FARE AND SERVICE HEADWAY FOR A HIGH SPEED RAIL SYSTEM WITH PRIVATE SECTOR INVOLVEMENT

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Abstract: In order to release the government financial loads and to implant the private enterprise vitalities, the Taiwan government decides to build its first high speed rail (HSR) system by a BOT (Build-Operation-Transfer) approach. As a private invested project, the operator usually sets the "maximum profit" as its business objective, while the government pursues the maximum social welfare. Comparing to other transport systems, the capital investment of high-speed rail project is massive and unique; therefore, the operator should have a different consideration regarding to fare and headway arrangements from public owned transport systems. Suitable price and level of services are the main factors not only to attract more users but also to benefit the operator. A multi-logit model is used to predict the high-speed rail demand in this study. An analytic optimization model with the objectives of maximum social welfare, maximum social welfare under break-even constraint, and the maximum profit are established to analyze the optimal fare and headways for the HSR system. Results of various objectives are compared. Policy implications and recommendations are discussed in this paper.

Keywords: High speed rail (HSR), BOT, Optimal fare, Headway, Break-even

1. INTRODUCTION

The speed advantages, service quality, market separation, and peak hour factors are taken into consideration for the present pricing mechanism of HSR system in Japan, Germany, and France. The competitive abilities of HSR systems and its service value always reflected on their pricing mechanism, and some pricing flexibilities for operators are set by these governments to make sure that operators can maintain their operation more efficient when needed.

The basic service HSR fare for Taiwan is set by the Bureau of Taiwan High-Speed Rail

(BOTHSR) according to the study of MVA Asia Limited in 1997. While the suggested HSR fare (3.11 NTD/passenger-km, value of year 1994) from the study is simply calculated as the 75% of air transport fare of city pairs in the Taiwan west corridor in 1994, without any detailed calculations and considerations. Therefore, this fare could not entirely reflect the true needs of both operator and users of HSR system in Taiwan.

If the HSR fare is set higher, the return of private investment may higher than expected, and it may make the private operator gets extra profit from the users and even the whole society. In contrast, strictly fare constraints will decrease the private operator's willingness and interests of investment. Thus this research aims to optimize the fare and service headway of the HSR system at the same time, and to satisfy both operator and users by a reasonable and acceptable pricing mechanism.

2. MODEL DEVELOPMENT

All the variables and parameters used for the model establishment in this study are listed in Table 1.

Table 1. Variables and Parameters

A_1	Model parameter. Utility of taking high speed rail will decrease as the running time of high speed rail (in-vehicle time and waiting time) rises, thus A_1 should be negative
a	Model parameter. Utility of taking high speed rail will decrease as the headway rises, thus "a" should be negative
α^k_{ij}	Model parameter, where k=1~m. Standing for the other m specific parameters in utility function excepting travel cost
B	Social welfare (NTD/day)
B_1	Model parameter. Utility of taking high speed rail will decrease as the travel cost rises, thus B_1 should be negative
C_f	Fixed cost of operator (NTD/day)
C_{ij}	A constant consisting by other variables, model parameters and specific constants of the model
C_L	Travel cost of passengers (NTD/trip), it could be presented as $C_L=PD_{ij}$ under the standard fare assumption, where P is the fare per passenger-km
D_{ij}	Travel distance between station i and j (km)
g^k_{ij}	Other variables in utility function
h_{ij}	Service headway of high speed rail from station i to j (min)
K	Capacity constrain of high speed rail system (seat-km)
Q_{ij}	Total HSR demand of west corridor of Taiwan from station i to j (trip/day)

r	Unit cost of operation (NTD/seat-km)
S_p	Operator's surplus, e.g., profit (NTD/day)
t_{ij}	Running time of high speed rail from station i to j (min)
T_{ij}	Total demand (derived demand not included) of west corridor of Taiwan from station i to j (trip/day)
T'_{ij}	Total demand of west corridor of Taiwan from station i to j (trip/day)
TC	Total operator's cost (NTD/day)
TR	Total operator's revenue (NTD/day)
U_{HSRij}	Utility of taking high speed rail from station i to j
X_{ij}	Derived rate of high speed rail demand from station i to j
ρ_{ij}	Load factor between station i and j (passenger-km/seat-km)

2.1 Demand and Cost Functions

The utility function for passengers taking high-speed rail from station i to j can be expressed as Equation (1). It is assumed that utility is affected by travel cost, travel time, service headway, and other variables.

$$\begin{aligned} U_{HSR_{ij}} &= B_1 \times C_L + A_1(t_{ij} + ah_{ij}) + \sum_k (\alpha_{ij}^k g_{ij}^k) \\ &= B_1 \times C_L + A_1(t_{ij} + ah_{ij}) + C_{ij} \end{aligned} \tag{1}$$

According to the Logit model, probability of taking high-speed rail from station i to j can be expressed as Equation (2).

$$Y_{ij} = \frac{e^{U_{HSR_{ij}}}}{e^{Uair_{ij}} + e^{Urail_{ij}} + e^{Ubus_{ij}} + e^{Ucar_{ij}} + e^{U_{HSR_{ij}}}} \tag{2}$$

It is assumed that level of service of HSR, level of service, and fare of competitive modes will be constants when the fare of HSR changes. Thus Equation (2) can be expressed into Equation (3), where $E = e^{Uair_{ij}} + e^{Urail_{ij}} + e^{Ubus_{ij}} + e^{Ucar_{ij}}$, is a constant.

$$Y_{ij} = \frac{e^{U_{HSR_{ij}}}}{E + e^{U_{HSR_{ij}}}} \tag{3}$$

From the utility and Logit model, demand function of HSR thus obtained as Equation (4), where X_{ij} stands for derived rate of high-speed rail demand from station i to j.

$$\sum_i \sum_j Qij = \sum_i \sum_j TijYij \times (1 + X_{ij}) \tag{4}$$

The total cost function of HSR is obtained as Equation (5), where C_f is fixed cost, r is variable cost, and ρ_{ij} is the load factor between station i and station j.

$$TC = C_f + r \times \sum_i \sum_j \frac{T_{ij} \times Y_{ij} \times (1 + X_{ij}) \times D_{ij}}{\rho_{ij}} \tag{5}$$

2.2 Modeling

The total social welfare is consisted by users' surplus and operator's surplus, as shown in Equation (6), where users' surplus is expressed as Equation (7). The detail computing process of users' surplus S_c will not be discussed here. Operator's surplus S_p is shown as Equation (8).

$$B = S_c + S_p \tag{6}$$

$$Sc = \sum_i \sum_j \frac{T'_{ij}}{B_1} LN\left(\frac{E}{E + e^{B_1 C_L + A_1 (t_{ij} + ah_{ij}) + C_{ij}}}\right) \tag{7}$$

$$S_p = TR\text{-}TC \tag{8}$$

Since TR=C_L* Equation (4), operator's surplus S_p then becomes Equation (9).

$$\begin{aligned} S_p &= \sum_i \sum_j T_{ij} \times Y_{ij} \times (1 + X_{ij}) C_L - C_f - r \times \sum_i \sum_j \frac{T_{ij} \times Y_{ij} \times (1 + X_{ij}) \times D_{ij}}{\rho_{ij}} \\ &= \sum_i \sum_j T'_{ij} \times Y_{ij} \times C_L - C_f - r \times \sum_i \sum_j \frac{T'_{ij} \times Y_{ij} \times D_{ij}}{\rho_{ij}} \\ &= \sum_i \sum_j T'_{ij} Y_{ij} D_{ij} (P - \frac{r}{\rho_{ij}}) - C_f \end{aligned} \tag{9}$$

Finally, social welfare B is obtained as Equation (10).

$$B = \sum_i \sum_j \frac{T'_{ij}}{B_1} LN\left(\frac{E}{E + e^{B_1 C_L + A_1 (t_{ij} + ah_{ij}) + C_{ij}}}\right) + \sum_i \sum_j T'_{ij} Y_{ij} D_{ij} (P - \frac{r}{\rho_{ij}}) - C_f \tag{10}$$

From the social welfare, users' surplus, operator's surplus, and capacity constraint of HSR system, three objectives of optimization can be analyzed as shown below.

Maximum Social Welfare

$$\text{MAX} \quad B=\sum_i \sum_j \frac{T'_{ij}}{B_1} LN\left|\frac{E}{E+e^{B_{ij}P+A_1(t_{ij}+ah_{ij})+Cij}}\right| + \sum_i \sum_j T'_{ij} Y_{ij} D_{ij} (P-\frac{r}{\rho_{ij}})-C_f$$

$$\text{S.T.} \quad T'_{ij} Y_{ij} D_{ij} \leq K \tag{11}$$

Maximum Social Welfare Under Break-even Condition

$$\text{MAX} \quad B=S_C+S_P$$

$$\text{S.T.} \quad S_P=0 \; (TR=TC)$$

$$T'_{ij} Y_{ij} D_{ij} \leq K \tag{12}$$

Here the Laplace multiplier λ could be used to solve Equation (12), and it could be transformed into Equation (13), as shown below.

$$\text{MAX} \quad Z=S_C+\lambda\,(S_P-0)$$

$$Z=\sum_i \sum_j \frac{T'_{ij}{}'}{B_1} LN\left|\frac{E}{E+e^{B_{ij}P+A_1(t_{ij}+ah_{ij})+Cij}}\right| + \lambda\left[\sum_i \sum_j T'_{ij} Y_{ij} D_{ij} (P-\frac{r}{\rho_{ij}})-C_f\right]$$

$$\text{S.T.} \quad T'_{ij} Y_{ij} D_{ij} \leq K \tag{13}$$

Maximum Operator's Profit

$$\text{MAX} \quad S_p=\sum_i \sum_j T'_{ij} Y_{ij} D_{ij} (P-\frac{r}{\rho_{ij}})-C_f$$

$$\text{S.T.} \quad T'_{ij} Y_{ij} D_{ij} \leq K \tag{14}$$

3. NUMERICAL STUDY

3.1 Assumptions

The latest operation arrangement of Taiwan HSR stations is over 10 stations, while in this research the primary operation condition with 10 stations will be applied, shown as Figure 1 and Table 2. Assume that full line operation will be carried out in 2003 without delay. The data of total demand of west Taiwan corridor Tij comes from the survey of Ministry of Transportation and Communication of Taiwan (MOTC). Average load factor is set as 0.74, and capacity constraint is 300,000 seats × 340 km. Unit cost (1995 as base year) is calculated as 1.0575 NTD/seat-km (detailed calculation not shown here). Loan payback is 90,899,149.68 NTD/day, while it is zero in the year 2003, 2023, 2028, and 2033. The franchised operation revenue feedback to government is 9,863,013.699 NTD/day, and zero in 2003. Self owned capital of private operator is 127.9 billion NTD, and total construction cost is assumed to be 325.9 billion NTD. The revenue from subordinate business will not be considered here.

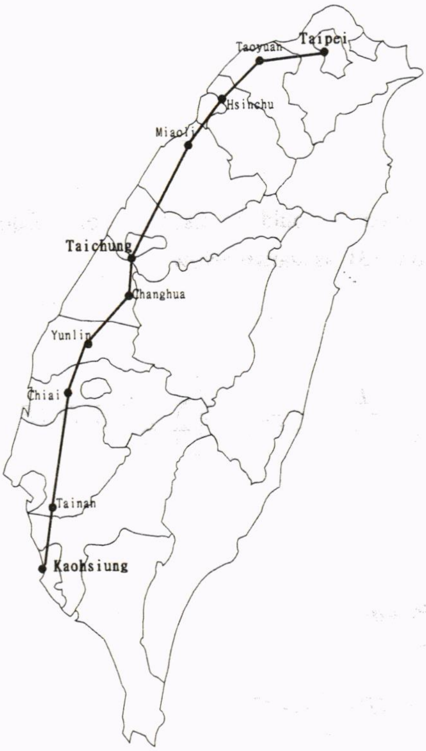

Figure 1. Operation Map of Taiwan High Speed Rail System

Table 2. Route Schedules of Taiwan High Speed Rail System

Schedule	Stations
A	**Taipei- Kaohsiung**
B	**Taipei- Taichung-** Tainan- **Kaohsiung**
C	**Taipei-** Taoyuan- Hsinchu- Chiai- Tainan- **Kaohsiung**
D	**Taipei-** Miaoli- **Taichung-** Changhua- Yunlin- Chiai- Tainan- **Kaohsiung**
E	**Taipei-** Taoyuan- Hsinchu- Miaoli- **Taichung-** Changhua- Yunlin- **Kaohsiung**
F	**Taipei-** Taoyuan- **Taichung**

3.2 Optimization Results

The numerical results of maximum social welfare, break-even conditions, and maximum profit are shown in Table 3~8.

Maximum Social Welfare

Table 3. Numerical Results under Maximum Social Welfare

Year	2003	2008	2013	2018	2023	2028	2033
Optimal fare (NTD/passenger-km) * Value of year 1995	1.007	1.008	1.009	1.011	1.012	1.013	1.014
Optimal fare (NTD/passenger-km)	1.264	1.504	1.788	2.127	2.529	3.007	3.574
Profit1 (NTD/day)	-114848	-126340	-107105	-45101	55353	232955	530047
Profit (NTD/day)	-114848	-100888503	-100869269	-100807265	-9807661	-9630058	-9332966
Users' surplus (NTD/day)	200761103	307565660	458576018	669420037	930088819	1272159401	1718906449
Social welfare (NTD/day)	200646255	206677157	357706750	568612772	920281159	1262529343	1709573484
Users' surplus/Social welfare	100.06%	148.81%	128.20%	117.73%	101.07%	100.76%	100.55%

Annotation: Loan payback and feedback for government are not included in profit1.

Table 4. Headway under Maximum Social Welfare (min)

Schedule/Year	2003	2008	2013	2018	2023	2028	2033
A	72.3973	70.9482	70.928	70.1743	70.9228	70.9255	70.9352
B	23.4475	22.981	22.9698	22.7286	22.9703	22.97	22.9733
C	13.5070	13.5422	13.5357	13.779	13.5359	13.5358	13.5377
D	38.7852	40.7121	40.6929	40.2655	40.6921	40.6921	40.6969
E	22.8702	22.585	22.7908	22.5509	22.7902	22.7908	22.7769
F	28.9633	27.0845	27.0713	27.5579	27.0718	27.0715	27.0754

It is shown that from year 2003~2033, the optimal fare changes from 1.007~1.014 NTD/passenger-km（value of the year 1995）. Social welfare changes from 73.24~623.99 billons NTD/year, and proportion of users' surplus over social welfare all exceeds 100%. While under the fare of maximum social welfare, private operator will not able to earn back the investment cost and get any profits. Thus the pricing of maximum social welfare will not be suitable for the private invested high-speed rail system.

Maximum Social Welfare Under Break-Even Condition

Table 5. Numerical Results under Break-even Condition

Year	2003	2008	2013	2018	2023	2028	2033
Optimal fare (NTD/passenger-km) * Value of year 1995	1.008	2.351	1.872	1.609	1.049	1.039	1.032
Optimal fare (NTD/passenger-km)	1.265	3.507	3.316	3.386	3.114	3.084	3.638
Profit1 (NTD/day)	0	100762163	100762163	100762163	9863014	9863014	9863014
Profit (NTD/day)	0	0	0	0	0	0	0
Users' surplus (NTD/day)	200661104	306565659	44596018	661220037	928988819	1253159401	1709206449
Social welfare (NTD/day)	200661104	306565659	44596018	661220037	928988819	1253159401	1709206449
Users' surplus/Social welfare	100%	100%	100%	100%	100%	100%	100%

Annotation: Loan payback and feedback for government are not included in profit1.

Table 6. Headway under Break-even Condition (min)

Schedule/Year	2003	2008	2013	2018	2023	2028	2033
A	72.3973	71.0406	70.9279	70.1743	70.9227	70.9254	70.9350
B	23.4475	23.0109	22.9698	22.7286	22.9703	22.9700	22.9733
C	13.5070	13.5599	13.5356	13.7790	13.5359	13.5357	13.5377
D	38.7852	40.7651	40.6928	40.2655	40.6920	40.6920	40.6968
E	22.8702	22.6144	22.7907	22.5509	22.7901	22.7908	22.7769
F	28.9633	27.1197	27.0713	27.5579	27.0717	27.0715	27.0753

It is shown that from year 2003~2033, the optimal fare changes from 1.008~2.351 NTD/passenger-km（value of the year 1995）. Social welfare increases from 16.28~623.86 billons NTD per year, and all the proportion of users' surplus over social welfare is 100% since the break-even condition. While under the fare of break-even condition, profits of private operator will be negative because the self-owned capital from private operator is not taken into calculation in this study. Thus the pricing under break-even conditions will not be suitable for the private investment high-speed rail system.

Maximum Operator's Profit

Table 7. Numerical Results under Maximum Operator's Profit

Year	2003	2008	2013	2018	2023	2028	2033
Optimal fare (NTD/passenger-km) * Value of year 1995	3.614	3.677	3.742	3.795	3.838	3.879	3.921
Optimal fare (NTD/passenger-km)	4.539	5.485	6.629	7.985	9.591	11.513	13.822
Profit1 (NTD/day)	81792721	125920804	177013533	2767514542	324870960	529081812	691274959
Profit (NTD/day)	81792721	25158641	76251370	2666752379	315007946	519218799	681411945
Users' surplus (NTD/day)	763283116	116825976	163094360	2537777920	296858282	482210738	628336988
Social welfare (NTD/day)	845075837	141984618	239345730	5204530298	611866229	1001429537	1309748933
Users' surplus/Social welfare	90.32%	82.28%	68.14%	48.76%	48.52%	48.15%	47.97%

Annotation: Loan payback and feedback for government are not included in profit1.

Table 8. Headway under Maximum Operator's Profit (min)

Schedule/Year	2003	2008	2013	2018	2023	2028	2033
A	72.3973	71.0501	70.9277	70.1743	70.9225	70.9252	70.9349
B	23.4475	23.014	22.9697	22.7286	22.9702	22.9699	22.9732
C	13.5070	13.5617	13.5356	13.7790	13.5358	13.5357	13.5376
D	38.7852	40.7706	40.6927	40.2655	40.6919	40.6919	40.6967
E	22.8702	22.6174	22.7906	22.5509	22.7901	22.7907	22.7768
F	28.9633	27.1234	27.0712	27.5579	27.0717	27.0714	27.0753

It is shown that from year 2003~2033, the optimal fare changes from 3.614~3.921 NTD/passenger-km（value of year 1995）.Take year 2008 for example, deduct the loan payback and feedback for government, the private operator can still makes the profit of 25 millions NTD per day. Although the social welfare under this condition is not the best, but the average proportion of users' surplus over social welfare still over 60%. Thus we may say that the maximum profit pricing will be suitable for the private investment high-speed rail system, and makes good for both users and operator.

3.3 Discussion

Government Assessed Fare

Since the governmental assessed fare is the basic fare for Taiwan HSR system, it should be considered in this study too. The cumulative revenue, cumulative cost, and cumulative profit of operator under governmental assessed fare are analyzed as Figure 2 (value of year 1995). It is shown that operator's revenue and expenditure will reach break-even and start to make profit in year 2013 under the governmental assessed fare.

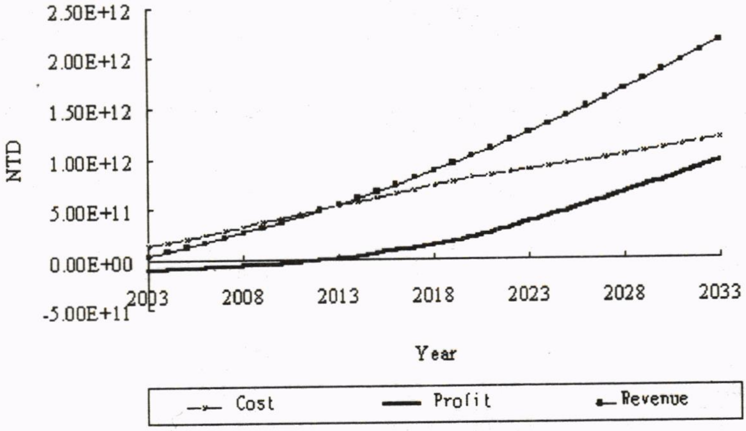

Figure 2. Relations between Cumulative Cost, Cumulative Revenue, and Cumulative Profit under Government Assessed Fare

Investment Effectives

Comparison for effectives from the fare of maximum profit pricing and governmental assessed is listed on Table 9. It is shown that under the government assessed fare, users' surplus will higher than that under maximum profit conditions. While the results of maximum profit still seems to be acceptable.

Table 9. Effects under Maximum Profit and Government Assessed Fare (billion NTD)

	Maximum profit	Government assessed fare
Total operator's cost	1,030.43	1,180.45
Total social welfare	2,653.67	3,084.42
Total users' surplus	1,536.25 (57.89%)	2,120.92 (68.76%)
Total operator's surplus	1,117.41 (42.11%)	963.50 (31.24%)

Annotation: Numbers in the brackets are proportions on social welfare

4. SENSITIVITY ANALYSIS

Load Factor

The load factors are assumed to be 80% of minimum level of service for all OD pairs in this study. In order to make sensitivity analysis, the percentage of level of service changes from 50%~100% here, and the average load factors then changes from 0.46~0.92. Relation of fare and load factor is shown in Figure 3. It is shown that optimal fare changes from 3.6~4.1 NTD/passenger-km, with a maximum difference of 0.5. Taking the distance from Taipei to Kaohsiung for example (340 km), the maximum difference of fare will about 170 NTD, thus the headway seems not to make a significant influence on fare.

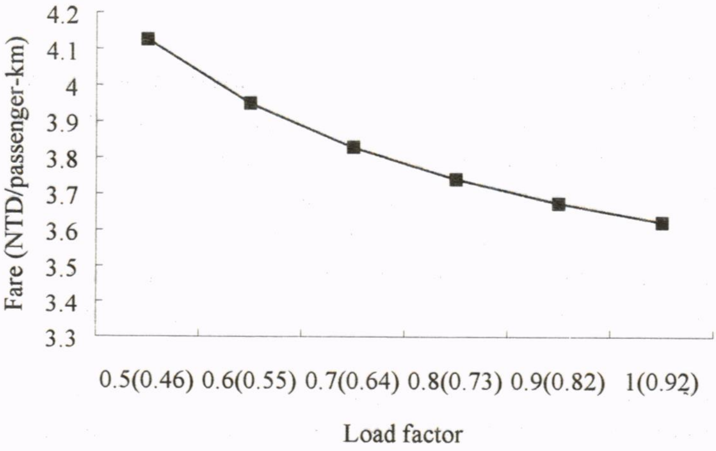

Figure 3. Relation between Optimal Fare and Load Factor

Unit Cost

Take the year 2013 for example, relation of optimal fare and unit cost is shown in Figure 4. It is shown that optimal fare changes from 3.6~4.1 NTD/passenger-km when increase rate of unit cost changes from –20%~50%, only makes the difference of 0.5 NTD/passenger-km. And it is found that the relation between them is almost linear, when unit cost increases 10%, the optimal fare will makes an increase about 1.5%.

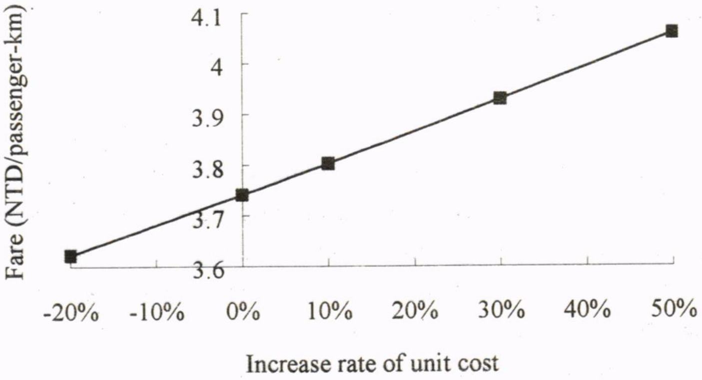

Figure 4. Relation between Optimal Fare and Unit Cost

Average Headway

System average headway is analyzed here to clearly understand the relation between service headway, profit, and fare. The average headway of system optimization is about 34 min, and relation of profit and average headway is shown in Figure 5. It is shown that profit will decrease about 5 billion NTD when average headway changes from 34~50 min; in contrast, profit will increase about 10 billion NTD when average headway changes from 34~17 min. Thus the decrease of headway increases the operator's profit rather than decreases it, and the decrease of headway seems to bring significant benefits to private operator.

Relation between average headway and optimal fare is shown in Figure 6. It is shown that optimal fare decreases from 4.01~3.61 when average headway increases from 17~59 min, and makes a difference of 0.4 NTD/passenger-km. While the decreasing rate of fare becomes smaller as the average headway increases. Taking the distance from Taipei to Kaohsiung for example, the maximum difference of fare will about 136 NTD, thus the headway seems not to make significant influences on fare.

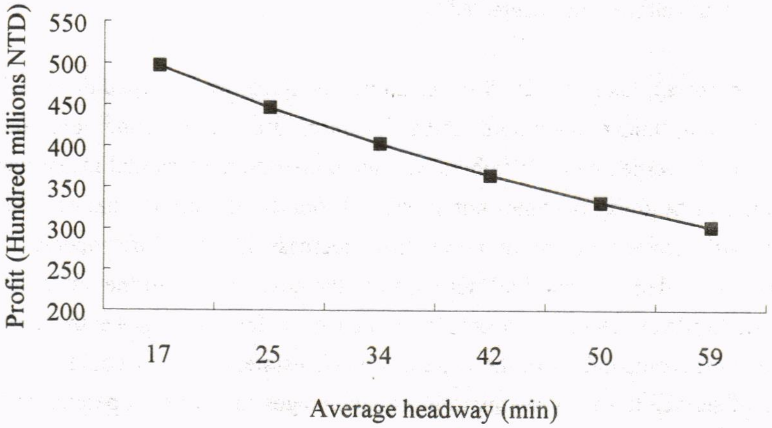

Figure 5. Relation between Profit and Average Headway

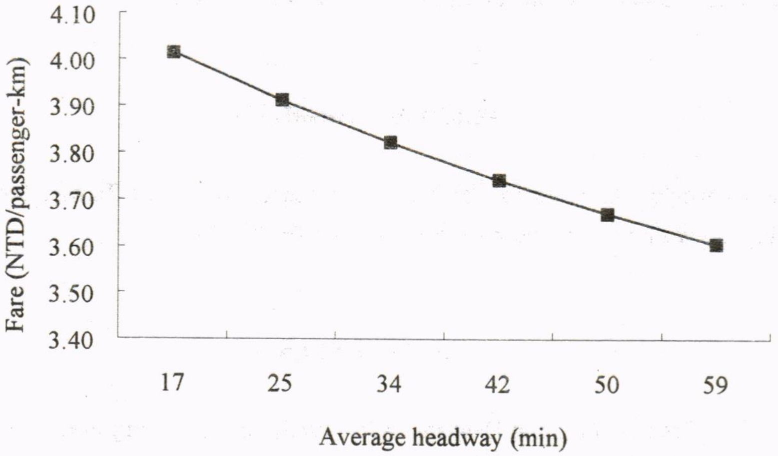

Figure 6. Relation between Optimal Fare and Average Headway

5. CONCLUSIONS AND RECOMMENDATIONS

The mathematical models are developed and used to analyze the optimal fare and service headway in this study. From this study, government assessed fare and maximum profit fare are thought to be suitable for Taiwan high-speed system. It is also found that variance of fare makes a greater influence on users' surplus than operator's profit. Thus the first step of HSR pricing mechanism is to set some lower bound of social welfare, for it is necessary to make sure that social welfare is not neglected.

In service headway, take year 2008 for example, the headway for schedule A is 70.95, 71.04, and 71.05 min, under maximum social welfare, break-even condition, and maximum operator's profit respectively. It's obviously that under the huge capital investment, cost from the variance of headway becomes not sensible. From the sensitivity analysis, the decrease of headway will increase the profit rather than decrease it. Therefore, operator will tend to satisfy all users' demand, and will not consider the cost rise due to the decrease of headway. Change of headway seems to make no influence on fares too, since the cost variance is relatively small compared with the huge capital investment. Thus it could be concluded that decrease of service headway seems to bring advantages to both the operator and users under the private invested HSR system.

The peak hour factors are not considered in this study, thus the further study can include a "Multiple periods" model. Also, service headway of A~F schedules are obtained by the allocation of system average headway simply according to the maximum demand of each schedule route, thus a further and precise model is worth exploring.

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