A SERVICE DESIGN MODEL FOR A HIGH-SPEED RAIL LINE

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Abstract: The paper presents a service design process of passenger railway for a regular interval. Two types of decisions are considered in the process. They are the passenger's choice of train service, i.e. a train demand model, and the operator's choice of service plan, i.e. a service design model. In the study, the train demand model is to solve the service choice problem with elastic origin/destination demand for a regular interval. It is a nonlinear programming model, representing a route choice problem on a service network with a generalized cost function. In the study, the service design problem is a mixed integer programming model to find a good service plan, in terms of service line, stop pattern, and train frequency, for the regular interval. In order to reflect the relationship between the operator and the passenger as well as to solve the highly related two problems together, the service design process is formulated as a bi-level program, where the operator's service design is the upper level problem, and the passenger's service choice is the lower level problem. A numerical example of Taiwan high-speed rail shows the function and performance of the bi-level programming model.

Key Words: Passenger railway, Service design, Train demand, Bi-level programming.

1. INTRODUCTION

A sequential process is practically used in planning railway service. First, in demand analysis, passenger's service choice behavior is considered to build train demand model, so as to estimate passenger volume for each train service [Nuzzolo, et al, 2000]. After that, service plan is developed to obtain high objective value with consideration of the estimated train demand [Bussieck, et al., 1997]. Therefore, in the two-step sequential process, the service design is not sensitive to passenger's choice, and the assumption of train service used in the first step may not be consistent with the result in the second step. In this study, we will integrate the two steps into one model. By solving the model, we estimate train demand and design service plan in the same time. That is, the model is to design the service plan with responsive train demand.

Some studies have shown the advantages in dealing with both operator's and passenger's objectives in the service planning for a transportation operator [Fu, et al., 1994]. In general, a passenger makes his service choice on the given service plan to maximize his utility or minimize his traveling cost. In other words, passenger is shortsighted on his choice, and will not consider other passengers or the railway operator. On the contrary, the railway operator usually makes his service plan with consideration of passenger's possible response. In order to reflect the asymmetric information relationship between operator and passenger, it is appropriate to solve the passenger's service choice and the operator's service design together as a Stackelberg game.

Bilevel programming describes a Stackelberg game as the problem, where two decision-makers, each with one's objective, act and react in a non-cooperative manner [Bard, 1998]. The most famous example in transportation planning is the equilibrium network design problem; in which the government's system optimization is the upper level problem, and the road users' route choice is the lower level problem [Boyce, 1984]. Recently, bilevel programming is proposed for the design problem with variable demand [Boyce, 1986]. In this study, the operator's design model for planning service type and train frequency is the upper-level problem, and a demand model for passenger's choice of train services is the lower level problem.

The organization of the paper is the following. In section 2, we develop a train demand model. It is a service choice model with elastic origin/destination demand. In section 3, we present a bilevel programming model for planning train service with variable demand. A sensitivity analysis based algorithm is proposed to solve the model. In section 4, we first describe the basic characteristics of Taiwan high-speed rail system. Then, a numerical example is presented to show how the model works and to examine its effects under various planning scenarios.

2. A SERVICE CHOICE PROBLEM

2.1. Choice Criteria

A service choice model is to find the passenger volume for a given service plan. The structure of the model is a passenger's choice problem. We assume the passenger is a cost minimizer, and his choice criterion is a linear generalized cost function, $c(x, f)$; where IVT is in vehicle travel time, $OVT(f)$ is out of vehicle travel time, OPC is out of pocket cost, and $CDC(x,f)$ is crowding or discomfort condition; in which x is passenger volume in the train, and f is train frequency in the regular interval.

$C(x,f) = a_0 + OPC + a_1\, IVTT + a_2\, OVTT(f) + a_3\, CDC(x,f);$

Moreover, a_1 and a_2 are the time value of IVT and OVT respectively. IVT equals to the sum of the train running time and dwell time. It is usually fixed for a specific system. OVT equals to the sum of access time, waiting time, transfer time, and egress time. It is dependent on train frequency f. OPC is the product of distance and fare rate, and the fare rate is fixed for each service type. CDC is an index of crowing condition, and it is defined as follows, where Q' is the practical capacity of the train.

$$CDC = IVT[\omega(\tfrac{x}{Q'})^{\theta}]$$

It is evident that CDC is a penalty associated with IVT. If the flow x equals to the practical capacity Q', CDC represents an increase of IVT by ω. The practical seating capacity is dependent on train frequency f. If the train is very crowded, the load factor is much bigger than 1 and the value of CDC is very big.

2.2. Service Network

Consider the example shown in Figure 1. There are five stations and four service types.

SA, SC, and SE are terminal stations for train services, and there are three service lines, (SA→SC), (SA→SE), and (SC→SE). For the service line (SA→SE), there are two stop-patterns: all stop train-service type 2, and express train-service type 3. Nodes 1 to 5 are origin and destination nodes. For example, link 1 is an access link, link 11 is an egress link, and link 19 is a transfer link. Therefore, a path from the origin to the destination represents one alternative for the passenger's choice of train service. For example, the five alternatives from SA to SE are 1-6-11-14-19-5 (service type 1 and service type 4), 1-11-13-15-16-17-5 (service type 1 and service type 2), 1-7-9-10-12-13-15-16-17-5 (service type 2), 1-7-9-10-12-14-19-5 (service type 2 and service type 4), and 1-8-18-5 (service type 3). In brief, the passenger's choice is represented as a path from the origin node to the destination node on the service network.

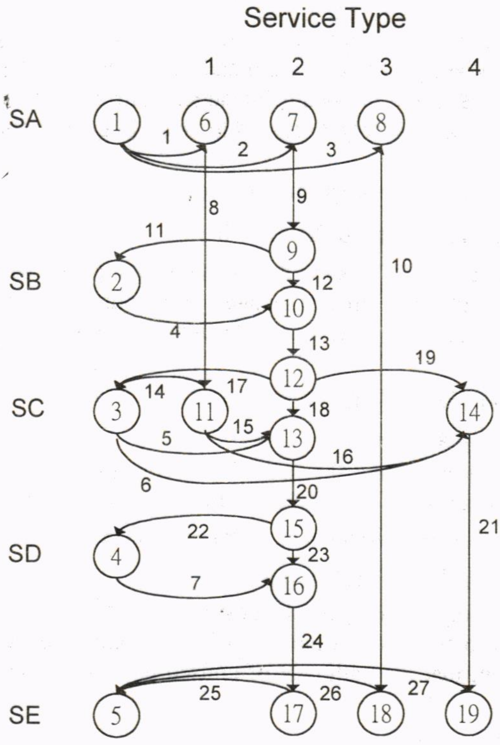

Figure 1: An Example Service Network

If the generalized traveling cost is assigned to each link appropriately, the cost of the path is exactly the cost of the choice. For example, the cost associated with the access link 1-6 is GC = $a_0 + 0 + a_1\ 0 + a_2$ OVT + a_3 0, where OVT equals to the access time and waiting time; the cost associated with the link 6-11 is GC = a_0 + OPC + a_1 IVT + a_2 0 + a_3 CDC, where OPC equals the product of distant rate and distance of link (6,11); and so on. Therefore, the passenger's service choice problem is exactly the route choice problem on the service network and with the generalized cost function.

2.3. A Train Demand Model with Elastic O/D Demand

Assuming cost minimizing behavior and independent choice for each road user, Nash equilibrium flow pattern is commonly used for the route choice problem. The user equilibrium network model deals with the interaction among the road users' route choices. For a rail line, one passenger's choice is dependent on other passengers' choices, when the seating capacity or crowding effect is considered. Because the cost function used in this study is dependent on flow variable x, the route choice problem is then written as a nonlinear programming problem.

$$\textbf{min} \quad \sum_{l} \int_0^{x_l} c_a(x)dx - \sum_{(i,j)} \int_0^{t_{ij}} d_{ij}^{-1}(x)dx \tag{1}$$

s.t.

$$\mathbf{A} \cdot \mathbf{h} = \mathbf{x} \tag{2}$$

$$\mathbf{B} \cdot \mathbf{h} = \mathbf{t} \tag{3}$$

$$\mathbf{h} \geq 0 \tag{4}$$

Where a is link index, **x** is link flow vector, *(i,j)* is origin destination index, *d(.)* is demand function, **h** is path flow vector, **t** is the origin and destination (O/D) matrix, **A** is the link-path incidence matrix, and **B** is the O/D-path incidence matrix. The model characteristics and solution algorithm have been widely discussed in detail in the literature [Bell, 1997].

The Lagrangian of the nonlinear programming model is

$$\mathbf{L}(\mathbf{h},\mathbf{t},\mathbf{z},\mathbf{k}) = \mathbf{f}(\mathbf{x} = \mathbf{A}\mathbf{h},\mathbf{t}) + \mathbf{z}^{\mathrm{T}}(\mathbf{t} - \mathbf{B}\cdot\mathbf{h}) + \mathbf{k}^{\mathrm{T}}\cdot\mathbf{h} \tag{5}$$

f(.) is the objective function (1), **z** is the dual variable associated with O/D demand constraint (3), and **k** is the Lagrange multiplier associated with constraint (4). At optimality, the first order condition for path flow and origin/destination demand are equations (6) and (7) respectively.

$$\nabla_{\mathbf{h}}\mathbf{L}(\mathbf{h},\mathbf{t},\mathbf{z},\mathbf{k}) = \nabla_{\mathbf{h}}\mathbf{f}(\mathbf{x} = \mathbf{A}\mathbf{h},\mathbf{t}) - \mathbf{B}^{\mathrm{T}}\cdot\mathbf{z} + \mathbf{k} = 0 \tag{6}$$

$$\nabla_{\mathbf{t}}\mathbf{L}(\mathbf{h},\mathbf{t},\mathbf{z},\mathbf{k}) = \nabla_{\mathbf{t}}\mathbf{f}(\mathbf{x} = \mathbf{A}\mathbf{h},\mathbf{t}) + \mathbf{z} = 0 \tag{7}$$

By the objective function (1) and the definitional constrain (2), we obtain equation (8) for path cost **g** and link cost **c.** In addition, link cost is further written as sum of flow independent cost and flow dependent cost, $c(\mathbf{x}) = c_0 + c_1(\mathbf{x})$.

$$\nabla_{\mathbf{h}}\mathbf{f}(\mathbf{h},\mathbf{t}) = \mathbf{A}^{\mathrm{T}}\cdot\mathbf{c}(\mathbf{x}) = \mathbf{A}^{\mathrm{T}}\cdot(\mathbf{c}_0 + \mathbf{c}_1(\mathbf{x})) = \mathbf{g} \tag{8}$$

At optimality, we have equation (9) for the positive path flow by equation (6). The path cost for used path equals to the dual variable **z**, which is only dependent on origin/destination. Moreover, it is evident that the path cost of unused path is greater than or equal to the dual variable **z**.

$$\nabla_{\mathbf{h}}\mathbf{f}(\mathbf{h},\mathbf{t}) = \mathbf{g} = \mathbf{B}^{\mathrm{T}}\cdot\mathbf{z} \tag{9}$$

Moreover, we get the optimal origin/destination traveling cost (10) or the optimal origin/destination demand (11), by the first order condition (7).

$$\mathbf{d}^{-1}(\mathbf{t}) = \mathbf{z} \tag{10}$$

$$\mathbf{d}(\mathbf{z}) = \mathbf{t} \tag{11}$$

2.4. Sensitivity Analysis of the Train Demand Model

Given a service plan in terms of train frequency (*f*) for each service type, the train demand model estimates equilibrium passenger volume (x) and origin destination demand (T). The sensitivity relationship between the model input (*f*) and model outputs (x, T) is written as (12) and (13).

$$\nabla_f x = \frac{\Delta x}{\Delta f} = \frac{\Delta x}{\Delta z} \cdot \frac{\Delta z}{\Delta c} \cdot \frac{\Delta c}{\Delta f} \tag{12}$$

$$\nabla_f t = \frac{\Delta t}{\Delta f} = \frac{\Delta t}{\Delta z} \cdot \frac{\Delta z}{\Delta c} \cdot \frac{\Delta c}{\Delta f} \tag{13}$$

The relationship between train frequency (*f*) and the generalized cost (*c*) is clear, so we can compute $\Delta c / \Delta f$ accordingly. In the following, we will discuss the derivation of $\Delta t / \Delta z$, $\Delta x / \Delta z$, and $\Delta z / \Delta c$.

First, by the demand function (11), we get $\Delta t / \Delta z = \nabla \mathbf{d}(\mathbf{z})$ at optimal. Secondly, the total derivative of $\mathbf{AA}^{\mathrm{T}}(\mathbf{c}_0 + \mathbf{c}_1(\mathbf{x})) = \mathbf{AB}^{\mathrm{T}}\mathbf{z}$ is written as equation (14), by equations (8) and (9).

$$\mathbf{AA}^{\mathrm{T}} \nabla \mathbf{c}_1(\mathbf{x}) \Delta \mathbf{x} = \mathbf{AB}^{\mathrm{T}} \Delta \mathbf{z} \tag{14}$$

If the inverse of $\mathbf{AA}^{\mathrm{T}}$ exists or the rank of $\mathbf{AA}^{\mathrm{T}}$ equals to the number of used path [Searle, 1971], we obtain $\Delta x / \Delta z$ as equation (15).

$$\frac{\Delta \mathbf{x}}{\Delta \mathbf{z}} = (\mathbf{AA}^{\mathrm{T}} \nabla \mathbf{c}_1(\mathbf{x}))^{-1} \mathbf{AB}^{\mathrm{T}} \tag{15}$$

Thirdly, by equations (8) and (9), we obtain the total derivative of the path cost **g**, with respect to flow variable **x** and dual variable **z**, as equation (16).

$$\Delta \mathbf{g} = \frac{\Delta \mathbf{g}}{\Delta \mathbf{x}} \Delta \mathbf{x} + \frac{\Delta \mathbf{g}}{\Delta \mathbf{z}} \Delta \mathbf{z} = \mathbf{A}^{\mathrm{T}} \nabla c_1(\mathbf{x}) \Delta \mathbf{x} + \mathbf{B}^{\mathrm{T}} \Delta \mathbf{z} \tag{16}$$

By the equation (8) of path cost and link cost, $\Delta \mathbf{g} = \mathbf{A}^{\mathrm{T}} \Delta \mathbf{c}$. If the inverse of $\mathbf{AA}^{\mathrm{T}}$ exists or the rank of $\mathbf{AA}^{\mathrm{T}}$ equals to the number of used path, it follows equation (17).

$$\Delta \mathbf{c} = \nabla \mathbf{c}_1(\mathbf{x}) \Delta \mathbf{x} + (\mathbf{AA}^{\mathrm{T}})^{-1} \mathbf{AB}^{\mathrm{T}} \Delta \mathbf{z} \tag{17}$$

Moreover, by constrain (3) and equation (11), we have the total derivative of origin/destination demand as equation (18),

$$\begin{aligned} \Delta \mathbf{t} &= \mathbf{B} \Delta \mathbf{h} + \nabla \mathbf{d}(\mathbf{z}) \Delta \mathbf{z} = \mathbf{BA}^{\mathrm{T}} (\mathbf{AA}^{\mathrm{T}})^{-1} \Delta \mathbf{x} + \nabla \mathbf{d}(\mathbf{z}) \Delta \mathbf{z} \\ &= \mathbf{BG} \Delta \mathbf{x} + \nabla \mathbf{d}(\mathbf{z}) \Delta \mathbf{z} \end{aligned} \tag{18}$$

where, $\mathbf{G} = \mathbf{A}^{\mathrm{T}} (\mathbf{AA}^{\mathrm{T}})^{-1}$.

By the equations (17) and (18), we get equation (19).

$$\begin{bmatrix} \Delta \mathbf{c} \\ \Delta \mathbf{t} \end{bmatrix} = \begin{bmatrix} \nabla \mathbf{c}_1(\mathbf{x}) & (\mathbf{AA}^{\mathrm{T}})^{-1} \mathbf{AB}^{\mathrm{T}} \\ \mathbf{BG} & \nabla \mathbf{d}(\mathbf{z}) \end{bmatrix} \begin{bmatrix} \Delta \mathbf{x} \\ \Delta \mathbf{z} \end{bmatrix} \tag{19}$$

By the inverse operation of the partitioned matrix, it follows that

$$\begin{bmatrix} \Delta \mathbf{x} \\ \Delta \mathbf{z} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{c} \\ \Delta \mathbf{t} \end{bmatrix} \tag{20}$$

$$\text{where, } \mathbf{J}_{11} = (\nabla\mathbf{c}_1(\mathbf{x}))^{-1}(\mathbf{I} - \mathbf{F}(-(\nabla\mathbf{T}(\mathbf{z}) - (\mathbf{BG})(\nabla\mathbf{c}_1(\mathbf{x}))^{-1}\mathbf{F})^{-1}\mathbf{F})^{-1}(\mathbf{BG})(\nabla\mathbf{c}_1(\mathbf{x}))^{-1})$$

$$\mathbf{J}_{12} = (\nabla\mathbf{c}_1(\mathbf{x}))^{-1}\mathbf{F}(\nabla\mathbf{T}(\mathbf{z}) - (\mathbf{BG})(\nabla\mathbf{c}_1(\mathbf{x}))^{-1}\mathbf{F})^{-1}$$

$$\mathbf{J}_{21} = -(\nabla\mathbf{T}(\mathbf{z}) - (\mathbf{BG})(\nabla c_1(\mathbf{x}))^{-1}\mathbf{F})^{-1}(\mathbf{BG})(\nabla c_1(\mathbf{x}))^{-1}$$

$$\mathbf{J}_{22} = (\nabla\mathbf{T}(\mathbf{z}) - (\mathbf{BG})(\nabla c_1(\mathbf{x}))^{-1}\mathbf{F})^{-1}$$

$$\mathbf{F} = (\mathbf{AA}^{\mathrm{T}})^{-1}\mathbf{AB}^{\mathrm{T}}$$

Therefore, we find $\Delta z / \Delta c = \mathbf{J}_{22}$.

3. A SERVICE DESIGN PROCESS

3.1. A Service Design Problem

Service design is one of the most important tasks in the strategic and tactical planning process for a railway operator. The design of passenger train service is generally considered for the regular interval, or a periodic timetable [Hooghiemstra, et al., 1996]. In a fixed interval (e.g. in one hour), service decisions are selections of service line, stop pattern, train length, and service frequency. In a rail network system, a line plan determines the routes connecting two terminal stations, and it is the basis of a timetable [Bussieck, et al., 1997]. For a service line, the main concern is selection of stop pattern, which specifies a set of stations where the train stops [Eisele, 1968]. A number of stop-patterns, such as all-stop, skip-stop, and zone-stop, have been identified and extensively studied, primarily for the many-to-one problem on a commuter rail line [Eisele, 1968; Sone, 1992]. Some results show that the pattern of zone-stop has the advantage over the others [Ghoneim, 1986]. However, for an inter-city rail line, research results show that there is no the best stop pattern [Chang, et al, 2000]. In a rail line, the decision of service line is simple, and it can be considered with stop pattern together. In this study, for a many-to-many rail line problem, the design variables are service type (service line and stop-pattern) and train frequency.

As discussed in section 1, the structure of the model is written as a bilevel programming because of the asymmetric relationship between the operator and the passenger. The upper level problem is the operator's service design model. Its decision variables are service type s and train frequency f. The lower level problem is a demand model, and its decision variables are passenger volume of each train service x, and passenger origin/destination demand t.

Maximize: Operator's objective (s, f, x, t)
(s, f)
s.t. Fleet size
Line capacity
Seating capacity
etc.
Minimize: Passenger's objective (x, t)
(x, t)
s.t. A given choice set (or service plan)

In other words, the operator makes his service decision with consideration of passenger's behavior, but the passenger is shortsighted in choosing train service. Moreover, there is no cooperation between the operator and the passenger.

3.2.A Bi-Level Programing Model

The objective of the design problem is usually to minimize the total operating cost, or to maximize the profit. The model for maximizing profit is written as follows.

$$\text{Max} \quad \sum_{l} \mathbf{P}_l \cdot \mathbf{x}_l - \left[\mathbf{C}_1 \cdot \mathbf{N} + \sum_{s} \mathbf{C}_2 \cdot \mathbf{D}_s \cdot \mathbf{f}_s + \sum_{s} \mathbf{C}_3 \cdot \mathbf{R}_s \cdot \mathbf{s}_s \right] \tag{21}$$

s.t.

$$\sum_{s} \mathbf{R}_s \cdot \mathbf{f}_s \leq \mathbf{N} \cdot \mathbf{R}_a \tag{22}$$

$$\sum_{s \in S} \mathbf{f}_s \cdot \gamma_{bs} \leq \frac{60}{\varphi_1 \cdot \dfrac{\sum_{s \in S1} \mathbf{f}_s \cdot \gamma_{bs}}{\sum_{s \in S} \mathbf{f}_s \cdot \gamma_{bs}} + \varphi_2 \cdot \dfrac{\sum_{s \in S2} \mathbf{f}_s \cdot \gamma_{bs}}{\sum_{s \in S} \mathbf{f}_s \cdot \gamma_{bs}}}, \quad \forall b \tag{23}$$

$$Q_s \cdot f_s \geq x_l \cdot \varepsilon_{ls}, \quad \forall l, s \tag{24}$$

$$\sum_{s} \alpha_{is} \cdot \mathbf{s}_s \geq 1, \quad \forall i \tag{25}$$

$$\sum_{s} \beta_{js} \cdot \mathbf{s}_s \geq 1, \quad \forall j \tag{26}$$

$$\mathbf{f}_s \leq \mathbf{M} \cdot \mathbf{s}_s, \quad \forall s \tag{27}$$

$$\mathbf{f}_s \in \mathbf{Z}^+, \quad \mathbf{s}_s \in \{0,1\} \tag{28}$$

$$\min \quad f(x = Ah, t) \tag{29}$$

s.t.

$$\mathbf{B} \cdot \mathbf{h} = t \tag{30}$$

$$\mathbf{h} \geq 0 \tag{31}$$

Notation:

P_l: Price or fare for link l.

C_1: fixed overhead cost (\$ / train).

N: fleet size (trains).

C_2: distance dependent variable cost (\$ / train-km).

D_s: running distance of service type s (km).

C_3: time dependent variable cost (\$ / train-minute).

R_s: running time of service type s (minute).

R_a: average available running time for the planning interval (minute).

φ_1, φ_2: parameters for speed group 1 and group 2.

γ_{bs}: a zero-one index. It is one if the section b is a part of the service s.

S: the set of service type.

$S1$: the set of service type with high average speed.

$S2$: the set of service type with relative low average speed.

Q_s: the seating capacity of a train.

ε_{ls}: a zero-one index. It is one if the link l is a part of the service s.

α_{is}: a zero-one index. It is one if the service s coves the origin i.

β_{js}: a zero-one index. It is one if the service s covers the destination j.

M: a big positive number.

As the example illustrated in Figure 1, a service type represents a combination of service line and stop pattern. S denotes the set of service type. The service design model is to find the optimal subset of service types, and the frequency for each service type. A zero-one variable s_s represents selection of service type s, and an integer variable f_s represents train frequency of service type s. By the constraint (27), it is evident that $f_s > o$ only if $s_s = 1$.

Profit is the difference between revenue and operating cost. Assume the fare rate is a constant ($/km) and it is not O/D dependent, the price for each link P_l is a constant, and the revenue can be computed by link. For each service type, the operating distance D_s, and operating time T_s are given, because the running and dwell times are fixed for a specific system. Moreover, the following cost parameters are given: fixed overhead cost C_1, distance dependent operating cost C_2, and time dependent variable cost C_3. With the parameters of price, cost, and service characteristics, the profit function is written as (21).

The inequality (22) is fleet size constraint for the regular interval. The inequality (23) is line capacity constrain used in practice. A section of rail line is the place between two consecutive stations. The line capacity is dependent on the speed difference among trains. In constrain (23), it is assumed that there are two speed groups. In the service planning stage, the seating capacity constraint (24) provides enough seats for passengers at each service link. Hence, it is not necessary to have a capacity constraint in the passenger's choice problem. Moreover, constraint (25) and constraint (26) make sure that there is at least one service type for each origin and each destination.

The passenger's choice model is one constrain of the service design model. The service type variable s, in the upper level problem, will decide the structure of service network for the lower level problem. As described in section 2.4, the frequency variable f, in the upper level problem, has direct effect on the link cost, and indirect effect on link passenger volume and origin/destination demand, in the lower level problem. Moreover, the decision variable x in the lower level problem is a variable of the objective function (21) and the seating capacity constraint (24) in the upper level problem.

3.3. Solution Algorithm

There are several approaches to solve a bilevel programming problem [Bard, 1998]. For an equilibrium network design problem, sensitivity analysis based methods are usually suggested for a Stackelberg solution [Bell, 1997; Yang, et al., 1998; Cho, et al., 1999]. Several sensitivity analysis methods for the equilibrium network problem have been studied [Tobin, et al, 1988; Cho, et al., 2000]. The iterative sensitivity algorithm procedure used in the study is listed in the following.

Step 0 (Initialization)

Set k=0, and find an initial solution f^k and s^k with the set of service type S.

Step 1 (Lower level problem)

Solve the passenger's choice problem for $x,^k$ and t^k, given f^k and s^k. The nonlinear programming problem is solved by the Frank-Wolfe algorithm or the method of successive averages.

Step 2 (Sensitivity analysis)

Compute the sensitivity of train frequency f to link flow x as $\nabla_f x^k$ and $\nabla_f t^k$.

Step 3 (Linear reaction function)

Set the linear reaction functions, $x^{k+1} = x^k + \nabla_f x^k (f^{k+1} - f^k)$ and $t^{k+1} = t^k + \nabla_f t^k (f^{k+1} - f^k)$.

Step 4 (Upper level problem)

Solve the upper level problem for f^{k+1} given the linear reaction functions. The upper level problem is solved by the branch and bound method.

Step 5 (Convergence test)

If it is converged, stop; otherwise, *k*=*k*+*1* and go to step 1.

4. CASE STUDY

4.1. Data

The model developed in the study was motivated by the high-speed rail project in Taiwan [Lin, 1995]. The HSR system is about 340-kilometer long, and located along the western corridor of Taiwan. It connects three metropolitans, and 7 cites of medium size. In the paper, we use a test example of 5 stations and 7 service types to demonstrate the effectiveness of the model. Tables 1 and 2, and Figure 2 show the relevant data inputted to the model. Table 1 is the input parameters of the bilevel model. In Table 2, the figures in parenthesis are respectively the distance (km) and train running time (minute). The train running time is the direct running without any intermediate stop. The origin/destination demand function, $t_{ij} = a - b \cdot c_{ij}$, is estimated using the forecasted demand data for 8 a.m. to 9 a.m. in 2003. The parameter used in the demand model is listed in Table 3.

Table 1: Input Parameters of the Model

Fare rate：3.54 (NT$/km)

C_1 ：fixed overhead cost in the regular interval = 4551(NT$/train)

C_2 ：operating distance dependent variable cost = 91.4536(NT$/train-km)

C_3 ：operating time dependent variable cost =825.5 (NT$/train-hour)

S ：the set of feasible service types $S = \{s1, s2, s3, s4, s5, s6, s7$

N ：fleet size =30 trains

Q ：seating capacity =800 seats/train

Cost function: $a_1=a_3=5.5$, $a_2=16.23$, $\omega=0.15$, $\theta=4$.

Table 2: Distance and Train Running Time

Destination Origin	SA	SB	SC	SD	SE
SA	(- , -)	(66, 27)	(159.5 ,42)	(308.4 , -)	(338.1 ,82)
SB	(66 , 27)	(- , -)	(93.6 , 24)	(242.4 , -)	(272.8 , -)
SC	(159.5 ,42)	(93.6,24)	(- , -)	(148.8, -)	(178.6, 47)
SD	(308.4 , -)	(242.4 , -)	(148.8, -)	(- , -)	(30.4 , 11)
SE	(338.1 ,82)	(272.8 ,-)	(178.6, 47)	(30.4, 11)	(- , -)

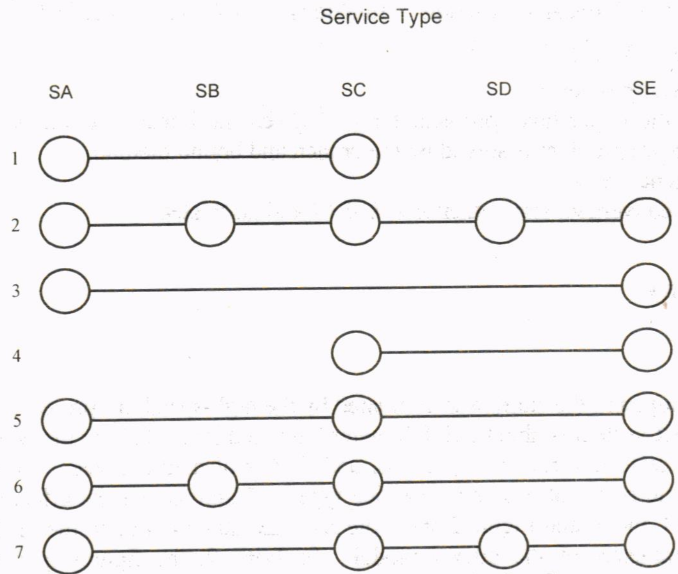

Figure 2: The Service Type in the Set S

Table 3: Origin/Destination Demand Function

Origin/Destination	a	b
SA-SB	1156.6	-0.39
SB-SC	827.3	-0.42
SA-SC	3715.1	-1.28
SC-SD	2859.8	-1.17
SB-SD	719.9	-0.22
SA-SD	3582.8	-0.80
SD-SE	1498.1	-0.60
SC-SE	4461.2	-1.51
SB-SE	2126.1	-0.66
SA-SE	3416.4	-0.44

4.2. Testing Results and Discussion

The major findings of the study are stated in the following.

1. Service Plan

The optimal service plan obtained by the model is illustrated in Figures 3 for profit maximum, or in Figure 4 for cost minimum. The number associated with each link is passenger volume or passenger trips, and the number in parenthesis is load factor. For both profit and cost objectives, the same service types are selected. They are service type 2 of all-stop train, service type 3 of express train, service types 6 and 7 of limited express train. The total train frequency is 12-train per hour for profit maximum, and 11-train per hour for cost minimum. There is one more all-stop train for profit maximum than cost minimum.

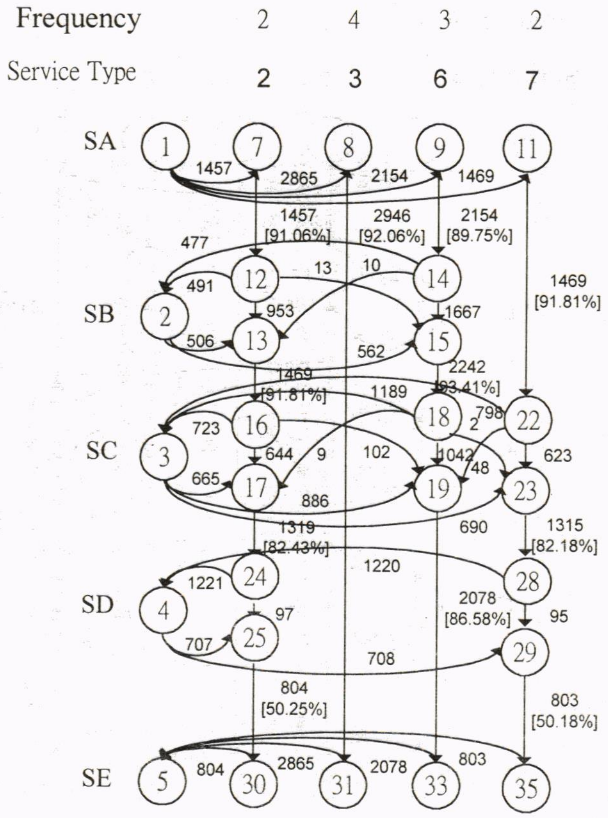

Figure 3: The Flow Pattern for Minimizing Total Operating Cost

2. Equilibrium Flow Pattern

As the equilibrium origin/destination demand shown in Table 4 and Table 5, maximum profit operation approaches a high demand value and cost minimum operation approaches a low demand value. The passenger volume of each train service is shown in Figure 3 and Figure 4. The number of passengers of all-stop train for profit maximum is higher than that for cost minimum. However, the load factor patterns illustrated in Figure 3 and Figure 4 are not much different, partly because the train frequency for cost minimum is one train few.

Table 4: Equilibrium Origin/Destination Demand for Cost Minimum

Destination / Origin	SA	SB	SC	SD	SE
SA	0	968	2,378	1,587	3,012
SB	968	0	332	317	419
SC	2,378	332	0	537	1,704
SD	1,587	317	537	0	1,415
SE	3,012	419	1,704	1,415	0

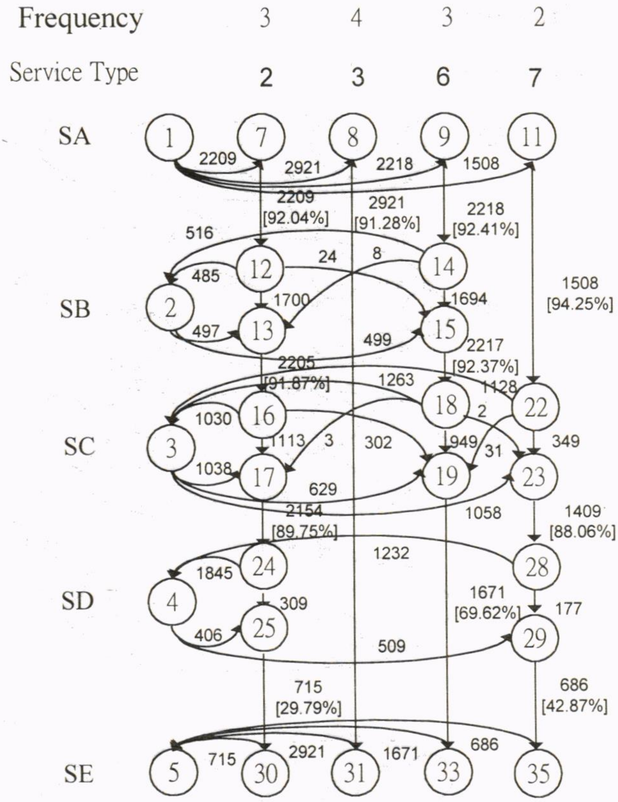

Figure 4: The Flow Pattern for maximizing Profit

Table 5: Equilibrium Origin/Destination Demand for Profit Maximum

Destination / Origin	SA	SB	SC	SD	SE
SA	0	1001	2,978	1,645	3,232
SB	1001	0	443	254	299
SC	2,978	443	0	1,178	1,547
SD	1,645	254	1,178	0	915
SE	3,232	299	1,547	915	0

3. The Sensitivity of Train Frequency

In the case of minimizing total operating cost, the change of passenger flow in response to an increase of the frequency of each is shown in Table 6. An increase of direct express train of service type 3 from SA to SE results in an increase of 248 passengers, and a decrease of the passenger flow for other service types, e.g. a decrease of 147 trips for service type 7. Hence, the sensitivity of train frequency to the passenger flow pattern is evident. The sensitivity results of train frequency are not only essential for solving the bi-level mathematical programming model, but also useful in timetable construction process. When we extend a regular peak hour timetable to be one-day timetable, the sensitivity results of stop pattern and train frequency are used for modifying the timetable for off-peak periods. In addition, many types of sensitivity results for service characteristics, e.g. sensitivity of fare level and fare structure, can be computed for detail and practical discussion.

Table 6: Sensitivity of Train Frequency for Cost Minimum

Passenger Flow	Frequency						
	S1	s2	s3	s4	s5	s6	s7
SA→SE	-51	-27	248	-27	-71	-32	-64
SA→SC	154	-56	-147	95	135	-57	137
SC→SE	88	-51	-72	131	134	107	-48
SA→SB	-103	83	-101	-73	-64	89	-73
SB→SC	-103	83	-101	-73	-64	89	-73
SC→SD	-37	78	-176	-104	-63	-75	112
SD→SE	-37	78	-176	-104	-63	-75	112

4. The sensitivity of O/D demand

In the case of maximizing profit, the change of passenger flow in response to one trip increase of an O/D demand is shown in Table 7. For example, 71.3% of the increased trip from SA to SE will use service type 3 from SA to SE directly. 18.2% will use service type 7 from SA to SC. Besides, many types of sensitivity analysis for demand characteristics, e.g. sensitivity of O/D demand pattern, can be computed for detail and practical discussion.

Table 7: Sensitivity of O/D Demand

Passenger Flow Change Rate	The Sensitivity of O/D Demand			
	SA/SE	SB/SE	SA/SC	SC/SE
SA→SE	0.713	0	0	0
SA→SC	0.182	0	0.658	0
SC→SE	0.18	0.587	0	0.724
SA→SB	0.105	0	0.342	0
SB→SC	0.105	1	0.342	0
SC→SD	0.142	0.413	0	0.276
SD→SE	0.142	0.354	0	0.276

5. Solution Algorithm

The convergence of the iterative sensitivity algorithm for solving the bilevel program with the numerical example is acceptable. The method does not get a descent or ascent direction at each iteration, however, it converges to the same optimal solution with different initial solutions.

5. CONCLUDING REMARKS

The paper develops a bilevel model for planning train service of a high-speed rail line. The design variables in the upper problem are service type and train frequency, and the decision variable in the lower problem is passenger flow for each origin/destination and train service. An iterative sensitivity algorithm is proposed to solve the model with the data from Taiwan high-speed rail project. In brief, the paper demonstrates a way for railway service planning problem, to solve the interaction between demand and supply quantitatively.

The paper is only an initial step for railway planning problem with variable train demand. Various further studies have to be done so as to clarify the relationship between the operator's marketing variables (e.g. price level and price structure) and the passenger's choice behavior (e.g. stochastic choice, and mode choice).

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Reference

Assad, A. A.(1982) A Class of Train-scheduling Problems. **Transportation Science,** Vol.16, pp.281-310.

Bell, M. G. H. and Y. Iida.(1997) **Transportation Network Analysis**. John Wiley & Sons, Inc., U.S.A.

Brebbia, R.J. Hill, G. LeBlanc, L. J., E. K. Morlok and W. P. Pierskalla.(1975) An Efficient Approach to Solving the Road Network Equilibrium Traffic Assignment Problem. **Transportation Research**, Vol. 9, pp.309-318.

Bussieck, M.R., P. Kreuzer and U. T. Zimmerman.(1997) Optimal Lines for Railway Systems. **European Journal of Operational Research**, Vol.96, pp.54-63.

Chang, Y-H, Yeh, C-H, and Shen, C-C.(2000) A Multiobjective Model for Passenger Train Services Planning: Application to Taiwan's High-speed Rail Line. **Transportation Research,** Vol.34B, pp.91-106.

Cho, H. J. and S.C. Lo.(1999) Solving Bilevel Network Design Problem Using a Linear Reaction Function without Nondegeneracy Assumption. **Transportation Research Record** 1667, pp.96-106.

Cho, H-J, T. E. Smith, and T. L. Friesz.(2000) A Reduction Method for Local Sensitivity Analyses of Network Equilibrium Arc Flows. **Transportation Research**, Vol. 34B, pp.31-51.

Eisele, D. O.(1968) Application of Zone Theory to a Suburban Rail Transit Network. **Traffic Quarterly**, Vol. 22, pp.49-67.

Fu, Z., and M. Wright.(1994) Train Plan Model for British Rail Freight Services through the Channel Tunnel. **Journal of the Operational Research Society**, Vol. 45, pp.384-391.

Ghoneim, N. S. and S. C. Wirasinghe.(1986) Optimum Zone Structure During Peak Periods for Existing Urban Rail Lines. **Transportation Research**, Vol. 20B, pp.7-18.

Hooghiemstra, J. S.(1996) Design of Regular Interval Timetables for Strategic and Tactical Railway Planning. In **Computers in Railway V**, Vol. 1, Edited by. J. Allan, C. A. pp.393-402,

Lin, C-Y. The high-speed rail project and its privatization in Taiwan. *Rail International*, 1995. pp.175-179.

Nuzzolo, A., Crisalli, U., and Gungemi, F.(2000) A Behavioral Choice Model for the Evaluation of Railway Supply and Pricing Policies. **Transportation Research**, Vol. 34A, pp.

395-404.

Tzeng, G-H and Shiau, T-A.(1988) Multiple Objective Programming for Bus Operation: a Case Study for Taipei City. **Transportation Research**, Vol. 22B, pp. 195-206.

Searle, S. R., (1971), Linear Models. New York: Wiley.

Sone, S.(1992) Novel Train Stopping Patterns for High-frequency, High-speed Train Scheduling. **Computers in Railways III**- Volume 2 eds. T.K.S. Murthy, J. Allan, R.J. Hill, G. Sciutto, S. Sone, Computational Mechanics Publications, U.K, pp.107-118.

Tobin, R. L. and Friesz, T. L.(1988) Sensitivity Analysis for Equilibrium Flow. **Transporation Science**, Vol. 22, pp.242-250.

Yang, H., and Bell, G. H.(1998) Models and Algorithms for Road Network Design: a Review and Some Developments. **Transport Reviews**, Vol. 18, pp.257-278.