# A MULTI-CLASS TRAFFIC FLOW SIMULATOR FOR ASSESSING NETWORK RELIABILITY UNDER ADVANCED TRAVELER INFORMATION SYSTEM

Gang XU
Research Student
Department of Civil and Structural Engineering
The Hong Kong Polytechnic University
Hung Hom, Kowloon
Hong Kong
Fax: +852-23346389
E-mail: gang.xu@polyu.edu.hk

William Hing-Keung LAM
Professor
Department of Civil and Structural Engineering
The Hong Kong Polytechnic University
Hung Hom, Kowloon
Hong Kong
Fax: +852-23346389
E-mail: cehklam@polyu.edu.hk

**Abstract:** Intelligent Transportation System (ITS) practice has been implemented in many countries aiming to improve transport network performance. Advanced Traveler Information System (ATIS) is one of the critical components of ITS. Therefore the effectiveness of the ATIS should be assessed before the adoption of ITS. Average travel time was widely used in the previous researches to assess the benefit of ATIS. However, drivers now pay additional attention on individual travel time or the probability of arriving at a destination within a specified time interval, which leads to the recent research interest in network reliability. In this paper a multi-class traffic flow simulator (MTFS) is proposed to assess the ATIS in terms of travel time reliability. The MTFS is empowered with the capability of modelling the route choice behaviour under ATIS based on the input of an outdated total OD matrix, a constant market penetration of ATIS and partial link traffic counts by user classes. The total OD matrix can also be updated by MTFS. A numerical example is given to show the effectiveness of the MTFS.

**Key Words:** ATIS, network reliability, multi-class, SUE

## 1. INTRODUCTION

With the rapid development and implementation of Information technology (IT), many new systems have been proposed to improve the performance of transportation network, especially for those networks with heavy traffic load. Intelligent Transportation System (ITS) is one of the options currently under implementation in many countries. Advanced Traveler Information System (ATIS) is an important component of ITS. It is expected that the introduction of ATIS can improve drivers' route choice by provision of real-time traffic information. Therefore more attention has recently been given to the impacts of ATIS by modelling drivers' route choice behaviour under ATIS (Van Vuren and Watling, 1991; Yang *et al.*, 1993; Jayakrishnan *et al.*, 1994; Maher and Hughes, 1996; Emmerink, 1998).

Three principles are generally used in modelling route choice behaviour under the context of ATIS, named user equilibrium (UE), stochastic user equilibrium (SUE) and system optimum (SO). UE condition can be achieved when all drivers have perfect knowledge of the whole network traffic condition and choose the path with shortest travel time. However, some drivers can not possess the complete information concerning travel and route alternatives. They will make route choice decision according to their perceived travel time, which is usually modeled by actual travel time plus an item of perceived error. That leads to SUE assignment. It is believed that the perceived error will be reduced by the ATIS and the drivers' route choice decision making can be improved (Ben-Akiva *et al.*, 1991). As the route choice behaviour of drivers with ATIS is different with that of drivers without ATIS, multi-class dimension should be introduced in modelling route choice under ATIS. The objective of SO is the minimisation of total travel time for all drivers. It is usually used for traffic control purposes.

As for the multi-class problem, SUE is usually used for the drivers without ATIS. As far as to those drivers with ATIS, the three principles had been used in prior researches. Yang (1998) modeled ATIS-equipped drivers with UE principle. SUE was used by Koutsopoulos and Lotan (1990), Maher and Hughes (1996). SO principle is adopted for drivers with ATIS by Kanafani and Al-Deek (1991). In the model proposed by Van Vuren and Watling (1991), all drivers were classified into three classes following three principles respectively.

In the previous researches, the average travel time was often used as the performance indicator. With the rapid economic development within the last decade, drivers are concerned not only about the average travel time, but also on their individual travel time or the probability of finishing their trips within a given period of time. Therefore, network travel time reliability should be taken into account. Moreover, the OD matrices in the previous researches were fixed. However, it is believed that fluctuations do exist in OD flows due to seasonal or daily variations. Hence, variation in OD matrix is considered in this paper.

The path flow estimator (PFE) was introduced based on an equivalent logit-based stochastic user equilibrium (SUE) problem (Bell and Iida, 1997; Bell *et al.*, 1997) with the use of traffic count data. The results of PFE include path flows and link flows together with travel times and the OD matrix. The variance of estimated travel times can be obtained by analytical sensitivity expressions or finite differencing approximation (Bell *et al.*, 1999), which can then be used for calculation of travel time reliability. It should be noted that the logit-type SUE model adopted in the PFE is based on the assumption that the perceived error of travel time is a random variable which follows Gumbel distribution.

The single-class traffic flow simulator (TFS) was proposed by Lam and Xu (1999). The TFS combines the general SUE assignment model and OD estimation from traffic counts. Based on partial traffic count data, the traffic flows and travel times for the links without detectors can be estimated by the TFS, together with their variance and covariance. The prior OD matrix can be updated at the same time. The travel time reliability can be assessed by using the link travel time and variance/covariance. A method based on a genetic algorithm has been proposed for calibration of the TFS (Lam and Xu, 2000).

In this paper, a multi-class traffic flow simulator (MTFS) is proposed on the basis of the TFS to assess the network reliability under the ATIS. The paper is structured as below. The notations that used throughout this paper is firstly presented before MTFS is formulated. The network reliability indices that can be assessed by the proposed MTFS are then discussed followed by the solution algorithm. Some test results with an example network are shown before drawing conclusions.

## 2. NOTATIONS

$\mathbf{B}^m$ : variance/covariance matrix of link flows by class $m$.

$C_{k,m}^{rs}$ : perceived travel time on path $k$ between OD pair $rs$ by class $m$.
$c_k^{rs}$ : actual travel time on path $k$ between OD pair $rs$.
$d_m$ : market penetration by class $m$.
$k$ : parameter in BPR function.
$p$ : parameter in BPR function.
$p_{a,m}^{rs}$ : link choice proportion for link $a$ of OD pair $rs$ (i.e. proportion of flow from $r$ to $s$ using link $a$) by class $m$.
$Q_{rs}$ : OD flow between OD pair $rs$.
$\hat{q}_{rs}$ : prior OD flow between OD pair $rs$.
$S_{rs}$ : expected minimum travel time between OD pair $rs$.
$s_a$ : capacity of link $a$.
$t_a(0)$ : free flow travel time on link $a$.
$t_a$ : travel time on link $a$.
$\mathbf{v}^{\mathbf{m}}$ : vector of link flow by class $m$.
$\mathbf{v}_{\mathbf{a}}^{\mathbf{m}}$ : vector of mean link flow by class $m$.
$\mathbf{v}_{\mathbf{d}}^{\mathbf{m}}$ : random variable of flow on links with detectors by class $m$.
$\bar{\mathbf{v}}_{\mathbf{d}}^{\mathbf{m}}$ : mean flow on links with detectors by class $m$.
$\hat{\mathbf{v}}_{\mathbf{d}}^{\mathbf{m}}$ : detected link flow by class $m$.
$\mathbf{v}_{\mathbf{e}}^{\mathbf{m}}$ : random variable of flow on links without detectors by class $m$.
$\bar{\mathbf{v}}_{\mathbf{e}}^{\mathbf{m}}$ : mean flow on links without detectors.
$\tilde{\mathbf{v}}_{\mathbf{e}}^{\mathbf{m}}$ : estimated flow on links without detectors based on the detected data by class $m$.
$\beta$ : coefficient of variance for OD variation.
$\varepsilon_{k,m}^{rs}$ : the perceived error of travel time on path $k$ between OD pair $rs$ by class $m$.
$\omega_m$ : coefficient of variance for perceived error of travel time by class $m$.
$\xi_{rs}$ : variation of OD flow between OD pair $rs$.
$\lambda$ : weighting factor for prior OD matrix in objective function of MTFS.
$\lambda_{rs}$ : OD factors used to update OD matrix.
$\delta$ : tolerance parameter of OD flows.
$\tau$ : stopping criteria parameter.

## 3. MODEL FORMULATION

### 3.1 Stochastic Perceived Path Travel Time

In this paper, SUE principle will be used for both drivers with and without ATIS. The relationship between the perceived and the actual travel time on path $k$ from origin $r$ to destination $s$ for class $m$ drivers can be expressed as

$$C_{k,m}^{rs} = c_k^{rs} + \varepsilon_{k,m}^{rs} \tag{1}$$

in which $\varepsilon_{k,m}^{rs}$ is the perceived error of travel time on path $k$ for OD pair $rs$ by class $m$ drivers. In order to model the effect of ATIS on reduction of perceived error, the perceived error for drivers with ATIS is assumed to be smaller than that for drivers without ATIS.

Different assumptions made for the distribution of $\varepsilon_{k,m}^{rs}$ forms different SUE models. Gumbel distribution adopted by PFE leads to a logit-type SUE assignment. In the proposed MTFS, there is no limitation on the assumption of this distribution. In other words, the MTFS can model various patterns of route choice behaviour, which is more flexible than the previous models.

### 3.2 Fluctuated Travel Demand and Market Penetration of ATIS

Travel demand is often considered to be constant in many of the previous OD estimators. But there must be some fluctuations in OD flow due to the seasonal or daily factors. Taking account of the OD flow variations, the OD flow between OD pair $rs$ can be expressed as

$$Q_{rs} = q_{rs} + \xi_{rs} \tag{2}$$

where $\xi_{rs} \sim N(0, \beta q_{rs})$ is the variation of OD flow for OD pair $rs$ and $\beta$ is a constant coefficient.

The market penetration of ATIS is defined as the percentage of road users equipped with the system (Yang, 1998). In this paper, the market penetration of ATIS is assumed to be constant for all the OD pairs. Given a total OD matrix and the market penetration of ATIS, the OD matrix of drivers with ATIS can be determined. Consequently the OD matrix for drivers without ATIS can be derived. Therefore, in the proposed MTFS, it is the total OD matrix that will be updated, not the OD matrices by user classes.

### 3.3 Multi-class Traffic Flow Simulator

Based on the single class TFS, the multi-class traffic flow simulator (MTFS) can be formulated as the following optimization problem:

$$\text{Min} \quad -\sum_m \sum_{rs} d_m q_{rs} S_{rs}(c_{rs}) + \sum_m \sum_a v_a^m t_a - \sum_a \int_0^{v_a} t_a(w)dw + \lambda \sum_{rs} (q_{rs} - \hat{q}_{rs})^2 \tag{3}$$

s.t.

$$\sum_{rs} d_m q_{rs} p_{d,m}^{rs} = \hat{v}_d^m, \quad \forall m, d \tag{4}$$

$$\sum_{rs} d_m q_{rs} p_{e,m}^{rs} = \tilde{v}_e^m, \quad \forall m, e \tag{5}$$

$$(1-\delta)\hat{q}_{rs} \le q_{rs} \le (1+\delta)\hat{q}_{rs}, \quad \forall rs \tag{6}$$

$$v_a = \sum_m v_a^m, \quad \forall a \tag{7}$$

$$\tilde{v}_e^m = \bar{v}_e^m + B_{21}^m (B_{11}^m)^{-1} (\hat{v}_d^m - \bar{v}_d^m) \tag{8}$$

The objective function (3) of the MTFS is a generalized multi-class SUE model with the additional consideration of the prior total OD matrix as the target total OD matrix. It is guaranteed that the assigned link flows corresponding to the updated OD matrix satisfy the SUE condition. The constraint (4) forces the assigned flows on detected links to reproduce the detected data. Equation (5) is employed to guarantee that the assigned flows on links without detectors to be the consistent with the resultant link flows estimated by Equation (8).

It is assumed in Equation (8) that the link flows by user classes follow multivariate normal (MVN) distribution of $\mathbf{v}^{\mathbf{m}} \sim MVN(\overline{\mathbf{v}}^{\mathbf{m}}, \mathbf{B}^{\mathbf{m}})$. $\overline{\mathbf{v}}^{\mathbf{m}}$ is the vector for the mean value of link flows and $\mathbf{B}^{\mathbf{m}}$ is variance/covariance matrix, which can be obtained from SUE assignment. As partial links can be installed with detectors. The link flows and variance/covariance matrix can be partitioned as

$$\mathbf{v}^{\mathbf{m}} = \begin{bmatrix} \mathbf{v}_{\mathbf{d}}^{\mathbf{m}} \\ \mathbf{v}_{\mathbf{e}}^{\mathbf{m}} \end{bmatrix} \sim MVN\left( \begin{bmatrix} \overline{\mathbf{v}}_{\mathbf{d}}^{\mathbf{m}} \\ \overline{\mathbf{v}}_{\mathbf{e}}^{\mathbf{m}} \end{bmatrix}, \begin{bmatrix} \mathbf{B}_{11}^{\mathbf{m}} & \mathbf{B}_{12}^{\mathbf{m}} \\ \mathbf{B}_{21}^{\mathbf{m}} & \mathbf{B}_{22}^{\mathbf{m}} \end{bmatrix} \right) \tag{9}$$

according to whether the links are installed with detectors or not (Lam and Xu, 1999).

If the OD matrix is not the actual one, the detected link flows $\hat{\mathbf{v}}_{\mathbf{d}}^{\mathbf{m}}$ may be different with the mean of assigned flows on links with detectors, $\overline{\mathbf{v}}_{\mathbf{d}}^{\mathbf{m}}$. The difference between $\hat{\mathbf{v}}_{\mathbf{d}}^{\mathbf{m}}$ and $\overline{\mathbf{v}}_{\mathbf{d}}^{\mathbf{m}}$, can be used to update the estimated flows on other links without detectors, $\overline{\mathbf{v}}_{\mathbf{e}}^{\mathbf{m}}$, with Equation (8). The derivation of the equation (8) is given in the appendix.

## 4. NETWORK RELIABILITY

### 4.1 Link Travel Time Reliability

Link travel time reliability is defined as the probability that a trip on link $a$ can be finished within the threshold value (Bell and Iida, 1997). If the link travel time $T_a$ is a normally distributed random variable as $T_a \sim N(\mu_a, \sigma_a^2)$, where $\mu_a$ is the mean and $\sigma_a$ is the standard deviation of travel time on link $a$. Then the probability that the travel time on link $a$ is less than some threshold $t$ can be expressed as

$$\Pr\{T_a \le t\} = \Phi\left(\frac{t - \mu_a}{\sigma_a}\right) \tag{10}$$

where $\Phi(x)$ is the integral of the unit normal distribution from $-\infty$ to $x$.

### 4.2 Path Travel Time Reliability

Similar to the definition of link travel time reliability, path travel time reliability refers to the probability that a trip on path $k$ connecting OD pair $rs$ can be finished within the threshold value. The path travel time $T_k^{rs}$ can be assumed as a normally distributed random variable as $T_k^{rs} \sim N(\mu_k^{rs}, (\sigma_k^{rs})^2)$, where $\mu_k^{rs}$ is the mean and $\sigma_k^{rs}$ is the standard deviation of travel time on path $k$ connecting OD pair $rs$. Then the probability that the travel time on the specified path is less than some threshold $t$ can be expressed as

$$\Pr\{T_k^{rs} \le t\} = \Phi\left(\frac{t-\mu_k^{rs}}{\sigma_k^{rs}}\right) \tag{11}$$

### 4.3 Network Travel Time Reliability

Network travel time reliability is defined as the probability of a trip within a transportation network can be finished in less than a threshold value. Based on the simulation data, the mean and standard deviation of travel times for all OD pairs in the network can be calculated. If the network travel time $T_n$ is a normally distributed random variable as $T_n \sim N(\mu_n, \sigma_n^2)$, where $\mu_n$ is the mean and $\sigma_n$ is the standard deviation of network travel time calculated. The probability that the network travel time within the threshold $t$ can be expressed as

$$\Pr\{T_n \le t\} = \Phi\left(\frac{t-\mu_n}{\sigma_n}\right) \tag{12}$$

## 5. SOLUTION ALGORITHM

The MTFS can be solved by the following algorithm, which is extended from the Monte Carlo simulation based algorithm for TFS (Lam and Xu, 1999). It can be described below.

Step 1. Initialize the OD factors $\lambda_{rs}$ to 1.

Step 2. Repeat the following procedure until convergence.

1. Mulit-class SUE assignment
   Input: OD flow $q_{rs} = \lambda_{rs}\hat{q}_{rs}$, market penetration $d_m$
   Output: $p_{a,m}^{rs}$, $\mathbf{\bar{v}_a^m}$, $\mathbf{B^m}$, link/path travel times with variance and covariance
2. Update link flows by class with detector data
   $\mathbf{\tilde{v}_e^m} = \mathbf{\bar{v}_e^m} + \mathbf{B_{21}^m}\left(\mathbf{B_{11}^m}\right)^{-1}\left(\mathbf{\hat{v}_d^m} - \mathbf{\bar{v}_d^m}\right)$
3. Check convergence.
   If $\mathrm{Max}\left[\left|\frac{\tilde{v}_e^m}{\bar{v}_e^m}-1\right|, \left|\frac{\hat{v}_d^m}{\bar{v}_d^m}-1\right|\right] \le \tau$, then stop; else, continue.
4. Calculate the OD factors $\lambda_{rs}$ by solving the following linearly constrained general least squares problem.

$$\text{Min } \sum_m \left[\sum_d \left(\hat{v}_d^{m} - \sum_{rs} d_m \lambda_{rs} \hat{q}_{rs} p_{d,m}^{rs}\right)^2 + \sum_e \left(\tilde{v}_e^m - \sum_{rs} d_m \lambda_{rs} \hat{q}_{rs} p_{e,m}^{rs}\right)^2\right] + \sum_{rs}\left(\lambda_{rs}\hat{q}_{rs} - \hat{q}_{rs}\right)^2 \tag{13}$$

$$1-\delta \le \lambda_{rs} \le 1+\delta, \ \forall rs \tag{14}$$

The performance of the proposed model together with the solution algorithm is tested with the numerical example in the following section.

## 6. NUMERICAL EXAMPLE

The Tuen Mun Corridor Network of Hong Kong used by Lam and Xu (1999) is used as the example network, as shown in Figure 1. The network consists of 3 zones, 4 nodes and 10 links. The BPR (Bureau of Public Roads) type function of

$$t_a = t_a(0) + k(v_a / s_a)^p \tag{15}$$

is adopted as the link travel time function. In Equation (15), $s_a$ is the capacity of link $a$, $t_a(0)$ is the free flow travel time and $p$ and $k$ are parameters of link $a$. The link characteristics are given in Table 1. The drivers are classified into two classes. The drivers with ATIS are named as class 1 while those without ATIS are named as class 2. The coefficients of OD variation are set arbitrarily as $\beta_1 = \beta_2 = 1.0$. The normal distribution is adopted for perception error as $\varepsilon^{rs}_{k,m} \sim N(0, \omega_m c^{rs}_k)$. In other words, the probit-type SUE assignment model is adopted in this example. The coefficients are set as $\omega_1 = 0.1$ and $\omega_2 = 0.25$ to represent the effect of ATIS on reduction of perceived error. The stopping criteria parameter is set to $\tau = 0.005$.

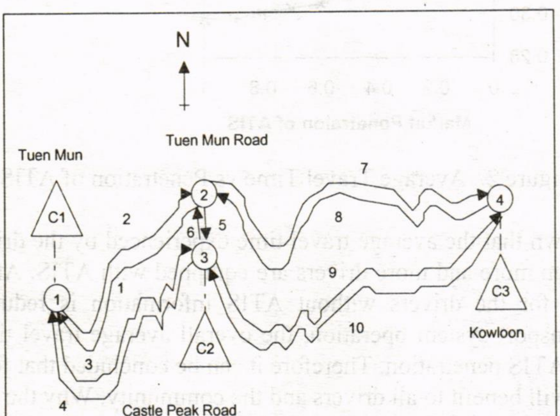


Figure 1. Tuen Mun Corridor Network

Table 1. The Link Data of the Network

| Link No. | $t_a(0)$ (hrs) | $s_a$ (pcu/hr) | Parameters $p$ | Parameters $k$ |
|---|---|---|---|---|
| 1,2 | 0.0900 | 5175 | 3.5 | 0.1050 |
| 3,4 | 0.1106 | 850 | 3.6 | 0.1408 |
| 5,6 | 0.0056 | 1150 | 3.6 | 0.0071 |
| 7,8 | 0.0335 | 4800 | 3.6 | 0.0335 |
| 9,10 | 0.0767 | 1000 | 3.6 | 0.1073 |

The actual and prior OD matrices are given in passenger car unit per hour (pcu/hr). The actual OD matrix is $\begin{bmatrix} 0 & 220 & 4952 \\ 313 & 0 & 127 \\ 4492 & 78 & 0 \end{bmatrix}$ and the prior OD matrix is $\begin{bmatrix} 0 & 172 & 4999 \\ 266 & 0 & 121 \\ 4843 & 83 & 0 \end{bmatrix}$

In this example, the detected link flows are the actual link flows obtained by allocation of the actual OD matrix onto the study network using the SUE assignment method.

### 6.1 Average Travel Time vs Market Penetration

Before the assessment of travel time reliability, the average travel time under different level of market penetration of ATIS is studied. It was reported by Maher and Hughes (1996) that around 40% penetration of ATT system resulted in the minimum total travel time for the whole network. The average travel times of two classes and overall average travel time are presented in Figure 2.

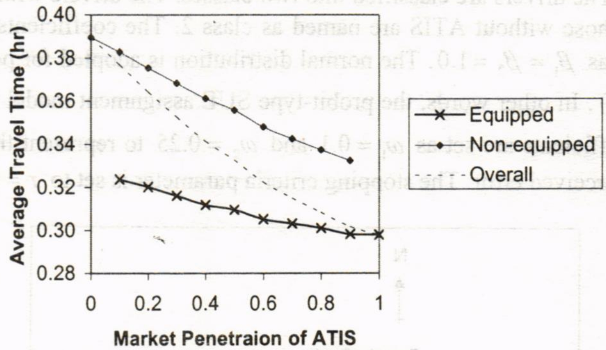


Figure 2. Average Travel Time vs Penetration of ATIS

In Figure 2, it is shown that the average travel time experienced by the drivers equipped with ATIS is reduced when more and more drivers are equipped with ATIS. At the same time, the average travel time for the drivers without ATIS information is reduced too. From the viewpoint of the transport system operation, the overall average travel time is also reduced with the increase of ATIS penetration. Therefore it can be concluded that for this example, the promotion of ATIS will benefit to all drivers and the community. Why the results are different from that found by Maher and Hughes (1996)? Let's look further into the total link flows in Figures 3(a) and 3(b).

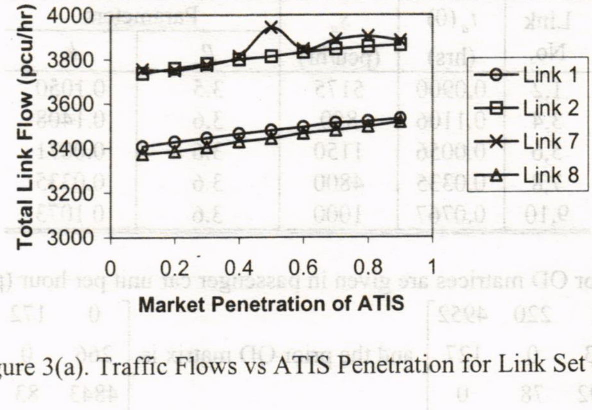


Figure 3(a). Traffic Flows vs ATIS Penetration for Link Set (I)

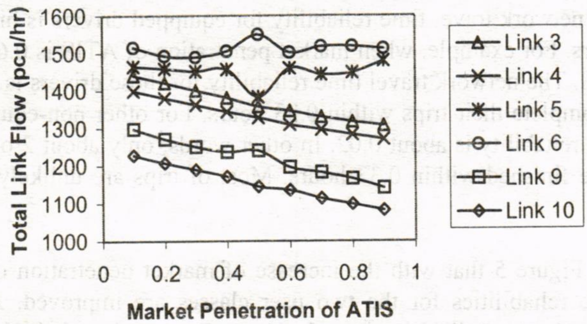


Figure 3(b). Traffic Flows vs ATIS Penetration for Link Set (II)

From the above two figures, it can be seen that with the increase of penetration of ATIS, more and more drivers know better the traffic condition in the whole network. And the drivers equipped with ATIS will turn to choose the shortest path for their travel.

## 6.2 Network Reliability vs Market Penetration

Figure 4 shows the mean ($\mu_n$) and standard deviation ($\sigma_n$) of network travel time under different level of ATIS market penetration.

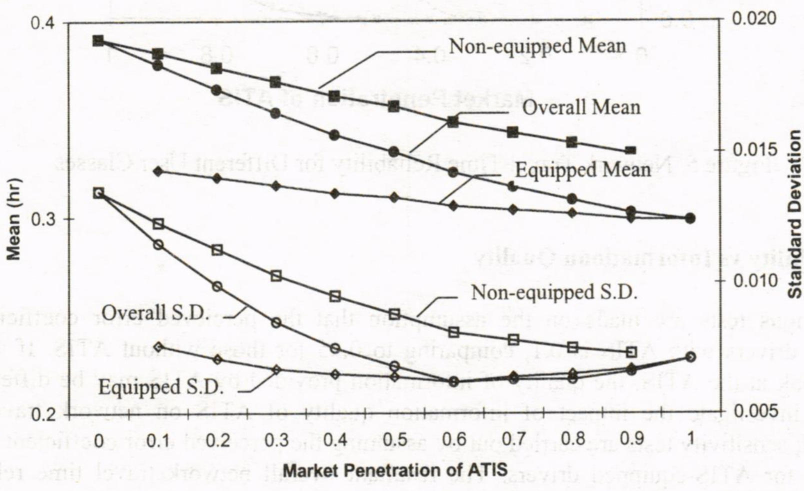


Figure 4. Mean and Standard Deviation of Network Travel Time

It is clear in the Figure 4 that the mean of network travel time for both equipped and non-equipped drivers are reduced with the increasing penetration of ATIS. Therefore the overall mean travel time shows the same trend. As far as the standard deviation is concerned, all the standard deviations for equipped, non-equipped and overall drivers decrease when the market penetration of ATIS increases from 0 to 0.6. If the market penetration of ATIS falls between 0.6 and 1, there are some slight increases in the standard deviation for equipped drivers and for all drivers.

According to Equation (12), Figure 5 shows the network travel time reliability curves for equipped, non-equipped and overall drivers with the threshold value $t$=0.33hr. It can be seen in Figure 5 that the network travel time reliability for equipped drivers is higher than that for non-equipped drivers. For example, when market penetration of ATIS is 0.6, 60% drivers are equipped with ATIS. The network travel time reliability for these drivers is 1.0. It means that these drivers can complete their trips within 0.33 hours. For other non-equipped drivers, the network travel time reliability is about 0.02. In other words, only about 2 out of 100 trips by these drivers can be finished within 0.33 hours. Most of trips are unlikely to be completed within 0.33 hours.

It is also shown in Figure 5 that with the increase of market penetration of ATIS, both the network travel time reliabilities for the two user classes are improved. And the resultant overall network travel time reliability has also been improved with higher penetration of ATIS.

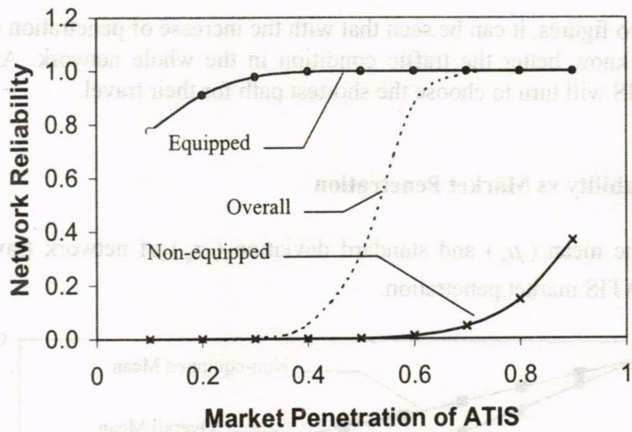


Figure 5. Network Travel Time Reliability for Different User Classes

### 6.3 Reliability vs Informationn Quality

The previous tests are made on the assumption that the percieved error coefficient for equipped drivers with ATIS is 0.1, comparing to 0.25 for those without ATIS. If we take further look at the ATIS, the quality of information provided by ATIS may be different. In order to investigate the impact of information quality of ATIS on network travel time reliability, sensitivity tests are carried out by assuming the perceived error coefficient of 0.15 and 0.05 for ATIS-equipped drivers. The resultant overall network travel time reliability curves correponding to the threshold value $t$=0.33hr are presented in Figure 6.

From this figure, it can be seen that when 'worse' information (corresponding to $\omega_1 = 0.15$) is provided by the ATIS, the network travel time reliability is lower than that corresponding to $\omega_1 = 0.1$. For example, when the market penetration of the ATIS is 0.8, the travel time reliability obtained from $\omega_1 = 0.1$ is almost 1.0. It means that all the trips can be made within the threshold of 0.33 hours. However, if there is more error in the ATIS information (represented by increasing $\omega_1$ from 0.1 to 0.15), the network travel time reliability is reduced to about 0.1. In other words, only 1 out of 10 trips can be completed within 0.33 hours.

In comparison, 'better' information will improve the network travel time reliability. The improvement of ATIS information is modeled by the smaller value taken by $\omega_1$. In Figure 6, it can be found that when $\omega_1 = 0.1$ and the market penetration of ATIS is 0.4, only a few trips (less than 10%) can be made within 0.33 hours. When more accurate ATIS information is provided ($\omega_1 = 0.05$), the travel time reliability correponds to the market penetration of 0.4 is about 0.9. This means almost 9 out of 10 trips can be finished within the threshold value of 0.33 hours.

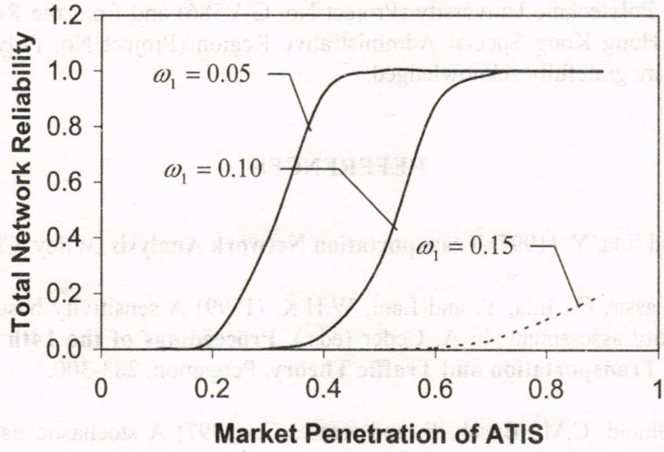


Figure 6. Network Travel Time Reliability under Different ATIS Information Quality

## 7. CONCLUSIONS

In this paper, the single-class traffic flow simulator proposed by Lam and Xu (1999) is extended to a multi-class traffic flow simulator (MTFS) in order to assess the network travel time reliability under ATIS. With the Monte Carlo simulation based heuristic solution algorithm developed, the impact of ATIS market penetration on the network travel time reliability can be assessed. Through an application of the proposed model to an example network, it was found that the introduction of ATIS does improve the network travel time reliability as well as the average travel time. Sensitivity tests have also been carried out to assess the impact of information quality provided by ATIS on network reliability. The more accurate the information provided by ATIS, the higher is the network travel time reliability.

The relationship between the average travel time and the market penetration of ATIS presented in this paper is different with that reported by Maher and Hughes (1996). Therefore further work will be carried out to investigate the impact of ATIS in depth by tests with different structure of networks and various OD demand level. In this paper, the ATIS is assumed to be operated by only one supplier. It will be more practical to consider the competition of several ATIS suppliers by introducing more than two user classes. The extension of the proposed model for practical application using real data is one of the tasks of further study. As the purpose of this paper is to assess the impacts of ATIS market penetration on network reliability, the effect of the network travel time reliability on route choice behaviour is not explicitly considered in the proposed model. However, this effect can be incorporated in the coefficient of perceived error, which can be calibrated by a stated preference survey on drivers' perceived travel times and route choice behaviour in response to additional information of travel time reliability. It is worthy to extend the proposed model to explicitly consider the effect of the network travel time reliability on route choice behaviour in further study.

## ACKNOWLEDGEMENTS

The work described in this paper was supported by grants from the Research Committee of the Hong Kong Polytechnic University (Project No. G-V586) and from the Research Grants Council of the Hong Kong Special Administrative Region (Project No. PolyU 5046/00E). These supports are gratefully acknowledged.

## REFERENCES

Bell, M.G.H. and Iida, Y. (1997) **Transportation Network Analysis**, Wiley, Chichester.

Bell, M.G.H., Cassir, C., Iida, Y. and Lam, W.H.K. (1999) A sensitivity based approach to network reliability assessment. In A. Ceder (eds.), **Proceedings of the 14th International Symposium on Transportation and Traffic Theory.** Pergamon, 283-300.

Bell, M.G.H., Shield, C.M., Busch, F. and Kruse, G. (1997) A stochastic user equilibrium path flow estimator. **Transportation Research, Vol. 5C**, 197-210.

Ben-Akiva, M., De Palma, A. and Kaysi, I. (1991) Dynamic network models and driver information systems. **Transportation Research, Vol. 25A**, 251-266.

Emmerink, R.H.M. (1998) **Information and Pricing in Road Transportation**. Springer-Verlag, Berlin.

Jayakrishnan, R., Mahmassani, H.S. and Hu, T.Y. (1994) An evaluation tool for advanced traffic information and management systems in urban networks. **Transportation Research, Vol. 2C,** 129-148.

Kanafani, A. and Al-Deek, H. (1991) A simple model for route guidance benefits. **Transportation Research, Vol. 25B,** 191-201.

Koutsopoulos, H.N. and Lotan, T. (1990) Motorist information systems and recurrent traffic congestion: sensitivity analysis of expected results. **Transportation Research Record, 1281**, 145-158.

Lam, W.H.K. and Xu, G. (1999) A traffic flow simulator for network reliability assessment. **Journal of Advanced Transportation, Vol. 33**, 159-182.

Lam, W.H.K. and Xu, G. (2000) Calibration of traffic flow simulator for network reliability assessment. In M.G.H. Bell and C. Cassir (eds.), **Reliability of Transport Networks.** Research Studies Press, 139-157.

Maher, M.J. and Hughes, P.C. (1996) Estimation of the potential benefits from an ATT system using a multiple user class stochastic user equilibrium assignment model. In Y.J. Stephanedes and F. Filippi (eds.), **Applications of Advanced Technologies in Transportation Engineering: Proceedings of the 4th International Conference.** ASCE, New York, 700-704.

Van Vuren, T. and Watling, D. (1991) Multiple user class assignment model for route guidance. **Transportation Research Record, 1306**, 22-32.

Yang, H. (1998) Multiple equilibrium behaviors and advanced traveler information systems with endogenous market penetration. **Transportation Research, Vol. 32B**, 205-218.

Yang, H., Kitamura, R., Jovanis, P., Vaughn, K.M. and Abdel-Aty, M.A. (1993) Exploration of route choice behavior with advanced travel information using neural network concept. **Transportation, Vol. 20**, 199-223.

## APPENDIX. PROOF OF EQUATION (8)

The following proof of Equation (8) is extracted from Lam and Xu (1999).

If the link flows are multivariate normally distributed random variable denoted as $\mathbf{v} \sim MVN(\overline{\mathbf{v}}, \mathbf{B})$, it can be partitioned as

$$\mathbf{v} = \begin{bmatrix} \mathbf{v_d} \\ \mathbf{v_e} \end{bmatrix} \sim MVN\left( \begin{bmatrix} \overline{\mathbf{v}}_\mathbf{d} \\ \overline{\mathbf{v}}_\mathbf{e} \end{bmatrix}, \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix} \right) \tag{A1}$$

according to whether the link is installed with or without detector.

In order to calculate the conditional distribution based on observed $\mathbf{v_d}$, i.e. $\hat{\mathbf{v}}_\mathbf{d}$, the following linear transformation can be made.

$$\begin{cases} \mathbf{Y_d} = \mathbf{v_d} \\ \mathbf{Y_e} = \mathbf{D}\mathbf{v_d} + \mathbf{v_e} \end{cases} \tag{A2}$$

If $\mathbf{Y} = \begin{pmatrix} \mathbf{Y_d} \\ \mathbf{Y_e} \end{pmatrix}$, then the above transformation can be written as $\mathbf{Y} = \begin{pmatrix} \mathbf{I_m} & \mathbf{0} \\ \mathbf{D} & \mathbf{I_{n-m}} \end{pmatrix} \mathbf{v}$, where $\mathbf{I_r}$ represents a unit diagonal matrix with dimension of $r \times r$. The objective is to obtain a constant matrix **D** that ensure the $\mathbf{v_e}$ and $\mathbf{v_d}$ mutually independent.

Considering that **Y** is the linear transformation of **v**, **Y** follows normal distribution too. If $\mathbf{v_e}$ and $\mathbf{v_d}$ are mutually independent, it means that $\mathbf{E}\left[(\mathbf{Y_d} - \mathbf{E}(\mathbf{Y_d}))(\mathbf{Y_e} - \mathbf{E}(\mathbf{Y_e}))^T\right] = \mathbf{0}$.

$$\begin{aligned} &\mathbf{E}\left[(\mathbf{Y_d} - \mathbf{E}(\mathbf{Y_d}))(\mathbf{Y_e} - \mathbf{E}(\mathbf{Y_e}))^T\right] \\ &= \mathbf{E}\left[(\mathbf{v_d} - \mathbf{E}(\mathbf{v_d}))(\mathbf{D}\mathbf{v_d} + \mathbf{v_e} - \mathbf{D}\mathbf{E}(\mathbf{v_d}) - \mathbf{E}(\mathbf{v_e}))^\mathbf{T}\right] \end{aligned}$$

$$= \mathbf{E}\left[(\mathbf{v}_d - \mathbf{E}(\mathbf{v}_d))(\mathbf{D}\mathbf{v}_d - \mathbf{D}\mathbf{E}(\mathbf{v}_d))^T\right] + \mathbf{E}\left[(\mathbf{v}_d - \mathbf{E}(\mathbf{v}_d))(\mathbf{v}_e - \mathbf{E}(\mathbf{v}_e))^T\right]$$
$$= \mathbf{E}\left[(\mathbf{v}_d - \mathbf{E}(\mathbf{v}_d))(\mathbf{v}_d - \mathbf{E}(\mathbf{v}_d))^T\mathbf{D}^T\right] + \mathbf{E}\left[(\mathbf{v}_d - \mathbf{E}(\mathbf{v}_d))(\mathbf{v}_e - \mathbf{E}(\mathbf{v}_e))^T\right]$$
$$= \mathbf{B}_{11}\mathbf{D}^T + \mathbf{B}_{12} \qquad \text{(A3)}$$

Let it equal to zero, we can get $\mathbf{D}^T = -\mathbf{B}_{11}^{-1}\mathbf{B}_{12}$. Therefore, $\mathbf{D} = -\mathbf{B}_{21}\mathbf{B}_{11}^{-1}$.

$$\mathbf{E}(\mathbf{Y}) = \mathbf{E}\begin{pmatrix}\mathbf{Y}_d\\ \mathbf{Y}_e\end{pmatrix} = \begin{pmatrix}\mathbf{E}(\mathbf{Y}_d)\\ \mathbf{E}(\mathbf{Y}_e)\end{pmatrix} = \begin{pmatrix}\mathbf{I}_m & \mathbf{0}\\ -\mathbf{B}_{21}\mathbf{B}_{11}^{-1} & \mathbf{I}_{n-m}\end{pmatrix}\begin{pmatrix}\mathbf{E}(\mathbf{v}_d)\\ \mathbf{E}(\mathbf{v}_e)\end{pmatrix} = \begin{pmatrix}\bar{\mathbf{v}}_d\\ -\mathbf{B}_{21}\mathbf{B}_{11}^{-1}\bar{\mathbf{v}}_d + \bar{\mathbf{v}}_e\end{pmatrix} \qquad \text{(A4)}$$

$$\operatorname{cov}\begin{pmatrix}\mathbf{Y}_d\\ \mathbf{Y}_e\end{pmatrix} = \begin{pmatrix}\mathbf{I}_m & \mathbf{0}\\ -\mathbf{B}_{21}\mathbf{B}_{11}^{-1} & \mathbf{I}_{n-m}\end{pmatrix}\operatorname{cov}\begin{pmatrix}\mathbf{v}_d\\ \mathbf{v}_e\end{pmatrix}\begin{pmatrix}\mathbf{I}_m & \mathbf{0}\\ -\mathbf{B}_{21}\mathbf{B}_{11}^{-1} & \mathbf{I}_{n-m}\end{pmatrix}^T$$
$$= \begin{pmatrix}\mathbf{I}_m & \mathbf{0}\\ -\mathbf{B}_{21}\mathbf{B}_{11}^{-1} & \mathbf{I}_{n-m}\end{pmatrix}\begin{pmatrix}\mathbf{B}_{11} & \mathbf{B}_{12}\\ \mathbf{B}_{21} & \mathbf{B}_{22}\end{pmatrix}\begin{pmatrix}\mathbf{I}_m & -\mathbf{B}_{11}^{-1}\mathbf{B}_{12}\\ \mathbf{0} & \mathbf{I}_{n-m}\end{pmatrix} = \begin{pmatrix}\mathbf{B}_{11} & \mathbf{0}\\ \mathbf{0} & \mathbf{B}_{22} - \mathbf{B}_{21}\mathbf{B}_{11}^{-1}\mathbf{B}_{12}\end{pmatrix} \qquad \text{(A5)}$$

As $\mathbf{Y}_d$ and $\mathbf{Y}_e$ are mutually independent, the distribution of $\mathbf{Y}_e$ under the condition that $\mathbf{Y}_d = \mathbf{y}_d$ for any $\mathbf{y}_d$ is the same as the distribution of $\mathbf{Y}_e$ without any condition. The distribution of $\mathbf{Y}_e$ follows

$$MVN_{n-m}\left(-\mathbf{B}_{21}\mathbf{B}_{11}^{-1}\bar{\mathbf{v}}_d + \bar{\mathbf{v}}_e, \mathbf{B}_{22} - \mathbf{B}_{21}\mathbf{B}_{11}^{-1}\mathbf{B}_{12}\right). \qquad \text{(A6)}$$

Considering that $\mathbf{Y}_d = \mathbf{v}_d$ and $\mathbf{Y}_e = -\mathbf{B}_{21}\mathbf{B}_{11}^{-1}\mathbf{v}_d + \mathbf{v}_e$, the distribution of $-\mathbf{B}_{21}\mathbf{B}_{11}^{-1}\mathbf{v}_d + \mathbf{v}_e$ under the condition that $\mathbf{v}_d = \tilde{\mathbf{v}}_d$ is $MVN_{n-m}\left(-\mathbf{B}_{21}\mathbf{B}_{11}^{-1}\bar{\mathbf{v}}_d + \bar{\mathbf{v}}_e, \mathbf{B}_{22} - \mathbf{B}_{21}\mathbf{B}_{11}^{-1}\mathbf{B}_{12}\right)$. So the distribution of $\mathbf{v}_e$ under the condition that $\mathbf{v}_d = \tilde{\mathbf{v}}_d$ can be written as

$$MVN_{n-m}\left(\mathbf{B}_{21}\mathbf{B}_{11}^{-1}\tilde{\mathbf{v}}_d - \mathbf{B}_{21}\mathbf{B}_{11}^{-1}\bar{\mathbf{v}}_d + \bar{\mathbf{v}}_e, \mathbf{B}_{22} - \mathbf{B}_{21}\mathbf{B}_{11}^{-1}\mathbf{B}_{12}\right) \text{ or} \qquad \text{(A7)}$$
$$MVN_{n-m}\left(\bar{\mathbf{v}}_e + \mathbf{B}_{21}\mathbf{B}_{11}^{-1}(\tilde{\mathbf{v}}_d - \bar{\mathbf{v}}_d), \mathbf{B}_{22} - \mathbf{B}_{21}\mathbf{B}_{11}^{-1}\mathbf{B}_{12}\right). \qquad \text{(A8)}$$