EXPERIENCES WITH THE DYNAMIC USER-OPTIMAL ROUTE CHOICE MODEL FOR CHUNGLI CITY

Huey-Kuo CHEN
Professor
Department of Civil Engineering
National Central University
Chung-Li, Taiwan,
32054
Fax: +886-3-4252960
E-mail:
ncutone@cc.ncu.edu.tw

Mei-Shiang CHANG
Post-doctor Research
Department of Civil Engineering
National Central University
Chung-Li, Taiwan,
32054
Fax: +886-3-4252960
E-mail:
mschang@trans.cv.ncu.edu.tw

Chung-Yi LEE
Research Associate
Department of Civil Engineering
National Central University
Chung-Li, Taiwan,
32054
Fax: +886-3-4252960
E-mail:
kublia@ trans.cv.ncu.edu.tw

Abstract: The dynamic user-optimal route choice problem takes into account the temporal dimension with which real world situations can be better represented. Though quite a few dynamic user-optimal route choice models appear in the literature, the actual application of dynamic user-optimal route choice models into a real network still needs more research and is indeed of much interest to researchers and practitioners.

In this paper, the dynamic user-optimal route choice model formulated by Chen and Hsueh (1998) is tested with the Chungli-Pincheng network. The results show for each time-dependent O-D pair that all the used route travel times are equal and minimal, which exactly comply with the Wardrop's equilibrium conditions. More importantly, it is demonstrated that the dynamic user-optimal route choice model can be even solved by a personal computer for a medium size real network. To shorten turn-around computation times, better solution algorithms will be explored using more powerful computer facilities such as supercomputers.

1. INTRODUCTION

The dynamic user-optimal route choice problem takes into account the temporal dimension with which real world situations can be better represented. Though quite a few dynamic user-optimal route choice models appear in the literature, experiences with medium or large scale networks have not yet reported. On a somewhat different line, implementing quasi-dynamic user-optimal route choice model into a real network (Janson, 1991), or simply adopting a heuristic for a large network can be observed (Boyce *et al*., 1996). However, without knowing mathematical properties of analytic solution algorithms, the model's results have less practical implications. Thus, it is a *must* to validate dynamic user-optimal route choice models and associated analytic solution algorithms with a real size network problem, which is indeed of much interest to researchers and practitioners.

Though dynamic travel choice models are theoretically better than their static counterparts and have great potential to be applied in real-time on-line route guidance systems. However, many inherent properties such as first-in-last-out phenomenon and non-convergence problem attributed to an accounting problem are indeed difficult to handle. Moreover, lacking actual experience with dynamic travel choice models on large scale transportation networks prevents their full scale deployment. In this paper, not all issues pertaining to the dynamic route choice models are of interests. Rather, only the median size

Chungli-Pincheng network (hereafter abbreviated as Chungli network) is tested with the dynamic user-optimal (DUO) route choice model formulated by Chen and Hsueh (1998). This DUO route choice model can be characterized by the dynamic extension of Wardrop's equilibrium conditions and their equivalence can be proved under the estimated actual link travel times. For easier reference, the equilibrium conditions and model formulation are rewritten in Section 2. An augmented Lagrangian method which embeds the gradient projection method is described in Section 3. The case study is presented in Section 4. Concluding remarks are finally provided in Section 5.

2. DYNAMIC USER-OPTIMAL ROUTE CHOICE PROBLEM

2.1 Equilibrium Conditions

Given O-D demands that are fixed, the equilibrium conditions for the DUO route choice problem state that for each origin-destination pair *rs*, the actual route travel times experienced by travelers departing during the same interval is equal and minimal. At the same time, the actual route travel time of any unused route for each O-D pair is greater than or equal to the minimal actual route travel time. In other words, at equilibrium, if the flow departing origin r during interval k over route p toward destination s is positive, i.e. $h_p^{rs}(k) > 0$, then the corresponding actual route travel time is minimal. On the contrary, if no flow occurs on route p during interval k, i.e. $h_p^{rs}(k) = 0$, then the corresponding actual route travel time is at least as great as the minimal actual route travel time. These dynamic user-optimal equilibrium conditions can be expressed mathematically as follows:

$$c_p^{rs*}(k)\begin{cases} = \pi^{rs}(k) & \text{if } h_p^{rs*}(k) > 0 \\ \geq \pi^{rs}(k) & \text{if } h_p^{rs*}(k) = 0 \end{cases} \tag{1}$$

2.2 Variational Inequality Formulation

Theorem 1: The DUO route choice problem is equivalent to finding a solution $\mathbf{u}^* \in \Omega$ such that the following VIP holds.

$$\sum_a \sum_t c_a^*(t)\left[u_a(t) - u_a^*(t)\right] \geq 0 \quad \forall \mathbf{u} \in \Omega^* \tag{2}$$

where Ω^* is a subset of Ω with $\delta_{apk}^{rs}(t)$ being realized at equilibrium, i.e., $\delta_{apk}^{rs}(t) = \delta_{apk}^{rs*}(t), \forall r,s,a,p,k,t$. The symbol Ω denotes the feasible region that is delineated below by flow conservation, flow propagation, nonnegativity, and definitional constraints.

Flow conservation constraint:

$$\sum_p h_p^{rs}(k) = \bar{q}^{rs}(k) \quad \forall r,s,k \tag{3}$$

Flow propagation constraints:

$$u_{apk}^{rs}(t) = h_p^{rs}(k)\delta_{apk}^{rs}(t) \quad \forall r,s,a,p,k,t \tag{4}$$

$$\sum_t \delta_{apk}^{rs}(t) = 1 \quad \forall r,s,p,a \in p,k \tag{5}$$

$$\delta^{rs}_{apk}(t) = \{0,1\} \quad \forall r,s,a,p,k,t \tag{6}$$

Nonnegativity constraint:

$$h^{rs}_{p}(k) \geq 0 \quad \forall r,s,p,k \tag{7}$$

Definitional constraints:

$$u_a(t) = \sum_{rs}\sum_{p}\sum_{k} h^{rs}_{p}(k)\delta^{rs}_{apk}(t) \quad \forall a,t \tag{8}$$

$$c^{rs}_{p}(k) = \sum_{a}\sum_{t} c_a(t)\delta^{rs}_{apk}(t) \quad \forall r,s,p,k \tag{9}$$

Equation (3) conserves the time-dependent O-D demands. Equation (4) describes the flow propagation along route p through the use of the indicator variables. If the indicator variable, $\delta^{rs}_{apk}(t)$, is equal to 1, then the route flow from origin r departing during interval k over route p toward destination s will get on link a during interval t. On the contrary, if the indicator variable, $\delta^{rs}_{apk}(t)$, is equal to zero, it implies that link a during interval t is not in route p that is associated with O-D pair rs and departure interval k.

Equation (5) indicates that the flow departing origin r during interval k over route p toward destination s can be incident to link a, at most once, during a specific time interval t. If the route flow is not presented on link a, then it must get on one of other links in the network, unless the destination has been reached. Equation (6) designates that indicator variables are integer-valued; implying that flow deformation is not possible in our model.

Equation (7) ensures that all route inflows are nonnegative. Equation (8) expresses the link flows in terms of the route flows through the use of the indicator variables. Equation (9) expresses the actual route travel time in terms of the actual link travel times, and computes the actual route travel time by adding up the actual travel times on those links along that route.

3. SOLUTION ALGORITHM

An augmented gradient projection method is proposed below for solving the DUO route choice problem.

3.1 Augmented gradient projection method

Augmented Gradient Projection Method

Step 0: Initialization.

Step 0.1: Let m=0. Set $\tau^0_a(t) = NINT\left[c_{a_0}(t)\right], \forall a,t$.

Step 0.2: Let n=1. Find an initial feasible solution $\{u^1_a(t)\}$. Compute the associated link travel times $\{c^1_a(t)\}$.

Step 1: *First Loop* Operation.

Let m=m+1. Update the estimated actual link travel times by

$$\tau^m_a(t) = NINT\left[(1-\gamma)\tau^{m-1}_a(t) + \gamma c^n_a(t)\right] \quad \forall a,t \tag{10}$$

Construct the corresponding feasible time-space network based on the estimated actual link travel times.

Step 2: ***Second Loop*** **Operation.**

Step 2.1: Let n=1. Compute and reset the initial feasible solution $\{u_a^n(t)\}$, based on the time-space network constructed by the estimated actual link travel times $\{\tau_a^m(t)\}$.

Step 2.2: Fix the inflows for each physical link other than on the subject time-space link at the current level, yielding the optimization problem defined by equations (11)~(16).

Step 3: ***Third Loop*** **Operation.**

Solve for the solution $\{u_a^{n+1}(t)\}$ using the gradient projection (GP) method. Compute the resulting link travel times $\{c_a^{n+1}(t)\}$.

Step 4: Convergence Check for the ***Second Loop*** **Operation.**

If $u_a^{n+1}(t) \approx u_a^n(t), \forall a, t$, go to Step 5; otherwise, set n=n+1, go to Step 2.2.

Step 5: Convergence Check for the ***First Loop*** **Operation.**

If $\tau_a^m(t) \approx c_a^{n+1}(t)\ \forall a, t$, stop; the current solution is optimal. Otherwise, set n=n+1, and go to Step 1.

In short, our augmented gradient projection solution procedure contains three levels, with the actual travel times being estimated in the first (outermost) level, inflows other than those on the subject time-space link being fixed in the second level, and the GP method (which is described in the following section) being applied in the third (innermost) level.

3.2 Gradient projection algorithm

The gradient projection (GP) algorithm iterates between the original master problem (MP) and the restricted master problem (RMP). In the MP, for each O-D pair and time interval, a new shortest path is searched over the time-space network and added, if appropriate, to the shortest path set. The RMP is then invoked. In the RMP, the path flows associated with all shortest paths that are stored in the shortest path set are optimally determined. This procedure continues until the convergence criterion is met. The steps of the GP algorithm can be described as follows:

Gradient Projection Algorithm

Step 0: Initialization.

Let n=0. For each O-D pair and time interval, search for a shortest route based on the free flow travel times $\{c_{a_0}(t)\}$. Create a path set to store all shortest routes with path flows denoted as $\{h_p^{rs}(k)^{(1)}\}$

Step 1: Master Problem (Column Generation).

Step 1.1: Let n=n+1. Update the link travel times $\{c_a(t)^{(n)}\}$ based on path flows $\{h_p^{rs}(k)^{(n)}\}$.

Step 1.2: For each O-D pair and time interval, search for a new shortest path over the network based on $\{c_a(t)^{(n)}\}$. The path set is augmented by the routes not contained in the set already. For each O-D pair and time interval, label the newest found/rediscovered shortest route as $\hat{p}$.

Step 2: Restricted Master Problem.

Use the gradient projection algorithm to solve the following restricted master problem.

$$\min \phi(\mathbf{u},\mathbf{h}) = \sum_a \sum_t \int_0^{u_a(t)} c_a\left(u_a(1)^{(n)}, u_a(2)^{(n)}, \ldots, u_a(t-1)^{(n)}, \omega\right) d\omega \quad (11)$$

S.t.

Flow conservation constraint:

$$\sum_p h_p^{rs}(k) = \bar{q}^{rs}(k) \quad \forall r,s,k \quad (12)$$

Nonnegativity constraint:

$$h_p^{rs}(k) \geq 0 \quad \forall r,s,p,k \quad (13)$$

Definitional constraints:

$$u_a(t) = \sum_{rs} \sum_p \sum_k h_p^{rs}(k) \bar{\delta}_{apk}^{rs}(t) \quad \forall a,t \quad (14)$$

$$c_p^{rs}(k) = \sum_a \sum_t c_a(t) \bar{\delta}_{apk}^{rs}(t) \quad \forall r,s,p,k \quad (15)$$

$$\bar{\delta}_{apk}^{rs}(t) = \{0,1\} \quad \forall r,s,a,p,k,t \quad (16)$$

Step 2.1: For each O-D pair and time interval, update the path flows $\left\{h_p^{rs}(k)^{(n+1)}\right\}$ and the associated inflow pattern $\left\{u_a(t)^{(n+1)}\right\}$ by the following formulas:

$$h_p^{rs}(k)^{(n+1)} = \max\left\{0, h_p^{rs}(k)^{(n)} - \alpha_p^{rs}(k)^{(n)} d_p^{rs}(k)^{(n)}\right\} \quad \forall r,s,p \neq \hat{p},k \quad (17)$$

$$h_{\hat{p}}^{rs}(k)^{(n+1)} = \bar{q}^{rs}(k) - \sum_{p \neq \hat{p}} h_p^{rs}(k)^{(n+1)} \quad \forall r,s,k \quad (18)$$

$$u_a(t)^{(n+1)} = \sum_{rs} \sum_p \sum_k h_p^{rs}(k)^{(n+1)} \bar{\delta}_{apk}^{rs}(t) \quad \forall a,t \quad (19)$$

where

$$d_p^{rs}(k) = c_{\hat{p}}^{rs}(k) - c_p^{rs}(k) \quad \forall r,s,p \neq \hat{p},k \quad (20)$$

$$\alpha_p^{rs}(k) = \frac{1}{\sum_a \sum_t \left(c_a'(t)\bar{\delta}_{apk}^{rs}(t) + c_a'(t)\bar{\delta}_{a\hat{p}k}^{rs}(t)\right) - \sum_{a \in p \cap \hat{p}} \sum_t 2c_a'(t)} \quad \forall r,s,p \neq \hat{p},k \quad (21)$$

Step 2.2: Calculate the difference between the link inflow patterns in two successive iterations by the following formula:

$$\varepsilon = \max_{a,t} \left| \frac{u_a(t)^{(n+1)} - u_a(t)^{(n)}}{u_a(t)^{(n)}} \right| \leq 0.0001 \quad (22)$$

If the difference ε is less than a predetermined tolerance, say 0.0001, the updated solution is deemed optimal. Otherwise, go to Step 1.

In Step 2, the opposite search direction $\left(c_p^{rs}(k) - c_{\hat{p}}^{rs}(k)\right)$ in equation (20) is determined by the first derivative of the objective function (11) with respect to route flow, and the move size $\alpha_p^{rs}(k)$ in equation (22) is determined by the inverse of the second derivative of the objective function (11) with respect to route flow.

4. CASE STUDY

To demonstrate that the DUO route choice model and the augmented gradient projection method are capable of being implemented in the real world. The Chungli-Pincheng network graphed in Figure 1 is taken for study. Chungli-Pincheng is an urban area with about 35 kilometers away from the southeastern part of Taipei metropolitan. The associated land area and population for Chungli-Pincheng urban area are roughly 7 thousand acres and 300 thousand residents, respectively. For the purpose of transportation planning, the entire network is represented by 139 nodes and 457 links, covered by 16 traffic zones.

The time interval is defined as 15 minutes (or 4 intervals per hour). Therefore, the time-dependent O-D demands are dimensioned with 16×16×4, which is not shown here due to space limitation. The dynamic travel time function is hypothesized as an FHWA-type formula:

$$c_a(t) = c_{a_0}(t)\left[1 + 1.5\left(\frac{u_a(t)}{CAP_a(t)}\right)^4\right] \quad \forall a, t \tag{23}$$

The augmented gradient projection method was coded with Borland C++5.01 and then executed on a Pentium-150 personal computer with 64MB RAM. The results with more than 200 pages computer output was obtained in about 20 hours. For illustration, only the route information (including links composing the route, route travel times and route flows) for origin 4 departing during interval 1 is summarized in Table 1.

Each route travel time is computed by summing link travel times along that route. The following example shows that the first route travel time between O-D pair (4,10) departing during interval 1 is the sum of travel times on link 4→152 during interval 1, link 152→140 during interval 1, link 140→128 during interval 4, link 128→129 during interval 6, link 129→121 during interval 7, link 121→122 during interval 9, link 122→107 during interval 10, link 107→103 during interval 12, link 103→104 during interval 14, link 104→102 during interval 15, link 102→219 during interval 16, link 219→220 during interval 25, link 220→10 during interval 28, as follows:

$$\begin{aligned}
c_{p_1}^{4-10}(1) &= c_{4\to152}(1) + c_{152\to140}(1) + c_{140\to128}(4) + c_{128\to129}(6) + c_{129\to121}(7) \\
&+ c_{121\to122}(9) + c_{122\to107}(10) + c_{107\to103}(12) + c_{103\to104}(14) + c_{104\to102}(15) \\
&+ c_{102\to219}(16) + c_{219\to220}(25) + c_{220\to10}(28) \\
&= 0.00 + 2.89 + 2.08 + 0.99 + 1.63 \\
&+ 1.08 + 1.63 + 1.54 + 0.72 + 0.72 \\
&+ 9.49 + 3.44 + 0.00 = 26.21
\end{aligned} \tag{24}$$

It can be observed for each time-dependent O-D pair that all the used route travel times are equal and minimal, which exactly comply with the Wardrop's equilibrium conditions.

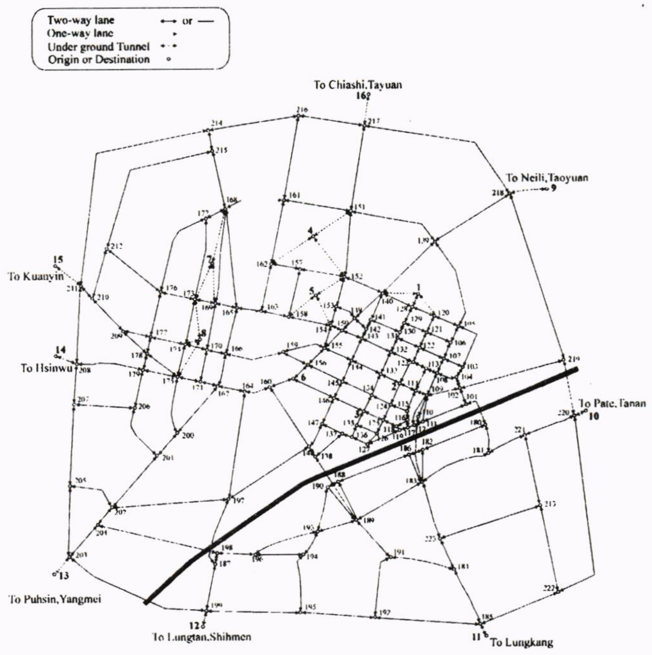

Figure 1. Chungli-Pincheng Network

Table 1. Route Information for Origin 4 Departing During Interval 1

O-D Pair	O-D Demands	Route	Travel Time	Flow
4-1	1	4→152→140→1	2.89	1.00
4-2	10	4→152→153→154→155→144→133→ 2	10.33	10.00
4-3	32	4→152→153→154→155→144→145→146→3	11.68	32.00
4-6	25	4→152→153→154→155→156→ 6	7.70	25.00
4-7	46	4→162→163→165→169→ 7	6.60	46.00
4-8	60	4→162→163→165→169→173→ 8	8.50	60.00
4-9	30	4→151→139→218→ 9	11.02	30.00

Table 1. (Continued) Route Information for Origin 4 Departing During Interval 1

O-D Pair	O-D Demands	Route	Travel Time	Flow
4-10	33	4→152→140→128→129→121→122→107→1 03→104→102→219→220→10	26.21	8.25
		4→152→140→128→120→105→103→104→1 02→219→220→10	26.21	15.52
		4→152→140→128→129→121→106→107→1 03→104→102→219→220→10	26.21	9.23
		4→152→153→154→155→144→145→146→1	34.90	7.00

4-11	7	47→148→138→189→191→184→185→11		
4-12	6	4→152→153→154→155→156→6→160→164 →197→187→199→12	36.08	6.00
4-13	3	4→162→163→165→166→167→200→201→202→204→203→13	32.36	3.00
4-14	6	4→162→163→165→166→170→174→175→179→208→14	15.81	3.59
		4→162→163→165→166→170→171→175→179→208→14	15.81	2.41
4-15	29	4→162→163→165→166→170→171→175→179→208→211→15	21.77	17.68
		4→162→163→165→166→170→174→175→179→208→211→15	21.77	11.32
4-16	37	4→151→217→16	6.23	37.00

5. CONCLUDING REMARKS

The DUO route choice model has be tested with a medium size real network. Although the selected real problem can be handled by a personal computer, the computation time is rather long. To expedite the deployment of the dynamic travel choice models, turn-around computation times must be shortened by developing better solution algorithms or adopting more powerful computer facilities such as supercomputers.

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REFERENCES

1. Boyce D. E., Lee D. H. and Janson B. N. (1996) A variational inequality model of an ideal dynamic user-optimal route choice problem. Paper will be published as a book chapter by the 4th meeting of the EURO working group on Transportation, Newcastle upon Tyne, UK.
2. Chen, H.K. (1998) **Dynamic Travel Choice Models : A Variational Inequality Approach**. Springer-Verlag, Berlin.
3. Chen, H.K. and Hsueh, C.F. (1998) A model and an algorithm for the dynamic user-optimal route choice problem. **Transportation Research 32B(3)**, 219-234.
4. Chen, H.K., Chang C.W. and Chang, M.S. (1999). A comparison of link-based versus route-based algorithms with the dynamic user-optimal route choice problem. **Transportation Research Record** (forthcoming).
5. Janson B. N. (1991) Dynamic traffic assignment for urban road networks. **Transportation Research 25B**, 143-161.

APPENDIX

Symbols used in this paper are summarized as follows:

a	: link designation
$c_a(t)$	: travel time for link a during time interval t
$c_p^{rs}(k)$	: travel time for route p between O-D pair rs during time interval k
$CAP_a(t)$	: capacity for link a during time interval t
$d_p^{rs}(k)$	: search direction
$h_p^{rs}(k)$	: departure flow rate on route p from origin r toward destination s during time interval k
k	: time interval designation which usually denotes the departure time interval for a route
p	: route designation
r	: origin designation
s	: destination designation
$\mathbf{u}$	: vector of link inflow rates
$u_a(t)$	: inflow rate into link a during time interval t
$u_{apk}^{rs}(t)$	: part of inflow rate for link a during time interval t that is departing origin r over route p toward destination s during time interval k
$x_a(t)$	: number of vehicles on link a at the beginning of time interval t
$\delta_{apk}^{rs}(t)$	: 1, if inflow rate on link a during time interval t departs from origin r over route p toward destination s during time interval k; otherwise, 0
$\alpha_p^{rs}(k)$	: move size
γ	: weight
$\tau_a(t)$	: actual travel time for link a during time interval t
$\pi^{rs}(k)$	: minimal route travel time between O-D pair rs during time interval k
Ω	: feasible region