

TRAFFIC EQUILIBRIUM PROBLEMS WITH ENVIRONMENTAL CONCERNS

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Abstract: In this paper, the traffic equilibrium problems with environmental concerns, traffic emissions specifically, are discussed. A minimal traffic emission model (MTE) is proposed. With a numerical example, the routing strategy in the route guidance system (RGS) provided by this model is compared with those under the traditional system performance objectives, such as minimal network travel time. Furthermore, A bi-level minimal traffic emission model (BMTE) for the electronic road pricing system (ERPS) is formulated. The road toll pattern will be determined to minimize traffic emissions when users make their route choices in a user equilibrium manner. This minimal traffic emission pricing strategy is also compared with alternative road pricing strategies, such as traditional marginal-cost pricing strategy.

1. INTRODUCTION

In recent years, urban pollution has emerged as the most acute problem, because of its negative effects on health and deterioration in living conditions. Traffic emission is a dominant source of urban air pollution. For instance, the sources contributing to fine particulate, namely dust, heating, traffic emission and industrial emission, were 15.9%, 28.3%, 54.3% and 1.5% respectively in autumn in area of Beijing in 1995. In the CBD of the city, the traffic emission contributes 66.4% in autumn, 66.9% in winter to the fine particulate. (Zhang *et al.*, 1998).

As Rilett and Benedek (1994) state, reducing traffic emissions has been an ongoing goal of many authorities over the past 20 years. Most of the programs implemented may be defined as passive in nature because these relatively stringent pollution control programs always regulate the emission levels from the vehicles. The more active measures, such as incorporating the environmental concerns into transportation planning, traffic management and control should be applied to reduce the pollution. Recently, the advent of intelligent transportation system (ITS) offers the system authority more active measures to deal with urban pollution problem. The route guidance system (RGS) and the electronic road pricing system (ERPS) are two components or sub-systems of ITS. The system authority can determine the routing strategy provided to travelers, or set the road pricing pattern to achieve the goal of air pollution reduction.

Traditional traffic modeling approaches used for evaluating and analyzing the traffic operation and management are predominately travel time (cost) based. It is obvious that these approaches are not appropriate for use in traffic network where air quality concerns prevail. Thus, traffic equilibrium analysis with environmental concerns becomes an open topic for transportation professionals.

Stopher (1993) examines the ability of conventional travel-forecasting models to respond to forecasting needs created by the Clean Air Act Amendments (CAAA) of 1990 and the air-quality lawsuit brought against the Metropolitan Transportation Commission, San Francisco. It is concluded that only those transportation-control measures (TCMs) result in a change in travel mode, auto occupancy, or destination can be modeled. Most remaining TCMs require significant model changes. For emissions modeling, the inputs cannot be obtained with the required specificity. Rilett and Benedek (1994) discuss the traffic assignment problems under environmental and equity objectives. In the traffic systems that will operate under ITS with environmental objectives, traffic may follow routes that are based on equitable rather than equilibrium or optimal consideration. The fact that historic traffic assignment techniques may be inadequate for the modeling such traffic systems is illustrated in Rilett and Benedek (1994). It is also shown that when ITS policies that attempt to reduce system travel time are implemented, other objectives such as reducing environmental pollution may actually be increased.

In this paper, the traffic equilibrium problems with environmental concerns, traffic emissions specifically are discussed. A minimal traffic emission model (MTE) is proposed, similar as Rilett and Benedek (1994). With a numerical example, the routing strategy in the route guidance system (RGS) provided by this model is compared with those under the traditional system performance objectives, such as minimal network travel time. Furthermore, based on the work by Yang and Lam (1996), we propose a bi-level minimal traffic emission model (BMTE) for the electronic road pricing system (ERPS). The road toll pattern will be determined to minimize traffic emissions when users make their route choices in a user equilibrium manner. This minimal traffic emission pricing strategy is also compared with alternative road pricing strategies, such as traditional marginal-cost pricing strategy.

The remainder of the paper will be organized as follows. In the next section, we discuss the traffic emission estimation models. Section 3 proposes the minimal traffic emission model (MTE). The model is solved in an example road network and the comparative analyses are also given. Section 4 presents the bi-level programming model (BMTE). The optimal road toll pattern is determined and compared with other pricing strategies. The final section provides a summary and identifies directions for future research.

2. TRAFFIC EMISSIONS ESTIMATION MODELS

With the rapid increase in mobility, the major current urban air pollutants are particulate matter, nitrogen oxides, and ozone from traffics, rather than those from industrial activity

and domestic heating, such as sulphur dioxide. It is necessary to quantify traffic emission levels as accurately as possible.

Attempts have been made to develop computation methods for the determination of traffic emissions. Taylor and Yong (1996) investigate the pollutant emissions and fuel consumption characteristics of mixed traffic streams under different levels of congestion. Based on the hierarchical family of models for fuel consumption and vehicle emission rates presented by Biggs and Akcelik (1986), Taylor and Yong focus on the elemental models and calibrate them with experimental data. The Biggs-Akcelik model family comprises four levels: (1) an instantaneous model, that indicates the rate of fuel usage or pollutant emission of an individual vehicle continuously over time. (2) an elemental model, that relates fuel use or pollutant emissions to traffic variables such as deceleration, acceleration, idling and cruising, etc. over a short road distance; (3) a running model, that gives emissions or fuel consumption for vehicles traveling over an extended length of road, and (4) an average speed model, that indicates level of emissions or fuel consumption over an entire journey.

Sturm *et al.* (1997) present an overview of methods to describe the emission behavior of road transport. They also hold the same idea that a single methodological approach will not be capable of estimating traffic emissions with adequate accuracy. As a result, different calculation methods have to be used for different ranges of application. In their paper, they classify the computational methods as (1) emission calculations based on actual driving behavior (modal modeling), (2) emission calculations for specific streets, and (3) emission calculations based on vehicle miles traveled.

All the pioneer works present a number of alternatives that can be used at a variety of levels of detail in an analysis, and thus offers considerable flexibility for use in transportation planning and traffic engineering. For simulation models, such as interaction analysis, the elemental model in Biggs-Akcelik family, or calculation methods based on actual driving behavior in Sturm *et al.* (1997) may be chosen. For the environment impact analysis of transport policies and transportation planning, the average speed model or calculations based on the specific streets may be preferred. The calculations based on vehicle miles traveled are more suitable for the national scale strategy examination.

In this paper, we adopt the traffic emissions estimation model used in Rilett & Benedek (1994). The model is also used in the TRANSYT 7-F and belongs to the domain of average speed models. It is noted that only Carbon Monoxide (CO) emission is considered in this paper. The choice of CO is based on the reason that CO may be considered as the best tracer for determining the traffic contribution to the overall atmospheric pollution of the area since it is almost solely emitted by vehicles (Alexopoulos *et al.*, 1993).

The CO emission function is described as follow:

$$ROP_a = 3.3963 \cdot \frac{e^{0.01456v_a}}{1000v_a} \quad (1)$$

Where, ROP_a is the rate of production of CO (grams/veh-ft), and v_a is the average vehicular velocity on link a (ft/sec). Then, we can obtain the link CO emission function easily as:

$$e_a(x_a) = ROP_a \cdot l_a = 3.3963 \times 10^{-3} \cdot e^{0.01456 \frac{l_a}{t_a(x_a)}} \cdot t_a(x_a) \quad (2)$$

Where, $e_a(x_a)$ denotes the CO emission by traffic x_a on link a (grams/veh). It depends on the link travel time $t_a(x_a)$. Traditionally, we assume that link travel time $t_a(x_a)$ monotonously increases with traffic volume x_a . l_a is the length of link a (ft).

Although different variables impact CO emissions, the average driving speed is the decisive parameter (Sturm et al., 1997). The relationship between CO emissions and the average driving speed in Equation (1) can be illustrated in Figure 1.

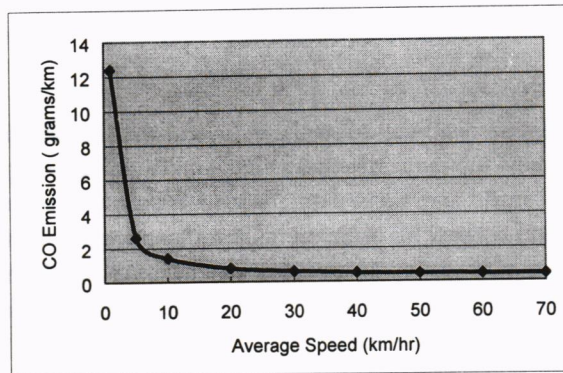


Figure 1 CO emissions as a function of average driving speed

It can be seen clearly that CO emissions decline as driving speed increases (<70 km/hr). With the assumptions of speed limit constraints (<70 km/hr) for urban arterial streets and the monotony of link travel time functions, the above function $e_a(x_a)$ satisfies the following conditions:

$$\frac{de_a(x_a)}{dx_a} > 0, \quad \frac{d^2e_a(x_a)}{dx_a^2} > 0 \quad \forall a \quad (3)$$

Thus, the link CO emission function $e_a(x_a)$ for urban arterial streets is a convex function, and increases with traffic volume x_a monotonously. This property is important for the models in the following sections.

3. THE MINIMAL TRAFFIC EMISSION MODEL FOR RGS

In a route guidance system (RGS), the travelers receive the route guidance through the in-vehicle receivers and variable message signs etc. It is assumed that each traveler can receive the route guidance and follow it. Thus their route choices will achieve the particular system optimum. There are alternative control objectives for the system authority. One of the traditional system performance objectives is to minimize the total network cost (MNC).

The routing strategy can be obtained by the well-known system optimum (SO) traffic assignment models (Sheffi, 1985). Here, we are concerned with the system performance criterion to minimize the CO emissions. Similarly, the model can be formulated as follows:

MTE:

$$\min Z(x) = \sum_a x_a \cdot e_a(x_a) \quad (4a)$$

subject to

$$\sum_k f_k^{rs} = q_{rs} \quad \forall r, s \quad (4b)$$

$$f_k^{rs} \geq 0 \quad \forall k, r, s \quad (4c)$$

$$x_a = \sum_{rs} \sum_k f_k^{rs} \delta_{ak}^{rs} \quad \forall a \quad (4d)$$

It is easy to see that MTE model can be solved by the standard Frank-Wolfe algorithm for the user equilibrium (UE) traffic assignment model, if the link travel time function is replaced by the following marginal link emission function.

$$\bar{e}_a(x_a) = e_a(x_a) + x_a \cdot \frac{de_a(x_a)}{dx_a} \quad (5)$$

From Section 2, we know $e_a(x_a)$ is a monotonic convex function. Thus, the optimal problem MTE will have the unique global optimum solution. The solution makes all the marginal emissions of all the routes used between OD pairs to be equal and minimal.

To illustrate and analyze the routing strategies induced by different control objectives, we perform MTE, MNC, UE assignment in an example network. The network is from Yang and Lam (1996), shown in Figure 2. The BPR link travel time function is used

$$t_a(x_a) = t_a^0 \left(1 + 0.15 \cdot \left(\frac{x_a}{s_a} \right)^4 \right) \quad (6)$$

Link free travel time t_a^0 , and link capacity s_a , are given in Table 1. It is assumed that there are only two OD pairs (1→3 and 2→4) and the demands are fixed to be $D_{13}=D_{24}=3000$ Veh/hr.

The resultant minimal CO emission flow pattern (MTE) is compared with the minimal network travel cost flow pattern (MNC) in Table 2. The corresponding total network cost and total CO emissions are given in Table 3, including that of user equilibrium (UE).

It is shown that for a route guidance system (RGS), the routing strategy provided to the travelers under the environmental objective is much different with those under the traditional system optimal objectives. It also proves the conclusion drawn by Rilett and Benedek (1994) that the reduction in system travel time and the reduction in environmental pollution may actually conflict. From Table 3, we know that the minimal travel time routing strategy (MNC) leads to the largest traffic emissions. In this numerical example, the MTE routing strategy may be the most suitable one to choose. In reality, it is necessary

and possible to make some tradeoffs among these system performance objectives. A weighted combination of these objectives is required, but it might not be enough simple to transform these differently measured and scaled objectives into comparable units. A better way to deal with this problem, just like the one illustrated by Yang and Bell (1998) for network design problems, is to apply some well-known weighting method of multi-objective mathematical programming to generate non-dominated or Pareto optimal alternatives. Then multiple-criteria decision-making is used to evaluate and select the compromise solution from those non-inferior alternatives.

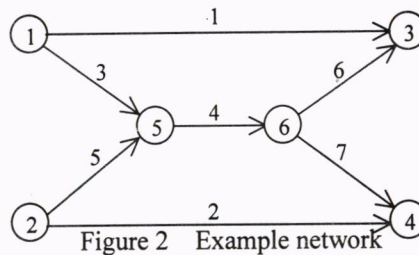


Table 1 Input data for example network

Link a	1	2	3	4	5	6	7
t_a^0 (min)	8.0	9.0	2.0	6.0	3.0	3.0	4.0
l_a (km)	8.0	9.0	2.0	6.0	3.0	3.0	4.0
s_a (veh/hr)	2000	2000	2000	4000	2000	2500	2500

Table 2 MTE, MNC flow patterns and CO emission patterns on example network

Link	Traffic Volume (veh/hr)			CO Emission (grams)		
	MNC	MTE	Difference	MNC	MTE	Difference
1	1791	2275	-21.3%	6618.63	8767.63	-24.5%
2	1847	2355	-21.6%	7701.71	10321.06	-25.4%
3	1209	725	66.7%	1096.68	655.45	67.3%
4	2362	1370	72.3%	6426.10	3716.10	72.9%
5	1153	645	78.7%	1568.10	874.90	79.2%
6	1209	725	66.7%	1640.91	982.87	67.0%
7	1153	645	78.7%	2086.50	1166.31	78.9%

Table 3 The main system performance of MNC, UE, MTE

	MNC	UE	MTE
Total Travel Time (hour)	1048	1200	1108
Total CO Emission (grams)	27139	26739	26484

4. MINIMAL TRAFFIC EMISSION MODEL FOR ERPS

In an electronic road pricing system (ERPS), the travelers make their route choices in a user equilibrium manner. It is assumed that each traveler has the perfect knowledge about the traffic and road tolls on the transportation network. The system manager attempts to optimize his or her objective through setting feasible road toll pattern on the network. Similarly, many alternative performance measures or system objective functions can be chosen. Some objectives, such as minimal total network travel cost (MNC), maximal total revenue (MR) etc., are fully discussed by Yang and Lam (1996). Based on their work, we formulate a bi-level programming model (BMTE) to determine the toll pattern with minimal CO emissions. The model is described as follows:

BMTE

$$\min Z(x) = \sum_a x_a \cdot e_a(x_a) \quad (7a)$$

subject to

$$u_a^{\min} \leq u_a \leq u_a^{\max} \quad \forall a \quad (7b)$$

where x_a and $e_a(x_a)$ are obtained by solving

$$\min F(x) = \sum_a \int_0^{x_a} c_a(\omega, u_a) d\omega$$

subject to

$$\sum_k f_k^{rs} = q_{rs} \quad \forall r, s \quad (7c)$$

$$f_k^{rs} \geq 0 \quad \forall k, r, s \quad (7d)$$

$$x_a = \sum_{rs} \sum_k f_k^{rs} \delta_{ak}^{rs} \quad \forall a \quad (7e)$$

where, u_a denotes the toll charges on link a , and u_a^{\min}, u_a^{\max} are the lower bound or upper bound of toll charges on link a respectively. These bounds should be predetermined with the revenue, fair and reasonable considerations. $c_a(x_a, u_a)$ is the link travel cost function, defined as $c_a(x_a, u_a) = t_a(x_a) + u_a$.

It is noted that model (7) is also non-convex, like any other form of bi-level mathematical programming problems, and hence it might be difficult to solve for a global optimum (Yang and Lam, 1996).

In the literature of transportation research, several solution algorithms are developed to deal with such bi-level problems. Recently, Yang and Yagar (1994) propose a sensitivity based algorithm (SBA) and have applied it successfully to optimal ramp metering in general freeway-arterial corridor system, traffic signal control in saturated road networks, congested OD matrix estimation problem and optimal road pricing (Yang and Lam, 1996). Unfortunately, the sensitivity analysis for equilibrium network in SBA is complex and needs more computation efforts. In this paper, we propose a simpler and robust method

based on genetic algorithms (GAs) to solve the BMTE model.

Genetic algorithms are search and optimization procedure motivated by natural principles and selection (Goldberg, 1989). Because of their simplicity, minimal problem restrictions, global perspective, and implicit parallelism, GAs have been applied to a wide variety of problem domains including engineering, sciences, and commerce. Our genetic algorithms based (GAB) method is outlines as:

GAB algorithm:

Step 1. Select at random the initial population $\mathbf{X}(1)$. Set $k=1$.

Step 2. Calculate the fitness functions for individuals $x_j(k), k=1,2,\dots,N$ by solving the lower level optimization problem, i.e. UE assignment by Frank-Wolfe algorithm, and reproduce the population $\mathbf{X}(k)$ according to the distribution of the fitness function values.

Step 3. By a random choice with probability P_c , carry out the one-point cross over operation.

Step 4. By a random choice with probability P_m , carry out the single-bit point mutation operation. Then we have a new population $\mathbf{X}(k+1)$.

Step 5. If k = maximal number of generation, the individual with the highest fitness is adopted as a suboptimal solution of the problem. Else, set $k = k+1$ and return to Step 2.

This GAB algorithm will be investigated in detail in a future paper. Now, We apply the proposed model and algorithm to a numerical example. The same network and input data are used as that in Section 3. For the present problem, an individual is defined as follows: first, we define

$$u = u_1 u_2 \cdots u_7 \quad (8)$$

where u_a denotes the toll employed on link a . Then u is coded by binary coding method to be an individual x . Meanwhile, we map the objective function (7a) to fitness form as following equation:

$$f(x) = C_{\max} - Z(x) \quad (9)$$

where C_{\max} is taken as 40000 grams in this example.

Following the recommendation by Goldberg (1989), GAB algorithm is performed with the following parameters:

- Population size is 50.
- Crossover probability is 0.6.
- Mutation probability is 0.0333
- The maximal number of generation is 50.
- The lower and upper bounds of link toll are: $0.0 \leq u_a \leq 5.0, a=1,2$ and $0.0 \leq u_a \leq 2.0, a=3-7$, same as Yang and Lam (1996).

The convergence of the algorithm is shown in Figure 2. It can be seen that the proposed

GAB algorithm converges quickly to the optimal solutions. The traffic flow pattern under the BMTE road pricing strategy is listed in Table 4. We can find that the flow pattern under the BMTE pricing strategy in ERPS is the same as that under the MTE routing strategy in RGS. Thus, it is proved that these two sub-systems of ITS can achieve identically the goal of reducing CO emissions.

In addition to the minimal traffic emission toll strategy (BMTE), traditional marginal-cost toll strategy (MCT) and minimal total network travel cost strategy (MNC) are also employed for comparing and verifying. The results are listed in Table 5. From Table 5, we can also draw the conclusion that the pricing strategy set on the network under the environmental objective is much different with those under the traditional system optimal objectives. We also need to find some tradeoffs among these objectives in realistic management.

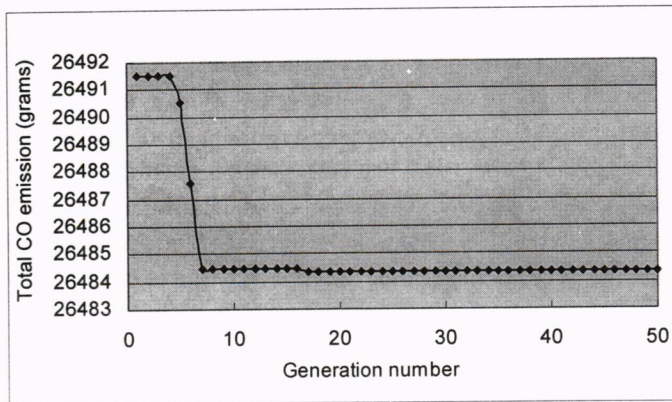


Figure 3 Total CO emissions versus generation number

Table 4 The optimum solution of BMTE

Link	Traffic volume (veh/hr)	Trip time (min)	Traffic Emission (grams)	Toll (min)
1	2276	10.014	8774.84	3.34
2	2353	11.588	10313.50	3.09
3	724	2.005	654.14	1.65
4	1370	6.012	3715.66	0.25
5	647	3.005	876.65	0.27
6	724	3.003	980.91	0.44
7	647	4.003	1168.63	1.14

Table 5 Solutions with alternative toll strategies

Toll Strategy	MCT	MNC*	BMTE
Toll pattern (min)			
Link 1	3.09	3.82	3.34
2	3.94	4.27	3.09
3	0.16	0.47	1.65
4	0.44	0.48	0.25
5	0.20	0.29	0.27
6	0.10	0.47	0.44
7	0.11	0.29	1.14
Total Network Cost (hour)	1048	1048	1108
Total CO emission (grams)	27139	27136	26484

Note: The MNC toll pattern is computed based on the model proposed by Yang and Lam (1996).

From the results presented in Table 5, it is easily seen that different toll strategy would generally lead different toll pattern. It is also observed that the MCT and MNC toll strategies generate the identical total network cost (minimal total network cost), which has been reported by Yang and Lam (1996). This demonstrates that there is usually more than one link toll pattern that leads to the system optimum (minimal total network cost). Similarly, it is shown that the flow pattern under the BMTE pricing strategy in ERPS is the same as that under the MTE routing strategy in RGS. Thus, it is proved that these two sub-systems of ITS can achieve identically the goal of reducing CO emissions.

5. CONCLUSIONS AND RECOMMENDATIONS

In this paper, the traffic equilibrium problems with environmental concerns, traffic emission specifically is discussed. The minimal traffic emission model (MTE) presented in Section 3 shows that for a route guidance system (RGS), the routing strategy provided to the travelers under the environmental objective is much different with those under the traditional system optimal objectives. The reduction in system travel time and the reduction in environmental pollution may actually conflict. In reality, a weighted combination of these objectives is required. A better way to deal with this problem to apply some well-known weighting method of multi-objective mathematical programming to generate non-dominated or Pareto optimal alternatives. Then multiple-criteria decision-making is used to evaluate and select the compromise solution from those non-inferior alternatives.

In Section 4, the bi-level minimal traffic emission model (BMTE) for the electronic road pricing system (ERPS) shows that the flow pattern under the BMTE pricing strategy in

ERPS is the same as that under the MTE routing strategy in RGS. Thus, it is proved that these two sub-systems of ITS can achieve identically the goal of reducing CO emissions. We can also draw the conclusion that the pricing strategy set on the network under the environmental objective is much different with those under the traditional system optimal objectives.

There is a primary deficiency for models in this paper. Accurately speaking, average speed for emissions calculations is the average speed from the beginning to the end of the trip, which is not the same as average link speeds in the proposed model. To overcome this problem, the MTE and BMTE model can be rewritten into a path-based formulation. Importantly, a path-based algorithm is needed to determine the path flow for the models in this paper. The gradient projection algorithm proposed by Jayakrishnan et al. (1994) will be employed to develop an efficient algorithm in the future research.

In future research, the emissions fees will be investigated in our models. This strategy may result in nonadditive costs, where traditional additive path costs model can not apply. The traffic equilibrium model with nonadditive path costs by Gabriel and Bernstein (1997) is a good candidate to adopt.

The future extensions also include investigating the equity objectives proposed by Rilett and Benedek (1994). Meanwhile, other transportation analysis topics with environmental concerns may be discussed, such as transportation network design problem.

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