MICROSCOPIC SIMULATION OF THE DAY-TO-DAY ROUTE CHOICE AND CONTROL INTERACTION

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Abstract: In contrast to equilibrium models of traffic networks, this study develops a day-to-day dynamic model (D-Day) that enables a day-to-day evolution of the driver and traffic system environment to be simulated over time. D-Day generates the whole range of possible states of network performance and route choice. The model includes a microscopic model of individuals' route choice behaviour and a macroscopic traffic model, in which variations in traffic demand and network supply conditions are treated as random variables between days.

From several tests conducted, it can be reported that D-Day average flows are broadly similar to the user equilibrium (UE), although the differences are larger than those between UE and stochastic user equilibrium (SUE). D-Day may predict either more or less benefits from optimum road tolls than found under the static models and in certain cases the benefits may effectively disappear, depending on the network and controls configuration. The optimal charges also vary, but they decrease as variation is increased.

1. INTRODUCTION

This paper considers a stochastic approach of modeling drivers and traffic system environment in which drivers are explicitly modeled as an individual entity with behavioural attributes influencing the route choice decision. Traffic system environment refers to the road network with all its traffic, geometric and control. On each successive day, demands and supply characteristics are generated through a pseudo-randomisation process, forming a whole range of route choice set within the specified attributes' distributions, hence allowing for a stochastic route choice process to occur from one day to another.

Day-to-day models of this kind have been the subject of an active research area in recent years. They provide a flexibility in modeling various features of demand specifications and of control applications (e.g. Cascetta, 1989; Mahmassani & Herman, 1990; Ben-Akiva et al.,1994), and are still in their infancy.

The most established steady state network traffic equilibrium method is well documented in Sheffi (1985), which assumes a long term Wardrop equilibrium behaviour in which no user can reduce his/her travel cost by unilaterally changing routes. This method has been accepted as a standard practice in network traffic evaluation. A further extension on the technique, stochastic user equilibrium method, is also becoming more accepted in practice. But, often within these two models, important new developments in traffic planning and control cannot be modelled adequately, whether due to the reason that they lack of behavioural realism or due to the phenomenon of the day-to-day variability in network conditions and controls, etc.

In this paper we pursue the direction under a stochastic process approach demonstrated amongst the first by Horowitz (1984) and Cascetta (1989), and use Monte Carlo technique to simulate individual day-to-day route choice. In such simulation, problems that may rise such as the effect of different starting conditions and the way to handle it have been reported elsewhere, e.g. Sorah (1995a). Using an artificial simple two-link network as a test network, this paper reports the results based on experiments using D-Day, a day-to-day route choice model developed in the study. The primary objective is to examine the effect of introducing random variable on the flow pattern and total travel time, and contrast the results with equilibrium solution. Experiments were also conducted when pricing control is put in place to show how D-Day may predict benefits from optimal road tolls.

2. MODEL STRUCTURE

Our general model of day-to-day evolution is illustrated in Figure 1. On day-0, given the network and initial demand level, the link costs c_a's are initialised. These link costs are used to generate a set of shortest path with costs c_{ij}'s from each origin i to all destinations j. Following this trips are loaded based on a utility maximisation concept, i.e. individuals follow a least cost route. The loading is processed individual to individual, as it is assumed that each driver perceives network attributes differently. At an aggregate level, having finished the loading, an O-D route proportion or split p_{pij} is obtained by adding up the resulting route choices for each origin and destination.

On the following day, the aggregate O-D demand level and network conditions are assumed to be random variables, and the same process as above is repeated. Precise details of the randomisation processes are left out for the moment; however each random variable is assumed to follow a certain distribution with a given mean and variance.

Additional processes takes place as each drivers' travelling experiences accumulate. Habit formation and information updating schemes are among the concepts to be incorporated and described later.

Figure 2 displays the model structure in more detail. The algorithmic steps can be summarised as follows.

1. *[Initialisation]* For each potential traveller in the network, assumes an initial perceived travel cost for each link in the network. Set day counter k=0.
2. *[OD demand]* Increment day counter: k=k+1. Randomly select the set of travellers who will make a journey on day k, for each origin-destination pair.

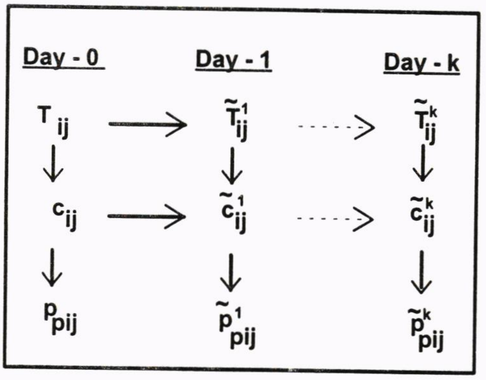


Figure 1. Effect of Randomisation Process

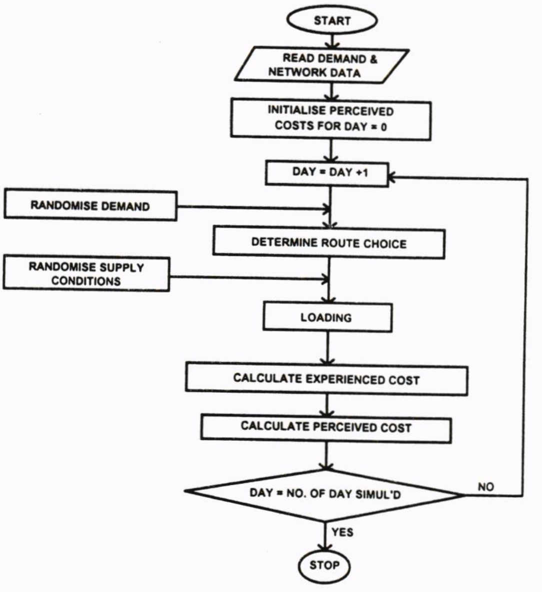


Figure 2. Model Structure

3. *[Route choice]* For each individual travelling on day k: Based on their currently perceived travel costs, and possibly on choices made in previous days, select a route according to some given choice mechanism.

4. *[Network conditions]* For each link in the network, characteristics such as the capacity is drawn randomly (to represent daily variability due to rain, parked vehicles, breakdowns, etc.), according to a given probabilistic law.

5. *[Loading]* The route choices made in step 3 are aggregated to form total link flows.

6. *[Supply + Control]* Average day k travel times are then computed from aggregate travel time - flow relationships, based on the characteristics generated in step 4. Added to this is a 'control' cost, e.g. charging, which is defined as an additional time that is perceived exactly the same as the time in the network supply condition.

7. *[Experience]* Each individual travelling on day k experiences the link travel times calculated in step 6, either only on the links on the route they choose to follow (self-learning model) or including the unchosen links (informed model), as specified in the next section.

8. *[Perceived]* Via some form of learning mechanisms, each individual updates a new perceived (day-averaged) travel cost for each link. Inputs to the learning process may include a combination of the travel times experienced in step 6, previous experiences and previous cost perceptions.

   If the maximum day simulated has not been reached return to step 2.

The resulting dynamic profile will certainly depend on how much variability is introduced into the model, and also on how drivers update their perceptions before making their route choice decision. Moreover, the choice mechanism becomes even more complex if the presence of information is taken into account.

In such a state of day-to-day demand and supply variability, a basic question is whether or not the system would end up at an equilibrium state. If all potential variables within the supply and the demand parameters are assumed to be random variables, it is very unlikely that the system would completely settle down to a fixed stationary state. However at this stage we can not give any prior judgement as to how this evolution may end up. Readers interested in theoretical review of stability issue in a day-to-day traffic assignment models are referred to Watling (1997).

3. EXPERIENCE, LEARNING AND PERCEPTIONS SPECIFICATIONS

The actual link and route costs calculated in the previous step are labelled as the 'experienced link costs' to drivers who may or may not travel along those links, depending on individuals' information system category. The experienced costs are accumulated from one day to another, and are used by drivers in their day-to-day learning about the most likely states of the network/traffic system environment. This will involve an updating process or a prediction of conditions, which we would expect to become more precise as more such experiences are accumulated.

The mechanisms involved while drivers perceive or update their travel cost estimation are not well understood at present. However, several theoretical models have been proposed in the literature, mainly based on the forecasting technique, i.e. time series method. For the present study the adjustment mechanism described as Model-1 and Model-3 in (Horowitz, 1984) are appropriate. We explain why this is so as follows.

Model-1 assumes that drivers acquire information or knowledge on both the chosen and unchosen links, although no mechanism as to how to achieve this was mentioned. This is a fundamental assumption in the equilibrium approach. Using the same assumption, but based on individual route choice simulation, we will examine how the resulting simulation differs from the equilibrium result. This model is hereafter called the *informed model*, in which the updated cost $\hat{C}_{i,m}$ on link i on day k is defined as a weighted sum of previous actual (measured) costs C_{ik}, plus a random component ε_{ik} denoting the "traveller's misperception" about the actual cost of travel:

$$\hat{C}_{i,m} = \sum_{k=1}^{m-1} a_k C_{ik} + \varepsilon_{ik} \qquad (1)$$

where: a_k denotes weighting parameters and m represents the memory ability of individual.

Model-3, which assumes that drivers can only obtain information through personal day-to-day experience, is used to test a more realistic information mechanism when not all trip makers can acquire information as given in (1). This model is hereafter called the *self-learning model*, defined by the following adjustment.

$$\hat{C}_{i,m} = \begin{cases} \hat{C}_{i,m-1} \text{ ,} \textit{if link } i \textit{ was not chosen in time period } m-1 \\ \hat{C}_{i,m-1} + \varepsilon_{i,m-1}, \textit{if link } i \textit{ was chosen for the first time in time period } m-1 \\ a_{m-1}(\hat{C}_{i,m-1} + \varepsilon_{i,m-1}) + (1-a_{m-1})\hat{C}_{i,m-2} \text{ ,} \textit{otherwise} \end{cases} \quad \textbf{(2)}$$

where on day 1 (m =1) $\hat{C}_{i,1}$ are identical to the given distribution of initial perceived cost $\hat{C}_{i,o}$. a_m denotes the weighting parameters which express how the previous experiences effect the current prediction.

Both adjustment mechanisms incorpate a level of perception error, ε terms in equations (1) and (2), in which the weighted previous experiences may be assumed to be subject to a global variability. Two variants of global variability can be investigated. One assumes that the between days perception error of an individual is typical, and the other assumes that the error is day dependent. In terms of route switching behaviour the former should give a more realistic situation, with the possibility of approaching a steady state condition, which can never occur in the later case. However, it might be desirable to totally neglect this global variability, in which case individuals would only update costs according to the specified self-learning model with no perception error. Sensitivity testing can be carried out with regard to the level of perception error for both models, in order to investigate the influence of the parameter on the flow and cost pattern. Other important specifications of the model are given in Sorah (1995b).

4. DATA AND THE EXPERIMENT

The network data is shown in Figure 1. It carries an average demand of 100 trips from an origin O to a destination D, which consists of two routes: (1) a bypass with a higher capacity but longer, and (2) a shorter city centre route. The cost-flow functions for the two routes are non-decreasing BPR-style functions:

$$c_i = t_i \left[1 + \alpha_i \left(\frac{f_i}{cap_i} \right)^{n_i} \right]$$

where: t_i = free-flow travel time on route i.

cap_i = capacity on route i.

f_i = flow on route i, and

α_i , n_i = cost function parameters, i.e. slope and power, dictating the sensitivity of cost to flows.

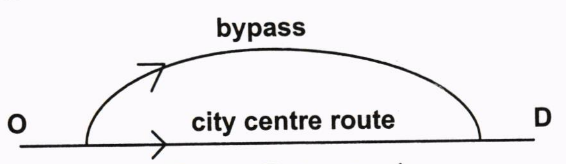


Figure 1. Test- network

Table 1 lists four sets network conditions to be examined in this section, each of which is characterised by the flow splits before and after an optimal control is implemented. The before situation refers to UE conditon under uncharge condition, while the after situation refers to SO condition which also satisfies the UE condition but with an optimal charge applied on the city centre route. The corresponding benefits and optimal charges are also indicated in the table. Table 2 shows the network attribute data set.

Table 1 The Test Network and its Static Equilibrium Solutions

	NETWORK A	NETWORK B	NETWORK C	NETWORK D
Before Situation (UE) (bypass:citycentre)	0.27:0.73	0.50:0.50	0.41:0.59	0.61:0.39
After Situation (SO) (bypass:citycentre)	0.44:0.56	0.62:0.38	0.59:0.41	0.72:0.28
UE costs (zero charge)	2332.2	1045.7	2190.9	683.9
SO costs	1844.3	943.9	1648.9	615.3
Benefits	20.9%	9.7%	24.7%	10.0%
Optimal charge (city centre)	14.86	4.23	15.23	3.0

Table 2 Network Data

Attributes	**Network A**		**Network B**		**Network C**		**Network D**	
	Bypass	City centre	Bypass	City centre	Bypass	City centre	Bypass	City centre
Free-flow time	18	6	8	5	20	3	6	3
Capacity	65	35	65	35	65	35	65	35
Slope (α)	0.72	0.15	0.40	0.26	0.15	0.80	0.15	0.80
Power (n)	1	4	1	4	1	4	1	4

5. D-DAY SIMULATION EXPERIMENT

Table 3 shows the list of possible experiments using D-Day, under 4 different networks with the parameters given in Table 2. Given the large number of potential combinations, a very large number of tests could be carried out with different ranges of variation levels and different sets of initial conditions. This would be impractical and complex. Instead, we first fix reasonable variation levels as standards for each random variable as listed in the table, together with the most useful combinations of parameters.

Table 3. Description of the Experiments

EXPERIMENTAL FACTOR		DESCRIPTION	STANDARD SETTING
4 Networks:	**A.**	Network A	Listed in Table 2
	B.	Network B	
	C.	Network C	
	D.	Network D	
2 Updating Models:	**i.**	Informed model	equal weight on each day with a set memory limit
	s.	Self-learning model	
6 Supply Variation Settings	**0.**	All parameters are fixed	
	1.	Free-flow time only	c.v. 0.1
	2.	Slope (α) only	c.v. 0.1
	3.	Power (n) only	c.v. 0.1
	4.	Capacity only	c.v. 0.2
	5.	All parameters 1 to 4 are randomised	As above settings
2 Demand Settings	**F.**	Fixed	100
	V.	Variable	111
1 Perception Error		Varied between days	c.v. 0.15
1 Control		Perceived as pure time	None

5.1 Effects of Level of Variations

Here we examine the effect of introducing random variables on flow and cost pattern. The parameters fall into three groups: trip demand (one parameter), network attributes (four parameters) and perception error. In order to keep the test manageable, we make the following arrangement. We have chosen network A as the test network in this particular experiment and use the informed model for the driver day-to-day cost adjustment, i.e. experiment Ai5V. The main reason for testing the informed model is that it provides a further background for comparisons with stochastic equilibrium solution.

We are primarily concerned with a 'typical' model setting in which all parameters are subject to random variation between days. In these experiments each variable including capacity has a fixed coefficient of variation of 0.1, except one that is varied over a range of coefficient of variation from 0.0 to 0.5. Each simulation is run for 1000 days, and the first 100 discarded to yield the sample mean.

Figures 2 and 3 show the changes in flow and cost as the coefficients of variations of the individual network attributes and of the demand level are varied independently. As shown in the figures, the coefficients of variations for the capacity and the power of the cost function have the greatest effect on flow and cost. In particular increased levels of variation lead to increased travel costs in a highly non-linear fashion. The free-flow travel time, the slope of the cost function and demand level all have a similar but much smaller effect.

The effects of perception error on flows and costs are displayed in Figures 4 and 5, respectively. Three models are examined: two from the day-to-day model and one from SUE. The SUE pattern is calculated based on the assumption that perceived cost is normally distributed with mean $c(v)$ and variance $\{\text{c.v. } c(v)\}^2$.

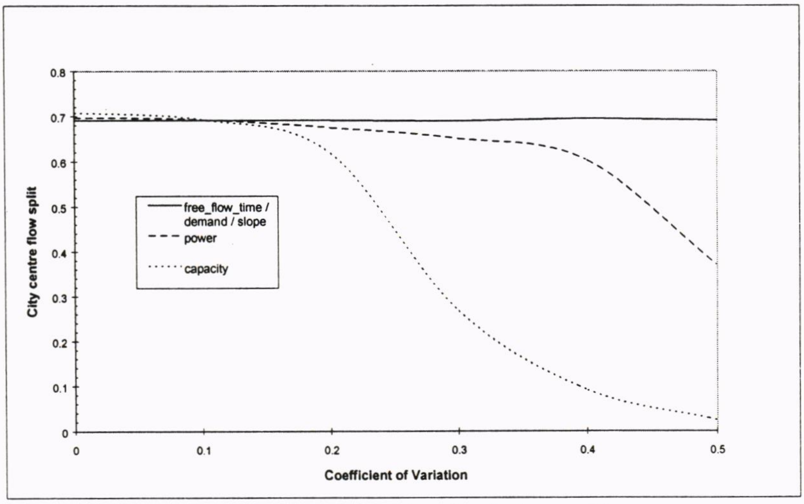


Figure 2. Effect of Level of Variation on Flows

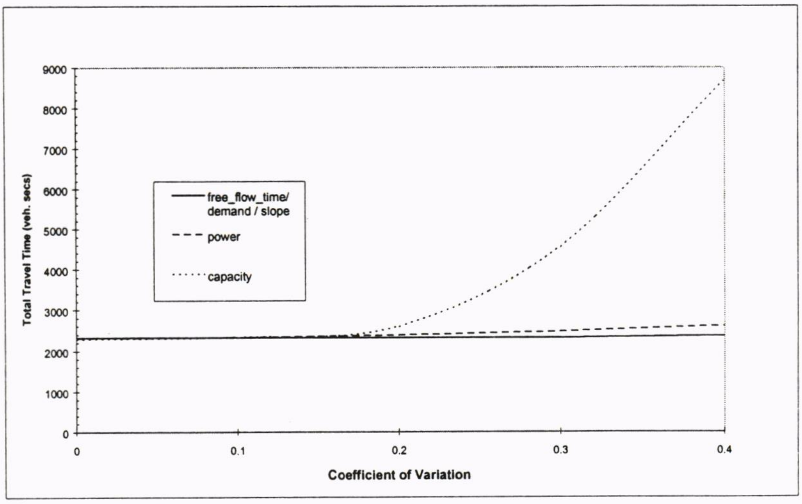


Figure 3. Effect of Level of Variation on Costs

This should provide the same comparative basis with the day-to-day model. Within the day-to-day model, in addition to the typical model Ai5V, another experiment (Ai0F) is also conducted in which only the perception error of individuals is variable between days. In theory experiment Ai0F is the closest to the SUE, as only the perception error comes into play.

As can be seen from the figures at c.v. values less than 0.02 the three models produce significantly different flow and cost patterns. Ai5V starts to approach the SUE at c.v. 0.02, while it is at c.v. 0.2 in the case of Ai0F. Below those levels the corresponding day-to-day models are characterised by all-or-nothing solutions, sometimes flip-flopping between successive days or sometimes remaining at one all-or-nothing solution for a number of days before switching. At higher levels the results from the three models become closer to each other, i.e. above c.v. = 0.2, Ai0F is very similar to SUE.

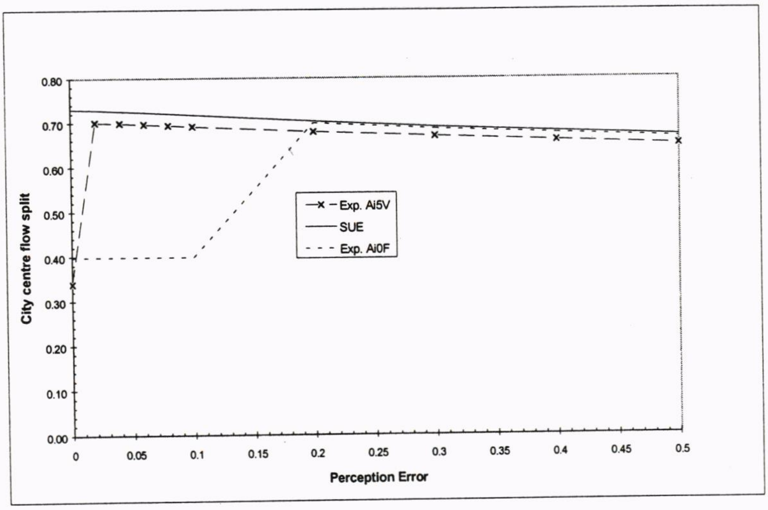


Figure 4. Effect of Level of Perception Error on Flows

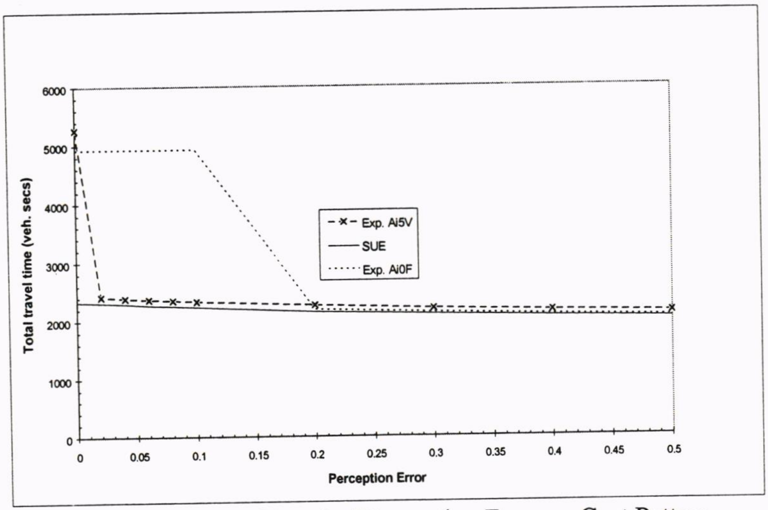


Figure 5. Effect of Level of Perception Error on Cost Pattern

5.2 Day-to-day Charging Control Experiments

In this section we implement the day-to-day model with a road user charging policy, where only the city centre route is subject to a charge. The day-to-day model is set to the "typical situation" described in Table 3 using informed cost adjustment with different levels of memory.

Let us first examine the results from network A. Figure 6 plots total travel time for the UE, SUE with two levels of perception error, 0.1 and 0.5, and the day-to-day model with unlimited memory over a range of charge levels. Each day-to-day data point is sampled from 25 initial conditions from 5 initial splits of flow with 5 sets of random number seeds each. As can be observed from the figure, there is no obvious optimality of charge resulting from the day-to-day model, whereas in UE and SUE there is a clear optimum charge.

Within SUE, the larger the perception error (c.v.) the smaller the benefit obtained. Although there may be a weak optimality in the day-to-day results, for this particular case it can be said that there is no significant benefit of optimal charging under the day-to-day model assesment. Figure 6 also plots the revenue profile from the day-to-day model. As in the static results, there is a maximum margin of revenue but at a level of charge that may worsen the network-wide travel cost.

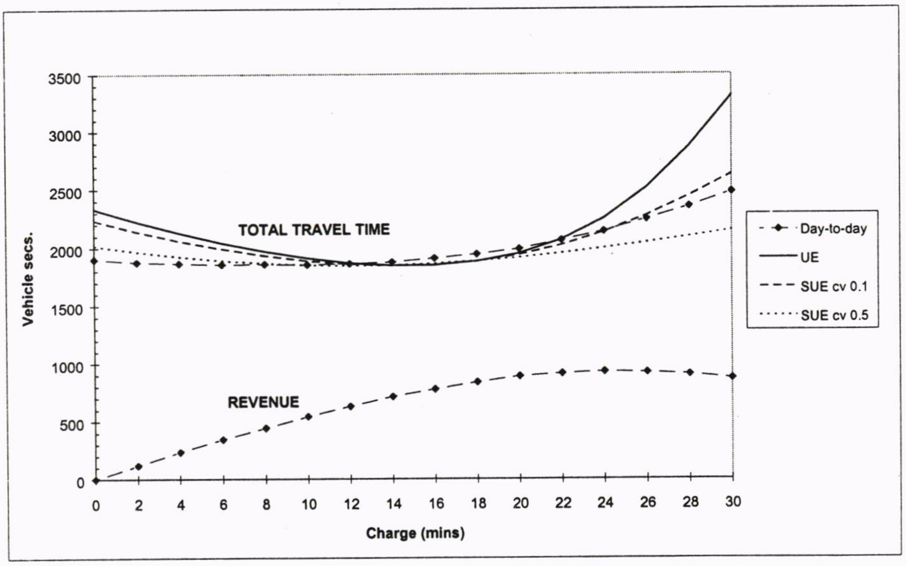


Figure 6. Total Travel Time & Revenue Profile
(Unlimited Memory Ai5V)

Figures 7 displays the corresponding flows and error bars from the day-to-day experiment. As expected, as the charge is increased the city centre route flow decreases.

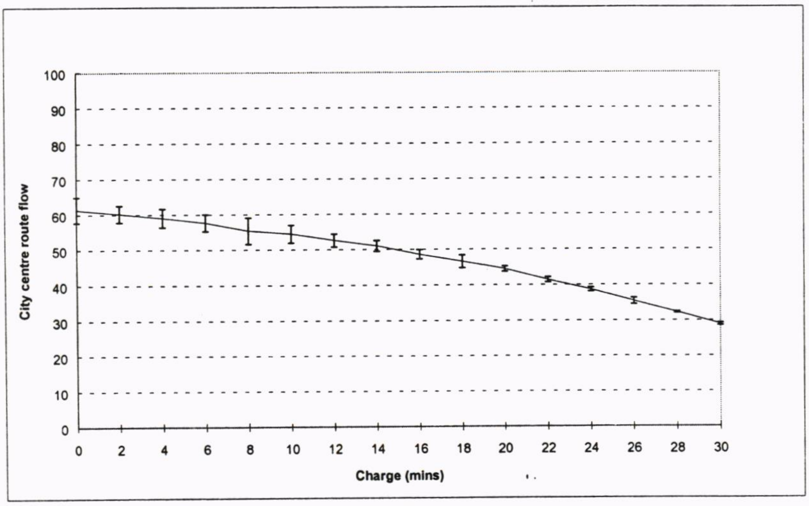


Figure 7. City Centre Flow Vs. Charge Level
(Unlimited Memory Ai5V)

Moreover, total travel times and error bars from a range of memory limits are displayed in Figure 8. At all charging levels, errors and therefore day-to-day variations were found substantial at medium memory limits. An optimal charge is hardly apparent at any of the memory limits tested, except for the weak optimum evident in the case of unlimited memory.

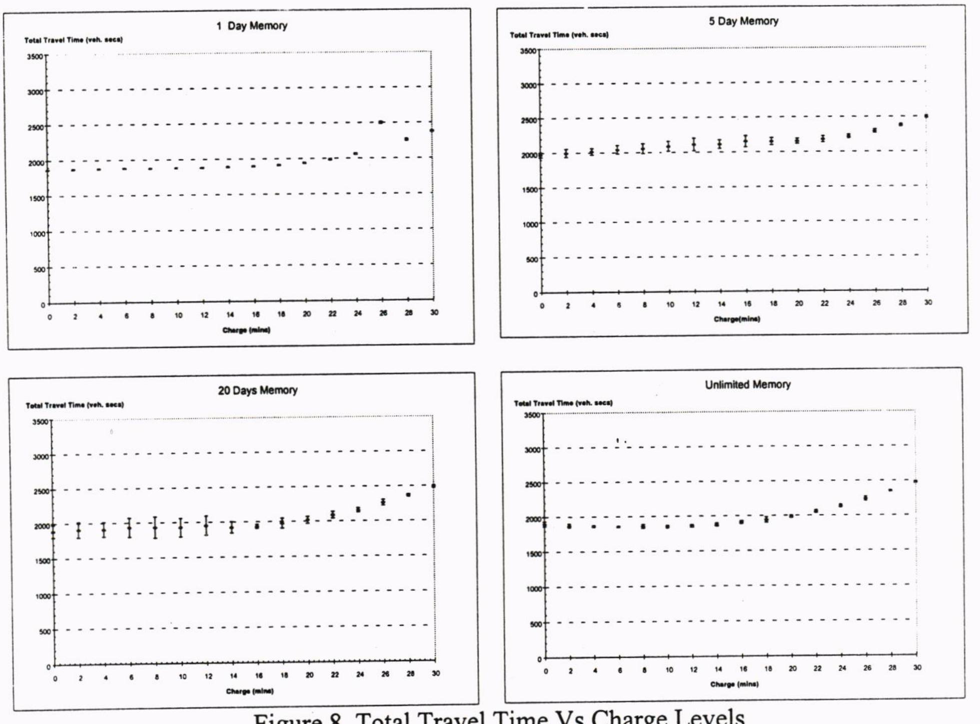


Figure 8. Total Travel Time Vs Charge Levels
(Experiment Ai5V)

From the foregoing tests of road charging using network A, there is therefore no strong indication that the day-to-day model does produce an optimal charge unlike the UE and SUE static models. However, this is a particular case with specific levels of variability; it is therefore necessary to further verify the effect by using different levels of variability over a variety of network conditions.

Thus the four networks are each tested with four levels of variability specified by scaling the level of all parameters in the typical situation with multiplying factors 0.25, 1.0, 2.0 and 4.0. Scaling factor 1.0 refers to the previous conditions using network A. The results are reported below, and are contrasted with the UE and SUE.

Figure 9 plots total travel times for the four networks against the level of charge. Each network is plotted separately together with the results from the static conditions: UE and SUE with c.v. 0.1.

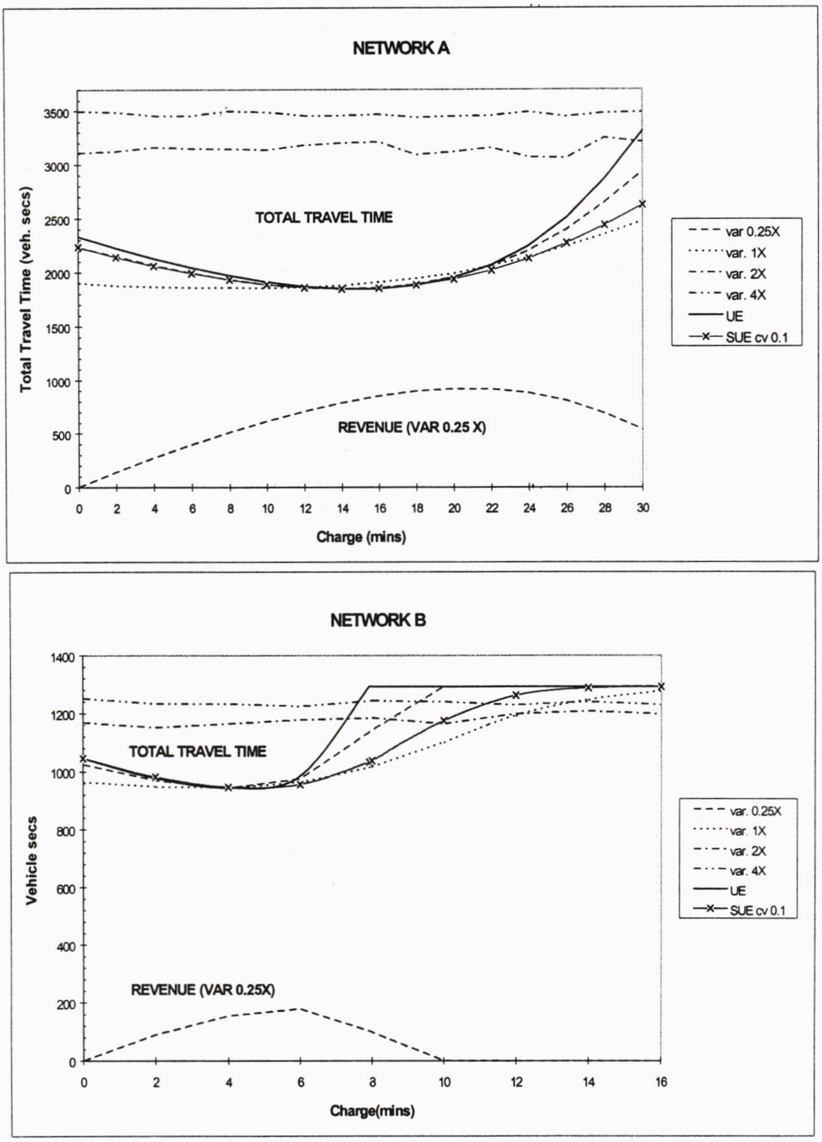
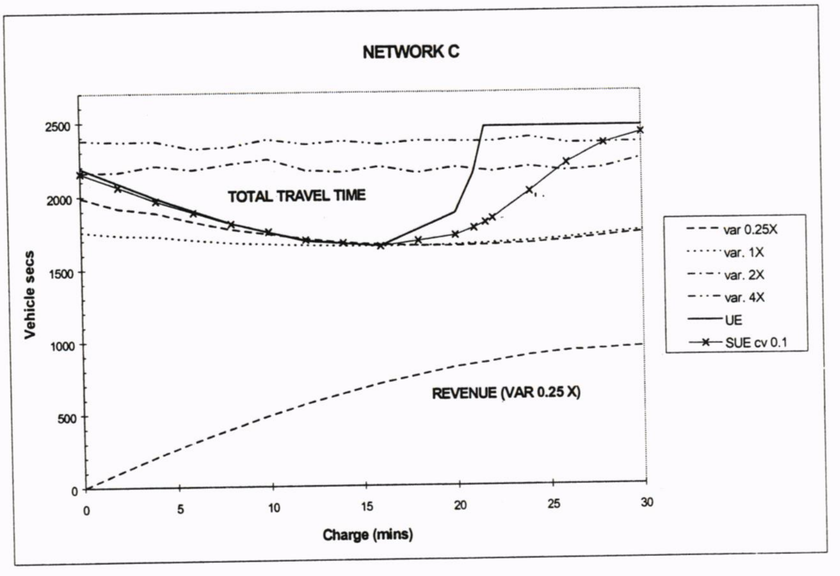


Figure 9. Costs Vs Charge in Four Networks



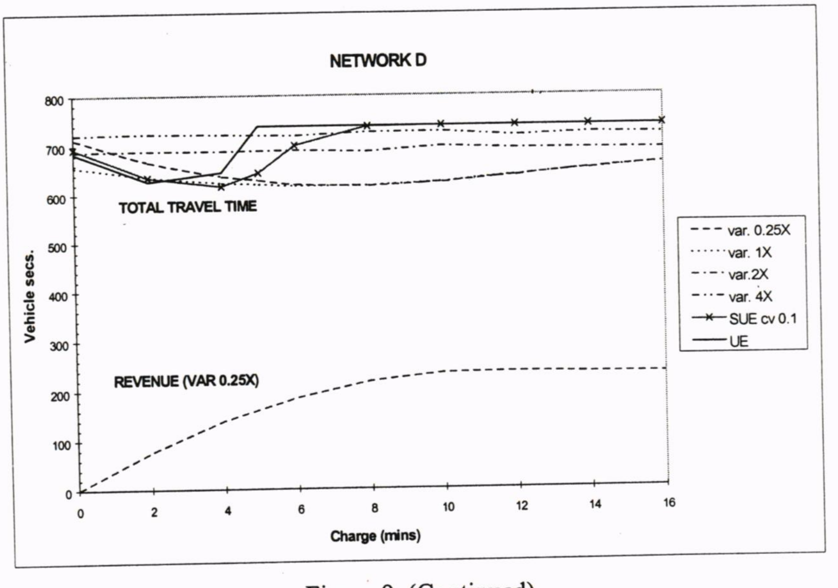


Figure 9. (Continued)

As can be seen, at the lowest levels of variability there is a strong indication that optimal charges exist in all networks. However at high variability the system costs level off over the whole range of charge levels at significantly higher costs. Hence, the benefits are only apparent at the lower levels of variability.

When an optimal charge level is apparent in the day-to-day model, the optimal charges are always greater than those in the static SUE model, see Table 4 for the summary benefits and optimal charges from all models under comparison. As the variation increases in day-to-day models the optimal charges do not move further away from the UE level, as they did in the static results.

In sharp contrast to results in static analyses where as variations increase benefits may decrease, increase or remain constant depending on the network characteristics, see Table 4, benefits under the day-to-day results always decrease as variations increase in all networks. More specifically, as variations increase the trend of benefit changes in networks A and C is the same under both SUE and day-to-day results, i.e. benefits reduce as more variations are introduced. The opposite occurs in network D; the static results indicated an increase in benefits as variation increases, while in the day-to-day model they seem to decrease. In network B the levels of benefits remain the same in the static results, whereas they again decrease under day-to-day conditions.

Table 4. Benefits (%) and Optimal Charges from Three Models

Models		Network A	Network B	Network C	Network D
UE		20.9 (15)	9.7 (4)	24.7 (15)	10.0 (3)
SUE	cv = 0.1	17.5 (15)	9.7 (5)	23.5 (16)	11.0 (4)
	cv = 0.3	12.1 (15)	9.7 (6)	21.3 (16)	13.4 (5)
	cv = 0.5	8.6 (15)	9.7 (7)	19.9 (20)	15.0 (6)
D-Day	var = 0.25 X	17.4 (14)	7.9 (4)	17.1 (20)	13.6 (8)
	var = 1 X (standard)	2.5 (10)	1.9 (4)	6.2(16)	6.6 (6)
	var = 2 X	n.a.	n.a.	n.a.	n.a.
	var = 4 X	n.a.	n.a.	n.a.	n.a.

Figures in brackets are the optimal charges.

Figure 9 also plots the revenue from the day-to-day results based on the 0.25 level of variability. Maximum revenues and the location of maximum revenues obtained from the three models are summarised in Table 5. Again, there is always a maximum margin of revenue that can be raised.

Table 5. Maximum Revenues and Their Locations

Models		Network A	Network B	Network C	Network D
UE		923.7 (20)	172.4 (5)	628.3 (16)	84.3 (3)
SUE	cv = 0.1	948.6 (22)	197.8 (6)	654.6 (16)	103.1 (4)
	cv = 0.3	1039.0 (28)	254.9 (>8)	757.9 (>21)	145.0 (6)
	cv = 0.5	>1154.0 (>30)	>287.4 (>8)	>822.2 (>21)	>178.8 (>6)
D-Day	var = 0.25 X	917.9 (20)	178.1 (6)	945.3 (30)	235.9 (12)
	var = 1 X (standard)	927.7 (24)	196.6 (8)	920.1 (30)	234.0 (14)
	var = 2 X	n.a.	n.a.	n.a.	n.a.
	var = 4 X	n.a.	n.a.	n.a.	n.a.

Figures in brackets are approximate location of maximum revenues.

The foregoing results imply very important conclusions. If variations such as the ones introduced into the day-to-day model do exist in the system, the potential to obtain system benefits from an optimal charging policy may no longer exist. Such conditions may well exist in real life networks and traffic systems. The presence of a number of bad occasions such as incidents or bad weather, etc. may counteract the purpose of optimal charging control.

6. CONCLUSIONS

Some generalisations and test specific findings can be summarised as follows.
The advantages of D-Day, a day-to-day dynamic model of driver route choice, have been demonstrated throughout. In contrast to the static models, D-Day can represent more behavioural aspects of drivers route choice decisions and explicitly incorporates day-to-day variability in traffic conditions. D-Day provides statistical data related to distribution of the performance of the system so as to provide better information of the state of the network.

In the data processing stage, amongst the most important factors to be decided are the number of initial days to be discarded (warm-up period) and the length of the simulation period. The decisions differ from case to case and therefore simple sensitivity testing should first be carried out to examine their effects on the results.

Two main sets of outputs need to be processed: the day-to-day flows and costs over the simulation period. In flow, the model produces a complete set of flows that might be expected if the route choice is regarded as a stochastic process. In terms of total day-to-day costs, the distribution is highly skewed as very poor network conditions occur on very rare occasions, but contribute quite significantly towards the means. The true mean costs are therefore significantly higher than the "ordinary" levels, depending on the variability parameter settings and the characteristics of the network tested. Given such a distribution the interquartile range is a more robust measure of dispersion than the standard deviation.

The effects of initial conditions on output flows and costs have been examined by varying the initial perceived cost distributions and the random number seeds. Tests using the very stable model of full information indicated that the mean flows from different initial conditions differ to some degree, whereas the costs do not significantly differ. This suggests that a sufficient number of runs should be repeated, as cpu time allows, in order to obtain representative results.

By contrast, in the self-learning model the flow patterns depend critically on the initialisation states and the flow patterns that result are highly counter-intuitive. These observations suggest that the self-learning cost adjustment does not perform as realistically as the informed cost adjustment.

In examining the relative effect of different sources of variation it was found that link capacity and the power of the cost function have the greatest effect on flows and costs. Free flow travel time, the slope of the cost function and demand level all have much smaller effects.

An important conclusion from tests on charging is that the effects of variability at reasonable levels may cause the benefits of optimal control to disappear and the total system cost to rise significantly above the UE level. This is a specific conclusion relating to the present model specification.

In the future it is planned to conduct an empirical test and validation of the D-Day model under an urban street network of developing country, as it was thought that apart from the existence of the network wide congestion, in such network travel time is very unreliable due to a considerable side friction along the links.

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REFERENCES

Ben-Akiva, M.E., Koutsopoulos, H.N., Mishalani, R.G., Yang, Q. (1994) **Integrated simulation framework for evaluating dynamic traffic management systems,** Poceedings of the First World Congress on Applications of Transport Telematics and Intelligent Vehicle-Highway Systems. pp 863-870

Cascetta, E. (1989). **A stochastic process approach to the analysis of temporal dynamics in transportation networks,** *Transportation Research*, 23B (1), 1-17.

Horowitz, J.L. (1984). **The stability of stochastic equilibrium in a two-link transportation network,** *Transportation Research*, 18B, pp.13-28.

Mahmassani, H., Herman, R. (1990). **Interactive experiment for the study of tripmaker behaviour dynamics in congested commuting systems,** *In Development in Dynamic and Activity-Based Approach to Travel Analysis.*, ed. Jones,P.

Sorah, H.A. (1995a) **MODELLING THE DAY-TO-DAY DYNAMICS OF ROUTE CHOICE : Effect of Initial Conditions,** Paper presented at World Conference on Transport Research, Sydney, Australia, 16-21 July 1995

Sorah, H.A. (1995b). **Modelling the Day-to-day Dynamics of Driver Route Choice and Traffic Control,** Draft PhD dissertation, University of Leeds (unpublished)

Wating, D.P (1997) **The stability of day-to-day dynamic traffic assignment models, Pre-prints the 8-th Meeting of the International Association for Travel Behaviour Research (IATBR),** Austin, Texas, September 1997.