MODELLING NETWORK VULNERABILITY AT THE LEVEL OF THE NATIONAL STRATEGIC TRANSPORT NETWORK

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Abstract: Transport network reliability has been the topic of a substantial body of recent research. This research has mostly focused on congested urban road networks and the probability that the network will deliver a required standard of performance. At the national level and outside the major urban centres, accessibility, regional coverage and inter-urban connectivity are the primary considerations. In these sparse networks, vulnerability of the network is more important than 'reliability' because of potentially severe consequences to transport services if specific links are cut. This paper reviews previous research on network reliability, and discusses extensions and adaptations to the reliability concepts that are more appropriate for strategic-level multi-modal transport systems. It also discusses techniques for identifying specific 'weak spots' in a network, where failure of some part of the transport infrastructure would have the most serious effects on access to specific locations and overall system performance.

Key Words: Network, Reliability, Modelling

1. BACKGROUND

Transport networks form the pathways for movement of goods and people. They allow convenient access to work and leisure in urban areas; they underpin the national and urban economy; and in rural and remote areas they are vital lifelines connecting isolated communities to essential services. Unfortunately transport networks are not 100 per cent reliable. Their performance can be degraded by freak events (such as earthquakes, blizzards and floods); or by incidents (including severe traffic crashes, special events and construction works); or simply by day-to-day congestion. These disruptions cause delays and detours with significant social, economic and environmental consequences. For critical components in other types of systems, redundancy is often used as a way of reducing the risk of system failure. For various reasons, this is not generally the case for transport networks. A more common response to falling performance or perceived risk is to upgrade key links by adding more capacity, but this simply makes the network more vulnerable to failure of those key links.

The importance of the adverse impacts of network degradation has stimulated substantial international research interest in transport network reliability, that is, the ability of degraded transport networks to cope with travel demand. As discussed in Section 2 of this paper, most of the recent research effort has focused on the reliability of urban passenger transport networks, in terms of the probability that the network will deliver a required standard of performance. This situation is characterised by high levels of congestion, a dense road network, and quantifiable probability of degradation of the network. Outside major urban centres, the situation is very different. In this case, the dominant consideration in transport network infrastructure provision is accessibility - linking urban centres, providing regional coverage, and basic levels of accessibility for the non-urban community and economy. The network is sparse; congestion is not a significant issue; travel distances and times are much longer; and access to essential community services and to markets is the major driving force underlying network development. In this context, the **vulnerability** of the network is perhaps more important than 'reliability'.

In broad terms, a network can be described as vulnerable if degradation of a small number of links (possibly a single link) has severe adverse consequences for accessibility between specific locations. For example, the road links to certain remote rural communities in Australia are occasionally cut by flood. The frequency of occurrence is low and the number of people affected is small, nevertheless the impact on these persons is substantial. The amenity of these communities is highly vulnerable to network failure. The purpose of analysing network vulnerability is to anticipate points of weakness where the network is highly vulnerable and network failures will have substantial adverse effects; and then to suggest remedial measures such as adding links to the network to make it more robust.

This paper develops the concept of network **vulnerability**. It begins by reviewing the current state of research into network reliability, then proposes extensions and adaptations to the reliability concepts that are more appropriate for strategic-level multi-modal transport systems. Several alternative definitions for vulnerability are proposed. The paper also discusses the development of algorithmic and visualisation tools that may be used to identify specific 'weak spots' in a network, where failure of some part of the transport infrastructure would have the most serious effects on access to specific locations and on overall system performance. Finally, the paper describes potential applications of network vulnerability concepts, and proposes directions for further research.

2. NETWORK RELIABILITY

Road network reliability became an important research topic in transport planning during the 1990s, although some elements have been the subject of research interest for some time before that. The Kobe earthquake of 1995 and its aftermath stimulated an interest in connectivity reliability. This is the probability that a pair of nodes in a network remain connected (that is, there continues to exist a connected path between them) when one or more links in the network have been cut. Bell and Iida (1997, pp.179-185) provided an analytical procedure for assessing connectivity reliability, and a summary of the procedure is given in Iida (1999).

Subsequent research was directed at degraded networks, usually urban road networks subject to traffic congestion, in which the network remained physically intact but the performance of one or more links could be so severely affected by congestion that their use by traffic is curtailed. This has led to the definition of two additional forms of reliability: travel time reliability and capacity reliability.

Travel time reliability considers the probability that a trip between an origin-destination pair can be completed successfully within a specified time interval (Bell and Iida, 1997, pp.191-192). This can be affected by fluctuating link flows and imperfect knowledge of drivers when making route choice decisions (Lam and Xu, 2000). One measure of link travel time variability is the coefficient of variation of the distribution of individual travel times (Asakura and Kashiwadani, 1991). The general assumptions are that this distribution is normal and that the travel times on adjacent links are statistically independent (Iida, 1999). Earlier empirical research (Richardson and Taylor, 1978, Taylor, 1982) supported these assumptions, although with the qualification that in uncongested networks the distribution of individual travel times may be positively skewed and is better represented by a log-normal distribution. Measures of travel time variability are useful in assessing network performance in terms of service quality provided to travellers on a day-to-day basis (Yang *et al*, 2000). Thus travel time variability can be seen as a measure of demand satisfaction under congested conditions (Asakura, 1999).

A supply-side measure of network performance in congested networks is capacity reliability, introduced by Chen *et al* (1999) and applied by Yang *et al* (2000). Capacity reliability is defined as the probability that a network can successfully accommodate a given level of travel demand. The network may be in its normal state or in a degraded stated (say due to incidents). Chen *et al* (1999) defined this probability as equal to the probability that the reserve capacity of the network is greater than or equal to the required demand for a given capacity loss due to degradation. Yang *et al* (2000) indicated that capacity reliability and travel time reliability together could provide a valuable network design tool.

Further research on network reliability is being pursued to develop this design tool. In addition, there is a need for further research to properly specify travellers' responses to uncertainty (Bonsall, 2000) so that reliability research can be used to properly inform developments of new driver information systems and to influence the design of new traffic control systems. Taylor (2000) has suggested that network reliability concepts can be applied in the design and evaluation of traffic calming schemes.

3. NETWORK VULNERABILITY

Standard approaches to network reliability focus on network connectivity and travel time and capacity reliability. This provides valuable insights into certain aspects of network performance but reliability arguments based on probabilities and absolute connectivity may not adequately diagnose potential network problems. Consider the example of road transport connections between Perth and Adelaide in Australia. This is a major trade and tourism link. Figure 1 shows the preferred route from Perth to Adelaide superimposed on the major road network of Australia. Figure 2 shows the best alternative route if the preferred route is cut at the point marked with a cross.

As shown in Figure 2, there is an alternative route so the network is still connected. Further, available records indicate that the probability that the preferred route is cut by flood or other cause is extremely small, so the travel time reliability is high. In addition, all of the diverted traffic can be easily accommodated on the alternative route. Therefore standard measures of network reliability would not indicate any major problem with the network. However the consequence of the failure of one or more specific links of the preferred route is a detour of some 6,000 kilometres. Clearly, the transport link between Perth and Adelaide is vulnerable to link failure and the consequences are substantial.

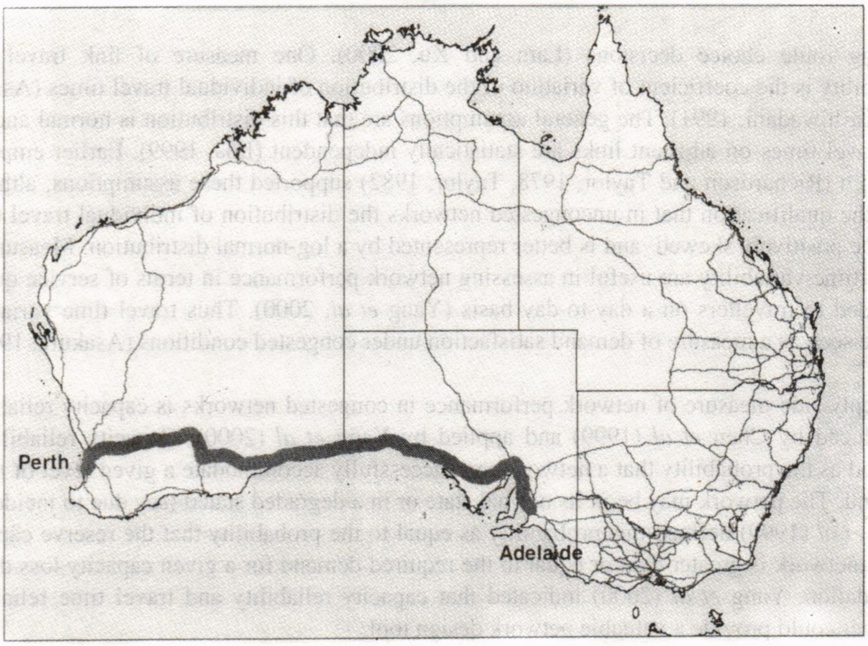

Figure 1: Preferred route from Perth to Adelaide

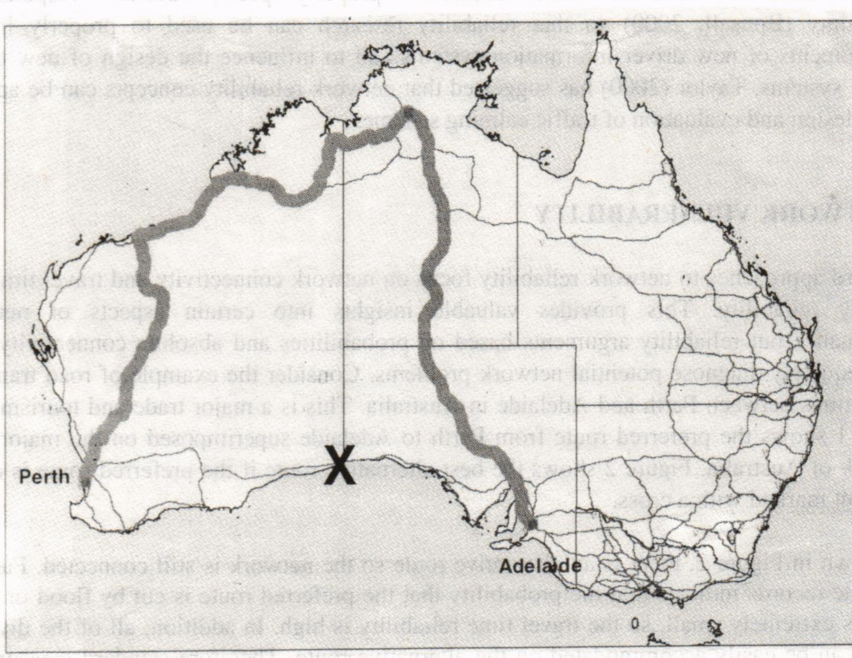

Figure 2: Alternative route from Perth to Adelaide with degraded network

- the opportunity presented to an individual to take part in a particular activity or set of activities
- the number of opportunities reached within a given travel time or distance
- the ease by which activities can be reached from a given location using a particular transport system
- the possibility to reach a given location within an acceptable amount of time, money and effort with respect to a specific policy

The formal definition of specific accessibility indicators may best be seen through a binary classification of the indices as either 'relative accessibility indices' or 'integral accessibility indices' (Wigan *et al*, 1978). Relative accessibility describes the degree of connection between two given points (e.g. a given location and a specific activity), which may be assessed in terms of distance, travel time or travel cost. It is commonly used, for instance, in determining the locations of emergency services (e.g. fire, ambulance or police) where the location of the nearest facility is of prime importance. The relative accessibility A_{ij} between two points i and j may be given as

$$A_{ij} = C_{ij} \tag{1}$$

where C_{ij} is the separation (distance, time or cost) between the points.

On the other hand, integral accessibility describes the interconnections between a given point and all other points or activities within a region. In essence integral accessibility (AI_i) is given by the summation of the relative accessibility of the location i over all points j

$$AI_i = \sum_j A_{ij} \tag{2}$$

Integral accessibility may then be assessed by a simple measure of travel separation, as implied by equations (1) and (2), which is suitable for tasks such as the location of emergency services (where the closest service location is often of most importance). However, the measure does not include any allowance of the level of provision of the facility at any one site, or allow for user choice in the selection of a given facility (Parolin *et al*, 1994). Composite measures of integral accessibility that include supply demand measures and spatial separation have been devised, with two such measures being:

(1) the Hansen index (e.g. Davidson, 1977), given by

$$AI_i = \sum_j B_j f(C_{ij}) / \sum_j B_j \tag{3}$$

in its normalised form, where $f(C_{ij})$ is an impedance function, and B_j is the amount or intensity of an activity at j, and

(2) the Black-Conroy cumulative distribution index, given by

$$AI_i(T) = \int_0^T N_i(t)dt \tag{4}$$

This example illustrates the concept of network vulnerability and the difference between network reliability and vulnerability. The concept of vulnerability is related to the consequences of link failure, irrespective of the probability of failure. In some cases the probability of link failure may be low and the effect on overall network performance may be small, but the social and economic impact on particular sections of the community may be sufficiently large to indicate a major problem that may warrant remedial action. For example, consider the impact on a rural community of loss of access to vital human services (such as a hospital) and to markets for their produce. These are examples of cases where low probability of occurrence, the availability of alternative routes and overall network performance does not offset the consequences of a network failure. In summary, network reliability and vulnerability are related concepts but whereas reliability focuses on connectivity and probability, vulnerability is about network weakness and consequences of failure.

The next step is to formulate a working definition of vulnerability. The concept of vulnerability could be applied to the connection between a particular origin and destination; or to access from a particular location to other parts of the network; or to the network as a whole. The following definitions provide a starting point for applying the concept to network analysis and diagnosis.

> *DEFINITION 1 - Connective Vulnerability (First Order)*
> *A connection between two nodes is vulnerable if loss (or substantial degradation) of a single network link significantly increases the (generalised) cost of travel between the two nodes.*

For a given origin and destination, network vulnerability is concerned with the consequences of network degradation. This definition uses loss of utility as the measure of vulnerability, as illustrated in Figures 1 and 2. Note that the definition has been termed *first order* connective vulnerability because it relates to the loss of a single link. Similar definitions could be stated for loss of two links (second order vulnerability), and so on.

> *DEFINITION 2 - Access Vulnerability*
> *A node is vulnerable if loss (or substantial degradation) of a small number of links significantly diminishes the accessibility of the node, as measured by a standard index of accessibility.*

This definition relates to the overall quality of access from a given node to all other parts of the network. Accessibility to facilities, goods, services and opportunities has long been recognised as a significant outcome of transport, and the concepts of accessibility have been explored over many years. In broad terms accessibility may be used as a measure of the spatial distribution of activities about a given point in a region, adjusted for the ability, need and desire of the population to overcome spatial separation. Several alternative definitions of accessibility have been proposed, and while there are some differences between these, their common characteristic is to identify accessibility as a measure of the spatial distribution of activities about a point, adjusted for the ability, need and desire of individuals and groups to overcome separation. Some of the possible definitions include:

- observed or expected travel cost between two points where $AI_i(T)$ is the cumulative number of opportunities within T travel cost units from location i and $N_i(t)$ is the absolute number of opportunities at t travel cost units from i (Black and Conroy, 1977). This index has been rarely used, perhaps because of the possibly arbitrary choice of the limiting travel cost units (T) (Neimeier, 1997), but it has the advantage of providing a continuous measure of the 'accessibility surface' from a given location. This surface could be used to assess the relative accessibility of a given location over a region, and choice of different T values would provide contours of the surface. Variations of these contours under different conditions of network degradation could form the basis for assessing overall changes in network vulnerability and the relative effects for different locations within the network.

4. IDENTIFYING NETWORK WEAKNESSES

The purpose of analysing network vulnerability is firstly, to be able to anticipate points of weakness where the network is vulnerable and network failures will have substantial adverse effects; and secondly, to be able to suggest remedial measures such as adding links to the network to make it more robust. It is tempting to suggest that a transport network is most vulnerable simply where observed link flows are greatest but

- alternative routes may be available providing a new equilibrium pattern of flows at little reduction in overall network performance; and
- considering aggregate flows may obscure significant vulnerabilities in connections between particular origins and destinations.

Therefore observed link volume is not necessarily a reliable indicator of vulnerability. This section of the paper addresses the development of analytical tools for identifying network weaknesses.

The definitions of network vulnerability emphasise the consequences of degradation of the network. In other words, if the 'best' path through the network is no longer available, how much worse is the second-best option, or the third best, and so on. This suggests an approach based on *n*th best paths though the network or on constrained shortest path algorithms, but in general, algorithms for these problems are inefficient and are not included in standard transport network modelling software packages. For a review of algorithms for *n*th-best and constrained path problems, see D'Este (1997).

An alternative starting point is probabilistic route choice algorithms, such as those based on the logit model. According to this model, a traveller will choose a particular path from the set of available paths from the required origin to destination on the basis of the utility of that path compared to the alternatives. The measure of utility is travel time or other appropriate generalised cost. The probability of using a particular path will then depend on its relative utility.

This argument can be extended to individual links. The probability of using a particular link is a measure of the utility of paths through that link compared to paths through alternative links. Note that for a network without loops, the probabilities for links that comprise a network cut will sum to unity. Therefore if the probability of using a particular link is low then there exist other links with similar or 'better' paths. However if the probability is high then paths through alternative links are inferior. The higher the probability of using a particular link, the greater the difference between its utility and the utility of paths through alternative links. If that link is cut then the network performance will degrade significantly. In other words, the link probabilities provide a measure of the relative performance of alternative paths and hence of the consequences of network failure.

It follows that logit-based assignment algorithms, such as Dial (1971) can form the basis of an heuristic method for identifying vulnerable links in a transport network. An alternative to Dial's formulation of logit-based assignment has been proposed by Bell (1995). Bell showed that the Floyd-Warshall shortest path algorithm (Floyd 1962, Warshall 1962) can be extended to logit-based multi-path assignment. Bell proposed two algorithms, the second and more general can be summarised as follows. Consider a matrix of weights **W**, initialised as follows:

if a link joins node m to n then $w_{mn} = \exp(-\alpha\, c_k)$, otherwise $w_{mn} = 0$.

where c_k is the cost of travel from m to n and α is a choice parameter that reflects the traveller's sensitivity to path cost. With a value of α close to zero, trips are assigned widely across available paths with little regard to cost, while as α increases, trips are increasingly assigned to the shortest path. Having constructed the matrix of initial weights, the next step is to calculate

$$\mathbf{U} \leftarrow (\mathbf{W} - \mathbf{I})^{-1} - \mathbf{I} \tag{5}$$

then **U** is the final matrix of link weights for the choice process. These weights can then be converted into link probabilities using the Van Vliet (1981) method. The probability that a trip from i to j uses the link joining node or centroid r to node or centroid s

$$P_{ijrs} = u_{ir}\, w_{rs}\, u_{sj} \,/\, u_{ij} \tag{6}$$

Using this basic relationship, similar formulas can be constructed for turning proportions or probabilities of using other combinations of links. The advantages of Bell's formulation are that it:

- calculates link weights for trips between all origins and all destinations simultaneously, and
- is valid whether or not there are loops in the network.

The matrix of link probabilities $\mathbf{P}_{ij}$ derived using relationship (6) provides an indicator of the overall vulnerability of trips between the given origin and destination and also of the location of key links. If $\mathbf{P}_{ij}$ has elements close to unity then this is a good indicator of potential network vulnerability because it suggests that there is a significant difference in amenity between the best and next best routes.

The individual link probabilities P_{ijrs} also provide an indicator of where to look for the key links where the connection is most vulnerable. In general, the higher the link probability, the greater the adverse impact if that link is broken. This suggests that candidates for the source of network vulnerability will be links with probability higher than a prescribed threshold λ. In general, the loss of any link with $p_{ijrs} > 0.5$ will adversely affect network performance but the effect may be minimal. In practice, a higher value of λ may be more appropriate. Recommendations for efficient values of the threshold λ and the choice parameter α are topics for further research.

Having assessed the vulnerability of connections between a particular origin and destination, the vulnerability of overall access of a particular node can then be evaluated by repeating the process for all destinations reachable from that node. Therefore calculating link probabilities using a logit-based multi-path assignment algorithm provides an heuristic technique for identifying both types of nodal vulnerabilities.

This discussion has considered the analysis of network vulnerability in terms of the spatial or topological configuration of the network, by defining a measure of vulnerability based on access of a given node to all other nodes. This measure may be seen as similar to the simple measure of integral accessibility defined by equation (2). Further research is needed to consider the use of accessibility-based measures of network vulnerability, which introduce some measure of access to opportunities located in different amounts at different locations in the network, perhaps using measures similar to the integral accessibility measures defined by equations (3) and (4).

5. EXAMPLE

The following example illustrates the identification of links where a network is potentially vulnerable. Consider the network shown in Figure 3. The number shown alongside each link is a measure of the link cost.

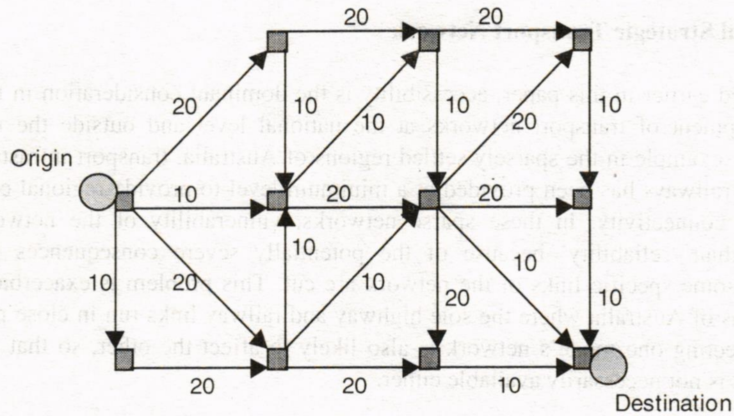

Figure 3: Example Network

Figure 4 shows the link probabilities after applying the method described in Section 4 with α=0.1. Probabilities less than 0.3 are shown with a thin dashed line, between 0.3 and 0.6 with a solid line and link probabilities greater than 0.6 are shown with a thick line. As shown in this figure, there is only one link with a high probability. According to the heuristic method developed in Section 4, the connection between the indicated origin and destination is likely to be vulnerable to degradation of this link, but relatively insensitive to loss of other links. If this key link is cut then the minimum and expected cost of travel increases by around 25 per cent, whereas if any other link is cut, there is little (if any) impact on network performance.

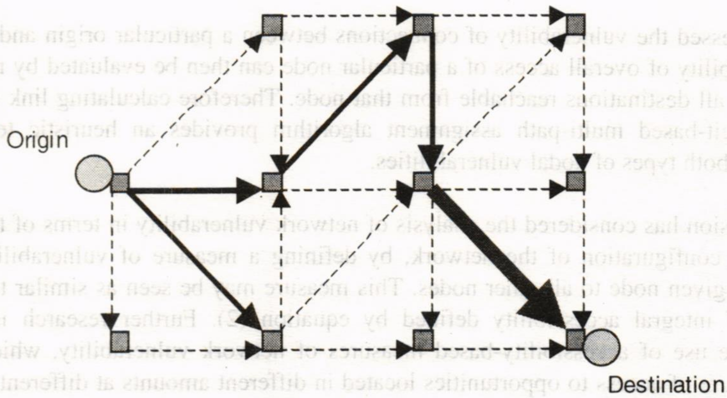

Figure 4: Link Probabilities

This example illustrates the application of the vulnerability concepts and the technique described in Section 4. It also illustrates the important phenomenon that all links on the shortest path are not necessarily points of vulnerability. If there is sufficient redundancy in the network then diversion to alternate paths can take place with little degradation of network performance.

6. APPLICATIONS

6.1 National Strategic Transport Network

As described earlier in this paper, accessibility is the dominant consideration in the planning and development of transport networks at the national level and outside the major urban centres. For example in the sparsely settled regions of Australia, transport infrastructure such as roads or railways has been provided at a minimum level to provide regional coverage and inter-urban connectivity. In these sparse networks, vulnerability of the network is more important than 'reliability' because of the potentially severe consequences to transport services if some specific links of the network are cut. This problem is exacerbated in some remote areas of Australia where the sole highway and railway links run in close proximity. A disaster affecting one mode's network is also likely to affect the other, so that cross-modal substitution is not necessarily available either.

The techniques developed in this paper provide insights into the robustness of a transport network, or conversely to its vulnerability to failure of key links. The ability to identify where the network is most vulnerability highlights locations where the network might be modified to make it more robust. In practical terms, the level of urgency and specific network adaptation will depend on the consequences of failure and the likely source and probability of failure. Therefore vulnerability analysis provides a mechanism for targeting risk assessments, with the ultimate aim of designing appropriate counter-measures. In the long run, if potential weaknesses in the network can be anticipated and adaptations made to the network, then the

7. CONCLUSIONS

The purpose of this paper has been to introduce the concept of network vulnerability and describe some preliminary research on definitions, techniques and applications. A network is considered to be vulnerable if degradation of a small number of network links, perhaps a single link, causes significant reduction in network performance. The aftermath of earthquakes, floods, and traffic incidents illustrate the vulnerability of transport network.

It is important to understand that vulnerability is a supply-side measure and is not so concerned with actual volume of usage of the network (the distribution or intensity of travel demand or the effects of congestion). The objective of vulnerability research is to develop tools for scanning network designs to indicate and rank the points or sections of weakness in terms of their potential to degrade network performance, in terms of accessibility to opportunities and activities.

There are three key questions to be addressed in the context of network vulnerability.

- Is the link between two locations or the overall accessibility of a location vulnerable to network degradation?
- If so, where is the network most vulnerable?
- What can be done to reduce the degree of vulnerability?

Answers to these questions provide the basis for making transport networks more robust in terms of delivering more consistent performance. This paper has developed a framework for addressing these questions and proposed an heuristic technique based on logit-based multi-path assignment for identifying potential vulnerability. The question of techniques for developing efficient response, perhaps by adding new links to reduce the vulnerability of a network, remains for future research. Key topics for further research, include

- the development of improved indicators of network vulnerability.
- the development of more precise techniques for identifying 'weak' links that make the network most vulnerable.
- tools for network diagnosis and auditing of transport networks, including visualisation tools.
- tools for identifying the best response to identified vulnerabilities.

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consequences of failure of key links can be substantially reduced. The result would be a transport network that provides a better and more robust level of performance.

6.2 Incident Risk Assessment

Although the development of the concept of network vulnerability has focused mainly on sparse non-urban networks, the techniques can be applied in an urban context, specifically in relation to vulnerability to degradation of network performance as a result of incidents, such as

- accidents, breakdowns and disabled vehicles, road blockages by emergency services operations, severe weather, natural disasters or distractions caused by events taking place along the roadside ('rubber-necking'); and
- planned events such as road works and special events.

In North America, it has been estimated that incidents cause 40-60 per cent of all urban traffic congestion. The effects of incidents are compounded by secondary accidents and by exponentially growing traffic delays.

Central to the objective of managing the road network to ameliorate the effect of incidents is the ability to identify locations with high incident vulnerability. The value of anticipating and hence avoiding potential problems has been recognised in many disciplines. In particular, safety auditing has become a standard feature of road design and assessment. Safety auditing involves examining all aspects of the road environment to identify safety risks and recommend measures to rectify the problems. This principle can be extended to incident risk assessment.

The aim of incident risk assessment is to identify locations or road links that are especially vulnerable to congestion caused by incidents. The degree of vulnerability will be influenced by a combination of factors, notably

- traffic volume per lane and volume to capacity ratio; and
- availability of alternative routes and permeability of the network (Taylor 1999, 2000).

Therefore there are two inter-connected sources of incident vulnerability. The first is the sheer volume of traffic. Locations with this type of vulnerability are easy to identify. The second source of vulnerability is associated with the inherent structure of the transport network and/or the way that it is managed. Locations with this sort of vulnerability can be more difficult to identify and the effects may be harder to predict. Techniques described in Section 4 provide a framework for diagnosing this second type of vulnerability.

Incident risk assessment can also be used to evaluate the expected vulnerability of proposed road links to traffic disruption caused by incidents. Incident risk assessment during the planning stages of major road projects should become a routine component of the planning process. It will allow incident management plans to be developed in advance where required, and may lead to design changes to reduce vulnerability to incidents. For example, extra exit ramps may be constructed or an automated incident detection system installed in strategic road links assessed to have a high incident risk.

REFERENCES

Asakura, Y. (1999) Evaluation of network reliability using stochastic user equilibrium. **Journal of Advanced Transportation, Vol. 33, No. 2**, 147-158.

Asakura, Y. and Kashiwadani, M. (1991) Road network reliability caused by daily fluctuation of traffic flows. **Proceedings 19th PTRC Summer Annual Meeting**, University of Sussex, Brighton, 73-84, July 1991.

Bell, M.G.H. (1995) Alternatives to Dial's logit assignment algorithm. **Transportation Research B, Vol. 29B**, 287-295.

Bell, M.G.H. and Iida, Y. (1997) **Transportation Network Analysis**. John Wiley and Sons, Chichester.

Black, J.A. and Conroy, M. (1977) Accessibility measures and the social evaluation of urban structure. **Environment and Planning A, Vol. 9**, 1013-1031.

Bonsall, P.W. (2000). Travellers' response to uncertainty. In M.G.H. Bell and C. Cassir (eds.), **Reliability of Transport Networks**. Research Studies Press, Baldock, Herts. 1-10.

Chen, A., Yang, H., Lo, H.K. and Tang, W.H. (1999) A capacity related reliability for transportation networks. **Journal of Advanced Transportation, Vol. 33, No. 2**, 183-200.

Davidson, K B (1977). Accessibility in transport/land use modelling and assessment. **Environment and Planning A, Vol. 9**, 1401-1416.

D'Este, G.M. (1997) Implementing a hybrid route choice procedure in a network modelling context. **Journal of the East Asian Society for Transport Studies, Vol. 2 No. 3**, 737-720.

Dial, R.B. (1971) A probabilistic multipath traffic assignment model which obviates path enumeration. **Transportation Research, Vol. 5**, 83-111.

Floyd, R.W. (1962) Algorithm 97: shortest path. **Communications of the ACM, Vol. 5**, 345.

Goodwin, P.B. (1992) A quality margin in transport. **Traffic Engineering and Control, Vol. 33 No. 12**, 661-665.

Iida, Y. (1999) Basic concepts and future directions of road network reliability analysis. **Journal of Advanced Transportation, Vol 33 No. 2**, 125-134.

Lam, W.H.K. and Xu, G. (2000) Calibration of traffic flow simulator for network reliability assessment. In M.G.H. Bell and C. Cassir (eds.), **Reliability of Transport Networks**. Research Studies Press, Baldock, Herts. 139-157.

Morris, J.M., Dumble, P.L. and Wigan, M.R. (1978) Accessibility indicators for transport planning. **Papers of the Australian Transport Research Forum, Vol. 4, No. 1**, 41-73.

Neimeier, D.A. (1997) Accessibility: an evaluation using consumer welfare. **Transportation, Vol. 24 No. 4**, 377-396.

Nicholson, A. and Zhen-Ping Du (1997) Degradable transportation systems: an integrated equilibrium model. **Transportation Research B, Vol. 31B**, 209-223.

Parolin, B., McDougall, W. and Nichols, M. (1994) Modelling rural accessibility. **Papers of the Australasian Transport Research Forum, Vol. 19**, 45-62.

Richardson, A.J. and Taylor, M.A.P. (1978) A study of travel time variability on commuter journeys. **High Speed Ground Transportation Journal, Vol. 12**, No. 1, 77-99.

Taylor, M.A.P. (1982). Travel time variability – the case of two public modes. **Transportation Science, Vol. 16, No. 2**, 517-521.

Taylor, M.A.P. (1999) Dense network traffic models, travel time reliability and traffic management II: application to reliability. **Journal of Advanced Transportation, Vol 33 No. 2**, 235-251.

Taylor, M.A.P. (2000) Using network reliability concepts for traffic calming - permeability, approachability and tortuosity - in network design. In M.G.H. Bell and C. Cassir (eds.), **Reliability of Transport Networks**. Research Studies Press, Baldcock, 217-242.

Van Vliet, D. (1981) Selected node-pair analysis in Dial's assignment algorithm. **Transportation Research B Vol. 15B**, 65-68.

Warshall, S. (1962) A theorem on Boolean matrices. **Journal of the ACM, Vol. 9**, 11-12.

Yang, H., Lo, H.K. and Tang, W.H. (2000) Travel time versus capacity reliability of a road network. In M.G.H. Bell and C. Cassir (eds.), **Reliability of Transport Networks**. Research Studies Press, Baldock, Herts. 119-138.

Zhen-Ping Du and Nicholson, A. (1997) Degradable transportation systems: sensitivity and reliability. **Transportation Research B, Vol. 31B**, 225-237.