A PROGRESSION SIGNAL OPTIMIZATION MODEL REFLECTING LANE USE ALLOCATIONS AT INTERSECTIONS ON AN ARTERIAL

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Abstract: Most progression signal timing models just have handled signal timing variables and left turn treatments under a specified lane use allocation. Therefore this paper proposes a signal optimization model for a coordinated arterial that considers lane use allocations as well as traffic conditions and signal timing plans. The optimization solutions show that the proposed model increases through bandwidths as well as objective functions for all tested cases, compared with those of the model that does not optimize lane use allocations. Simulation results also show that the model is able to reduce delay at transportation network. The more the traffic volume increases, the larger the effect shows. It is noticed that the effects are superior in cases with restricted capacity for traffic demands, to those with reserved capacity. Since the proposed model can optimize left turn phase sequences, combined usage of this model and TRANSYT-7F is recommend significantly both to make the best use of the capacity for turn volumes at intersections.

Key Words: progression signal timing plans, lane use allocation, mixed integer linear programming, time-space diagram, through bandwidth

2. INTRODUCTION

The capacity and level of service of signalized intersections mainly depend upon geometric conditions as well as signal timing plans. The factors are so interactive that variation of one may influence others. Lane use allocation would be the primary variable among the geometric conditions which are the number of lanes of an approach entering an intersection, the lane use allocation for movements on the approach, and the number of lanes on a leg exiting from the intersection for each movement. The lane use allocation needs to be adjusted to meet demand volumes for each movement. Most optimization models of progression signal-timing plans have handled only signal timing variables and left turn treatments under the specified lane use allocation. Since the lane use allocation, therefore, has not been considered as the decision variable in the models but prescribed as it was, the solution would not be the best signal plan. To obtain the best signal plan in terms of appropriate capacity, the geometric conditions have to be included in the decision variables in the optimizing procedure of the models. The optimal selection of lane use allocations is dependent upon the conditions of turning volumes and the geometric variables such as the number of lanes both on departure approach and on arrival one for movements.

This paper develops a method of mixed integer linear programming (MILP) for a progression signal optimization plan that considers geometric conditions as well as traffic and signal operating conditions. Haruo Ozaki (1991) discussed a method of analyzing a signalized intersection that considers prevailing geometric, traffic, and signal operating conditions simultaneously. Based upon or modifying the method by Haruo Ozaki, the new methodology is developed to optimize simultaneously the three variables and extended to a coordinated arterial. This model for lane use allocation is formulated by a binary integer variable for lane use and the demand volume ratio variable on each lane satisfying demand volume for each movement. Unlike traditional bandwidth models, the proposed model maximizes single or multi weighted sum of bandwidths multiplied by the number of through lanes at intersections in the both directions on an arterial. The proposed model optimizes simultaneously both progression signal timing plans (cycle, green split, offset, phase sequences) and lane use allocations along a coordinated arterial.

2. MODEL FORMULATION

2.1 Lane Use Model

There are three movements on an approach; right turn traffic, left turn traffic, and through traffic. The layouts for approaches and movements are defined in Figure 1.

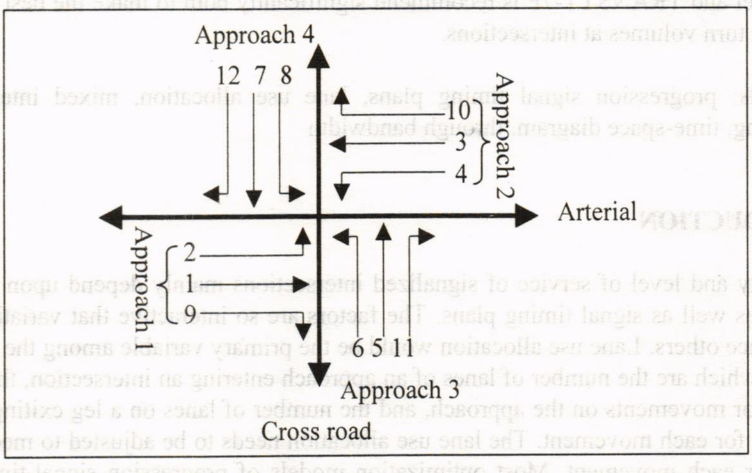

Figure 1. Definition for Approaches and Movements at An Intersection

Each movement on an approach uses some of lanes among the limited number of lanes. The number of lanes for a movement should be equal to or less than the number of lanes on a leg receiving the movement. In case that a movement uses more than one lane, it is assumed that flow ratios of the movement on the lanes are equal. If through and left turn traffic shares one lane, flow rates of all lanes are supposed to be equal on an approach. It is also assumed that through traffic include right turn traffic. Lane grouping layout for movements on an approach, which are through movement with right turn traffic and left turn movement, is depicted in Figure 2. It is assumed that the approach has $NL+NT+1$ lanes, where NL lanes and NT lanes are reserved for left turn and through movement respectively, as shown in Figure 2.

Consequently, the usage of one lane (the 2nd lane in Figure 2) remains undecided. The problem is how this one lane should be assigned.

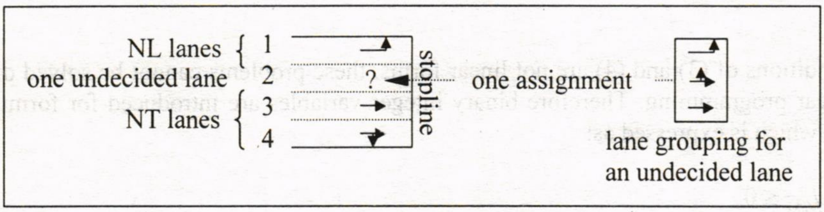

Figure 2. Layout for lane allocation on an approach

The following variables are defined to describe the lane use allocation for a movement i, which enters an intersection from approach j, as follows:

N_{kx} = total number of entrance lanes on approach k at intersection x (N_{kx} = $NL+NT+1$ in Figure 2)

N_{ikx} = total number of movements on approach k at intersection x

K_{ijkx} = proportion of demand volume for movement i which uses j-th lane on approach k at intersection x

L_{ijkx} = binary integer, which indicates the use or not of j-th lane on approach k at intersection x by movement i, to meet $\sum_{j=1}^{N_{kx}} L_{ijkx} \leq N_{iokx}$.

where:

N_{iokx} = number of lanes on a leg, which is exit approach, receiving movement i;

$$L_{ijkx} = 0, \text{ if } K_{ijkx} = 0; \text{ and} \tag{1}$$

$$L_{ijkx} = 1, \text{ if } K_{ijkx} > 0. \tag{2}$$

In order to meet the demand volume of the movement i:

$$\sum_{j=1}^{N_{kx}} K_{ijkx} = 1 \tag{3}$$

If the reserve capacity should be evaluated, the equation above should be slightly modified as follows.

$$\sum_{j=1}^{N_{kx}} K_{ijkx} = u \tag{4}$$

where, u is an adjustment factor for designed demand volume rate of each movement, usually greater than 1.

Lane usage of j-th lane on approach k at intersection x by movement i should be expressed as follows:

$$\sum_{i=1}^{N_{ikx}} L_{ijkx} = 1\text{, if lane is used exclusively,} \tag{5}$$

$$\sum_{i=1}^{N_{ikx}} L_{ijkx} > 1\text{, if lane is shared lane.} \tag{6}$$

Since conditions of (3) and (4) are not linear forms, these problems cannot be solved directly using linear programming. Therefore binary integer variables are introduced for formulation of MILP, which is expressed as:

$$K_{ijkx} - L_{ijkx} \leq 0 \tag{7}$$

$$B \times K_{ijkx} - L_{ijkx} \geq 0 \tag{8}$$

where, B is sufficiently large such that $B \times K_{ijkx} \geq 1$ is redundant with respect to any active constraint.

K_{ijkx} can be calibrated in two ways. Haruo Ozaki (1991) treated K_{ijkx} as a decision variable defined by (2). In this method, there may be big differences among flow rates of the lanes used by identical movement. There is also an irrelevant assumption that traffic dissipates into saturation flow rate all over green interval for all movements.

In order to cope with problems above, K_{ijkx} is calibrated according to lane use allocation of an undecided lane in this paper. If the undecided lane is exclusive lane, flow rates of the lanes used by identical movement are assumed to be equal. If the undecided lane is shared lane used by through and left turn traffic, flow rates of all lanes are supposed to be equal on an approach. The lane use allocation for an undecided lane depends upon signal conditions as well as volumes of through and left movements. The flow ratios of the movements, depending upon the lane use allocations for the undecided lane, are calibrated as one of the following three cases:

Let us define the 2nd lane as the undecided lane, as shown in Figure 2, then the cases are expressed by:

a. Exclusive lane use allocation for the through movement

Number of allocated lanes for the through movement is $NT+1$, and that for the left turn movement is NL, then:

$$K_{i(j=1)kx,1} = \frac{1}{NL}\text{, where } i = 2, 4, 6, 8 \text{ for left turn movement;} \tag{9}$$

$$\sum_{j=1}^{N_k} K_{i(j\geq 2)kx,1} = \frac{1}{NT+1}\text{, where } i = 1, 3, 5, 7 \text{ for through movement;} \tag{10}$$

b. Exclusive lane use allocation for the left turn movement

Number of allocated lanes for the through movement is *NT*, and for the left turn movement is *NL+1*, then:

$$K_{i(j=1)kx,2} = \frac{1}{NL+1}, \text{ where } i = 2, 4, 6, 8 \text{ for left turn movement;} \quad (11)$$

$$K_{i(j=2)kx,2} = \frac{1}{NL+1}, \text{ where } i = 2, 4, 6, 8 \text{ for left turn movement;} \quad (12)$$

$$\sum_{j=1}^{N_k} K_{i(j\geq 3)kx,2} = \frac{1}{NT}, \text{ where } i = 1, 3, 5, 7 \text{ for through movement;} \quad (13)$$

c. Shared lane use allocation for the through and left turn movement

One characteristic of shared lane depends upon how green interval is indicated to sharing movements. In order to avoid blocking effects on each other, sharing movements are supposed to be grouped and given identical green intervals. Therefore the traffic flow volumes for all lanes are assumed to be equal. When movement *i1* and movement *i2* share the *j*-th lane on an approach *k*, $K_{ijkx,3}$ are:

$$K_{(i2)(j=1)kx,3} = \frac{V_{1kx} + V_{2kx}}{(NT+2)V_{2kx}} \quad (14)$$

$$K_{(i2)(j=2)kx,3} = \frac{(NT+1)V_{2kx} - V_{1kx}}{(NT+2)V_{2kx}} \quad (15)$$

$$K_{(i1)(j=2)kx,3} = \frac{2V_{1x} - NT \times V_{2kx}}{(NT+2)V_{1kx}} \quad (16)$$

$$K_{(i1)(j\geq 3)kx,3} = \frac{V_{1kx} + V_{2kx}}{(NT+2)V_{1kx}} \quad (17)$$

where, V_{ikx} is traffic volume of movement *i* on approach *k* at intersection *x*, and K_{ijkx} is nonnegative. Therefore shared lane use allocation for the undecided lane has to be excluded, if K_{ijkx} is negative. (14)-(17) are derived from constraints to indicate identical green for both movements in Appendix 1.

L_{ijkx} for an undecided lane depends upon the lane use allocation for the lane. The linear forms are shown in Appendix 2.

2.2 Signal Timing Model

All movements are supposed to depart and clear an intersection during green interval under under-saturated traffic flow. It is assumed that the departure traffic flows within one cycle be modeled as two parts with saturation flow departure (s_{ijkx}) and arrival flow departure ($K_{ijkx} \times V_{ikx}$) as shown in Figure 3.

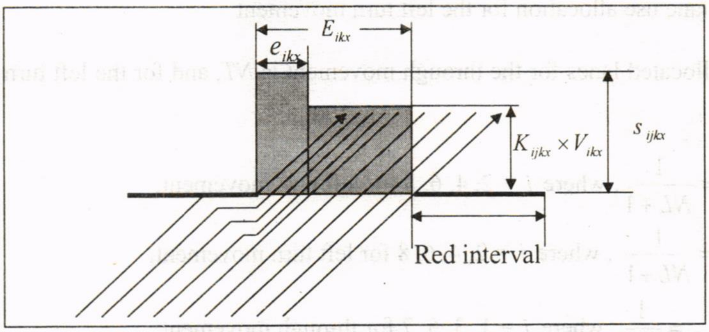

Figure 3. Departure Flow Model for All Movements

The following variables are defined in the Figure 3:
E_{ikx} = green interval for movement i on approach k at intersection x, normalized by cycle length,
e_{ikx} = green interval for movement i with saturation flow departure on approach k at intersection x, normalized by cycle length.

In the arterial with well-designed progression signal plans, it could be said that queues on through lanes built during red time would be inflows (Q_{kx}) from right turning and left turning during one cycle length at upstream intersection. From the premise, e_{ikx} of through movement is equal to or greater than clearance time for the queues. The condition is explained by:

$$e_{ikx} \geq \frac{Q_{kx}}{3{,}600 \times NT_{ikx}} \times sh_{ikx} \tag{18}$$

where:
sh_{ikx} = the saturation flow headway of movement i on approach k at an intersection x
NT_{ikx} = number of lanes for movement i on approach k at intersection x
NT_{ikx} and e_{ikx} depends upon the lane use allocation for an undecided lane, linear forms meeting the conditions for them are shown in Appendix 3.

Green interval for movement i has to be long enough to meet the distributed volume for each lane used by the movement. Based upon this constraint, green interval for movement i is calibrated by:
a. Exclusive lane use allocation for through movement
- Green interval for through movement on exclusive through lane

$$K_{ijkx,1} \times V_{ikx} \leq s_{ijkx} \times e_{ikx} + K_{ijkx,1} \times V_{ikx}(E_{ikx} - e_{ikx}), \text{ where, i =1, 3, 5 or 7.} \tag{19}$$

- Green interval for left turn movement on exclusive left turn lane

$$K_{ijkx,1} \times V_{ikx} \leq s_{ijkx} \times e_{ikx} + K_{ijkx,1} \times V_{ikx}(E_{ikx} - e_{ikx}), \text{ where, i =2, 4, 6 or 8.} \tag{20}$$

b. Exclusive lane use allocation for left turn movement
- Green interval for through movement on exclusive through lane

$$K_{ijkx,2} \times V_{ikx} \le s_{ijkx} \times e_{ikx} + K_{ijkx,2} \times V_{ikx}(E_{ikx} - e_{ikx}), \text{ where, i =1, 3, 5 or 7.} \tag{21}$$

- Green interval for left turn movement on exclusive left turn lane

$$K_{ijkx,2} \times V_{ikx} \le s_{ijkx} \times e_{ikx} + K_{ijkx,2} \times V_{ikx}(E_{ikx} - e_{ikx}), \text{ where, i =2, 4, 6 or 8.} \tag{22}$$

c. Shared lane use allocation for the through movement (*i*) and left turn movement (*j*)
- Green interval for through and left turn movement on all lanes

$$K_{ijkx,3} \times (V_{ikx} + V_{jkx}) \le s_{ijkx} \times e_{ikx} + K_{ijkx,3} \times (V_{ikx} + V_{jkx})(E_{ikx} - e_{ikx}), \text{ where, i = 1, 2, 3, 4, 5, 6, 7, or 8.} \tag{23}$$

Since conditions above for three cases are not linear forms, these problems cannot be solved directly using MILP. The equivalent linear forms for them are presented in Appendix 4.

2.3 Progression Control Model

The time-space diagram for a coordinated arterial of this model follows that of MAXBAND (Little & Kelson, 1980) shown in Figure 4. All time variables are in units of the cycle length. In the MAXBAND model, the queue clearance time (τ) is specified by user, but the time are calibrated automatically as saturation flow departure time (e_{ikx}) in this model.

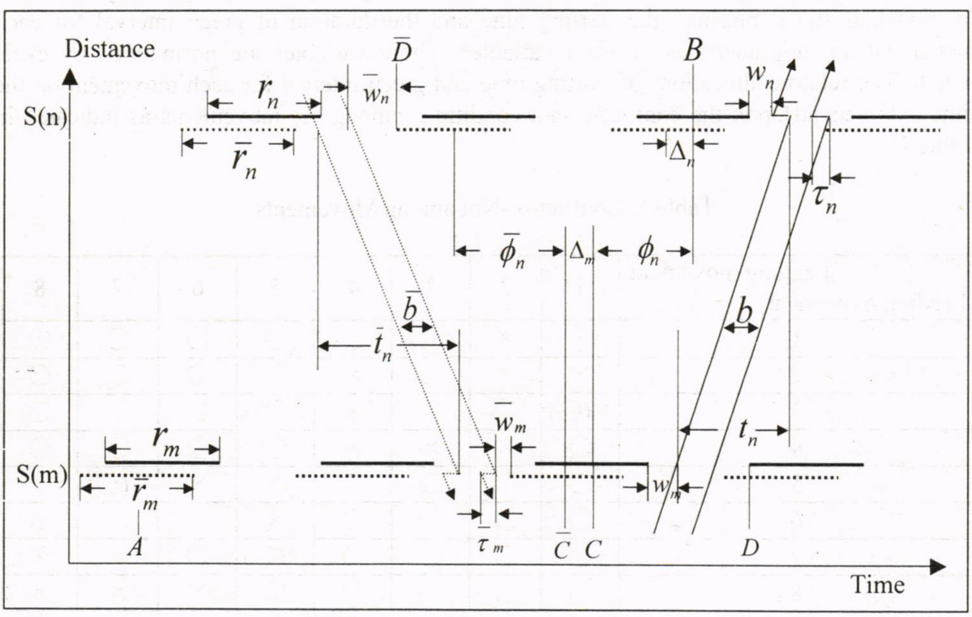

Figure 4. Basic Time-Space diagram

The following variables are defined with outbound reds drawn as solid and inbound reds as dashed lines:
b = outbound through bandwidth
$\bar{b}$ = inbound through bandwidth
w_n = interval between ending point of outbound red time and starting point of outbound through bandwidth
$\bar{w}_n$ = interval between starting point of inbound red time and ending point of inbound through bandwidth
$t_n[(\bar{t}_n)]$ = outbound [inbound] travel time from $S(m)[(S(n)]$ to $S(n)[(S(m)]$
$\phi_n[(\bar{\phi}_n)]$ = time from center of an outbound(inbound) red at $S(m)[(S(n)]$ to the center of a particular outbound [inbound] red at $S(n)[(S(m)]$
Δ_n = time from center of $\bar{r}_n$ to nearest center of r_n

The loop equation derived from Figure 4 is as follows:

$$\Delta_n - \Delta_m + t_n + \bar{t}_n + w_m + \bar{w}_m - w_n - \bar{w}_n + 0.5r_m + 0.5\bar{r}_m - 0.5r_n - 0.5\bar{r}_n - e_{(i=1)n} - e_{(i=3)m} = m_n \quad (24)$$

where, m_n is an integer variable.
The detailed procedure for deriving (24) is recommended to refer MAXBAND (Little & Kelson, 1980).

2.4 Signal Phasing Model

It is assumed that the green interval of each movement is given once during one cycle length. In modeling signal phasing, the starting time and the duration of green interval for each movement are introduced as decision variables. These variables are normalized by cycle length. The relative allocations of starting time and green interval for each movement on the time axis depend upon the conflict-or-not conditions among the movements as indicated in Table 1.

Table 1. Conflict-or-Not among Movements

Leading movement \ Lagging movement	1	2	3	4	5	6	7	8
1	×	×	×(○)	○	○	○	○	○
2	×	×	○	×	○	○	○	○
3	×(○)	○	×	×	○	○	○	○
4.	○	×	×	×	○	○	○	○
5	○	○	○	○	×	×	×(○)	○
6	○	○	○	○	×	×	○	×
7	○	○	○	○	×(○)	○	×	×
8	○	○	○	○	○	×	×	×

In the Table 1, '×' means compatibility between leading movement and lagging movement, '○' does incompatibility. '( )' is the only case shared by both through and left turn traffic. The signal phasing for movement *i* and *j* is modeled as either of two pairs of clearance interval constraints of green interval suggested by Importa & Cantarella (1984) as follows:

$$S_{ikx} + E_{ikx} + Z \times Y_{ijkx} \le S_{jkx} \text{ and} \tag{25}$$

$$S_{jkx} + E_{jkx} + Z \times Y_{ijkx} \le S_{ikx} + 1 \tag{26}$$

or

$$S_{jkx} + E_{jkx} + Z \times Y_{ijkx} \le S_{ikx} \text{ and} \tag{27}$$

$$S_{ikx} + E_{ikx} + Z \times Y_{ijkx} \le S_{jkx} + 1 \tag{28}$$

where,
S_{ikx} = the starting time of green interval for movement *i* on approach *k* at intersection *x*
E_{ikx} = the green interval for movement *i* on approach *k* at intersection *x*
Z = the reciprocal of cycle length (*1/C*)
Y_{ijkx} = the minimum clearance interval to be reserved between the end of green interval for movement *i* (*j*) and the start of green for movement *j* (*i*), in seconds;
$Y_{ijkx} \ge 0$, if movement *i* and movement *j* are incompatible, otherwise, Y_{ijkx} is an arbitrary large negative value.
Since conditions of (25)-(28) are not linear forms, this problem should be transformed into following equivalent linear forms with a binary integer (α_{ijkx}) as follows:

$$S_{ikx} + E_{ikx} + Z \times Y_{ijkx} - \alpha_{ijkx} \le S_{jkx}, \text{ and} \tag{29}$$

$$S_{jkx} + E_{jkx} + Z \times Y_{ijkx} + \alpha_{ijkx} \le S_{ikx} + 1 \tag{30}$$

In the constraints of (29)-(30), there are 28 integer variables per intersection with four legs. In order to reduce the number of integer variables that increase the searching time for a optimal solution, we included all movements on cross streets into a common red time (R_x) for main arterial as follows:

$$S_{ikx} \ge S_{rx} \text{ and} \tag{31}$$

$$S_{ikx} + E_{ikx} \le S_{rx} + R_x, \text{ where, i =5, 6, 7 or 8 for movements on cross streets.} \tag{32}$$

where,
S_{rx} = the starting time of the common red time
S_{ikx} = the starting time for movements on cross streets
E_{ikx} = the green interval for movements on cross streets
All movements on main arterial are incompatible with the common red interval. In order to meet the condition, additional phasing constraints for movements on an arterial should be added as follows:

$$S_{ikx} + E_{ikx} + Z \times Y_{irx} - \alpha_{irx} \le S_{rx} \text{ and} \tag{33}$$

$$S_{rx} + R_x + Z \times Y_{irx} + \alpha_{irx} \le S_{ikx} + 1, \text{ where, i = 1, 2, 3 or 4.} \tag{34}$$

The undecided lane should be allocated most efficiently for traffic flow. The use allocation for a lane depends upon volumes of all movements. The characteristics of a shared lane depend upon how green interval is indicated to sharing movements. Here, sharing movements are supposed to be grouped and given identical green interval in order to avoid blocking effects on each other. When movement *i1* and movement *i2* share the *j*-th lane on an approach *k*, the constraints indicating identical green interval to the movements are described as:

$$S_{(i1)x} - S_{(i2)x} + L_{(i1)jkx} + L_{(i2)jkx} \le 2 \tag{35}$$
$$S_{(i1)x} - S_{(i2)x} - L_{(i1)jkx} - L_{(i2)jkx} \ge -2 \tag{36}$$
$$E_{(i1)x} - E_{(i2)x} + L_{(i1)jkx} + L_{(i2)jkx} \le 2 \tag{37}$$
$$E_{(i1)x} - E_{(i2)x} - L_{(i1)jkx} - L_{(i2)jkx} \ge -2 \tag{38}$$

2.5 Objective Function

This paper aims at developing MILP of progression signal plans that considers lane use allocations as well as traffic and signal conditions. Signal timing plans for conventional progression bandwidth models are calibrated to maximize a single or multi weighted sum (MULTI-BAND, 1991) of bandwidths. Unlike traditional bandwidth models, the proposed model maximizes the values of objective function, which are the bandwidths multiplied by the number of through lanes on approaches along an arterial with *n* intersections, as follows:

$$\text{Maximize } \sum_{x=1}^{n} (NT_{(k=1)x} \times b + NT_{(k=2)x} \times \bar{b}) \tag{39}$$

where:
$NT_{(k=1)x}$ = the number of outbound through lanes at intersection *x*.
$NT_{(k=2)x}$ = the number of inbound through lanes at intersection *x*.

Since $NT_{(k=1)x}$ and $NT_{(k=2)x}$ are decision variables depending on lane use allocation for an decided lane, the objective function is not linear form. Linearization of (39) described in Appendix 5. Providing minimum green intervals to meet demand volume for left turns and through movements on cross streets, the objective function makes green intervals for through movements on an arterial as large as possible.

3. MODEL APPLICATION AND ASSESSMENT

The proposed model accommodates signal timing plans (cycle length, green interval, offset, and phase sequences) and lane use allocations. The primal improvement of the proposed model is the capability to optimize lane use allocations by introducing them as decision variables into a progression signal optimization model on a coordinated arterial. To evaluate the effect of the ability, the model is applied to test arterial with 3 intersections that spaced 500m as Figure 5.

The optimal selection of lane use allocation is dependent upon the conditions of turning volumes and the geometric variables such as the number of lanes both on departure approach and on arrival one for movements. For the adequate evaluation of this model, the evaluation should be performed by varying left turn volumes and lanes. For the purpose of considering conditions above, the test corridor is classified into two cases: case 1 has twice left turn volume as many as case 2. To examine effects of the number of lanes, Case 2 is divided into two cases; case 2-1 with four lanes including one left turn bay, case 2-2 with three lanes without left turn bay. In any cases, all crossroads have two through-lanes and one left-turn bay each. Saturation flow rates for each movement are as follows; 1,800 pcphpl for through lane, 1,700 pcphpl for through-right turn lane and left-turn lane. The traffic volumes for test arterial are as shown in Table 2.

Table 2. Medium Traffic Volumes on Arterial (in vph)

Intersection	Direction	Volume			
		Left		Through	Right
		Case 1	Case 2		
1	Outbound	618	309	1,030	138
	Inbound	516	258	860	114
2	Outbound	632	316	1,043	141
	Inbound	530	265	870	118
3	Outbound	648	324	953	130
	Inbound	542	271	802	100
Cross street	Both	105		189	126

Lane use layouts for intersections on the arterial are depicted in Figure 5. This figure is designed to optimize signal timing plans on the fixed geometry. In the figure, the target lanes for optimization of lane use allocation are drawn as dashed lines.

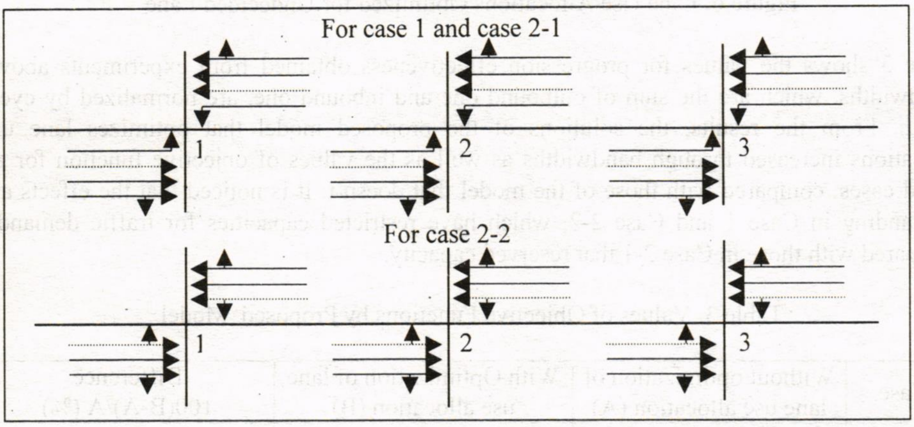

Figure 5. Lane Use Layouts in Present Arterials

For test problems above, we compare two models: one with optimization tool for lane use allocations but the other without that. The models for the experiments were performed by

MILP program used world-widely. To compare different models consistently, cycle length range and running speed were held same for all cases. The computer used in this study was a 700Mhz Pentium with 128 MB RAM. The computer running time to solve the MILP problems ranged from 14 to 20 minutes. Figure 6 shows optimal lane use allocations obtained from the model for the test corridor. In the Figure, lane use allocation for undecided lane is drawn as dashed lines. The lane use allocations are different from the fixed lane use in the Figure 5.

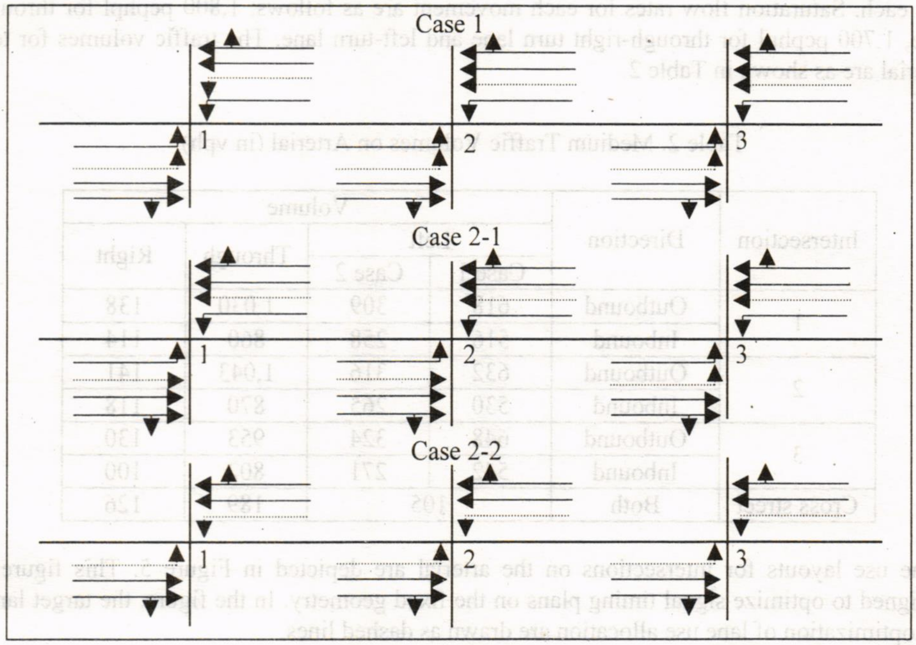

Figure 6. Lane Use Allocations Optimized for Undecided Lane

Table 3 shows the values for progression effectiveness obtained from experiments above. Bandwidths, which are the sum of outbound one and inbound one, are normalized by cycle length. From the results, the solutions of the proposed model that optimizes lane use allocations increased through bandwidths as well as the values of objective function for all tested cases, compared with those of the model that doesn't. It is noticed that the effects are outstanding in Case 1 and Case 2-2, which have restricted capacities for traffic demands, compared with those in Case 2-1 that reserved capacity.

Table 3. Values of Objective Functions by Proposed Model

Case	Without optimization of lane use allocation (A)	With Optimization of lane use allocation (B)	Difference 100(B-A)/A (%)
Case 1	5.3422	5.5530	3.9
Case 2-1	7.4422	7.4704	0.4
Case 2-2	3.7540	4.9614	32.2

To test the advantages of the proposed model over existing models without reflecting lane use allocations, average delay was used as a measure to evaluate the performances of the traffic models. To measure the effectiveness of performance on the common basis, the TRAFNETSIM network simulation package was employed to evaluate these signal-timing solutions. Due to its inherent variability, 3 simulation runs with different random number seeds were performed for each signal timing solution. It took about 85 seconds to simulate each case. The simulation period was 2 hours with 15-min periods. All simulated networks reached equilibrium during 15-min fill time prior to simulation.

Table 4 shows the results of average delay time calibrated from the simulations. Similarly to optimization results aforementioned, the proposed model optimizes lane use allocations is superior to the model that doesn't in reducing delay for all cases. It is also noticed that the effects are dazzling in some cases (Case 1 and Case 2-2) with restricted capacities for traffic demands, compared with those in ones (Case 2-1) with reserved capacity.

Table 4. Comparisons of Delay Time by Proposed Model (in sec/veh)

Case	Value	Without optimization for lane use allocation (A)	With optimization for lane use allocation (B)	Difference 100(B-A)/A (%)
Case 1	For arterial	53.4	47.0	12.0
	For system	101.3	79.2	21.8
Case 2-1	For arterial	43.97	42.53	3.3
	For system	73.01	71.88	1.5
Case 2-2	For arterial	71.6	39.5	44.8
	For system	132.6	91.3	31.1

4. CONCLUSIONS AND RECOMMANDATIONS

In this study, progression signal timing optimization model reflecting lane use allocations is developed. This model consists of a mixed integer linear optimization programming considering traffic volumes and capacities for movements, cycle length, green interval, phasing and offset to maximize the sum of directional bandwidths multiplied by the number of through lanes at intersections on the corridor. The optimization solutions show that the proposed model that optimizes lane use allocation increased through bandwidth as well as the value of objective function for all tested cases, compared with those of the model that doesn't. The more the traffic volume increases, the larger the effect comes. From the simulation results, it is found that the model is able to reduce delay at transportation network, as the optimization results are. It is noticed that the effects are outstanding in cases with restricted capacities for traffic demands, compared with those with reserved capacity for them.

Since the proposed model can optimize left turn phase sequences reflecting lane use allocation, combined usage of this model and TRANSYT-7F is recommend significantly both to make the best use of the capacity and to reduce delay for turn volumes at intersections. Therefore this model may be outstanding tool that optimize three important factors, which are traffic, signal, and geometric conditions, affecting level of service for coordinated corridor. Using objective function, maximizing the bandwidth for through movements on arterial, it is found that the proposed model assigned green splits for the movements as much as possible, satisfying the demand volumes for left turn movements and through movements on cross streets. It is also mainly due to green timing modeling. It is desirable to provide additional green intervals for the left turn movements and through ones on the cross streets to reduce the possibility of saturation or unexpected over-saturation. It can be satisfied by introduction of an adjustment factor for green splits for the movements. The factor should be determined to meet traffic conditions. The assumptions and simplifications applied in the proposed model, such as time-stationary flow rates, unsaturated traffic conditions, and queue clearance times, can be relaxed further to accommodate real-world situations

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APPENDICES

Appendix 1:

$$K_{(i2)(j=1)kx,3} + K_{(i2)(j=2)kx,3} = 1$$
$$\sum_{j=2}^{N_{kx}} K_{(i1)jkx,3} = 1$$
$$K_{(i2)(j=1)kx,3} \times V_{2kx} = K_{(i1)(j=3)kx,3} \times V_{1kx}$$
$$K_{(i2)(j=2)kx,3} \times V_{2kx} + K_{(i1)(j=2)kx,3} \times V_{1kx} = K_{(i1)(j=3)kx,3} \times V_{1kx}$$

Appendix 2:

In order to meet constraints of L_{ijkx} for an undecided lane, two binary variables (θ_{kx} & γ_{kx}) are introduced as follows:

a. L_{ijkx} for through movement;

$$L_{(i1)(j=2)kx} \geq -\theta_{kx} - \gamma_{kx} + 1$$
$$L_{(i1)(j=2)kx} \leq \theta_{kx} + \gamma_{kx} + 1$$
$$L_{(i1)(j=2)kx} \geq -\theta_{kx} + \gamma_{kx} - 1$$
$$L_{(i1)(j=2)kx} \leq \theta_{kx} - \gamma_{kx} + 1$$
$$L_{(i1)(j=2)kx} \geq \theta_{kx} - \gamma_{kx} - 1 + 1$$
$$L_{(i1)(j=2)kx} \leq -\theta_{kx} + \gamma_{kx} + 1 + 1$$
$$L_{(i1)(j=2)kx} \geq \theta_{kx} + \gamma_{kx} - 2 + 1$$
$$L_{(i1)(j=2)kx} \leq -\theta_{kx} - \gamma_{kx} + 2 + 1$$

b. L_{ijkx} for left turn movement;

$$L_{(i2)(j=2)kx} \geq -\theta_{kx} - \gamma_{kx}$$
$$L_{(i2)(j=2)kx} \leq \theta_{kx} + \gamma_{kx}$$
$$L_{(i2)(j=2)kx} \geq -\theta_{kx} + \gamma_{kx}$$
$$L_{(i2)(j=2)kx} \leq \theta_{kx} - \gamma_{kx} + 2$$
$$L_{(i2)(j=2)kx} \geq \theta_{kx} - \gamma_{kx} - 1 + 1$$
$$L_{(i2)(j=2)kx} \leq -\theta_{kx} + \gamma_{kx} + 1 + 1$$
$$L_{(i2)(j=2)kx} \geq \theta_{kx} + \gamma_{k} - 2 + 1$$
$$L_{(i2)(j=2)kx} \leq -\theta_{kx} - \gamma_{kx} + 2 + 1$$

From the constraints above, the lane use allocation for an undecided lane is calibrated by:
a. An exclusive lane use allocation for the through movement, if $\theta_{kx} = \gamma_{kx} = 0$.
b. An exclusive lane use allocation for the left turn movement, if $\theta_{kx} = 0$ and $\gamma_{kx} = 1$.
c. A shared lane use allocation for the through and left turn movement, if $\theta_{kx} = 1$.

Appendix 3:

$$e_{ikx} \geq -L \times \theta_{kx} - L \times \lambda_{kx} + \frac{Q_{kx}}{3{,}600 \times NT_{ikx,1}} \times sh_{ikx}$$

$$e_{ikx} \geq -L \times \theta_{kx} + L \times \lambda_{kx} - L + \frac{Q_{kx}}{3{,}600 \times NT_{ikx,2}} \times sh_{ikx}$$

$$e_{ikx} \geq L \times \theta_{kx} - L \times \lambda_{kx} - L + \frac{Q_{kx}}{3{,}600 \times NT_{ikx,3}} \times sh_{ikx}$$

$$e_{ikx} \geq L \times \theta_{kx} + L \times \lambda_{kx} - 2L + \frac{Q_{kx}}{3{,}600 \times NT_{ikx,3}} \times sh_{ikx}$$

where, L is a big integer, then:
a. $NT_{ikx,1}$ ($NT_{ikx,1} = NT + 1$ in the case of figure 2), if $\theta_{kx} = \gamma_{kx} = 0$,

b. $NT_{ikx,2}$ ($NT_{ikx,2} = NT$ in the case of figure 2), if $\theta_{kx} = 0$ and $\gamma_{kx} = 1$,
c & d. $NT_{ikx,3}$ ($NT_{ikx,3} = NT + NL + 1$ in the case of figure 2), if $\theta_{kx} = 1$.

appendix 4:

$$K_{ijkx,1} \times V_{ikx} \le s_{ijkx} \times e_{ikx} + K_{ijkx,1} \times V_{ikx}(E_{ikx} - e_{ikx}) + L \times \theta_{kx} + L \times \gamma_{kx}$$
$$K_{ijkx,2} \times V_{ikx} \le s_{ijkx} \times e_{ikx} + K_{ijkx,2} \times V_{ikx}(E_{ikx} - e_{ikx}) + L \times \theta_{kx} - L \times \gamma_{kx} + L \times 1$$
$$K_{ijkx,3} \times (V_{ikx} + V_{jkx}) \le s_{ijkx} \times e_{ikx} + K_{ijkx,3} \times (V_{ikx} + V_{jkx})(E_{ikx} - e_{ikx}) - L \times \theta_{kx} + L \times \gamma_{kx} + L \times 1$$
$$K_{ijkx,3} \times (V_{ikx} + V_{jkx}) \le s_{ijkx} \times e_{ikx} + K_{ijkx,3} \times (V_{ikx} + V_{jkx})(E_{ikx} - e_{ikx}) - L \times \theta_{kx} - L \times \gamma_{kx} + L \times 2$$

Where,
a. An exclusive lane use allocation for the through movement, if $\theta_{kx} = \gamma_{kx} = 0$,
b. An exclusive lane use allocation for the left turn movement, if $\theta_{kx} = 0$ and $\gamma_{kx} = 1$,
b. A shared lane use allocation for the through and left turn movement, if $\theta_{kx} = 1$.

Appendix 5:

In order to transform the objective function into linear one, the decision variables for the objective function should be replaced by variables (B for outbound and $\overline{B}$ for inbound) as follows:

Maximize $\sum_{x=1}^{n}(B + \overline{B})$

B is calibrated by:

$$B \ge (NT_{(k=1)x} + 1)b - \theta_{kx} - \gamma_{kx},$$
$$B \le (NT_{(k=1)x} + 1)b + \theta_{kx} + \gamma_{kx},$$
$$B \ge NT_{(k=1)x} \times b - \theta_{kx} + \gamma_{kx} - 1,$$
$$B \le NT_{(k=1)x} \times b + \theta_{kx} - \gamma_{kx} + 1,$$
$$B \ge (NT_{(k=1)x} + \beta_{kx})b + \theta_{kx} - \gamma_{kx} - 1,$$
$$B \le (NT_{(k=1)x} + \beta_{kx})b - \theta_{kx} + \gamma_{kx} + 1,$$
$$B \ge (NT_{(k=1)x} + \beta_{kx})b + \theta_{kx} + \gamma_{kx} - 2,$$
$$B \le (NT_{(k=1)x} + \beta_{kx})b - \theta_{kx} - \gamma_{kx} + 2,$$

where:

$\beta_{kx} = \dfrac{K_{(i1)jkx} \times V_{(i1)x}}{K_{(i1)jkx} \times V_{(i1)x} + K_{(i2)jkx} \times V_{(i2)x}}$, only for shared lane use allocation for an undecided lane;

where, $i1$ is a through movement and $i2$ is a left turn movement. $\overline{B}$ is obtained by substituting B and $NT_{(k=1)x}$ with $\overline{B}$ and $NT_{(k=2)x}$ respectively in the equations above.