in the original no toll case to maintain the equilibrium commuting cost (TC^e). For this purpose, all homogeneous commuters with the same values of β and γ therefore will not alter their original preferred arrival times at work in the no toll case if the bottleneck is tolled. All commuters' equilibrium departure time switching decisions then can be investigated by the above principle. We call it *"the permanent schedule delay costs principle"*. The detailed discussion of all commuters' departure time switching decisions will be made below accompanied with Figures 1 and 2.

First, group A commuters will not alter their original departure times in the no toll case when the bottleneck is priced with the optimal single-step toll. This is because the equilibrium early costs $\overline{\overline{AK}}$ in the no toll and optimal single-step toll cases coincide during the departure period (t_q,t'). The part of $t_q t'a'a$ in Figure 2 represents those commuters of group A. Obviously, their departure interval is indifferent in both the no toll and the optimal single-step toll cases. Next, because the equilibrium early cost ($\overline{\overline{W\hat{t}}}$) in the optimal single-step toll case and the equilibrium early cost ($\overline{K\tilde{t}}$) in the no toll case are two identical and parallel lines, group B that originally depart during the period ($t',\tilde{t}$) in the no toll case will shift all their departures to the period ($t^+,\hat{t}$) in the optimal single-step toll case. Therefore, the dotted line area $t'\tilde{t}ba'$ in Figure 2 will move to the shadowed part of $t^+\hat{t}dc$. Similarly, because the equilibrium late cost ($\overline{\overline{\hat{t}L}}$) in the optimal single-step toll case and the equilibrium late cost ($\overline{\tilde{t}J}$) in the no toll case are two identical and parallel lines, group C that originally depart during the period $[\tilde{t},t^{\#})$ in the no toll case will shift all their departures to the period $[\hat{t},t^{-})$ in the optimal single-step toll case. Therefore, the dotted line area $\tilde{t}t^{\#}ge$ in Figure 2 will move to the shadowed part of $\hat{t}t^{-}hf$. Finally, because both the equilibrium late costs $\overline{\overline{JG}}$ and $\overline{\overline{GB}}$ in the optimal single-step toll case coincide with the equilibrium late cost ($\overline{\tilde{t}B}$) in the no toll case, groups D and E will not alter their original departure times in the no toll case if the bottleneck is priced with the optimal single-step toll. This supports the conclusion, which we have made in Section 3.1, that the total equilibrium departure rate for the period $[t^{\#},t^{-})$ in the optimal single-step toll case is $\frac{2\alpha s}{\alpha+\gamma}$. Because the 1st floor of $[t^{\#},t^{-})$ has been occupied by group C, the shadowed part of *ghkj* in the 2nd floor of $[t^{\#},t^{-})$ and $t^{-}t_{q'}ih$ in Figure 2 indicate groups D and E commuters, respectively.

4. COMMUTER BEHAVIOR IN THE OPTIMAL MULTI-STEP TOLL CASES

Because the methods to derive departure time values, some equilibrium results and departure time switching decisions for the optimal multi-step toll cases are similar to the optimal single-step toll case that we have mentioned in Section 3, this Section shows only these results without providing detailed derivations.

PROPOSITION A, concerning to investigate how many commuters will or will not pay the toll to cross a queuing bottleneck, is developed as follows:

PROPOSITION A: $\frac{n}{n+1}$ *of all commuters (N) will pay the toll under the optimal n-step toll schemes. The number of toll payers who arrive at work early (including on-time) or late are* $\frac{n\cdot\gamma\cdot N}{(n+1)(\beta+\gamma)}$ *or* $\frac{n\cdot\beta\cdot N}{(n+1)(\beta+\gamma)}$, *respectively. On the other hand, the remaining* $\frac{1}{n+1}$ *of all commuters will not pay the toll to cross a queuing bottleneck. The number of these commuters who arrive at work early or late are* $\frac{\gamma\cdot N}{(n+1)(\beta+\gamma)}$ *or* $\frac{\beta\cdot N}{(n+1)(\beta+\gamma)}$, *respectively.*

Departure time shifts from the no toll case to the optimal multi-step toll schemes are shown as Table 3. Theses results are also obtained according to "the permanent schedule delay costs principle" that we have mentioned in Section 3. *PROPOSITION B*, concerning to investigate how many groups of and what kind of commuters will make departure time switching decisions, is developed as follows:

PROPOSITION B: n groups of early arrivals and also n groups of late arrivals will alter their original departure times in the no toll case if a queuing bottleneck is priced with the optimal n-step tolls. Moreover, commuters that will or will not alter their original departure times are the same as those who will or will not pay the tolls, respectively, to cross a queuing bottleneck.

Finally, *PROPOSITION C*, concerning to investigate how long will the toll payers (or departure time shifters) delay their original departure times in the no toll case, is developed as follows:

Table 3. Types of Departure Time Interval Shift from the No Toll Case to the Optimal Single- and Multi-Step Toll Cases

Types	No Toll → 1-Step Toll	No Toll → 2-Step Toll	No Toll → 3-Step Toll	No Toll → *n*-Step Toll
Type I $\left(\mathrm{DR} = \dfrac{\alpha \cdot s}{\alpha - \beta}\right)$	No Shift during $[t_q, t')$	No Shift during $[t_q, t')$	No Shift during $[t_q, t')$	No Shift during $[t_q, t')$
Type II $\left(\mathrm{DR} = \dfrac{\alpha \cdot s}{\alpha - \beta}\right)$	$[t', \tilde{t}) \rightarrow [t^{+}, \hat{t})$	$[t', t_1) \rightarrow [t^{+}, t'')$ $[t_1, \tilde{t}) \rightarrow [t^{++}, \hat{t})$	$[t', t_1) \rightarrow [t^{+}, t'')$ $[t_1, \dot{t}_2) \rightarrow [t^{++}, t''')$ $[t_2, \tilde{t}) \rightarrow [t^{+++}, \hat{t})$	$[t', t_1) \rightarrow [t^{+}, t'')$ $[t_1, t_2) \rightarrow [t^{++}, t''')$ $[t_2, t_3) \rightarrow [t^{+++}, t'''')$ $\vdots$ $[t_{n-2}, t_{n-1}) \rightarrow [t^{\overset{n-1}{+\cdots}}, t^{\overset{n}{\prime\cdots}})$ $[t_{n-1}, \tilde{t}) \rightarrow [t^{\overset{n}{+\cdots}}, \hat{t})$
Type III $\left(\mathrm{DR} = \dfrac{\alpha \cdot s}{\alpha + \gamma}\right)$	$[\tilde{t}, t^{\#}) \rightarrow [\hat{t}, t^{-})$	$[\tilde{t}, t_{1'}) \rightarrow [\hat{t}, t^{--})$ $[t_{1'}, t^{\#\#}) \rightarrow [t^{\#}, t^{-})$	$[\tilde{t}, t_{1'}) \rightarrow [\hat{t}, t^{---})$ $[t_{1'}, t_{2'}) \rightarrow [t^{\#}, t^{--})$ $[t_{2'}, t^{\#\#\#}) \rightarrow [t^{\#\#}, t^{-})$	$[\tilde{t}, t_{1'}) \rightarrow [\hat{t}, t^{\overset{n}{-\cdots}})$ $[t_{1'}, t_{2'}) \rightarrow [t^{\#}, t^{\overset{n-1}{-\cdots}})$ $[t_{2'}, t_{3'}) \rightarrow [t^{\#\#}, t^{\overset{n-2}{-\cdots}})$ $\vdots$ $[t_{(n-2)'}, t_{(n-1)'}) \rightarrow [t^{\overset{n-2}{\#\cdots}}, t^{--})$ $[t_{(n-1)'}, t^{\overset{n}{\#\cdots}}) \rightarrow [t^{\overset{n-1}{\#\cdots}}, t^{-})$
Type IV $\left(\mathrm{DR} = \dfrac{\alpha \cdot s}{\alpha + \gamma}\right)$	No Shift during $[t^{\#}, t_{q'}]$	No Shift during $[t^{\#\#}, t_{q'}]$	No Shift during $[t^{\#\#\#}, t_{q'}]$	No Shift during $[t^{\overset{n}{\#\cdots}}, t_{q'}]$

PROPOSITION C: New departure time periods to commuters who pay the first step toll ρ *under the optimal n-step toll schemes will be formed* $\frac{2\rho}{\alpha(n+1)}$ *hours later than their original departure time periods in the no toll case. In addition, departure time period delays to other toll payers who pay* 2ρ, 3ρ, ····· ,$n\rho$ *are 2, 3, ·····, n times longer than the first step toll payer.*

5. PRACTICAL IMPLICATIONS AND CONCLUSIONS

This paper has provided a methodological framework to forecast detailed commuter behavior if a queuing road bottleneck is priced with the optimal single- and multi-step toll schemes. This kind of research is rare to find in the field of transport economics because it is difficult to objectively forecast some uncertainties in commuters' alternative decisions if the congestion toll is considered to put into practice. These uncertainties including how many and what kind of commuters will or will not pay the toll to cross a queuing bottleneck, and what are the changes in commuters' departure patterns compared with the no toll case.

Some conclusions obtained from this paper, and practical implications of these conclusions are illustrated as follows:

(1) Table 3 has shown regular departure time shifts for types II and III commuters from the no toll to the optimal n-step toll cases. The total number of them is $\frac{n}{n+1}$ of all commuters (N). They will not only alter their original departure time in the previous no toll case, but also pay the tolls to cross a queuing bottleneck in the optimal n-step toll cases. On the other hand, the remaining $\frac{1}{n+1}$ of all commuters will neither alter their original departure time nor pay any tolls to cross a queuing bottleneck. The above information has two practical implications. Firstly, it allows policy makers to estimate the toll revenue, which are paid by all departure time shifters, before implementing the optimal step toll scheme. Therefore, it is a useful information for the authorities to budget the policy of levying the optimal step tolls to a queuing bottleneck. Secondly, it implies that the general characteristics of commuters (including percentages of different sexes, average ages, average wages, etc.) who will or will not pay the optimal step tolls become predictable in the present no toll case. Take the optimal 1-step toll case of Table 3 for example. The general characteristics of commuters who neither alter their original departure times nor pay any tolls to cross a queuing bottleneck can be investigated by surveying Types I and IV commuters who depart during the early arrival period $[t_q, t')$ and the late arrival period $[t^{\#}, t_{q'}]$, respectively, in the present no toll case. On the other hand, the general characteristics of commuters who are both the departure time shifter and the toll payer can be investigated by surveying Types II and III commuters who depart during the early arrival period $[t', \tilde{t})$ and the late arrival period $[\tilde{t}, t^{\#}]$, respectively, in the present no toll case.

(2) New departure times to all optimal n-step toll payers will be later than their original departure times in the no toll case. Departure time delay to the 1st step toll (ρ) payer equals $\frac{2\rho}{\alpha(n+1)}$. Departure time delays to other toll payers including the 2nd step toll (2ρ) payer, 3rd

step toll (3ρ) payer, ·····, n^{th} step toll ($n\rho$) payer are 2, 3, ·····, n times longer than the first step toll payer. This implies that the higher step tolls a commuter pays, the longer his new departure time will delay when compared with his original departure time in the no toll case. Therefore, one can enjoy more leisure time at home in the morning before he goes for work than he would be in the original no toll case if he pays higher tolls in the optimal multi-step toll cases.

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APPENDICES : Definition of All Notations

N :Total number of auto-commuters who have to cross a bottleneck to reach their workplaces;
s : Capacity of the bottleneck (measured by the traffic flow);
t : Departure time from home, also arrival time at the bottleneck;
t^* : A fixed work start time for all commuters;
t_q : Time at which queue first forms;
$t_{q'}$: Time at which queue disappears;
$\tilde{t}$: Departure time at which allows one to arrive at work on time in the no toll case;
$\hat{t}$: Departure time at which allows one to arrive at work on time under the optimal step toll scheme;
τ : The optimal fine toll;
ρ: The optimal single-step toll, or the 1st step toll (the lowest toll) under the optimal multi-step toll schemes;
t^+ : Time that starts the levying of the optimal single-step toll, or time that starts the levying of the 1st step toll under the optimal multi-step toll schemes;
t^{++} : Time when the 1st step toll stops and the 2nd step toll is levied under the optimal multi-step toll schemes;
t^{+++} : Time when the 2nd step toll stops and the 3rd step toll is levied under the optimal multi-step toll schemes;
$t^{\overset{n}{+\cdots}}$: Time when the $(n\text{-}1)^{th}$ step toll stops and the n^{th} step toll is levied under the optimal n-step toll schemes ;

t' : The start time when no one departs until t^{+} under both the optimal single- and multi-step toll schemes;

t'' : The 2nd start time when no one departs until t^{++} under the optimal multi-step toll schemes;

t''' : The 3rd start time when no one departs until t^{+++} under the optimal multi-step toll schemes;

$t^{\overset{n}{'\cdots}}$: The n^{th} start time when no one departs until $t^{\overset{n}{+\cdots}}$ under the optimal n-step toll schemes;

$t^{\overset{n}{-\cdots}}$: Time when the n^{th} step toll finishes and the $(n\text{-}1)^{th}$ step toll restarts under the optimal n-step toll schemes;

t^{---} : Time when the 3rd step toll finishes and the 2nd step toll restarts under the optimal multi-step toll schemes;

t^{--} : Time when the 2nd step toll finishes and the 1st step toll restarts under the optimal multi-step toll schemes;

t^{-} : Time when the optimal single-step toll finishes, or time when the 1st step toll finishes under the optimal multi-step toll schemes;

$t^{\#}$: Time when some departures start waiting at the temporary parking area until t^{-} to avoid paying the optimal single-step toll, or time when some departures start waiting at the temporary parking area until $t^{\overset{n}{-\cdots}}$ in order to pay the $(n\text{-}1)^{th}$ step toll under the optimal n-step toll schemes;

$t^{\#\#}$: Time when some departures start waiting at the temporary parking area until $t^{\overset{n-1}{-\cdots}}$ in order to pay the $(n\text{-}2)^{th}$ step toll under the optimal n-step toll schemes;

$t^{\#\#\#}$: Time when some departures start waiting at the temporary parking area until $t^{\overset{n-2}{-\cdots}}$ in order to pay the $(n\text{-}3)^{th}$ step toll under the optimal n-step toll schemes;

$t^{\overset{n}{\#\cdots}}$: Time when some departures start waiting at the temporary parking area until t^{-} in order to avoid paying any toll under the optimal n-step toll schemes;

$t_1, t_2, t_3 \cdots t_n$: Departure times during $[t_q, \tilde{t}]$ used to show early arrivals' departure time shifts in detail from the no toll to the optimal multi-step toll cases;

$t_{1'}, t_{2'}, t_{3'} \cdots t_{n'}$: Departure times after $\tilde{t}$ used to show late arrivals' departure time shifts in detail from the no toll to the optimal multi-step toll cases;

T_Q : Time period spent waiting in the queue;

T_E : Time period spent at the workplace before the work start time;

T_L : Time period by which the work arrival time exceeds the work start time;

α : Penalty cost per hour for the time period spent waiting in the queue;

β : Penalty cost per hour for the time period spent at the workplace before the work start time;

γ : Penalty cost per hour for the time period by which the arrival time at work exceeds the work start time;

TC : The commuting cost (per vehicle) incurred due to bottleneck queuing.

Superscript "e": *Equilibrium, e.g.,* TC^e , T_Q^e , T_E^e , T_L^e .