INITIAL NUMBER OF FACILITY IN A M/M/N(∞) PORT SERVICE QUEUEING SYSTEM

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Abstract: In this paper we considered many evaluation indices of port service queueing system , and to study the determinate process of initial value of optimal number in a M/M/N(∞) port service queueing system. Those indices including the degree of congestion(DC), average waiting time (W_q) and waiting time factor ($W_q\mu$). This paper present the initial number of berth in a M/M/N(∞) port service queueing system would be $N_o = (\lambda/\mu) + \beta\sqrt{(\lambda/\mu)}$, and we found the relationship between the traffic density(α) and "the initial parameter value of facility" (β) under each of the evaluation index , then the relationship can be categorized by general form equation. Therefore, we plotted the table and graphics, and it can be cited for the researcher and their application.

Key Words: Initial number of facility, Queueing system, Port planning, Traffic density, Level of service, Evaluation indices

1.INTRODUCTION

In general, we solve optimal number of servers in a M/M/N(∞) queueing system by using average service time ($1/\mu$) as a constant. Recently, the solution for optimal number of servers , always think of the factors about management side. The factor effect service facility including number of operating equipment, operating efficiency , interaction of operating equipments, the characteristic of operation system in the peak and the factor cause by dwell time. So ,we think about the variation of many complex factors, by using the system simulation to solve the optimal solution simply. In this time, to know an initial value of number of servers is the most important to effect the simulation time. In past time ,the process to decided initial value of facility servers (N) in a queueing system always take the utilization factor (ρ) less than one, i.e. $\rho = \lambda/(N\mu) < 1$. Newell (1982) applied the theory of normal distribution to estimate the number of service facility, but the result was an approximant not a really value. Taha (1987) employed the aspiration level model, but the model limits the decision maker's performance. Lawarence and Teng (1990) improvised the idea of normal distribution with traffic density (λ/μ) in addition to the method of standard deviation for 1 to 2 times as the minimum required number of servers (N), But this method never think of what level of service in the system was better. So that the initial value of servers always to be larger than the optimal numbers. Huang (1997) used the degree of congestion (DC), average waiting time in queue (Wq), waitint time factor (Wq · μ) and cost function to investigate the optimal number of berth and the number of crane for the container terminal, but the viability of this model depends on how well we can estimate the cost parameters. Generally, these parameters are difficult to estimate. The methods that determine each of the optimal service facility and the ceilig limit to its employment are shown in Table 1.

Table 1. The methods that determine the number of initial service facility.

Author & Year	The methods	Limit to employment
Newell(1982)	The theory of normal distribution	The result is an approximant not a really value.
Taha(1987)	The aspiration level model	The aspiration level model limits the decision maker's performance
Lawrence and Teng (1990)	The method of traffic density (λ/μ)+(1～2) standard deviation	The result is only applicable for local solution.
Huang (1997)	The method of cost function	The viability of this model depends on how well we can estimate the cost parameters, Generally, these parameters are difficult to estimate

The purpose of this paper would be found a method which determinate the number of initial facility value easily, such as estimating from a table or a graph. Based on the result of this paper, the planners could be determinate the number of initial facility from a table or a graph. Validity shorten the simulation time, and resolve the problem of the initial number of servers over the optimal number of servers.

2. THE PORT SERVICE QUEUEING SYSTEM

In this paper we considered many evaluation indices of M/M/N queueing system for port facilities planning, including degree of congestion(DC),berth occupancy factor(ρ),average number of ships in port(L),average number of ships in queue(L_q),average waiting time (W_q) and average waiting time factor ($W_q \cdot \mu$).

This study has formulated the evaluation indices into three categories in port queueing system, the type of probability index(such as DC), the type of queue length index (such as L_q and W_q) and the type of non-dimension index (such as $W_q \cdot \mu$). The type of probability index only shows the states of vessels in port system, it could not be expressive of the waiting time directly, and the type of queue length indices and the type of non-dimension indices would be change with the type of ship, the weight of ship and service tiome of ship. We define them as follow:

(1) degree of congestion (DC)

Degree of congestion is the probability to wait for berths when vessels arrive at the port.

$$DC = \sum_{j=N+1}^{\infty} P_N(j) \quad (1)$$

where N is the number of berth, and $P_N(j)$ is the probability of j vessels in the port within the period T.

(2) berth utilization factor (ρ)

Berth utilization factor is the ratio of berth actual utilization time to the gross berth time available.

$$\rho = 1-\left(\sum_{j=0}^{N-1}(N-j)P_N(j)/N\right) \quad (2)$$

$$= \left(\sum_{j=1}^{N} j\,P_N(j)/N\right)+DC \quad (3)$$

(3) average number of ships in port (L)

Average number of ships in port is expected number of ships in the port within the period T.

$$L = \sum_{j=0}^{\infty} j\,P_N(j) \quad (4)$$

(4) average number of ships in queue (L_q)

Average number of ships in queue is expected number of ships that are waiting for service in the port system

$$L_q = \sum_{j=N+1}^{\infty}(j-N)P_N(j) = L-(\lambda/\mu) \quad (5)$$

where λ is mean arrival rate of vessel ; μ is mean service rate

(5) average waiting time (W_q ; AWT)

Average waiting time is a vessel spends in the waiting line for service.

$$W_q = L_q/\lambda \quad (6)$$

(6) waiting time factor ($W_q \cdot \mu$; AWT/AST)

Waiting time factor is the ratio of the average waiting time(AWT ; Wq) to average service time(AST ; $1/\mu$).

$$W_q \cdot \mu = AWT/AST \quad (7)$$

The past relevant literatures about the port planning was application of M/M/N queueing theory, therefore, we define the steady-state probabilities and evaluation indices about M/M/N queueing theory under the steady-state situation as follow :

$$P_N(j) = P_N(0)(\lambda/\mu)^j/j! \qquad (0 \leqq j \leqq N)$$

$$= P_N(0)(\lambda/\mu)^j/(N!\cdot N^{(j-N)}) \qquad (j \geqq N)$$

where $$P_N(0) = 1/\left[\sum_{j=0}^{N-1}(\lambda/\mu)^j/j! + (\lambda/\mu)^N/(N!(1-\rho))\right]. \quad (8)$$

and the substitution of eq. (8) into eq. (1),(2),(5),(6),(7), then yields

$$DC = 1 - \sum_{j=0}^{N} P_N(0)(\lambda/\mu)^j / j! \tag{9}$$

$$\rho = 1 - \sum_{j=0}^{N-1} ((N-j) P_N(0)(\lambda/\mu)^j / (j! \cdot N)) \tag{10}$$

$$L_q = P_N(0)(\lambda/\mu)^j \rho / (j! \cdot (1-\rho)^2) \tag{11}$$

$$W_q = L_q/\lambda = P_N(0)(\lambda/\mu)^j \rho / (j! \cdot (1-\rho)^2 \cdot \lambda) \tag{12}$$

$$W_q \cdot \mu = P_N(0)(\lambda/\mu)^j / (j! \cdot (1-\rho)^2 \cdot N) \tag{13}$$

3.THE MODIFIED RESOLUTION METHOD FOR THE INITIAL NUMBER OF FACILITY

After having deduced relevant methods of research and their ceiling limits of employment, this study has employed the normal distribution theory by Newell (1982) and the traffic density by Lawrence and Teng (1990), with the addition of determining the initial number of its standard deviation by 1 to 2 times. With regard to the above-mentioned defects of research, some improvements have been made in the following:

(1) Various kinds of system evaluation indices are considered (such as DC, Wq, Wq.μ) in view of the differences under diverse traffic density.

(2) Range of application under diverse traffic density and level of service are considered after the standard deviation value has been multiplied by 1 to 2 times.

Therefore, the method of this study will consider the applicability of each and every kind of system evaluation index (DC, Wq, Wq.μ) under different traffic density and level of service, so that the designation of initial number of system facility can be conducted. Based on the Lawrence and Teng (1990) of the facility initial number with the addition of standard deviation by 1 to 2 times to resolve for the traffic density (λ/μ), and the equation can be written as in eq.(14). According to the aforementioned research method explicated in this research, when eq.(14) is utilized in higher traffic density the value will over the optimal number of facility. In other words, the method itself is only applicable for local solution. As a result, this study has modified the standard deviation value by 1 to 2 times in equation (14) to become the parameter for the initial number (β) as indicated in equation (15), and it's characteristics of application are then examined.

$$N_o = \frac{\lambda}{\mu} + (1 \sim 2)\sqrt{\frac{\lambda}{\mu}} \tag{14}$$

$$N_o = \frac{\lambda}{\mu} + \beta\sqrt{\frac{\lambda}{\mu}} \tag{15}$$

3.1 Range of port service indices under different level of service

This study has considered the system evaluation index of the queueing system characteristic to formulate the method to determine the initial number of facility in a M/M/N queueing system. As a whole, three kinds of evaluation indices as the degree of congestion(DC), average waiting time (W_q), and waiting time factor ($W_q \cdot \mu$) are being taken into account. Due to the fact that the initial number of the queueing system facility is largely relevant to the change of each of the evaluation index as well as to the level of service indicated by the evaluation, it is why that this study has formulated the level of service into three categories as A, B and C according to the evaluation indices. Take the degree of congestion as an example, the range of category A is between 0% to 20 %, that is the probability for waiting is only about 0% to 20% as of system congestion. As for the range of B category it is about 20% to 50%, while category C is over 50%. As a matter of course, category A offers higher level of service, while the classification of other evaluation indices are as shown in Table 2.

Table 2 Range of port service indices under different level of service

level of service / port service indices	level A	Level B	level C
degree of congestion (DC)	0-20%	20%-50%	>50%
average waiting time (Wq)	0-3 hrs	3-6 hrs	>6 hrs
waiting time factor ($W_q \cdot \mu$)	0-0.15	0.15-0.3	>0.3

3.2 The resolution method for the initial parameter values (β) of facility

The resolution for the initial number of port service facility is obtained primarily through using the optimal facility number under different traffic density ($A=\lambda/\mu$) and different service level in each of the forgoing port index. Take the degree of congestion (DC) as an example, when the traffic density (α=1.0), its optimal number of facility should be 2 in order to maintain its service level of DC=25%. Based on Lawrence and Teng(1990) using the initial number of the standard deviation by 1 to 2 times to its traffic density (λ/μ), the value to be 2 happens to be the optimal number of berth when the standard deviation being of 1 time is planted into the initial solution of the simulation. Should the standard deviation of 2 times is employed to resolve for the initial number of berth, the initial solution of the simulation being 3 is well over the optimal solution. As can be understood, the resolution for the initial number of facility for M/M/N queueing system put forward in the literature by Lawrence and Teng(1990) is only of a local solution, which is as circled in Figure 1. In order to solve the problem, this study has found out the smallest facility number of each range and its relationship value to the traffic density, which is as seen in Table 3. They are marked as found in columns (3) (4) (5) and are called "the initial parameter value of facility(β)".

Based on this relationship values, the relationship chart between the traffic density and initial parameter value of facility can be mapped. As seen in Figure 1, it represents the three curves of initial numbers of facility under three kinds of service levels. Furthermore, the circular curve as found in Figure 2 can be obtained after fitting of these three curves, while such curve stands for the relationship between traffic density and the initial parameter value of facility when considering the degree of congestion (DC). They can be indicated by equations (16) to (18):

$$\beta = 1.1 + 2.8 \cdot e^{-3\alpha^{0.5}} \tag{16}$$

$$\beta = 0.65 + 2.3 \cdot e^{-1.8\alpha^{0.5}} \tag{17}$$

$$\beta = 0.37 + 2.3 \cdot e^{-1.8\alpha^{0.5}} \tag{18}$$

Table3. The optimal initial number of facility under difference level of service and degree of congestion

(1)α	(3)β	(4)β	(5)β	No1	No2	No	MIN(N)	1%	3%	5%	10%	15%	20%	No	MIN(N)	25%	30%	35%	40%	45%	50%	No	MIN(N)	55%	60%	70%
0.1	2.80	2.70	2.70	0.42	0.73	0.99	1	2	2	2	1	1	1	0.95	1	1	1	1	1	1	1	0.95	1	1	1	1
0.2	2.20	1.70	1.70	0.65	1.09	1.18	1	2	2	2	2	2		0.96	1							0.96	1			
0.3	2.00	1.25	1.25	0.85	1.40	1.40	1	3	2	2	2	2		0.98	1							0.98	1			
0.4	1.70	1.20	1.20	1.03	1.66	1.48	2	3	2	2	2	2	2	1.16	1	1	1	1	1	1	1	1.16	1	1	1	1
0.5	1.70	1.15	1.15	1.21	1.91	1.70	2	3	3	2	2	2	2	1.31	1	2	1	1	1	1	1	1.31	1	1	1	1
0.6	1.70	1.15	1.10	1.37	2.15	1.92	2	4	3	3				1.49	1				1	1	1	1.45	1	1	1	1
0.7	1.70	1.15	0.95	1.54	2.37	2.12	2	4	3	3				1.66	2						1	1.49	1	1	1	1
0.8	1.65	1.10	0.95	1.6	2.	2.28	2	4	4	3	3			1.78	2							1.65	2			
0.9	1.65	1.10	0.95	1.85	2.80	2.47	2	4	4	3	3			1.94	2							1.80	2			
1.0	1.55	1.10	0.95	2.00	3.00	2.55	3	5	4	4	3	3	3	2.10	2							1.95	2			
1.2	1.45	1.10	0.95	2.30	3.39	2.79	3	5	4	4				2.40	2		2	2	2	2	2	2.24	2	2	2	2
1.4	1.45	1.10	0.90	2.58	3.77	3.12	3	5	5	4	4			2.70	2					2	2	2.46	2	2	2	2
1.6	1.45	1.10	0.90	2.86	4.13	3.43	3	6	5					2.99	3							2.74	3			
1.8	1.45	1.10	0.90	3.14	4.48	3.75	4	6	5	5				3.28	3	3	3	3	3	3	3	3.01	3	3	3	3
2.0	1.45	1.00	0.90	3.41	4.83	4.05	4	6	5	5				3.41	3			3	3	3	3	3.27	3	3	3	3
2.2	1.40	0.85	0.85	3.68	5.17	4.28	4	7	6					3.46	3				3	3	3	3.46	3	3	3	3
2.5	1.25	0.85	0.85	4.08	5.66	4.48	5	7	6	6				3.84	4		4	4	4	4	4	3.84	4	4	4	4
3.0	1.25	0.85	0.85	4.73	6.46	5.17	5	8	7	7				4.47	4				4	4	4	4.47	4	4	4	4
3.5	1.25	0.85	0.85	5.37	7.24	5.84	6	9	8					5.09	5		5	5	5	5	5	5.09	5	5	5	5
4.0	1.20	0.74	0.74	6.00	8.00	6.40	6	10	8	8				5.48	5				6	5	5	5.48	5	5	5	5
5.0	1.20	0.74	0.65	7.24	9.47	7.68	8	11	10					6.65	7	7	7	7	7	7	6	6.45	6	6	6	6
6.0	1.20	0.74	0.60	8.45	10.90	8.94	9	12	11	11				7.81	8		8	8	8	8	8	7.47	7	8	7	7
8.0	1.15	0.74	0.50	10.83	13.66	11.25	11	15	14					10.09	10			10	10	10	10	9.41	9	10	10	9
10.0	1.15	0.74	0.45	13.16	16.32	13.64	14	18	17					12.34	12	13	13	13	12	12	12	11.42	11	12	12	11
12.0	1.15	0.74	0.43	15.46	18.93	15.98	16	20	19					14.56	14		15	15	15	14	14	13.49	13	14	14	13
14.0	1.15	0.65	0.39	17.74	21.48	18.30	18	23	22					16.43	16		17	17	17	16	16	15.46	15	16	16	15
16.0	1.15	0.65	0.37	20.00	24.00	20.60	21	25	24					18.60	19			19	19	19	18	17.48	17	18	18	17
18.0	1.15	0.65	0.37	22.24	26.49	22.88	23	28	27					20.76	21	22	22	21	21	21	20	19.57	20	20	20	20
20.0	1.10	0.65	0.37	24.47	28.94	24.92	25	30	29					22.91	23	24	24	24	23	23	23	21.65	22	23	22	22

Note: (light grey) When β=1,the number of initial facility over the optimal solution
(dark) When β=2,the number of initial facility over the optimal solution

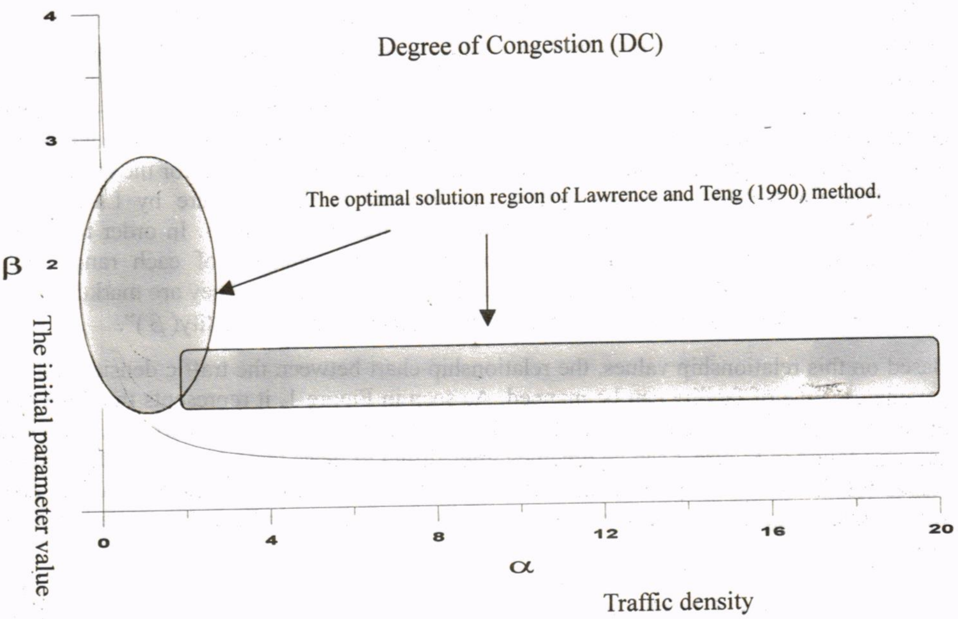

Fig 1. The relation between the traffic density and the initial parameter value at different level of service.

Likewise, other evaluation indices can also be obtained from using the same method, which is as shown in Figure 3 to Figure 5.

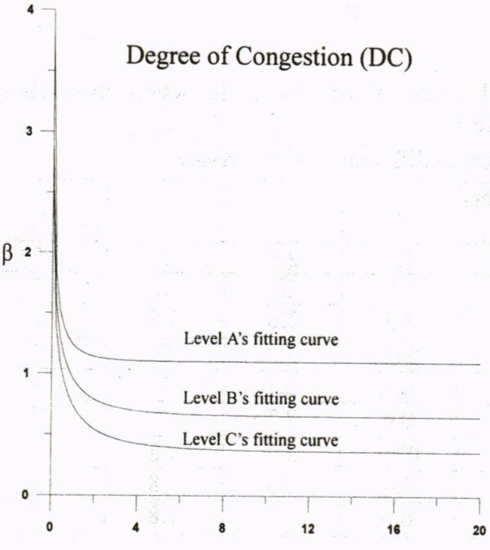

Fig2. Fitting DC curves of the relationship between the traffic density(α) and the initial parameter value(β)

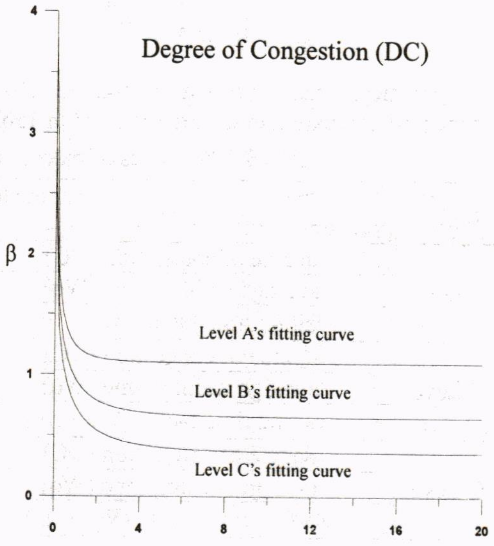

Fig3. The initial parameter value between the traffic density and the level of DC

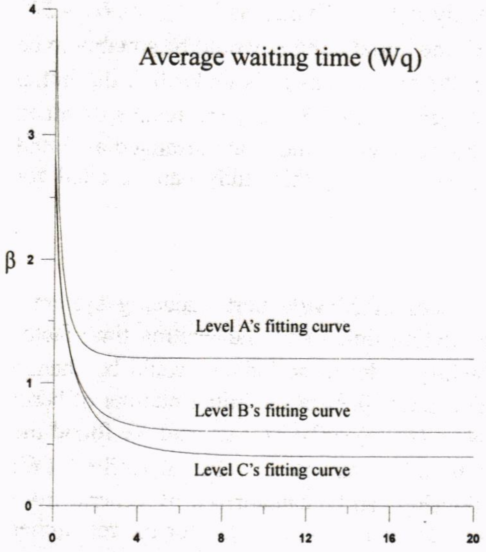

Fig4. The initial parameter value between the traffic density and level of W_q

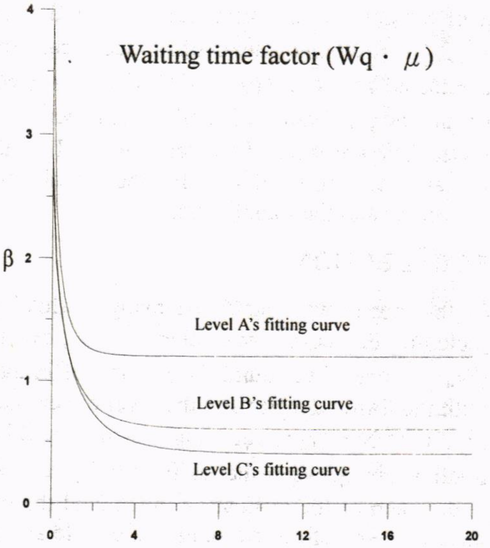

Fig5. The initial parameter value between the traffic density and the level of $W_q\mu$

4. DISCUSSION

According to the research results, the relationship between the traffic density and initial number of facility under each of the evaluation index can be summarized as follow:

$$\beta = a + b \cdot e^{-c\alpha^{d}} \tag{19}$$

The parameter changes found in equation (19) because of different service levels under each of the port evaluation index are arranged in Table 4.

Table 4. The values of parameters in different level of service

L.O.S.	parameters / port service indices	a	b	c	d
A Level	degree of congestion (DC)	1.1	2.8	3	0.5
	average waiting time (Wq)	1.5	2.8	2.5	0.7
	waiting time factor (Wq · μ)	1.5	2.8	2.5	0.7
B Level	degree of congestion (DC)	0.65	2.3	2	0.5
	average waiting time (Wq)	0.6	2.3	2	0.7
	waiting time factor (Wq · μ)	0.6	2.3	2	0.7
C Level	degree of congestion (DC)	0.37	2.3	1.8	0.5
	average waiting time (Wq)	0.4	2.3	1.8	0.7
	waiting time factor (Wq · μ)	0.4	2.3	1.8	0.7

As shown from the study results, the initial number of the port service facility will vary greatly in accordance to diverse level of service and port evaluation index. Take DC as an example, when the level of service is of category A, the determination of the initial number would be suitably to be conducted as[$\lambda/\mu+(1.1\sim2.8)\sqrt{(\lambda/\mu)}$]. When the level of service is of category B, the initial number would be suitably to be conducted as [$\lambda/\mu+(0.65\sim2.7)\sqrt{(\lambda/\mu)}$]. When the level of service is category C, the initial number would be suitably to be conducted as [$\lambda/\mu+(0.37\sim2.7)\sqrt{(\lambda/\mu)}$].When the traffic density is under 0.5, the initial parameter (β)value of facility will then be between 1.1 to 2.8. And the results obtained under different level of service from each of the port evaluation index are arranged as found in Table 4. The resolution for the initial parameter values in this study can be cited for researcher and their application.

5.CONCULSION

In this paper we considered many evaluation indices of M/M/N port queueing system , including degree of congestion (DC) , average waiting time (W_q) and waiting time factor ($W_q \cdot \mu$), and we found the number of initial facility under these indices would be change with the traffic density and the level of service. This paper present the initial number of berth in a M/M/N(∞) port queueing system would be $N_o = (\lambda / \mu) + \beta\sqrt{(\lambda / \mu)}$, and we found the relationship between the traffic density(α) and "the initial parameter value of facility" (β) under each of the evaluation index ,and the relationship can be categorized by general from equation. Therefore, we plotted the table and graphics, and it can be cited for the researcher and their application. Based on the result of this paper ,the planners could be estimate the number of initial facility from a table or a graph, so it is become easily and conveniently.

In the real world, the application of estimate the initial number of facility, such as the number of entrance of public parking facility, logistic center, supermarket and the number of ticket windows of railroad or bus station, movie theater, and so on. The results of this paper could be to reduce the system utilization time for the user. In this paper we only considered the case of M/M/N queueing system , so in the future we would be consider in addition cost function and the other queueing system such as $M/E_k/N$ or E_k /E_k/N system.

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