THE PROPOSAL OF FUZZY TRAFFIC ASSIGNMENT MODELS

Takamasa AKIYAMA
Professor
Department of Civil Engineering,
Gifu University
Yanagido, 1-1, Gifu,
501-1193, Japan.
Fax: +81-58-230-1528
E-mail: takamasa@cc.gifu-u.ac.jp

Tomoko NOMURA
Engineer
City Planning Division
Gifu Prefectural Government
Yabuta-mimani, 2-1-1, Gifu,
500-8570, Japan.
Fax: +81-58-271-7686

Abstract: The UE traffic assignment model is formulated with the crisp travel time even though perfect information may not be always given in real transport networks. This model might be called as a deterministic traffic assignment. Recently, the random number has been introduced to illustrate the distribution of the perceived travel times of individual drivers. This type of the model can describe SUE (stochastic user equilibrium) state. On the other hand, the travel time for the paths can be described as a fuzzy number in terms of human ambiguous perception. In this study, the possibility measure can be used to compare the fuzzy numbers. The route choice behaviour is modelled with the values of possibilities between fuzzy goal and the fuzzy travel time for the individual travel paths. It would be proposed that fuzzy user equilibrium situation (FUE) might be calculated with the similar method in the SUE assignment.

1. INTRODUCTION

Traffic assignment is the technique to estimate traffic flows on links in an urban transport network. The route choice bahaviour has been described with the assumption of perfect travel time information on the networks in the standard estimation. The crisp number has been commonly used to describe the travel times of drivers. Therefore, it is assumed that the travel time is perceived such as 10 minutes, 35 minutes in the exact numbers. The traffic is loaded on the shortest path between the origin and the destination in the traffic assignment. It corresponds to the assumption that all drivers chose the shortest paths in terms of crisp value of travel time. However, many uncertain factors may be involved in the perceived travel time because it might not be realized that drivers have got perfect information for all their possible routes.

Furthermore, the other factors as travel cost, safety and comfort may influence the route choice in the real world. This observation shows that some other routes which have larger travel time than the shortest path might be chosen by drivers. The stochastic traffic assignment is developed to consider the uncertain factors of perceived travel times. In this approach, the probability distribution of the perceived travel time is assumed because drivers just have imperfect information. The OD traffic is loaded on links of the network according to the route choice probability.

On the other hand, ambiguous factors are observed in subjective recognition of drivers for travel times. The fuzzy traffic assignment is proposed to consider these factors in this research. Particularly, the traffic assignment with fuzzy travel time is introduced. In this model, a perceived travel time of a driver can be quantified with certain range of perception. In this way, a realistic description of route selection would be proposed. The prediction of traffic on the network is discussed as well.

2. TRAVEL TIME DESCRIPTION WITH FUZZY NUMBERS

The idea of fuzzy travel time is introduced to formulate the fuzzy traffic assignment on the transport network. A fuzzy number is a ambiguous number on the real axis such as "about 15" (Sakawa, 1989) . Therefore, the subjective recognition of drivers can be illustrated by fuzzy number in the expression of travel time. It reflects that drivers chose routes with fuzzy traffic information in real situation.

2.1 Arithmetic Operation on Fuzzy Travel Times

A Fuzzy number is described by certain type of membership function. In the formulation of the study, triangular type fuzzy numbers (Triangular Fuzzy Numbers; Kaufmann & Gupta, 1988) are used in the description of travel time. Generally, a T. F. N. is expressed with a form illustrated in Figure 1. The fuzzy number might be labelled as "about a_2". It corresponds to the linguistic variables in this model.

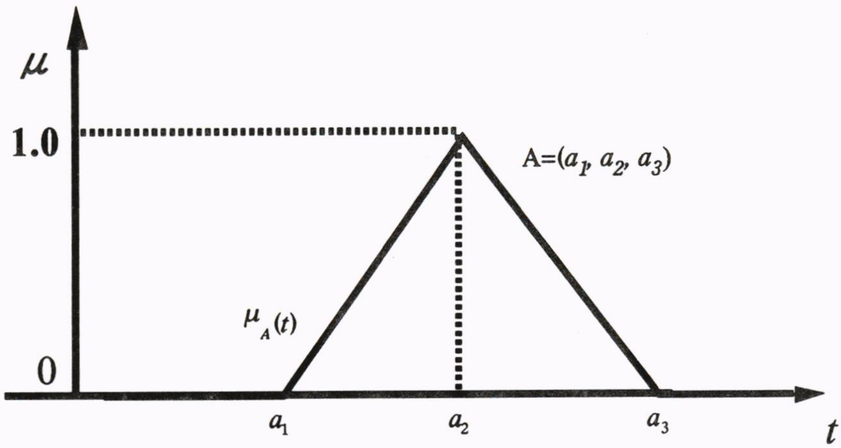


Fig. 1 The Outline of Triangular Fuzzy Number

A triangular fuzzy number can be defined by the triplet (a_1, a_2, a_3). Assume two T. F. N. s **A** and **B** by the triplets as **A**= (a_1, a_2, a_3) and **B**=(b_1, b_2, b_3), the extended sum operation on two T. F. N. s gives a T. F. N. as follows:

$$\mathbf{A}(+)\mathbf{B} = (a_1, a_2, a_3)\ (+)\ (b_1, b_2, b_3)$$

$$= (a_1+b_1,\ a_2+b_2,\ a_3+b_3) \tag{1}$$

The operation is based on the extension principle for fuzzy sets. It is known that the fuzzy number, **A(+)B** is described by a T. F. N. as well. If the link travel time is described as fuzzy number, a fuzzy path travel time can be obtained as a sum of fuzzy link travel times in the same manner. As a result, the path travel time can be illustrated as T.F.N.

2.2 Comparison Between Fuzzy Travel Times

Two concepts have been mentioned to order or rank path fuzzy travel times in the similar researches. There are the method with "the representative value" and the method with "the value of index" (Shao & Akiyama, 1994). In the first idea, the typical representation values, "the total time difference" and "the center of gravity value" have been introduced.

In this research, "possibility measure" is used as an index of distance between fuzzy travel times according to the second idea. Fuzzy measure is a sort of scale which considers ambiguous nature of individuals. First of all, a possibility measure and an inevitability measure are mentioned as basic fuzzy measures. After the definitions, the comparison between the fuzzy numbers are considered by using "possibility measures".

2.2.1 Definition of Possibility Measures

Define the crisp sets (non-fuzzy sets), **A** and **B**, which belong to the universe Ω. The function $\Pi_B(\mathbf{A})$ is defined :

$$\Pi_B(A) = \begin{cases} 1 & A \cap B \neq \varnothing \\ & \qquad\qquad \forall A \subseteq \Omega \\ 0 & others \end{cases} \tag{2}$$

The elements of subset B are possible to belong to subset A if $A \cap B \neq \varnothing$. Therefore, the measure can show the degree of belonging possibility with the value as 0 or 1.
Assume that x is an element in Ω., the function $\Pi_B(\mathbf{A})$ can be described with characteristic functions, $c_A(x)$ and $c_B(x)$ for subsets A and B as follows:

$$\Pi_B(A) = \sup_{x \in \Omega} \min\{c_B(x),\ c_A(x)\} \tag{3}$$

Similarly a necessity measure can be defined as a dual function to the possibility measure. These definitions are easily extended to measures on fuzzy numbers.

2.2.2 Comparison of Fuzzy Numbers with Possibility Measures

The index of ranking for fuzzy numbers have been proposed by Dubois and Prade[1), 2)]. The index can be recognized as extensions of the possibility measures and the necessity measures to fuzzy subsets. Define the membership functions, μ_M (u) and μ_N (v) of fuzzy subsets M and N. The measures are defined as follows:

$$\begin{aligned} Pos(M \geq N) &= \sup_{u \geq v} \min\{\mu_M(u),\ \mu_N(v)\} \\ Pos(M > N) &= \sup_{u} \inf_{v \geq u} \min\{\mu_M(u),\ 1-\mu_N(v)\} \\ Nes(M \geq N) &= \inf_{u} \sup_{v \leq u} \max\{1-\mu_M(u),\ \mu_N(v)\} \\ Nes(M > N) &= 1-\sup_{u \leq v} \min\{\mu_M(u),\ \mu_N(v)\} \end{aligned} \tag{4}$$

From the top to the bottom, there are "the possibility that M is greater or equal to N", "the possibility that M is greater than N", "the necessity that M is greater or equal to N", and "the necessity that M is greater than N" respectively.

2.2.3 Possibility Indices

The first possibility index as $Pos(B \geq A)$ in equation (4) indicates the degree that fuzzy number A is greater than or equal to fuzzy number B. This measure is formulated in the following equation.

$$\Pi_B^{(1)}(A) = \sup_{x \in \Omega} \min\{\mu_A(x),\ \mu_B(x)\} \tag{5}$$

As shown in Figure 2, the value of $\Pi^{(1)}$ is corresponding to the point of intersection of membership functions of fuzzy subset A and B. Similarly, the value of $\Pi^{(2)}$ can be given corresponding to the point of intersection of membership functions of fuzzy subset A and complementary subset of B.

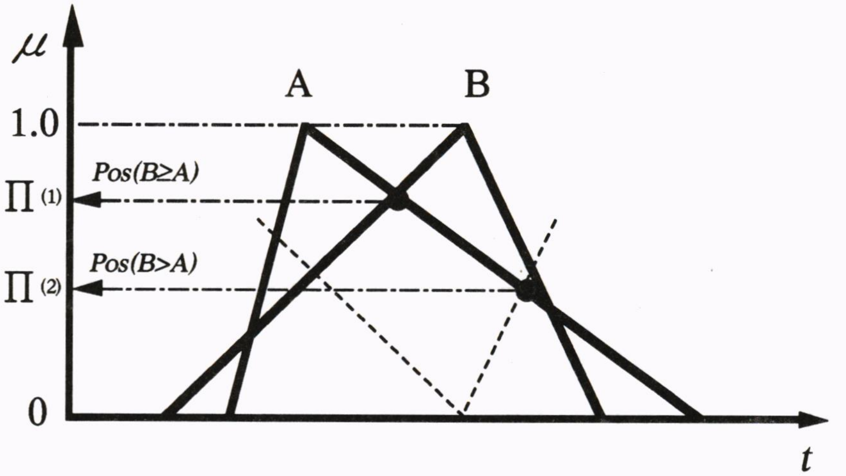


Fig. 2 Possibility Measures for Fuzzy Number B to A

The possibility index is used to describe route choice decision with fuzzy travel time information. Fuzzy goal is introduced because many values of the possibilities should be determined in multi-route choice models.

3. DESCRIPTION OF ROUTE CHOICE WITH FUZZY TRAVEL TIME

Flow independent situation is assumed to demonstrate the description of route choice with fuzzy travel time. The shortest path algorithm is commonly used for the description of route choice behaviour with crisp travel time in conventional assumption. Stochastic assignment models are proposed to determine the probability of route choice to deal with imperfect characteristics of traffic information (Sheffi & Powell, 1981). A route choice model with fuzzy information may be proposed by expressing in comparison with these methods in this research.

3.1 Route Choice Models

The route choice models are formulated corresponding to the assumptions for the behaviour of drivers. As it reflects the different provision of traffic information, the different user equilibrium might be deduced.

3.1.1 Shortest Path Model

In the user equilibrium (UE) model, it is assumed that all drivers have got perfect information on a network and always take the route to minimize their travel time (Infrastructure Planning

Committee, 1987). Therefore, the travel time for the path where certain OD traffic is loaded becomes minimum among the paths on the network. The shortest path algorithm is often used in practical applications. Dijkstra method is used in the study. The detailed algorithm can be seen in the references.

The deterministic model may not formulate all phenomena in the real world because there are many uncertain factors. The elements of uncertainty should be considered to describe realistic traffic flow on the networks. The random number in the stochastic approach represents the probability for the travel time because the perceived values of individual drivers distribute randomly. The occurrence of the events can be formulated by this manner. On the contrary, the fuzzy number can describe human perception for the travel time as a approximate number. The perceived range for the travel time of an individual driver can be treated by fuzzy set representation. Therefore, it should be memorized that stochastic approach and fuzzy approach are independently representing the different aspects of uncertainty to the same event.

3.1.2 Stochastic Assignment Model

It is assumed in UE model that perfect information is known by all drivers. However, it seems to be difficult to provide the precise information of overall network to the drivers. Because of this situation, the drivers do not chose only the shortest route between origin destination but the other route with certain proportion. Traffic flows on the links are determined according to the probability distribution of path travel time. The dial method is used in the study. The detailed algorithm can be seen in the references.

3.1.3 Fuzzy Assignment Model

The travel time is recognized as a fuzzy number which has subjective width in Fuzzy assignment model. An assignment method using "possibility index" is introduced to describe realistic route choice (Akiyama & Yamanishi, 1993) .

Fuzzy goal, G is designed as a fuzzy subset to compare the travel times for over the three routes or more simultaneously (Itoh & Ishii, 1996). The fuzzy goal G is regarded as a target value that the travel time between OD pairs is almost below the goal G.

A settlement of the fuzzy goal is mentioned. At the beginning, the standard travel time T is determined. The value corresponds to the most desirable time of the driver for OD trip. Practically, a travel time on the shortest path might be used for the value.

A fuzzy goal can be determined from parameters a and b for the left and right spreads of the fuzzy number on the basis of standard value T. The outline of the fuzzy goal is shown in Figure 3, where $T_{min} = aT$, $T_{max} = bT$.

$$\mu_G(t) = \begin{cases} 1, & t \le T_{\min} \\ \dfrac{T_{\max} - t}{T_{\max} - T_{\min}}, & T_{\min} < t < T_{\max} \\ 0, & T_{\max} \le t \end{cases} \tag{6}$$

The fuzzy goal in Figure 3 may represent a sufficient level of driver who estimate the path travel time between T_{min} and T_{max} Although the linear function is assumed here, non linear function would be available as well after some practical studies.

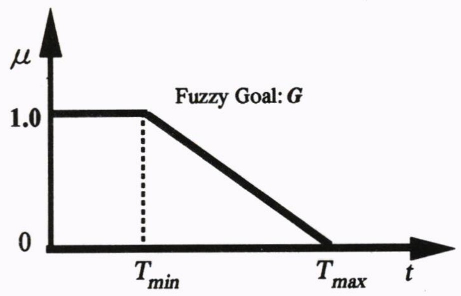


Fig. 3 Fuzzy Goal Illustrated by Linear Function

Assume that a fuzzy path travel time A and a fuzzy goal B are determined as shown in Fig. 4. The possibility measures of path travel time A to fuzzy goal B can be calculated according to the definitions. It is interpreted that the value of possibility corresponds to the achievement level of travel time to the predetermined goal B. In this sense, the value may indicate the degree of acceptance for the drivers who chose the path.

In other words, a sort of utility for the path would be calculated with fuzzy traffic information. Assume the utility function of the path i as $U_i=U_i(\Pi_i)$, the value of utility should be increased in proportion to the value of possibility, Π_i. In this sense, it might be assumed as:

$$U_i \geq U_j, \qquad \textit{if} \quad \Pi_i \geq \Pi_j \tag{7}$$

Therefore, it would be assumed that traffic flow occurs in proportion to the values of possibilities for the paths. Concerning with the utility, the similarity of possibility measure to the random utility in the logit model has been discussed in the related research (Henn, 1997, Akiyama, 1998).

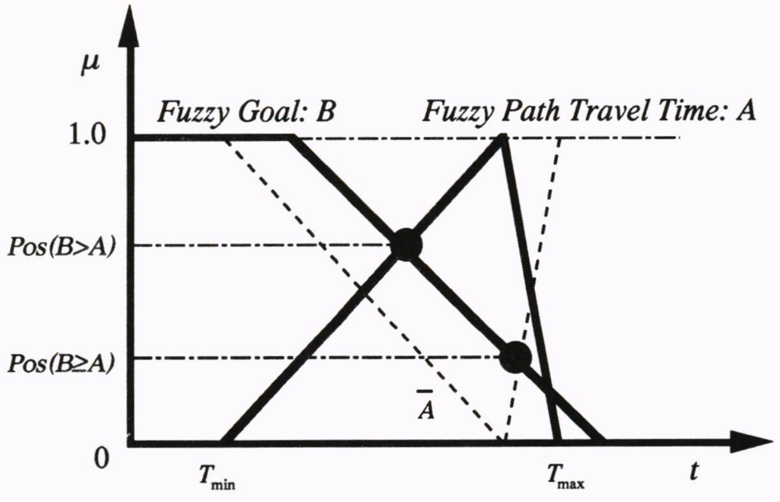


Fig. 4 Possibility Measures for Fuzzy Travel Time

In addition, it has been reported in earlier related studies that triangular fuzzy number (T.F.N.) is applicable to describe perceived travel time and the spread of T.F.N. changes in proportion to the crisp travel time as t (Akiyama & Yamanishi, 1993). Therefore, the description in Figure 4 might be accepted commonly.

According to the consideration, the calculation algorithm of the possibility method is summarized as follows:

【Algorithm】

Step 1: A fuzzy goal B is determined in advance.
Step 2: The fuzzy route travel times for individual OD pairs are determined as sums of fuzzy link travel times. In this calculation, the operation of extend sum can be introduced:
Step 3: A point of intersection between fuzzy goal B and fuzzy route travel time A is obtained as shown in Figure 4. The value of Π on the vertical axis shows a possibility index.
Step 4: Determine the choice probability for each route in proportion of the value of possibility index.
Step 5: OD traffic is loaded to the route with each proportion.

3.2 Numerical Example for Calculation

The simple network is illustrated in Figure 5 to consider the model calculation more realistically. There are five links with the BPR type cost functions as follows:

$$t(x) = t_0\left\{1+0.15\left(\frac{x}{Q}\right)^4\right\} \tag{8}$$

These link data for the network are summarized in Table 1. Calculation steps can be shown numerically with this example. Fuzzy link travel times are expressed as triangular fuzzy numbers shown in Figure 5. Therefore, fuzzy travel times are expressed by a triplet as *(αt, t, βt)* with parameters, α and β to determine the right and left spreads of fuzzy number. Let us consider the traffic flow from node 1 to node 4. Three paths are observed. Each methods becomes clear from comparison of each assignment procedure.

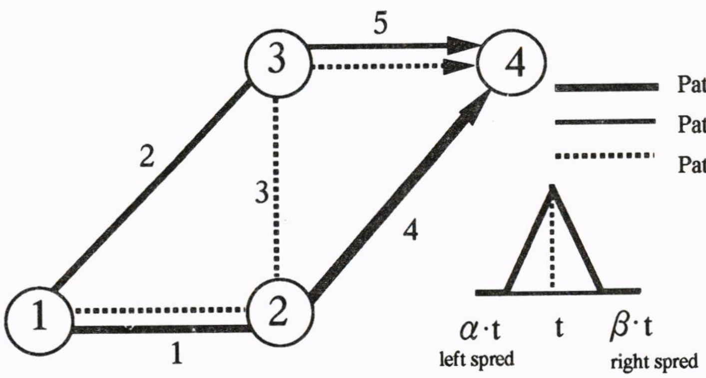


Fig. 5 Three Paths in The Example Network

Table 1 Link Data for The Network

Link No.	t_0	α	β	Q
1	4	0.3	1.2	100
2	5	0.8	1.8	100
3	3	0.8	1.6	100
4	5	0.2	1.1	100
5	4	0.6	1.7	100

Step 1: Path 1 or path 2 become the shortest path when the performance functions with crisp travel time are used. The shortest path travel time (T=9) gives a standard value T of a fuzzy goal. A fuzzy goal is determined by parameters a and b. In the example, a=0.8, b=1.5 respectively.

Step 2: Fuzzy route travel time can be determined. For instance, the fuzzy travel time for route 1 is calculated as follows:

$$t_{path\ 1} = t_{link\ 1} (+) t_{link\ 4}$$
$$= (1.2,\ 4.0,\ 4.8)(+)(1.0,\ 5.0,\ 5.5)$$
$$= (1.2+1.0,\ 4.0+5.0,\ 4.8+5.5)$$
$$= (2.2,\ 9.0,\ 10.3)$$

Similarly,

$$t_{path\ 2} = (6.4,\ 9.0,\ 15.8)$$
$$t_{path\ 3} = (6.0,\ 11.0,\ 16.4)$$

Step 3: A value of possibility measure is regarded as an achievement level of the route travel time in the previous step to the fuzzy goal. If the index as $Pos(B \geq A)$ is introduced, it becomes Π_1=0.863 as a intersection between membership functions for fuzzy travel time for route 1 and fuzzy goal G which is shown in Figure 6.

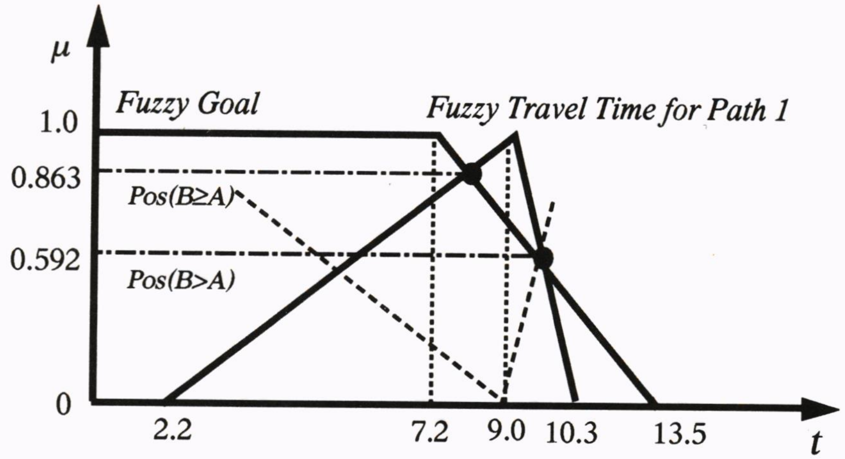


Fig. 6 Calculation of Possibility Index

Similarly, achievement levels of route 2 and route 3 to the fuzzy goal are counted as Π_2=0.798 and Π_3=0.664 respectively.

Step 4: Proportions of Route choice is calculated by using possibility index Π_i. For instance, the proportion for route 1 is counted:

$$w_1 = \frac{\Pi_1}{\Pi_1 + \Pi_2 + \Pi_3}$$
$$= \frac{0.863}{0.863 + 0.798 + 0.664}$$

In this manner, the weights for the path flows are obtained as w_1=0.371, w_2=0.343 and w_3=0.285 respectively. In this formulation, the linear function is assumed in terms of relation between the route choice rate and the scale of the utility.

Step 5:Each origin destination traffic is loaded on the path corresponding to the weight given in step 4. Thus $V_{\text{path 1}} = 600 \times 0.371$=223. Similarly, $V_{\text{path 2}} = 206$, $V_{\text{path 3}}$ = 171 are obtained.

The same algorithm should be applicable even though the index, $Pos(B \geq A)$ is replaced with $Pos(B > A)$. The different value of the index as 0.592 can be observed in Figure 6 for the same example.

3.3 Calculation Results

Assume that there are 600 unit of flow going from node 1 to node 4. Assignment results of different methods are compared. First of all, these calculation results are summarized in Table 2. In Dijkstra method, the crisp shortest route is chosen. Therefore, all 600 unit traffic are loaded to route 1 or route 2. On the contrary, the loading weights are calculated to all reasonable routes in Dial method. Therefore, there are 38 unit of flow going on path 3. Furthermore, the trip rates for three paths are mostly dispersed in the possibility method considering fuzzy travel times.

Table 2 Flow Independent Assignment of Each Method

	Dijkstra Method		Dial Method θ=0.5		Possibility Method			
					a = 0.8, b = 1.5		a = 1.0, b = 1.5	
	Trip Rate	Traffic Flow	weight	Traffic Flow	Π	Traffic Flow	Π	Traffic Flow
Path 1	1 (0)	600 (0)	0.422	253	0.715	230	1	215
Path 2	0 (1)	0 (600)	0.422	253	0.612	197	1	215
Path 3	0	0	0.155	93	0.536	173	0.789	170

The change of assigned traffic is observed by change of parameter a, b in the fuzzy goal. The possibility, $Pos(B \geq A)$ is introduced in the calculation. When the value of a is fixed as a=0.8, the value of b is changed. As a result of Table 3, it is known that the path flows are imbalance as a value of b is small. The values of the possibility index for path travel times are more dispersed when the gradient of a fuzzy goal is steep.

Table 3 Traffic Flow Change with Parameters for Fuzzy Goal

b	1.05	1.1	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5
Path 1	252	246	241	237	234	231	228	226	224	223
Path 2	198	200	202	203	204	205	205	206	206	206
Path 3	150	154	157	160	162	165	167	168	170	171

(a=0.8, b=1.5)

Similar results might be given from the other example with the possibility, $Pos(B > A)$.

The appropriate parameters of fuzzy goal might be determined trough the survey on the perception of drivers. Some results have been reported in the other researches (Akiyama, & Yamanishi, 1993, Akiyama & Nomura, 1998).

4. A Traffic Assignment with Fuzzy Travel Times

In this chapter, a assignment method with fuzzy travel times is proposed considering with the dependence between travel time and flow. The network loading mechanism underlying the UE assignment can be modelled by an all-or-nothing assignment process. Frank-Wolfe method is used here. For the stochastic assignment, the method of successive average (MSA) is introduced with a stochastic network loading of the dial method. In fuzzy equilibrium assignment, the MSA can be introduced as well with the network loading by the possibility method mentioned in the previous chapter.

4.1 Traffic Assignment Algorithm

The main purpose of traffic assignment is to estimate link flows on the transport network in equilibrium. The route choice models for the individual drivers have been formulated on the different assumptions. The traffic flow in each equilibrium condition may be formulated as a mathematical programming. Therefore, the proper optimization technique would be introduced to solve each problem (Sheffi, 1985).

4.1.1 User Equilibrium Assignment

The user equilibrium (UE) assignment with the equal travel times principle announced by Wardrop. The UE defines as "For each OD pair, at user equilibrium, the travel time on all used paths is equal, and (also) less than or equal to the travel time that would be experienced by a single vehicle on any unused path." An optimization method such as Frank-Wolfe technique is used to generate UE flow in this study. The detailed algorithm is well known in the references (Sheffi, 1985).

4.1.2 Stochastic User Equilibrium Assignment

The stochastic user equilibrium (SUE) is considered. At SUE, no driver can improve his or her perceived travel time by unilaterally changing route (Sheffi & Powell, 1981). The method of successive average (MSA) with the dial's network loading method is used as a calculation algorithm. This method is well known as a standard technique. This detailed algorithm can be seen in the references.

4.1.3 Fuzzy User Equilibrium Assignment

The MSA is utilized to generate stochastic user equilibrium flow on the network. Trip rates for the paths are determined by possibility measure in every calculation step (Akiyama & Kawahara, 1997). Therefore, the algorithm is summarized as follows:

Step 0: Define the parameters, a and b for fuzzy goal in advance.

Step 1: Perform an all-or-nothing assignment based on $t_a^0 = t_a(0)$.

Step 2: Determine $\{V_a^{(1)}\}$ based on the possibility measures. Set counter n:=1

Step 3: Update. Set $t_a^{(n)} = t_a(V_a^{(n)})$ (by BPR function) corresponding to $\{V_a^{(1)}\}$ with fuzzy travel times.

Step 4: Determine path flow pattern $\{Y_a\}$ with the possibility method based on $t_a^{(n)}$.

Step 5: Search direction. $V_a^{(n+1)} = V_a^{(n)} + \frac{1}{n}(Y_a - V_a^{(n)})$

Step 6: Convergence test. If a convergence criterion is met, stop (the current solution, $\{V_a^{(1)}\}$, is the set of fuzzy equilibrium link flows); otherwise, set n:=n+1 and go to step 3.

The MSA is well known as a solving algorithm for SUE problem. The convergence of the algorithm has been confirmed even though it may not be the most efficient way. Therefore, it might be confirmed that the FUE algorithm converges quite similarly because only the part of route choice in SUE algorithm is replaced by fuzzy route choice representation. It is reported in the earlier study (Akiyama & Shao, 1999).

4.2 Numerical Example for Fuzzy User Equilibrium

An numerical example calculation is carried out by using an example of general origin destination matrix shown in Table 4.

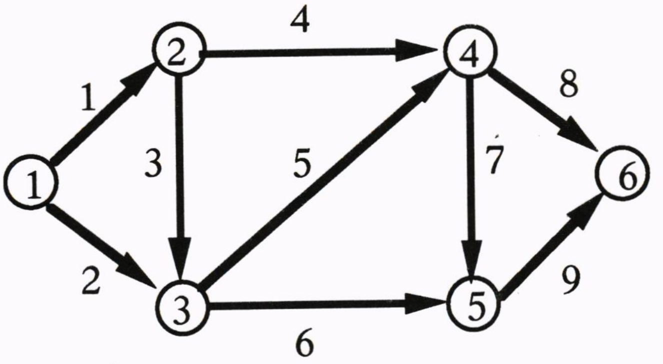


Fig. 7 The Example Network with Fuzzy Links

Table 4 Link Data for The Network

links	t_0	α	β	Q
1	4.0	0.2	1.5	100
2	3.0	0.5	1.8	100
3	2.0	0.3	1.2	100
4	5.0	0.8	1.1	100
5	7.0	0.4	1.6	100
6	8.0	0.6	1.7	100
7	7.0	1.0	1.0	100
8	5.0	0.8	1.5	100
9	3.0	0.7	1.6	100

The network has nine links and six nodes as shown in Figure 7. The parameters for each link performance function are listed in Table 4. The same type BPR functions as the previous example are used. Therefore, user equilibrium assignment is carried out under the same condition as the previous chapter.

Table 5 OD Matrix for The Example Network

2	3	4	5	6	D \ O
100	100	100	100	100	1
	100	100	100	100	2
		100	100	100	3
			100	100	4
				100	5

In the proposed algorithm for FUE, the incidence matrix between links and paths should determined to prepare the reasonable paths between each OD pairs before the calculation. As the path flows are considered as variables in the FUE problem, the algorithm demands more computation efforts comparing to UE and SUE algorithm. Therefore, this formulation should be modified in further study to apply the algorithm to the practical problem as traffic analysis in a large scale transport network.

In the example, every reasonable paths for a OD pair is assumed to be known. In other words, path flows are used as variables in the assignment. Therefore, the traffic on a link is obtained after the path flows are determined. In this sense, it can be seen that the proposed algorithm has worked properly. The results of different assignment are summarized in Table 6.

Table 6 Fuzzy User Equilibrium Flow compared with UE and SUE

	UE	SUE (θ=0.5)	Fuzzy UE (a=0.8, b=1.5)	
			$Pos(B \geq A)$	$Pos(B > A)$
link-1	230	235	283	261
link-2	270	263	217	239
link-3	214	224	301	252
link-4	316	310	282	310
link-5	282	389	328	298
link-6	302	396	290	293
link-7	225	235	265	254
link-8	273	265	244	253
link-9	227	232	256	247

As a result of fuzzy equilibrium assignment, the variation of link flow distribution seems to be larger in FUE with $Pos(B \geq A)$ comparing with UE and SUE results. On the other hands, the distribution of link flow seems to be rather uniform in FUE with $Pos(B > A)$. However, it cannot be concluded that this tendency is commonly observed in case of FUE. The further analysis for link flow would be recommended to reveal the characteristics in terms of practical applications.

There is another observation. As a path consists of more links, the path travel time tends to be described as a number with more fuzziness, which is expressed as the width of spreads in the fuzzy number. Thus, the different size of the left spread for fuzzy travel time generate different biased traffic. At the same time, assigned traffic may change according to the parameters for fuzzy goal as a and b. As the slope of membership function for fuzzy goal becomes steeper with a smaller value of a, the assigned traffic is dispersed. This tendency has been already observed in the result of fuzzy flow independent assignment in the previous chapter. It summarizes that the traffic flows are sensitive to change with small range of fuzzy goal in fuzzy user equilibrium (FUE).

It would be noted that the FUE can describe fuzzy perception of driver to the travel time which is different from the probability distribution assumed in SUE. Therefore, it may not be discussed that the FUE can be superior to SUE. Apart from the advantages of the methods, the difference of the description would be known from the calculation results. If the FUE approach is introduced, the fuzzy link travel times can be estimated as well as equilibrium link flows. For example, traffic flows for link 4, link 5 and Link 6 are estimated by the UE and the FUE according to the Table 6. The fuzzy times as well as crisp travel time are derived from these values. Three crisp numbers and three fuzzy numbers are illustrated respectively in Figure 8. It is observed that the FUE can determine not only different equilibrium flows but also fuzzy link travel times.

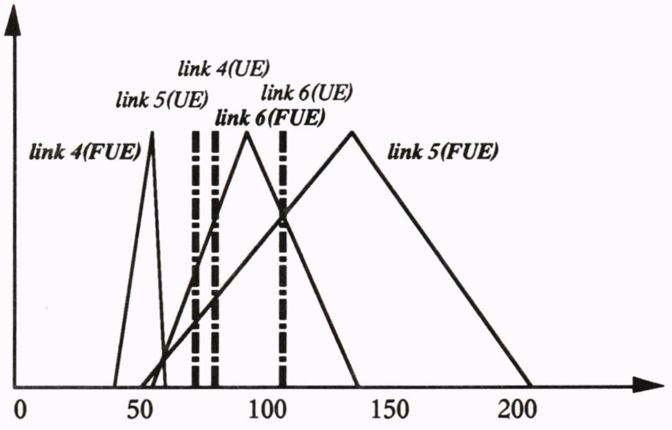


Fig. 8 The Crisp travel times and Fuzzy Travel Time for Some Links

5. CONCLUDING REMARKS

The traffic assignment with fuzzy travel time information is introduced to consider the perception of drivers on the network. The study aims to describe the bahaviour of drivers with traffic information realistically. Research results are summarized :

Firstly, a method of comparison between fuzzy numbers is proposed. Particularly, the possibility measures are taken to be an index of comparison. The possibility measure can reflect on ambiguous characteristics of fuzzy number and give a numerical value for magnitude of fuzzy number.

Secondly, the fuzzy assignment method was proposed by using possibility index. Individual possibility index value of each route is indicated comparing to a defined fuzzy goal. It is assumed that trip rate for the path is given in proportion of the value of possibility. The traffic flow is changed corresponding to the shape of fuzzy travel time because it reflects on the perception of drivers.

Thirdly, the algorithm of fuzzy user equilibrium (FUE) assignment is implemented. The method of successive average (MSA) can be applied to determine the traffic flow similarly to the stochastic assignment. In the numerical example, the FUE flow can be observed.

It is known that the proposed algorithm can be applicable to FUE assignment with $Pos(B \geq A)$ as well as $Pos(B > A)$.

Two issues are mainly raised for future subjects. Firstly, fuzzy traffic assignment with possibility measure has been proposed. Apart from the possibility measure, the necessity measures, $Nes(B \geq A)$ as well as $Nes(B > A)$ might be introduced to describe realistic situation. Therefore, this index reflects on more prudent judgment than the previous one. (The detail would be seen in Appendix).
Secondly, path flows are used as variables in the proposed algorithm for fuzzy equilibrium assignment. Link flows are determined after path flows are assigned. Therefore, the algorithm may not be applied to large scale networks because a link-path incidence matrix should be given in advance. Therefore, the development of the algorithm on the basis of link flows is recommended to apply to general networks (Akiyama, 1988).

Furthermore, the hybrid number that has both characteristics of random number and fuzzy number is planned to be applied to UE problem as well (Akiyama & Nomura, 1998).

REFERENCES

Akiyama, T. and Yamanishi, H. (1993) Travel Time Information Service Device Based on Fuzzy Sets Theory, **Proceedings of the Second International Symposium on Uncertainty Modeling and Analysis, ISUMA'93**, 238-245 .

Akiyama, T. and Kawahara, T. (1997) Traffic Assignment Model by using Possibility Measure, **Proceedings of the 13th Fuzzy System Symposium,** 221-222 (in Japanese).

Akiyama, T. and Nomura, T. (1998) The Techiniques of Traffic Assignment Model with Fuzzy Measures, **Proceedings of the 14th Fuzzy System Symposium,** 833-834 (in Japanese).

Akiyama, T. (1998) Traffic Assignment with Fuzzy User Equiribrium Conditions, **Paper Presented at Workshop in Deparment of Civil Enigineering, Kyoto University**, Kyoto, Japan.

Akiyama, T. and Shao, C. F. (1999) Traffic Information Service and Fuzzy Traffic Flow Analysis, **Journal of Japan Society for Fuzzy Theory and Systems,** Vol. 11, No. 2, 246-258 (in Japanese).

Henn, V. (1997) Fuzzy RouteChoice Model for Traffic Assignment, **Paper Presented at 9th mini EURO Conference on Fuzzy Sets in Traffic and Transportation Systems,** Budva, Yugoslavia.

Infrastructure Planning Committee (1987) Transport Network Analysis and Planning - Advanced Theory and Applications, **Japan Society for Civil Engineers** (in Japanese).

Itoh, T. and Ishii, H. (1996) A Model of Fuzzy Shortest Path by Possibility Measure, **Journal of Japan Society for Fuzzy Theory and Systems**, Vol. 8, No. 6, 134-142 (in Japanese).

Kaufmann, A. and Gupta, M. M. (1988) Fuzzy Mathematical Models in Engineering and Management Science, **Elsevier Science Publishers B. V.**

Sakawa, M. (1989) Fuzzy Sets Theory and Its Application, **Morikita Press co. Ltd.** (in Japanese).

Shao, C. and Akiyama, T. (1994) Empirical Based on Road Traffic Flow Analysis with Fuzzy Route Information, **Proceedings of the Second Symposium on Application of Fuzzy Sets Theory to Civil and Architecture Engineering,** 77-85, (in Japanese)

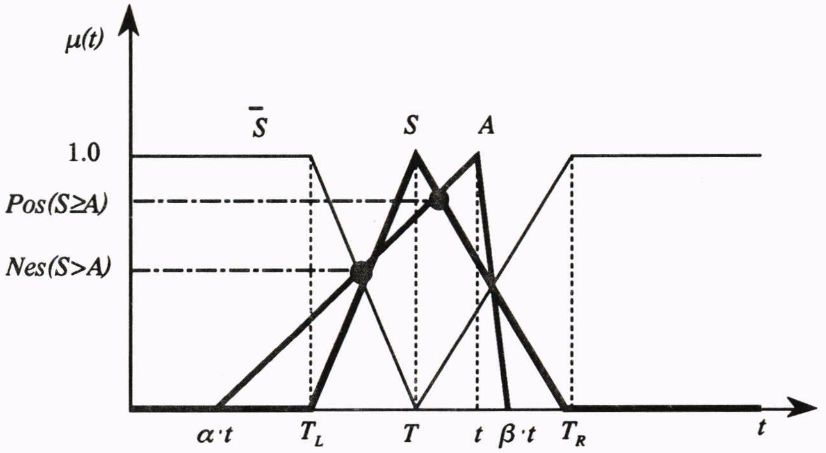
Sheffi, Y. and Powell, W. B. (1981) A comparison of stochastic and deterministic traffic assignment over congested networks, **Transportation Research,** Vol. 15B, No. 1, 53-64.

Sheffi, Y. (1985) Urban Transportation Network, **Prentice Hall.**

APPENDIX: The Route Choice Model with Standard Fuzzy Travel Time

In the proposed method with possibility measure, the OD flow is assigned in proportion to the possibility index as $Pos(B \geq A)$. In the modelling, the fuzzy goal is determined. The membership value of fuzzy goal is equal to 1 as $\mu_A(x) = 1.0$ when the travel time is less than the specific value. Therefore, the method with fuzzy goal can be named as a route choice model with fuzzy goal travel time **(FGT model)**.

On the other hand, another type of fuzzy measures as necessity, $Nes(B > A)$ might be used. In this case, it is assumed that driver would know the standard travel time for each path as a fuzzy number. Otherwise, the index of necessity does not make sense. The fact suggest another type of route choice model with Standard Fuzzy Travel Time **(SFT model)**. The outline of standard fuzzy travel time, S is illustrated in Figure 8. The fuzzy measures as $Pos(S \geq A)$ and $Nes(S > A)$ are indicated as well.



S: Fuzzy Standard Travel Time
A: Fuzzy Travel Time for the Path

Fig. 9 The Outline of Standard Fuzzy Travel Time and Fuzzy Measures