BOTTLENECK TRAFFIC CONGESTION UNDER ALTERNATIVE WORK SCHEDULES WITH VARIOUS STEP-TOLL SCHEMES

Lawrence W. LAN
Professor
Institute of Traffic and Transportation
National Chiao Tung University
4F, 114 Sec.1, Chung-Hsiao W. Rd.
Taipei, 10012, Taiwan
Fax: +886-2-2331-2160
E-mail: lwlan@sunwk1.tpe.nctu.edu.tw

Tien-Tsyh CHEN
Ph.D.
Institute of Traffic and Transportation
National Chiao Tung University
4F, 114 Sec.1, Chung-Hsiao W. Rd.
Taipei, 10012, Taiwan

Abstract: This paper develops deterministic queueing models, using user equilibrium theory, to explore queueing properties at a bottleneck with various step-toll schemes under fixed, flexible and staggered working schedules. The results have shown that the benefit brought by optimal step-tolls under flexible working hours is more than the ceiling of fixed working hours. The fees for optimal step-toll schemes under staggered working hours should be lower than those under fixed working hours. At optimal step-tolls, it appears that some commuters would be reluctant to pass through the toll station at the transitions from higher step-tolls to lower ones. Introducing suboptimal step-tolls will completely remove such "reluctant queues" at the cost of slight loss in systems performance.

1. INTRODUCTION

The idea that congestion tolls could reduce congestion appeared as early as in Pigou's literature (1920). In the past few years, some papers discussed the congestion tolls based on marginal cost pricing principles (e.g., Morrison,1986) which implicitly assuming that the length of peak hour is exogenous and that traffic flow is uniform over the peak hour. It fails to treat the commuter's departure time decision, which is one of the major factors for causing congestion. The other approach, based on user equilibrium principle, makes the departure time decision endogenous, was developed by Vickrey (1969). There were quite a few papers (e.g., Hendrickson *et al.*, 1981; de Palma *et al.*, 1986; Lan *et al.*, 1987; Braid, 1989; Arnott *et al.*, 1990, 1993; Laih, 1994; Chen *et al.*, 1995) using Vickrey's approach to deal with congestion toll issues, but in which a fixed working hours was assumed in the analysis.

Limited attention has been received on the impact of flexible/staggered working hours on traffic congestion. Although Henderson (1981), D'Este (1985), and Jovanis (1981) erected theoretical models or applied simulation techniques in exploring the benefits of flexible/staggered working hours on dispersing traffic congestion, their analytical assumptions are different from those used in the above-mentioned papers on congestion tolls and user equilibrium models. Consequently, their results are not easily compared. Lan *et al.* (1999) utilized a user equilibrium model to evaluate the effect of flexible/staggered working hours on traffic congestion, but did not factor congestion tolls into the analysis.

Theoretically, there are four types of road tolls: (i) no toll; (ii) a uniform toll; (iii) a coarse toll, or called "single step-toll" (Laih, 1994); and (iv) a time-dependent toll (Arnott *et al.*, 1993). Among these, uniform tolls utilize the same toll rate regardless of whether demand is at peak. Consequently, they neither affect commuters' departure times nor reduce the concentration of trips during peak hours. The optimal settings for time-dependent tolls have been investigated by Vickrey (1969), Hendrickson *et al.* (1981), and Arnott *et al.* (1990). However, their investigations did not address alternative work schedules. To fill such gaps, this study will examine the rates and types of congestion tolls under alternative work schedules. The main purpose is to gain some insights of the effectiveness of various tolls under alternative work schedule systems on the commuter departure times, dispersal of peak-hour trip demand, and traffic congestion reduction at a single bottleneck.

2. VARIOUS STEP-TOLLS UNDER FIXED WORKING HOURS

Following previous studies (e.g., Laih, 1994; Arnott *et al.*, 1993), the commuter's travel cost $U_i(t)$ composed of fixed costs (ρ), queueing delay costs incurred at the bottleneck, early schedule delay costs, late arrival costs, and congestion toll $F(t)$ are factored into the equation:

$$U_i(t) = \rho+\alpha r_i(t)+\beta h_i(t)+\gamma p_i(t)+F(t). \tag{1}$$

Where $r_i(t)$, α respectively denote queueing delay and its unit cost, $h_i(t)$, β denote early schedule delay and its unit cost, $p_i(t)$, γ denote late schedule delay and its unit cost. Because fixed travel cost (ρ) does not affect the results of the analysis, this paper sets $\rho=0$. When choosing departure times, commuters seek to reduce their travel costs to the lowest possible level. Thus, if each commuter's travel cost is identical, one will have no way to reduce one's travel cost by varying departure times to reach the desired user equilibrium.

2.1 Optimal Step-tolls

In the case of single step-toll, assume that the bottleneck in time period (t_j , t_j') is levying a flat congestion toll F_{a1} and no toll in other periods (t_q , t_j) and (t_j', t_q'). Under user equilibrium conditions, the travel cost for each commuter can be expressed as G_{a1}.

$$U_i(t)=\alpha r_i(t)+\beta h_i(t)+\gamma p_i(t)+ F_{a1} = G_{a1} \quad \forall\ i \tag{2}$$

Figure 1 depicts the equilibrium curve A(t) of cumulative arrivals at the bottleneck. At the earliest departure time t_q commuters experience the longest early schedule delay, while at the latest departure time t_q' commuters incur the largest late schedule delay, but at neither point do commuters experience queueing delays. They also avoid paying the toll. At departure times t_j and t_j' commuters suffer an early and late schedule delay, respectively. At both times, toll payments are required but there will exist no queueing delays. According to user equilibrium, the following simultaneous equations are obtained:

$$\begin{aligned}
&\beta(x_1+x_2+x_3+x_4+x_5)= G_{a1}\\
&\alpha x_2+\beta(x_3+x_4+x_5)= G_{a1}\\
&\beta(x_3+x_4+x_5)+ F_{a1}= G_{a1}\\
&\alpha(x_4+x_5)+ F_{a1}= G_{a1}\\
&\gamma x_6+F_{a1}= G_{a1}\\
&\alpha(x_5+x_6)+\gamma x_6= G_{a1}\\
&\gamma(x_6+x_7)= G_{a1}
\end{aligned}$$

$(x_1+x_2+x_3+x_4+x_5+x_6+x_7)\mu=N.$

The solutions are: $x_1=[(\alpha-\beta)/(\alpha\beta)\ F_{a1}$; $x_2= F_{a1}/\alpha$; $x_3=[(\alpha-\beta)/(\alpha\beta)](G_{a1}-F_{a1})$;
$x_4=\{[(\gamma/\alpha)+1][\beta/(\beta+\gamma)](N/\mu)\}-[(\alpha+2\gamma)/(\alpha\gamma)]F_{a1}$;
$x_5=[(\alpha+\gamma)/(\alpha\gamma)]F_{a1}-[\beta/(\beta+\gamma)](N/\mu)$; $x_6=[\beta/(\beta+\gamma)](N/\mu)-F_{a1}/\gamma$; $x_7=F_{a1}/\gamma$.

The equilibrium travel cost for each commuter is $G_{a1} = [\beta\gamma/(\beta+\gamma)](N/\mu)$, which is identical to the equilibrium cost under no toll conditions (G_p) by Lan *et al.* (1999). This result shows that commuters can trade off queueing delays against tolls, as long as congestion tolls do not increase overall travel costs. Moreover, the slope of the curve of cumulative early arrivals at the bottleneck is $m_1(t) = [\alpha/(\alpha-\beta)]\mu$, arriving late is $m_2(t) = [\alpha/(\alpha+\gamma)]\mu$. Total queueing delay, W_{al}, is defined as the area bounded by the cumulative arrival and departure curves in Figure 1, which is expressed as:

$$W_{a1} = (1/2)\{[2(\beta+\gamma)/(\alpha\beta\gamma)](F_{a1}^2\mu)-(2/\alpha)F_{a1}N+(\beta/\alpha)[\gamma/(\beta+\gamma)](N^2/\mu)\}. \tag{3}$$

Setting the first order condition of W_{al} to zero yields the optimal toll rate (F_{a1}^*):

$$F_{a1}^*=(1/2)[(\beta\gamma)/(\beta+\gamma)](N/\mu). \tag{4}$$

Note that the optimal toll F_{a1}^* is a half of the travel cost G_p under no toll conditions. Substituting F_{a1}^* into equation (3) yields the total queueing delay under optimal toll rate conditions:

$$W_{a1}^*=(1/4)(\beta/\alpha)[\gamma/(\beta+\gamma)](N^2/\mu). \tag{5}$$

Notice that W_{a1}^* is also a half of the total queueing delay under no toll conditions (W_p) by Lan *et al.* (1999). From the results displayed in Figure 1 it can be seen that the time toll collection commences at $t_j=t_q+(1/2)[\gamma/(\beta+\gamma)](N/\mu)$ and terminates at $t_j'= t_q+\{1-(1/2)\ [\beta/(\beta+\gamma)]\}(N/\mu)$. Our results are exactly the same as Laih (1994) who utilized completely different approach with complicated mathematics to derive optimal step-tolls under fixed working hours. This type of single step-toll will create three queueing peaks, as shown in Figure 2. The times of occurrence of the first twos are at $t_q+(1/2)(1-\beta/\alpha)[\gamma/(\beta+\gamma)](N/\mu)$ and $t_q+[1-(\beta/2\alpha)][\gamma/(\beta+\gamma)](N/\mu)$ and both have identical maximum queue lengths:

$$Q_{a1}^*=(1/2)(\beta/\alpha)[\gamma/(\beta+\gamma)]N. \tag{6}$$

Note that Q_{a1}^* is also a half of the maximum queue length under no toll conditions (Q_p) by Lan *et al.* (1999). The third queueing peak occurs at $t_q+\{1-(1/2)[\beta/(\beta+\gamma)]\}(N/\mu)$ which coincides the time of termination of toll collection. The queue length for the third peak is $(1/2)\{\beta\gamma/[(\alpha+\gamma)(\beta+\gamma)]\}N$ which is shorter than the first twos. The existence of third queueing peak is due to a portion of commuters unwilling to pay the toll waiting temporarily in front of the toll station until the toll has returned to no toll status before passing through. We refer such queues as "reluctant queues."

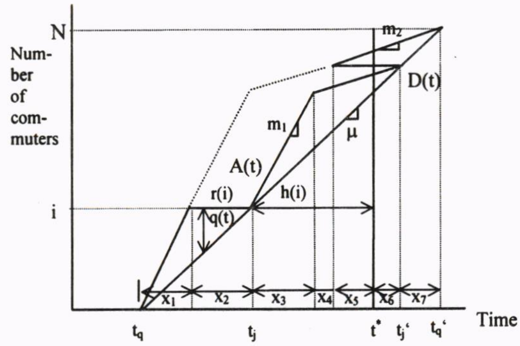


Figure 1. Equilibrium arrival pattern for optimal single step-toll under fixed working hours

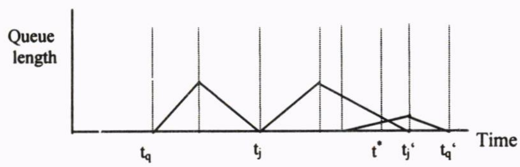


Figure 2. Queue length for optimal single step-toll under fixed working hours

In the case of two step-tolls, assume that in both periods (t_j, t_k) and (t_k', t_j') a flat congestion toll F_{a1} is levied, and in time period (t_k, t_k') a higher flat congestion toll F_{a2} is collected, while at all other times no toll is collected. The equilibrium curve of cumulative arrivals reaching the bottleneck A(t) is depicted in Figure 3. Using the same method, equilibrium travel cost is $G_{a2}=[\beta\gamma/(\beta+\gamma)](N/\mu)$ which is identical to G_p under no toll conditions. The slope of the curve of cumulative early arrivals is $m_1(t)=[\alpha/(\alpha-\beta)]\mu$, late arrivals is $m_2(t)=[\alpha/(\alpha+\gamma)]\mu$. The optimal toll rates are $F_{a1}^*=(1/3)G_{a2}=(1/3)[(\beta\gamma)/(\beta+\gamma)](N/\mu)$ and $F_{a2}^*=(2/3)G_{a2}=(2/3)[(\beta\gamma)/(\beta+\gamma)](N/\mu)$. Total queueing delay is $W_{a2}^*=(1/6)(\beta/\alpha)[\gamma/(\beta+\gamma)](N^2/\mu)=(1/3)W_p$. Toll rate F_{a1}^* begins at $t_j=t_q+\{(1/3)[\gamma/(\beta+\gamma)]\}(N/\mu)$ and F_{a2}^* begins at $t_k=t_q+\{(2/3)\ [\gamma/(\beta+\gamma)](N/\mu)\}$; then F_{a2}^* terminates at $t_k'=t_q+\{1-(2/3)[\beta/(\beta+\gamma)]\}\ (N/\mu)$ and F_{a1}^* terminates at $t_j'=t_q+\{1-(1/3)[\beta/(\beta+\gamma)]\}$ (N/μ). Such two step tolls produce five queueing peaks. The queue lengths of the first three peaks are identical and can be expressed as $Q_{a2}^*=(1/3)(\beta/\alpha)\ [\gamma/(\beta+\gamma)]N=(1/3)Q_p$, while for the other two peaks equal to $(1/3)\{\beta\gamma/[(\alpha+\gamma)(\beta+\gamma)]\}N$, which are shorter than the former three.

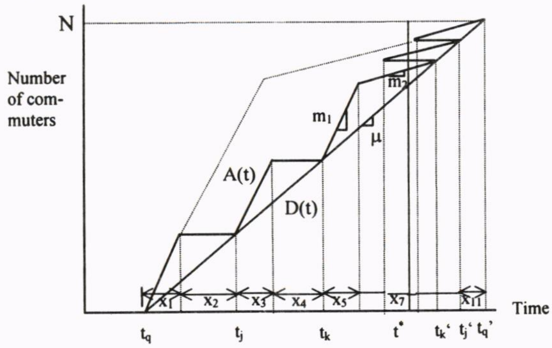


Figure 3. Equilibrium arrival pattern for optimal two step-tolls under fixed working hours

If we expand to n step-tolls, equilibrium travel cost is $G_{an}= [\beta\gamma/(\beta+\gamma)](N/\mu)$, total queueing delay and maximum queue length are, respectively, $W_{an}^{*}=(1/2)[1/(n+1)](\beta/\alpha)[\gamma/(\beta+\gamma)](N^2/\mu)$ and $Q_{an}^{*}=[1/(n+1)](\beta/\alpha)[\gamma/(\beta+\gamma)]$ which are $[1/(n+1)]$ of those for no toll conditions. The optimal toll schemes are $F_{a1}^{*}=[1/(n+1)][(\beta\gamma)/(\beta+\gamma)](N/\mu)$, $F_{a2}^{*}=2F_{a1}^{*}$, $F_{a3}^{*}=3F_{a1}^{*}$, ..., $F_{an}^{*}=nF_{a1}^{*}=[n/(n+1)][(\beta\gamma)/(\beta+\gamma)](N/\mu)$. Our results confirm the conclusions reached by Laih (1994), that for n step-toll schemes, the largest possible reduction in system queueing delay is $n/(n+1)$.

2.2 Suboptimal Step-Tolls

Notice that optimal step-tolls produce reluctant queues. This paper proposes "suboptimal" step-tolls, depicted in Figure 4, to eliminate the reluctant queues. As the toll rate reaches the nth step (the highest rate), it remains unchanged until time (t^{**}) and then instantly drops to no toll status. Assume that prior to time t_j no toll is collected at bottleneck, but after time t_j a flat congestion toll F_{b1} is collected. Under user equilibrium, the equilibrium curve of cumulative arrivals at the bottleneck A(t) is shown in Figure 5. At the earliest departure time t_q commuters experience the largest early schedule delay, while at the latest departure time t_q' commuters incur the highest late schedule delay. At both times, neither queueing delay nor toll payment is incurred. At t_j commuters avoid queueing delay, but they experience early schedule delay and must pay a toll. The equilibrium travel cost is $G_{b1}=\{[\beta\gamma/(\beta+\gamma)](N/\mu)+[\beta/(\beta+\gamma)]F_{b1}\}$ which is greater than that of no toll conditions by the amount of $[\beta/(\beta+\gamma)]F_{b1}$. Furthermore, the slopes of both early and late cumulative arrivals at the bottleneck can be obtained by $m_1(t)=[\alpha/(\alpha-\beta)]\mu$ and $m_2(t)=[\alpha/(\alpha+\gamma)]\mu$. The optimal toll rate for suboptimal step-tolls scheme is $F_{b1}^{*}=[(\beta\gamma)/(\beta+2\gamma)](N/\mu)$, higher than that of optimal step-tolls by the amount of $(1/2)\{(\beta^2\gamma)/[(\beta+\gamma)(\beta+2\gamma)]\}(N/\mu)$. Total queueing delay under suboptimal toll scheme is $W_{b1}^{*}=(1/2)\{(\beta\gamma)/[\alpha(\beta+2\gamma)]\}(N^2/\mu)=[(\beta+\gamma)/(\beta+2\gamma)]W_p$, lower than that of no toll conditions by a ratio of $[(W_p-W_{b1}^{*})/W_p]=\gamma/(\beta+2\gamma)$. Toll collection begins at time $t_j = t_q+[\gamma/(\beta+2\gamma)](N/\mu)$ and continues through the peak traffic period until $t^{**}= t_q'+F_{b1}^{*}/\gamma$. This will prevent the formation of reluctant queues. Additionally, as Figure 5 demonstrates, two queueing peaks exist with the same size equal to $Q_{b1}^{*}=(\beta\gamma)/[\alpha(\beta+2\gamma)]N=[(\beta+\gamma)/(\beta+2\gamma)]Q_p$.

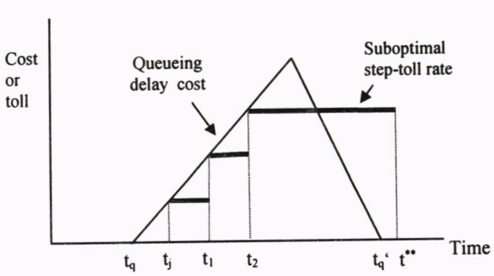


Figure 4. Illustration of suboptimal three step-tolls

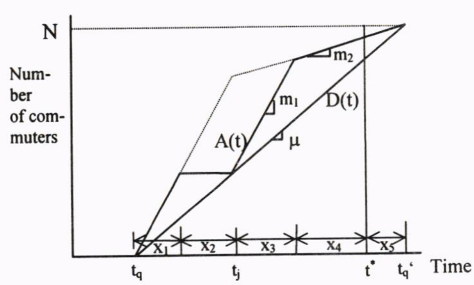


Figure5. Equilibrium arrival pattern for suboptimal single step-toll under fixed working hours

A suboptimal toll can be expanded to n steps by using the same approach. Total queueing delay and maximum queue length, respectively, are $W_{bn}^{*}=(1/2)\{(\beta\gamma)/[\alpha\beta+\alpha(n+1)\gamma]\}(N^2/\mu)$ and $Q_{bn}^{*}=(\beta\gamma)/[\alpha\beta+\alpha(n+1)\gamma]N$, lower than those of no toll conditions by a ratio of $n\gamma/[(\beta+(n+1)\gamma]$.

The suboptimal toll schemes are $F_{b1}^{*}=\{(\beta\gamma)/[\beta+(n+1)\gamma]\}(N/\mu)$, $F_{b2}^{*}=2F_{b1}^{*}$, $F_{b3}^{*}=3F_{b1},\ldots$, $F_{bn}^{*}=nF_{b1}^{*}=n\{(\beta\gamma)/[\beta+(n+1)\gamma]\}(N/\mu)$, while equilibrium travel cost is $\{[(n+1)\beta\gamma)]/[\beta+(n+1)\gamma]\}(N/\mu)$. In sum, compared with no toll conditions, a suboptimal single step-toll under fixed working hours can reduce total queueing delay and maximum queue length by a ratio of $\gamma/(\beta+2\gamma)$; for two step-tolls the ratio is $2\gamma/(\beta+3\gamma)$ and for n steps it is $n\gamma/[\beta+(n+1)\gamma]$. However, under suboptimal single step-toll, the equilibrium travel cost is higher than that of no toll conditions by a ratio of $\beta/(\beta+2\gamma)$, whereas for two step-tolls the ratio is $2\beta/(\beta+3\gamma)$, and for n step-tolls is $n\beta/[\beta+(n+1)\gamma]$. As n approaches infinity, the ratio turns out to be β/γ. The highest toll is then $F_{b\infty}^{*}=\beta(N/\mu)$, which is the ceiling of the suboptimal congestion toll.

Table 1 summarizes the system performance under fixed working hours with optimal and suboptimal step-tolls. Generally, system performance is the best under optimal step-tolls, but the problem of reluctant queues emerges during the transition from higher to lower congestion tolls. Suboptimal step-tolls can eliminate this problem, but the corresponding system performance is slightly less effective than the optimal ones. Under the optimal step-tolls, peak period at the bottleneck begins at time t_q and ends at t_q', just as under no toll conditions; however, under suboptimal step-tolls, peak period begins and ends earlier. Traffic peak will shift earlier if more steps of the toll schemes are introduced.

Table 1. System performance of various step-tolls under fixed working hours

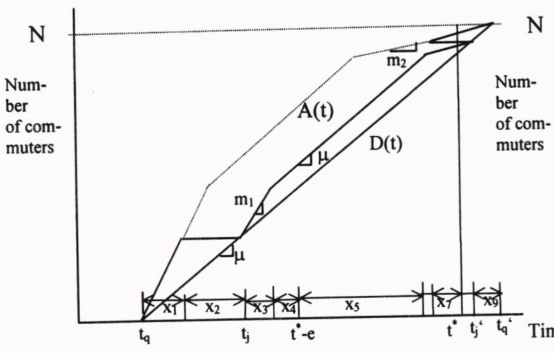
Toll schemes	Toll rate	Travel cost	Total queueing delay	Max. queue length
No toll (A1)	—	$[\beta\gamma/(\beta+\gamma)](N/\mu)$	$(1/2)(\beta/\alpha)[\gamma/(\beta+\gamma)]\times(N^2/\mu)$	$(\beta/\alpha)[\gamma/(\beta+\gamma)]N$
Optimal step-tolls (A2)	$F_{a1}^{*}=[1/(n+1)]\times[\beta\gamma/(\beta+\gamma)](N/\mu)$ $F_{a2}^{*}=2F_{a1}^{*}$ … $F_{an}^{*}=[n/(n+1)]\times[\beta\gamma/(\beta+\gamma)](N/\mu)$	$[\beta\gamma/(\beta+\gamma)](N/\mu)$	$(1/2)[1/(n+1)]\times(\beta/\alpha)[\gamma/(\beta+\gamma)](N^2/\mu)$	$[1/(n+1)]\times(\beta/\alpha)[\gamma/(\beta+\gamma)]N$
Suboptimal step-tolls (A3)	$F_{b1}^{*}=\{(\beta\gamma)/[\beta+(n+1)\gamma]\}(N/\mu)$ $F_{b2}^{*}=2F_{b1}$ … $F_{bn}^{*}=n\{(\beta\gamma)/[\beta+(n+1)\gamma]\}(N/\mu)$	$\{[(n+1)\beta\gamma]/[\beta+(n+1)\gamma]\}(N/\mu)$	$(1/2)(\beta/\alpha)\{\gamma/[\beta+(n+1)\gamma]\}(N^2/\mu)$	$(\beta/\alpha)/\{\gamma/[\beta+(n+1)\gamma]\}N$
Benefit from optimal step-tolls (A1-A2)/A1	—	0	$[n/(n+1)]$	$[n/(n+1)]$
Benefit from Sub-optimal step-tolls (A1-A3) /A1	—	$-n\beta/[\beta+(n+1)\gamma]$	$n\gamma/[\beta+(n+1)\gamma]$	$n\gamma/[\beta+(n+1)\gamma]$

3. VARIOUS STEP-TOLLS UNDER FLEXIBLE WORKING HOURS

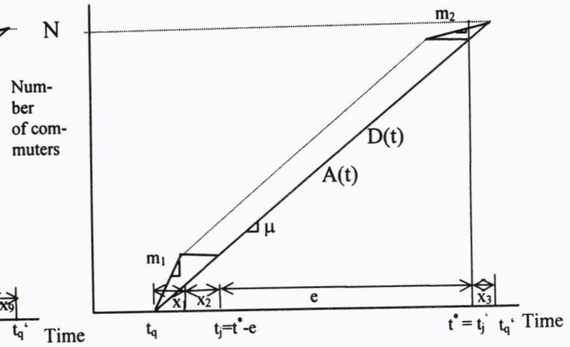
3.1 Optimal Step-Tolls

In the case of single step-toll, assume that a bottleneck between time period (t_j, t_j')is subject to a flat congestion toll F_{c1}, but at other time periods (t_q, t_j) and (t_j', t_q') no toll is collected. Let e represent the duration of flexible time period. If $e < (1/2)(N/\mu)$, the curve of cumulative arrivals at the bottleneck A(t) is shown in Figure 6(a). Using the same approach, we obtain the equilibrium travel cost $G_{c1} = [\beta\gamma/(\beta+\gamma)](N/\mu\text{-}e)$ which is identical to that of no toll conditions (G_f). The slopes of both of early and late cumulative arrivals at the bottleneck are $m_1(t) = [\alpha/(\alpha\text{-}\beta)]\mu$ and $m_2(t) = [\alpha/(\alpha+\gamma)]\mu$. The optimal toll rate is $F_{c1}^*=(1/2)[(\beta\gamma)/(\beta+\gamma)](N/\mu)$ which is a half of equilibrium travel cost under no toll conditions with fixed working hours (G_p). Notice that F_{c1}^* is independent of the duration of the flexible working time period. The total queueing delay and maximum queue length under optimal toll rates are respectively $W_{c1}^* =(1/2)(\beta/\alpha)[\gamma/(\beta+\gamma)]\ [(1/2)(N^2/\mu)\text{-}e^2\mu]$ and $Q_{c1}^*=(1/2)(\beta/\alpha)[\gamma/(\beta+\gamma)]N$. Additionally, there is still the phenomenon of reluctant queue with length $(1/2)\{\beta\gamma/[(\alpha+\gamma)(\beta+\gamma)]\}N$.

When $e \geq (1/2)(N/\mu)$, the curve of cumulative arrivals at the bottleneck is shown in Figure 6(b). The times at which toll collection begins and ends coincide with the starting and ending points of flexible working hours. Equilibrium travel cost is $G_{c1}= [\beta\gamma/(\beta+\gamma)][(N/\mu)\text{-}e]$, identical to either the optimal toll rate (F_{c1}^*) or the equilibrium travel cost under no toll conditions (G_f) obtained by Lan *et al.* (1999), implying that under flexible working hours a fixed congestion toll (F_{c1}^*) is a perfect surrogate for the cost of queueing delay under no toll conditions (G_f). Total queueing delay under optimal step-tolls is $W_{c1}^* =(1/2)(1/\mu)(\beta/\alpha)[\gamma/(\beta+\gamma)](N\text{-}e\mu)^2$, maximum queue length is $Q_{c1}^*=(\beta/\alpha)[\gamma/(\beta+\gamma)](N\text{-}e\mu)$ and reluctant queue length is $\{(\beta\gamma)/[(\alpha+\gamma)(\beta+\gamma)]\}(N\text{-}e\mu)$.



(a) When $e<(1/2)(N/\mu)$



(b) When $e\geq(1/2)(N/\mu)$

Figure 6. Equilibrium arrival pattern for optimal single step-toll under flexible working hours

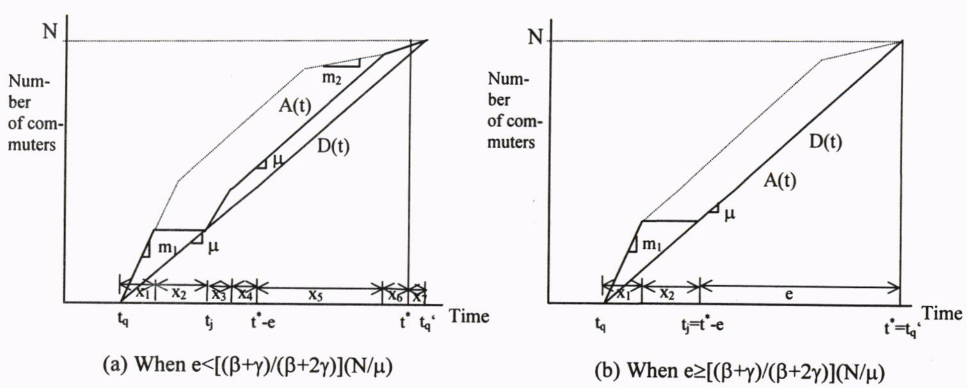
In the case of two step-tolls, two flat congestion tolls are collected, F_{c1}, F_{c2}, respectively, where $F_{c2} > F_{c1}$. When $e < (1/3)N/\mu$, equilibrium travel cost is $G_{c2}=[\beta\gamma/(\beta+\gamma)](N/\mu\text{-}e)$. The optimal toll rates are $F_{c1}^*=(1/3)[(\beta\gamma)/(\beta+\gamma)](N/\mu)$ and $F_{c2}^*=(2/3)[(\beta\gamma)/(\beta+\gamma)](N/\mu)$. Total queueing delay under the optimal step-tolls is $W_{c2}^*=(1/2)[(\beta\gamma)/(\alpha\beta+\alpha\gamma)][(1/3)N^2/\mu\text{-}e^2\mu]$, while maximum queue length is $Q_{c2}^*=(1/3)[(\beta\gamma)/(\alpha\beta+\alpha\gamma)]N$. When $e \geq (1/3)N/\mu$, the times at which congestion toll F_{2c}^* begins and ends again coincide with the starting and ending points of the flexible work hours, the equilibrium travel cost is $G_{c2}=[\beta\gamma/(\beta+\gamma)][(N/\mu)\text{-}e]= F_{c2}^*= G_f$. This means that F_{c2}^* is a perfect substitute for the cost of queueing delay under no toll conditions.

The optimal toll rate for F_{c1}^{*} is $(1/2)F_{c2}^{*}$. Total queueing delay is $W_{c2}^{*}=(1/4)(1/\mu)(\beta/\alpha)[\gamma/(\beta+\gamma)](N\text{-}e\mu)^2$, maximum queue length is $Q_{c2}^{*}=(1/2)(\beta/\alpha)[\gamma/(\beta+\gamma)](N\text{-}e\mu)$.

Expanding the step-toll schemes to n steps, and when $e < [1/(n+1)](N/\mu)$, the equilibrium travel cost is $G_{cn}=[\beta\gamma/(\beta+\gamma)](N/\mu\text{-}e)$, identical to that of no toll conditions. Optimal toll rates are $F_{c1}^{*}=[1/(n+1)][\beta\gamma/(\beta+\gamma)](N/\mu)$, $F_{c2}^{*}=2F_{c1}^{*}$,..., and $F_{cn}^{*}=nF_{c1}^{*}=[n/(n+1)]\ [\beta\gamma/(\beta+\gamma)](N/\mu)$. Total queueing delay under optimal toll schemes is $W_{cn}^{*}=(1/2)(\beta/\alpha)[\gamma/(\beta+\gamma)]\{[1/(n+1)](N^2/\mu)\text{-}e^2\mu\}$, and maximum queue length is $Q_{cn}^{*}=[1/(n+1)](\beta/\alpha)[\gamma/(\beta+\gamma)]N$. When $e \geq [1/(n+1)]N/\mu$, equilibrium travel cost is $G_{cn}=[\beta\gamma/(\beta+\gamma)](N/\mu\text{-}e)$. The optimal toll schedules are $F_{c1}^{*}=(1/n)[\beta\gamma/(\beta+\gamma)](N/\mu\text{-}e)$, $F_{c2}^{*}=2F_{c1}^{*}$,..., and $F_{cn}^{*}=nF_{c1}^{*}=[\beta\gamma/(\beta+\gamma)](N/\mu\text{-}e)$. Here the times at which the toll F_{cn}^{*} begins and ends coincide with the starting and ending points of flexible time period, thus F_{cn}^{*} is equivalent to the equilibrium travel cost. Total queueing delay under optimal toll schemes is $W_{cn}^{*}=[1/(2n\mu)](\beta/\alpha)[\gamma/(\beta+\gamma)](N\text{-}e\mu)^2$, maximum queue length is $Q_{cn}^{*}=(1/n)(\beta/\alpha)[\gamma/(\beta+\gamma)](N\text{-}e\mu)$.

3.2 Suboptimal Step-tolls

In the case of single step-toll, assume that at a bottleneck prior to time t_j no toll is collected, but after t_j a fixed congestion toll F_{d1} is collected. When $e < [(\beta+\gamma)/(\beta+2\gamma)](N/\mu)$, the equilibrium curve of cumulative arrivals at the bottleneck A(t) is depicted in Figure 7(a). Equilibrium travel cost is $G_{d1}=[\beta\gamma/(\beta+\gamma)](N/\mu\text{-}e)+[\beta/(\beta+\gamma)]F_{d1}$, slightly greater than that of no toll conditions by $[\beta/(\beta+\gamma)]F_{d1}$. The optimal toll rate for suboptimal toll scheme is $F_{d1}^{*}=[(\beta\gamma)/(\beta+2\gamma)](N/\mu)$ which is independent of the length of flexible work hours. Total queueing delay under the suboptimal toll scheme is $W_{d1}^{*}=(1/2)(\beta\gamma/\alpha)\{[1/(\beta+2\gamma)](N^2/\mu)\text{-}[1/(\beta+\gamma)]e^2\mu\}$ and maximum queue length is $Q_{d1}^{*}=(\beta\gamma)/\ [\alpha(\beta+2\gamma)]N$. Collection of the above suboptimal step-tolls begins at $t_j=t_q+[\gamma/(\beta+2\gamma)](N/\mu)$ and ends at $t^{**}=t_q'+[\beta/(\beta+2\gamma)](N/\mu)$. When $e\geq [(\beta+\gamma)/(\beta+2\gamma)](N/\mu)$, collection of the congestion toll begins at time $t_j=t_q+(N/\mu\text{-}e)$ and ends at $t^{**}=t_q'+F_{d1}/\gamma$. Under these conditions commuters will not be late for work, and the termination of flexible time period is concurrent with the end of peak traffic. The curve of equilibrium cumulative arrivals at the bottleneck A(t) is shown in Figure 7(b). Equilibrium travel cost is $G_{d1}=\beta[(N/\mu)\text{-}e]$, while the optimal toll rate is $F_{d1}^{*}=\beta[(N/\mu)\text{-}e]$. Total queueing delay is $W_{d1}^{*}=(1/2)(1/\mu)(\beta/\alpha)(N\text{-}e\mu)^2$ and maximum queue length is $Q_{d1}^{*}=(\beta/\alpha)(N\text{-}e\mu)$.



(a) When $e<[(\beta+\gamma)/(\beta+2\gamma)](N/\mu)$ (b) When $e\geq[(\beta+\gamma)/(\beta+2\gamma)](N/\mu)$

Figure 7. Equilibrium arrival pattern for suboptimal single step-toll under flexible working hours

In case of n step-tolls and when $e < \{(\beta+\gamma)/[\beta+(n+1)\gamma]\}(N/\mu)$, the equilibrium travel cost is $G_{dn}=[\beta\gamma/(\beta+\gamma)](N/\mu-e)+[\beta/(\beta+\gamma)]F_{dn}^{*}$, suboptimal tolls are $F_{d1}^{*}=\{\beta\gamma/[\beta+(n+1)\gamma]\}(N/\mu)$, $F_{d2}^{*}=2F_{d1}^{*},\ldots,$ and $F_{dn}^{*}=nF_{d1}^{*}=\{n\beta\gamma/[\beta+(n+1)\gamma]\}(N/\mu)$. Total queueing delay is $W_{dn}^{*}=(1/2)(\beta/\alpha)\{\gamma/[\beta+(n+1)\gamma]\}(N^2/\mu)-(1/2)(\beta/\alpha)/[\gamma/(\beta+\gamma)]e^2\mu$ and maximum queue length is $Q_{dn}^{*}=(\beta/\alpha)/\{\gamma/[\beta+(n+1)\gamma]\}N$. When $e \geq \{(\beta+\gamma)/[\beta+(n+1)\gamma]\}(N/\mu)$, the equilibrium travel cost will be $G_{dn}=\beta(N/\mu-e)$, suboptimal toll schemes are $F_{d1}^{*}=(1/n)\beta(N/\mu-e)$, $F_{d2}^{*}=2F_{d1}^{*},\ldots,$ and $F_{dn}^{*}=nF_{d1}^{*}=\beta(N/\mu-e)$. Here F_{dn}^{*} is equivalent to the travel cost, and the toll collection period and flexible time period start and end concurrently. Total queueing delay under suboptimal tolls is $W_{dn}^{*}=[1/(2n)](\beta/\alpha)(1/\mu)(N-e\mu)^2$ and maximum queue length is $Q_{dn}^{*}=(1/n)(\beta/\alpha)(N-e\mu)$.

The system performance under optimal and suboptimal step-toll schemes.are summarized in Table 2. Under the same condition, the optimal step-toll always results in better performance than the suboptimal toll schemes, but leads to the problem of reluctant queues. Such queues

Table 2. System performance of various step-tolls under flexible working hours

Toll schemes		Toll rate	Travel cost	Total queueing delay	Max. queue length
No toll (A1)		—	$[\beta\gamma/(\beta+\gamma)](N/\mu-e)$	$(1/2)(\beta/\alpha)[\gamma/(\beta+\gamma)] \times (N/\mu-e)(N+e\mu)$	$(\beta/\alpha)[\gamma/(\beta+\gamma)] \times (N-e\mu)$
Optimal step-tolls (A2) $e<[1/(n+1)](N/\mu)$		$F_{c1}^{*}=[1/(n+1)] \times [\beta\gamma/(\beta+\gamma)](N/\mu)$ $F_{c2}^{*}=2F_{c1}^{*}$ ... $F_{cn}^{*}=[n/(n+1)] \times [\beta\gamma/(\beta+\gamma)](N/\mu)$	$[\beta\gamma/(\beta+\gamma)](N/\mu-e)$	$(1/2)(\beta/\alpha)[\gamma/(\beta+\gamma)] \times \{[1/(n+1)](N^2/\mu)-e^2\mu\}$	$[1/(n+1)] \times (\beta/\alpha)[\gamma/(\beta+\gamma)]N$
Optimal step-tolls (B2) $e\geq[1/(n+1)](N/\mu)$		$F_{c1}^{*}=(1/n)[\beta\gamma/(\beta+\gamma)] \times (N/\mu-e)$ $F_{c2}^{*}=2F_{c1}^{*}$ ... $F_{cn}^{*}=[\beta\gamma/(\beta+\gamma)](N/\mu-e)$	$[\beta\gamma/(\beta+\gamma)](N/\mu-e)$	$[1/(2n\mu)](\beta/\alpha) \times [\gamma/(\beta+\gamma)](N-e\mu)^2$	$(1/n)(\beta/\alpha) \times [\gamma/(\beta+\gamma)](N-e\mu)$
Suboptimal step-tolls (A3) $e<\{(\beta+\gamma)/[\beta+(n+1)\gamma]\}(N/\mu)$		$F_{d1}^{*}=\beta\gamma/[\beta+(n+1)\gamma] \times (N/\mu)$ $F_{d2}^{*}=2F_{d1}^{*}$ ... $F_{dn}^{*}=n\beta\gamma/[\beta+(n+1)\gamma] \times (N/\mu)$	$[\beta\gamma/(\beta+\gamma)](N/\mu-e)+[\beta/(\beta+\gamma)]F_{dn}^{*}$	$(1/2)(\beta/\alpha)\{\gamma/[\beta+(n+1)\gamma]\}(N^2/\mu)-(1/2)(\beta/\alpha)/[\gamma/(\beta+\gamma)] \times e^2\mu\}$	$(\beta/\alpha)/\{\gamma/[\beta+(n+1)\gamma]\}N$
Suboptimal step-tolls (B3) $e\geq\{(\beta+\gamma)/[\beta+(n+1)\gamma]\}(N/\mu)$		$F_{d1}^{*}=(1/n)\beta(N/\mu-e)$ $F_{d2}^{*}=2F_{d1}^{*}$ ... $F_{dn}^{*}=\beta(N/\mu-e)$	$\beta(N/\mu-e)$	$[1/(2n\mu)](\beta/\alpha) \times (N-e\mu)^2$	$(1/n)(\beta/\alpha)(N-e\mu)$
Benefit from optimal step-tolls	(A1-A2) /A1	—	0	$[n/(n+1)] \times \{1/[1-(e\mu/N)^2]\}$	$1-[1/(n+1)] \times [N/(N-e\mu)]$
	(A1-B2) /A1	—	0	$1-(1/n)[(N-e\mu)/(N+e\mu)]$	$1-(1/n)$
Benefit from sub -optimal step-tolls	(A1-A3) /A1	—	$-\{n\beta/[\beta+(n+1)\gamma] \times [N/(N-e\mu)]\}$	$\{n\gamma/[\beta+(n+1)\gamma]\} \times \{1/[1-(e\mu/N)^2]\}$	$1-(\beta+\gamma)/[\beta+(n+1)\gamma][N/(N-e\mu)]$
	(A1-B3) /A1	—	$-\beta/\gamma$	$1-(1/n)(1+\beta/\gamma) \times [(N-e\mu)/(N+e\mu)]$	$1-(1/n)(1+\beta/\gamma)$

are completely eliminated through suboptimal step-toll schemes. Table 2 shows that under flexible working hours in comparison with no toll condition, when $e<[1/(n+1)]N/\mu$, total queueing delay for optimal tolls can be reduced by a ratio of $[n/(n+1)]\{1/[1-(e\mu/N)^2]\}$; when $e\geq[1/(n+1)]N/\mu$ it is reduced by a ratio of $\{1-(1/n)[(N-e\mu)/(N+e\mu)]\}$. Both ratios are greater than that of fixed working hours, $n/(n+1)$. If a suboptimal step-tolls is utilized, when $e<\{(\beta+\gamma)/[\beta+(n+1)\gamma]\}N/\mu$, total queueing delay will be reduced by the ratio of $\{n\gamma/[\beta+(n+1)\gamma]\}\{1/[1-(e\mu/N)^2]\}$; when $e\geq\{(\beta+\gamma)/[\beta+(n+1)\gamma]\}N/\mu$ the reduction ratio will be $1-(1/n)(1+\beta/\gamma)[(N-e\mu)/(N+e\mu)]$. Both reduction ratios are greater than that of fixed working hours, $(n\gamma)/[\beta+(n+1)\gamma]$. In other words, the effectiveness of a step-tolls in reducing queueing delay is more marked under a flexible work hours system than under a fixed work hours system.

4. VARIOUS STEP-TOLLS UNDER STAGGERED WORKING HOURS

4.1 Optimal Step-Tolls

There are two types of staggered working hours systems, step-type and uniform-type (Lan *et al.*, 1999). This paper will focus on the uniform-type staggered working hours system. Commuters accumulated by starting work time form a line with the slope $\omega(\omega>\mu)$. The first commuter starting work time is t_0 and the last starting work t^*. The time length between them is $d=t^*-t_0=N/\omega$. In the case of single step-toll, a flat toll F_{y1} is levied during time period (t_j, t_j') while at times (t_q, t_j) and (t_j', t_q') there is no toll. The equilibrium cumulative arrivals at the bottleneck A(t) is shown in Figure 8. The equilibrium travel cost is $G_{y1}=[\beta\gamma/(\beta+\gamma)](N/\mu)(1-\mu/\omega)$. The slope of cumulative early arrivals is $m_1(t)=\mu/[1-(\beta/\alpha)(1-\mu/\omega)]$; similarly, for the late arrivals is $m_2(t)=\mu/[1+(\gamma/\alpha)(1-\mu/\omega)]$. Both slopes are identical to those of no toll conditions. The optimal toll is $F_{y1}^*=(1/2)[\beta\gamma/(\beta+\gamma)](N/\mu)(1-\mu/\omega)=(1/2)G_u$. Total queueing delay under optimal tolls is $W_{y1}^*=(\beta/\alpha)[\gamma/(\beta+\gamma)](N^2/\mu)(1-\mu/\omega)=(1/2)W_u^*$ and maximum queue length is $Q_{y1}^*=(1/2)(\beta/\alpha)[\gamma/(\beta+\gamma)]N(1-\mu/\omega)=(1/2)Q_u^*$. Where G_u, W_u^*, Q_u^* are respectively expressed as equilibrium travel cost, total queueing delay and maximum queue length under no toll condition (Lan *et al.*,1999). Furthermore, there is still the problem of a reluctant queue with length $(1/2)\{\beta\gamma/[(\alpha+\gamma)(\beta+\gamma)]\}N(1-\mu/\omega)$. In the case of multiple step-tolls, equilibrium travel cost, total queueing delay and maximum queue length are $(1-\mu/\omega)$ times those of fixed working hours.

4.2 Suboptimal Step-Tolls

In the case of single step-toll, assume that at a bottleneck prior to time t_j no toll is collected and after t_j a flat congestion toll is collected. The equilibrium cumulative arrivals at the bottleneck A(t) is shown in Figure 9. The equilibrium travel cost is $G_{z1}=[\beta\gamma/(\beta+\gamma)](N/\mu)(1-\mu/\omega)+[\beta/(\beta+\gamma)]F_{z1}$, greater than that of no toll conditions by $[\beta/(\beta+\gamma)]F_{z1}$. The optimal toll rate is $F_{z1}^*=[\beta\gamma/(\beta+2\gamma)](N/\mu)(1-\mu/\omega)$. Total queueing delay is $W_{z1}^*=\{(\beta/\alpha)[\gamma/(\beta+2\gamma)](N^2/\mu)(1-\mu/\omega)$ and maximum queue length is $Q_{z1}^*=(\beta/\alpha)[\gamma/(\beta+2\gamma)]N(1-\mu/\omega)$. In case of multiple step-tolls, equilibrium travel cost, total queueing delay and maximum queue length are $(1-\mu/\omega)$ times those of fixed working hours also.

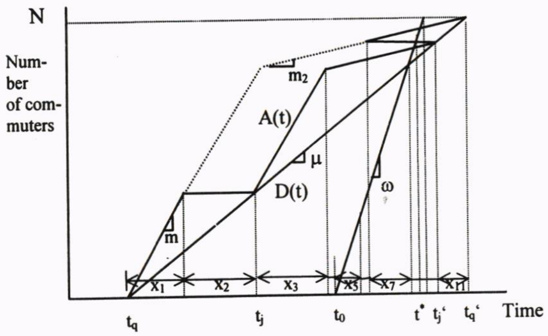


Figure 8. Equilibrium arrival pattern for optimal single step-toll under staggered working hours

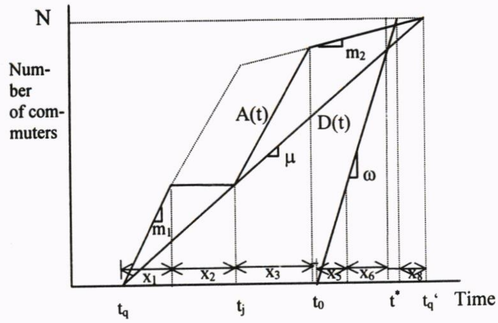


Figure 9. Equilibrium arrival pattern for suboptimal single step-toll under staggered working hours

5. SENSITIVITY ANALYSIS

Following previous studies (e.g. Small, 1982; Lan *et al.* 1999), assume α=\$6.4/hr, β=\$3.9/hr and γ=\$15.21/hr, N=1800 commuters, each commuter drives one car and passes through a bottleneck in order to reach the destination. The capacity of the bottleneck is μ=900 cars per hour. Under fixed work schedules, all commuters arrive at work at 9:00; under flexible working hours, between 8:30 and 9:00; and under uniform-type staggered system between 8:30 and 9:00. Utilizing the analytic models for step-tolls, the system performance under three different work schedules with various step-tolls are displayed in Table 3.

It is clear from Table 3 that optimal step-tolls generate reluctant queues. Though suboptimal step-tolls do not produce reluctant queues, their performance are inferior to optimal step-tolls scheme across the range of measures in this example. The results of these calculations are also shown in Figure 10. The charts (a), (b), (c) depict the changes in queues through the bottleneck at any moment throughout the morning rush hours for the fixed, flexible, and staggered working schedules, respectively, with no toll, optimal single step-toll and suboptimal single step-toll schemes.

The effects of bottleneck capacity, unit costs of queueing, early and late schedule delays on total queueing delay are similar as the results of Lan *et al.* (1999). Thus, the following two parameters including number of steps in step-tolls and the length of flexible/staggered time will be discussed for sensitivity analysis. The effect of number of steps (n) in optimal and suboptimal step-tolls on total queueing delay under alternative work schedules is shown in Figure 11. Note that total queueing delay decreases as n increases, but the level of effectiveness gradually drops off as n approaches infinity. Regardless of the work schedules system in question, optimal step-tolls are always superior to suboptimal ones in total queueing delay reduction, but the difference between the two schemes will become narrower as n increases. In fact, one can easily show that as n approaches infinity, total queueing delay will approach zero.

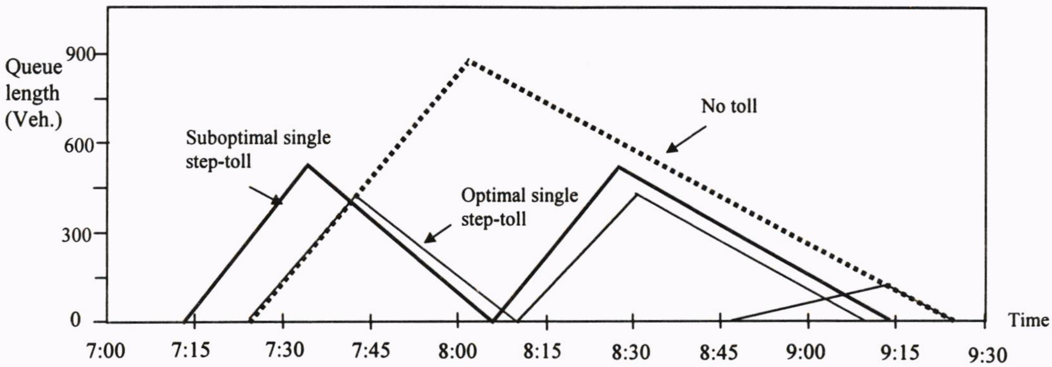
The results of varying e and d under single step-toll are shown in Figure 12. Under flexible working hours with optimal step-tolls, when $e<[1/(n+1)]N/\mu$, total queueing delay (W_{c1}^*) differentiated by e is $\partial W_{c1}^*/\partial e= -(\beta/\alpha)[\gamma/(\beta+\gamma)]e\mu$; and when $e>[1/(n+1)]N/\mu$ the differentiation is $\partial W_{c1}^*/\partial e = -(1/n)(\beta/\alpha)[\gamma/(\beta+\gamma)](N-e\mu)$. With suboptimal step-tolls the respective differentiations are when $e<\{(\beta+\gamma)/[\beta+(n+1)\gamma]\}N/\mu$ then $\partial W_{d1}^*/\partial e = -(\beta/\alpha)[\gamma/(\beta+\gamma)]e\mu$; when $e>\{(\beta+\gamma)/[\beta+(n+1)\gamma]\}N/\mu$ then $\partial W_{d1}^*/\partial e = -(1/n)(\beta/\alpha)(N-e\mu)$. Therefore, as the length of flexible time (e) increases, its effect on reducing total queueing delay grows larger.

Under staggered working hours of uniform-type with the optimal step-tolls, total queueing delay differentiated by d is $\partial W_{y1}^*/\partial d = -(1/2)[1/(n+1)](\beta/\alpha)[\gamma/(\beta+\gamma)]N$; with suboptimal step-tolls the differentiation yields $\partial W_{z1}^*/\partial d = -(1/2)(\beta/\alpha)\{\gamma/[1/(\beta+(n+1)\gamma]\}N$; both are independent of d. As d increases, the reduction rate of total queueing delay is greater under suboptimal step-tolls than under optimal ones.

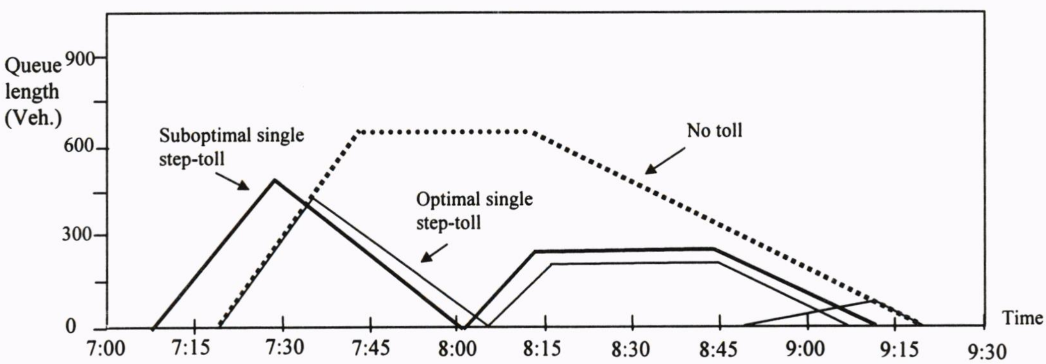
Finally, as e and d increase, total queueing delay will decrease accordingly. Regardless of the working schedules system in question, as $e=d=N/\mu$, total queueing delay will drop to zero.

Table 3. The results from examples

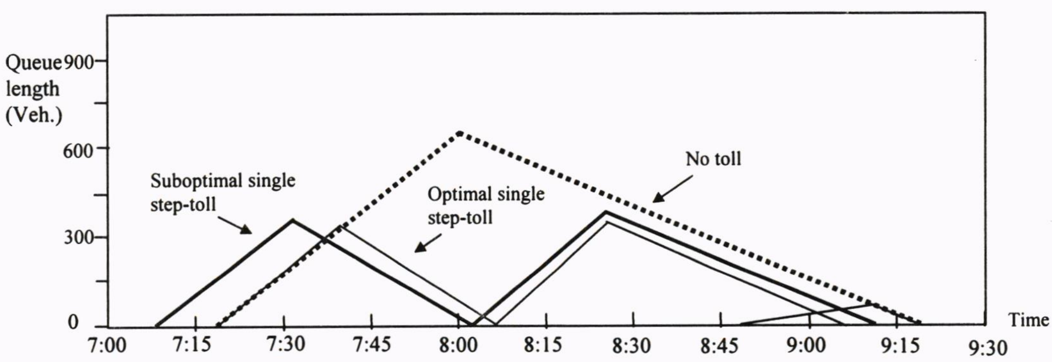
Work schedules	Items	No toll	Optimal step-tolls	Subptimal step-tolls
Fixed work hours	Starting work hour	9:00	9:00	9:00
	Peak hour period	7:24～9:24	7:24～9:24	7:14～9:14
	Toll rate ($)	—	3.1	3.5
	Period of toll collection	—	8:12～9:12	8:08～9:28
	Max. queueing delay (min.)	58	29	33
	Travel cost ($)	6.2	6.2	6.9
	Total queueing delay (veh-hr.)	873	437	486
	Max. queue length (veh.)	873	437	486
	Reluctant queue length (veh.)	—	129	—
Flexible work hours	Starting work hour (flexitime)	8:30～9:00	8:30～9:00	8:30～9:00
	Peak hour period	7:18～9:18	7:18～9:18	7:08～9:08
	Toll rate ($)	—	3.1	3.5
	Period of toll collection	—	7:54～9:09	8:01～9:22
	Max. queueing delay (min.)	44	22	32
	Travel cost ($)	4.7	4.7	5.4
	Total queueing delay (veh-hr.)	819	383	432
	Max. queue length (veh.)	654	437	486
	Reluctant queue length (veh.)	—	129	—
Staggered work hours	Starting work hour	8:30～9:00	8:30～9:00	8:30～9:00
	Peak hour period	7:18～9:18	7:18～9:18	7:10～9:10
	Toll rate ($)	—	2.3	2.6
	Period of toll collection	—	8:06～9:06	8:03～9:20
	Max. queueing delay (min.)	44	22	24
	Travel cost ($)	4.7	4.7	5.2
	Total queueing delay (veh-hr.)	655	327	364
	Max. queue length (veh.)	655	327	364
	Reluctant queue length (veh.)	—	97	—



(a) Fixed working hours



(b) Flexible working hours



(c) Staggered working hours

Figure 10 Changes in queue length under alternative working hours with various tolls

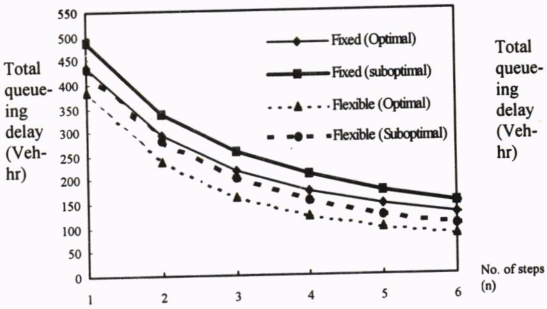


Figure 11. The effect of number of steps on total queueing delay

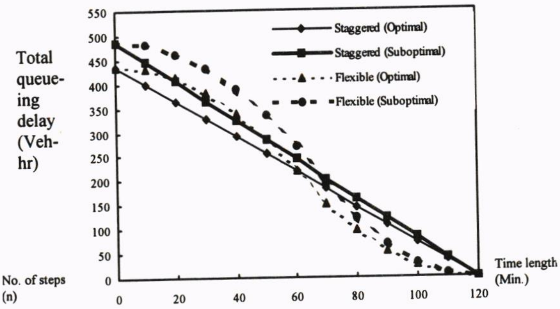


Figure 12. The effect of flexible and staggered time length on total queueing delay

6. CONCLUDING REMARKS

Theoretically, according to previous literature it is possible to attain a state in which the bottleneck queues are completely eliminated under user equilibrium. Practically, time-dependent tolls necessitate not only advanced technology such as electronic toll collection (ETC) employing automatic vehicle identification and contactless smart cards, but must take into account the privacy rights as well as the willingness of citizens to accept the system.

Although step-tolls cannot completely eliminate queueing delays, they are effective in ameliorating them. Laih (1994), in examining step-tolls, came to the conclusion that if single step-toll is utilized, at most a half of the total system queuing delay will be eliminated, while if n steps are utilized, the greatest possible effectiveness is n/(n+1). Laih *et al.* (1997) also pointed out when a step-toll is employed, it is necessary to construct a "waiting lane" in front of the toll-station to accommodate the "reluctant queues," those who are unwilling to enter the station during the transition period between higher and lower tolls. The design and operation of such toll-stations may be complex and difficult.

This paper has developed analytic queueing models to examine queueing properties at a single bottleneck under optimal and suboptimal step-tolls with fixed, flexible, and staggered working schedules. The results of fixed working hours under optimal step-tolls are exactly identical to those of Laih (1994) who used completely different approach, confirming the accuracy of our analytic models.

Regardless of the types of work schedules, it is found that system performance is the best under optimal step-tolls, but reluctant queues will appear as tolls transition from higher to lower levels. Under fixed working hours, optimal n step-tolls can reduce total queueing delay and maximum queue length by a ratio of n/(n+1) in comparison with no toll conditions. As the number of steps in step-tolls approaches infinity, congestion at the bottleneck disappears completely, equivalent to a time-dependent toll proposed by Vickrey (1969). To avoid reluctant queues, this paper recommends the adoption of suboptimal step-tolls, although they are not as effective as optimal step-tolls across the range of system performance measures. Under fixed working hours, suboptimal step-toll can reduce total queueing delay and maximum queue length by a ratio of $n\gamma/[\beta+(n+1)\gamma]$ in comparison with no toll conditions.

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