# ON THE EFFECTIVENESS OF VARIOUS SIGNAL-CONTROLLED POLICIES FOR EQUILIBRIUM TRAFFIC SIGNAL SETTINGS

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**Abstract**: In a recent paper, a capacity-maximizing policy was proposed for setting signal-controlled road networks taking into account the route changing behavior of road users. This paper attempts to compare the effectiveness of this policy with other existing signal-controlled policies for equilibrium traffic signal settings, including equisaturation policy, local delay-minimizing policy and Smith's $P_0$ policy. Two measures, the total travel time and reserve capacity, are employed to demonstrate the results using a small network.

## 1. INTRODUCTION

Consider a network consisting of a set of links $A$ and a set of junctions $N$, in which the set of links entering into a signal-controlled junction is $\overline{A}$ ($\overline{A} \subset A$) and the set of signal controlled junctions is $J$ ($J \subset N$). Let $\mathbf{q}$ be the OD matrix, $\lambda$ be the signal settings for all signal controlled junctions $J$, and $v_a$, $p_a$ and $c_a$ be respectively the equilibrium flow, the maximum acceptable degree of saturation and the capacity for link $a \in A$.

Two measures are useful to describe the operational performance of a signal-controlled network: total travel time comprising the link travel time and delay at intersection; and capacity of the network. While the definition of the former measures in a network is straight forward, the definition of the latter term needs to be strengthened. This is because in a general network where route changing is allowed, the traffic will arrange itself in accordance with the Wardrop's equilibrium principle (Wardrop, 1952). The capacity of those links entering into a signal-controlled junction (i.e. $\overline{A}$) depends dynamically on the signal-controlled policy at the junctions. In a recent paper, the concept of reserve capacity, first proposed by Allsop (1972) and then Wong (1996) for individual junction, is extended to the network case (Wong and Yang, 1997) as an alternative measures for the operational performance of a signal-controlled network.

Suppose that the OD matrix is multiplied by a factor $\mu$. Then the queues and delays at the junctions in the network at equilibrium condition will be acceptable provided that the resulting degree of saturation on any link $a$ is not exceeding the maximum acceptable value for the link, i.e.

$$v_a(\mu\mathbf{q},\lambda) \leq p_a c_a(\lambda), \quad a \in A, \lambda \in \Lambda \tag{1}$$

where $\Lambda$ is the set of feasible signal settings of the signal-controlled junctions in the network.

The largest multiple of the OD matrix that can be accommodated without unacceptable queues and delays is found by maximizing $\mu$ with respect to the above constraints for all links. Note that this assumes that the OD demand pattern is preserved. Let the resulting value of $\mu$ be $\mu^*$. Then if $\mu^* > 1$ the junction has reserve capacity amounting to $100(\mu^*-1)$ per cent of the OD matrix $\mathbf{q}$, and if $\mu^* < 1$ the junction is overloaded by $100(1-\mu^*)$ per cent of the OD matrix $\mathbf{q}$.

While the measures of total travel time is a useful indication of the operational performance of the signal-controlled network subject to the existing traffic demand pattern, the measures of reserve capacity is a good indication of the sustainability of the signal settings with respect to the fluctuation in traffic demand pattern. Since the change in traffic demand pattern is largely uncertain during operation, the philosophy of the concept of reserve capacity is to allocate the capacities in the network by setting the timings as to keep the traffic conditions as far away from the saturation as possible. This allows the maximum flexibility in catering for the likely fluctuation in traffic demand pattern in the network in future operation.

The problem for maximizing the factor $\mu$ while maintaining user equilibrium of traffic flows was solved by a bi-level programming approach (Wong and Yang, 1997). This paper attempts to compare the effectiveness of this policy with other existing signal-controlled policies for equilibrium traffic signal settings, including equisaturation policy, local delay-minimizing policy and Smith's $P_0$ policy (Van Vuren and Van Vliet, 1992). The two measures, total travel time and reserve capacity, are employed to demonstrate the results using a small network.

## 2. EQUILIBRIUM TRAFFIC SIGNAL SETTINGS

### 2.1 Capacity-Maximizing Policy

The problem is formulated as a bi-level programming problem and solved by a successive linear programming algorithm. The upper-level program is formulated as

$$\underset{\mu,\lambda}{Maximise} \quad \mu \tag{2a}$$

subject to

$$v_a(\mu,\lambda) \le p_a c_a(\lambda), \quad a \in \bar{A} \tag{2b}$$

$$\mathbf{G}_k \cdot \boldsymbol{\lambda}_k \ge \mathbf{b}_k, \quad k \in J \tag{2c}$$

where $\mathbf{G}_k$, $\mathbf{b}_k$ and $\lambda_k$ are respectively the coefficient matrix, constant vector and signal settings for junction $k \in J$. Such linear constraints on the signal settings in equation (2c) can be set by both stage-based and group-based methods (Allsop, 1972; Heydecker and Dudgeon, 1987)

The equilibrium flow $v_a(\mu,\lambda)$ is obtained by solving the following lower-level program

$$\underset{\mathbf{v}}{Minimise} \quad \sum_{a\in A} \int_0^{v_a} t_a(x,\lambda)dx \tag{3a}$$

subject to

$$\sum_{r\in R_{ij}} f_r = \mu q_{ij}, \quad i \in O, j \in D \tag{3b}$$

$$v_a = \sum_{r\in R} f_r \delta_{ar}, \quad a \in A \tag{3c}$$

$$f_r \ge 0, \quad r \in R \tag{3d}$$

where

| | |
|---|---|
| $R$ | the set of routes in the network |
| $R_{ij}$ | the set of routes between origin-destination pair $(i,j)$ |
| $O$ | the set of origin nodes |
| $D$ | the set of destination nodes |
| $\mu$ | the flow multiplier for the whole network (upper-level decision variables) |
| $\lambda$ | vector of all signal settings (upper-level decision variables) |
| $v_a$ | flow on link $a \in A$ |
| $\mathbf{v}$ | vector of all link flows (lower-level decision variables) |
| $t_a(v_a,\lambda)$ | link cost function consisting of two components: travel cost on the link and delay at the intersection |
| $q_{ij}$ | the demand between origin-destination pair $(i,j)$ |
| $\mathbf{q}$ | the origin-destination matrix |
| $f_r$ | flow on route $r \in R$ |
| $\delta_{ar}$ | 1 if route $r$ uses link $a$, and 0 otherwise |

For given $\mu^*$ and $\lambda^*$, the lower-level programming program (3) can be solved by standard convex combination method (Frank-Wolfe algorithm) suggested by LeBlanc et al (1975). The traffic flow $v_a^*$ so obtained represents the equilibrium flow on link $a$ when the demands of all OD pairs are increased by a factor of $\mu^*$. These results will then be fed into the upper-level programming problem to solve for the maximum flow multiplier and the corresponding signal settings. For the development of an efficient algorithm for this purpose, the derivatives of the equilibrium flow with respect to the flow multiplier and the signal settings are determined by a sensitivity analysis (Tobin and Friesz, 1988).

Using the equilibrium flow and the sensitivity analysis results in the previous two sections, the upper-level programming problem can be approximated by a linear program in which the flow multiplier is maximized subject to the set of linear constraints, and the decision variables at this level of program is the flow multiplier itself and the signal settings of all signal controlled junctions in the network. This problem can be solved by standard algorithm (for example, the simplex method). The maximum flow multiplier $\mu^*$ and the signal settings $\lambda^*$ are then used as the input for the lower-level programming problem.

### 2.2 Equisaturation Policy

In equisaturation policy, the determination of equilibrium traffic signal settings is divided into two stages: equisaturation signal settings (i.e. the degrees of saturation from the critical approaches are set to be identical); and equilibrium assignment (i.e. the program in equation (3) with $\mu = 1$). These two stages are solved alternately until certain convergence criteria are satisfied (Van Vuren and Van Vliet, 1992).

### 2.3 Delay-Minimizing Policy

The delay-minimizing policy is very similar to the equisaturation policy, in that the determination of equilibrium traffic signal settings is also divided into two stages: calculation of delay-minimizing settings at individual junctions; and equilibrium assignment (i.e. the program in equation (3) with $\mu = 1$). These two stages are solved alternately until certain convergence criteria are satisfied (Van Vuren and Van Vliet, 1992).

### 2.4 Smith's $P_0$ policy

Smith (1980, 1985) proposed an ad hoc procedure for the equilibrium traffic signal settings. Similar to the previous two approaches, the problem is divided into two stages. In the first stage, instead of equating the degree of saturation as in the equisaturation policy, the product of saturation flow and delay at the critical approaches are set to be identical to determine the signal settings. These settings are then fed into the second stage for equilibrium assignment. The equilibrium flows thus obtained are then used in the first stage for optimal signal settings. The procedure is repeated until certain convergence criteria are satisfied.

## 3. NUMERICAL EXAMPLE

Consider a network with 7 links and 6 nodes of which nodes E and F are signal controlled junctions as shown in Figure 1. The OD demand from node C to node D is 0.1 veh/s, and that from node A to node B is ranging from 0.3 veh/s to 0.6 veh/s for parametric analysis. For simplicity assuming that traffic flow on link 1 is not allowed to turn to link 6, there are only two paths AEB and AFB for the OD pair (A,B). For OD pair (C,D), the only path is CEFD. For the signal controlled junctions E and F, each of them are controlled by two phases: $\phi_1$ and $\phi_5$ for junction E, and $\phi_3$ and $\phi_6$ for junction F where $\phi_a$ is the proportion of a cycle that is effectively green for link $a$. Let the saturation flow of the links be 0.5 veh/s, the maximum degree of saturation for phases be 0.9, the lost time between phases be 5 seconds, the cycle time be 90 seconds, and the minimum green time for a phase be 5 seconds. The effective green time is 1 second longer than the actual green time.

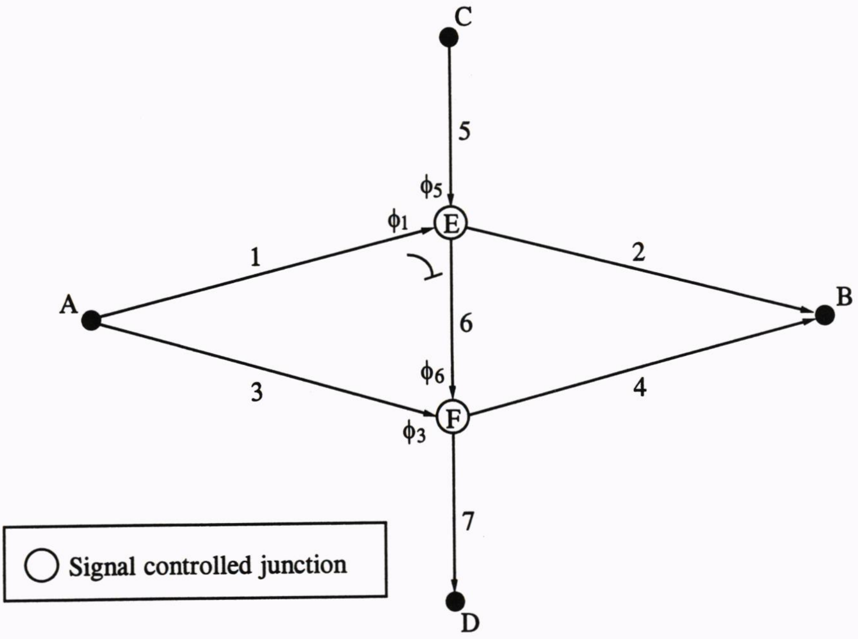


Figure 1. An Example Signal-Controlled Network

The link cost functions are given as

$$t_1 = 200+300v_1^2+\frac{9}{10}\left(\frac{45(1-\phi_1)^2}{1-2v_1}+\frac{(2v_1/\phi_1)^2}{2v_1(1-2v_1/\phi_1)}\right)$$

$$t_2 = 100+300v_2^2$$

$$t_3 = 150+100v_3^2+\frac{9}{10}\left(\frac{45(1-\phi_3)^2}{1-2v_3}+\frac{(2v_3/\phi_3)^2}{2v_3(1-2v_3/\phi_3)}\right)$$

$$t_4 = 200+100v_4^2$$

$$t_5 = 200+100v_5^2+\frac{9}{10}\left(\frac{45(1-\phi_5)^2}{1-2v_5}+\frac{(2v_5/\phi_5)^2}{2v_5(1-2v_5/\phi_5)}\right)$$

$$t_6 = 300+100v_6^2+\frac{9}{10}\left(\frac{45(1-\phi_6)^2}{1-2v_6}+\frac{(2v_6/\phi_6)^2}{2v_6(1-2v_6/\phi_6)}\right)$$

$$t_7 = 400+100v_7^2$$

where the first two terms in the expressions represent the traversing cost on the link and the third term (if any) is the delay at intersection estimated from Webster's formula (Webster and Cobbe, 1966).

## 4. RESULTS

When the demand for OD pair (A,B) gradually increases from 0.3 veh/s to 0.6 veh/s, the results from the four approaches described in Section 2 are summarized in Table 1, in which the total travel time is calculated as

$$T = \sum_a t_a v_a \tag{4}$$

where $t_a$ and $v_a$ are respectively the travel time and flow on link $a$. The maximum flow multiplier in the Table is a direct result from the capacity-maximizing policy. However, for the other three policies, the maximum multipliers are determined by gradually increasing the value of $\mu$ while fixing the signal settings determined for the existing traffic demand, until any of the approaches reaches the maximum acceptable degree of saturation.

The results are also plotted in Figures 2 and 3 for the total travel time and reserve capacity respectively. From Figure 2, it can be seen that the travel time increases almost linearly with respect to the increase in demand. The variations of total travel time from different policies are not very significant, although the capacity-maximizing policy is marginally better. As far as the operation of the signal-controlled network is concerned for the existing demand pattern, all policies performs equally well. However, from Figure 3, it can be seen that the reserve capacity for the capacity-maximizing policy is much better than the other policies, especially the P0 policy where the network tends to be overloaded with small increase in demand. Therefore, the capacity-maximizing policy is more sustainable to the fluctuation in traffic demand in the network.

## 5. CONCLUSION

In a recent paper, a capacity-maximizing policy was proposed for setting signal-controlled road networks taking into account the route changing behavior of road users (Wong and Yang, 1997). This paper attempts to compare the effectiveness of this policy with other existing signal-controlled policies for equilibrium traffic signal settings, including equisaturation policy, local delay-minimizing policy and Smith's $P_0$ policy. Two measures, total travel time and reserve capacity, are employed to demonstrate the results using a small network. It has found that while the total travel times from different policies are more or less the same, the capacity-maximizing policy is superior than the other policies in terms of reserve capacity in the network. The capacity-maximizing policy tends to be more sustainable in this small network. Larger size of networks will be tested for this conclusion to be drawn more reliably.

Table 1. The Results of Analysis from Four Signal-Control Policies

| $q_{AB}$ | | Capacity-maximizing policy | Equisaturation policy | Delay-minimizing policy | Smith's $P_0$ policy |
|---|---|---|---|---|---|
| 0.30 | $\phi_1$ | 0.58314 | 0.68333 | 0.67339 | 0.57720 |
| | $\phi_3$ | 0.58314 | 0.06667 | 0.06667 | 0.43057 |
| | $\phi_5$ | 0.32797 | 0.22778 | 0.23772 | 0.33391 |
| | $\phi_6$ | 0.32797 | 0.84444 | 0.84444 | 0.48054 |
| | $T$ | 203.4 | 208.4 | 208.1 | 204.1 |
| | $\mu^*$ | 1.476 | 1.329 | 1.011 | 1.191 |
| 0.35 | $\phi_1$ | 0.60768 | 0.68353 | 0.67287 | 0.58392 |
| | $\phi_3$ | 0.60768 | 0.30228 | 0.28576 | 0.45052 |
| | $\phi_5$ | 0.30343 | 0.22758 | 0.23824 | 0.32719 |
| | $\phi_6$ | 0.30343 | 0.60883 | 0.62535 | 0.46059 |
| | $T$ | 222.4 | 227.9 | 227.3 | 223.4 |
| | $\mu^*$ | 1.365 | 1.290 | 1.272 | 1.081 |
| 0.40 | $\phi_1$ | 0.62903 | 0.67727 | 0.66425 | 0.59312 |
| | $\phi_3$ | 0.62903 | 0.47802 | 0.49041 | 0.47317 |
| | $\phi_5$ | 0.28208 | 0.23384 | 0.24686 | 0.31799 |
| | $\phi_6$ | 0.28208 | 0.43309 | 0.42070 | 0.43794 |
| | $T$ | 242.0 | 244.6 | 243.8 | 243.4 |
| | $\mu^*$ | 1.269 | 1.223 | 1.203 | 0.987 |
| 0.45 | $\phi_1$ | 0.64773 | 0.67788 | 0.66508 | 0.60478 |
| | $\phi_3$ | 0.64773 | 0.55981 | 0.57033 | 0.49991 |
| | $\phi_5$ | 0.26338 | 0.23323 | 0.24603 | 0.30633 |
| | $\phi_6$ | 0.26338 | 0.35130 | 0.34078 | 0.41120 |
| | $T$ | 262.3 | 264.0 | 263.2 | 264.3 |
| | $\mu^*$ | 1.185 | 1.154 | 1.133 | 0.906 |
| 0.50 | $\phi_1$ | 0.66422 | 0.68116 | 0.66920 | 0.61897 |
| | $\phi_3$ | 0.66422 | 0.61119 | 0.61206 | 0.53268 |
| | $\phi_5$ | 0.24689 | 0.22995 | 0.24191 | 0.29214 |
| | $\phi_6$ | 0.24689 | 0.29992 | 0.29905 | 0.37843 |
| | $T$ | 283.8 | 284.8 | 284.1 | 286.4 |
| | $\mu^*$ | 1.111 | 1.090 | 1.068 | 0.837 |
| 0.55 | $\phi_1$ | 0.67885 | 0.68591 | 0.67508 | 0.63555 |
| | $\phi_3$ | 0.67885 | 0.64735 | 0.64048 | 0.57337 |
| | $\phi_5$ | 0.23226 | 0.22520 | 0.23603 | 0.27556 |
| | $\phi_6$ | 0.23226 | 0.26376 | 0.27063 | 0.33774 |
| | $T$ | 307.1 | 307.3 | 306.6 | 310.2 |
| | $\mu^*$ | 1.045 | 1.030 | 1.009 | 0.777 |
| 0.60 | $\phi_1$ | 0.69191 | 0.69168 | 0.68254 | 0.65482 |
| | $\phi_3$ | 0.69191 | 0.67432 | 0.66451 | 0.62095 |
| | $\phi_5$ | 0.21920 | 0.21943 | 0.22857 | 0.25629 |
| | $\phi_6$ | 0.21920 | 0.23679 | 0.24660 | 0.29017 |
| | $T$ | 334.1 | 332.7 | 331.7 | 337.0 |
| | $\mu^*$ | 0.986 | 0.975 | 0.954 | 0.725 |

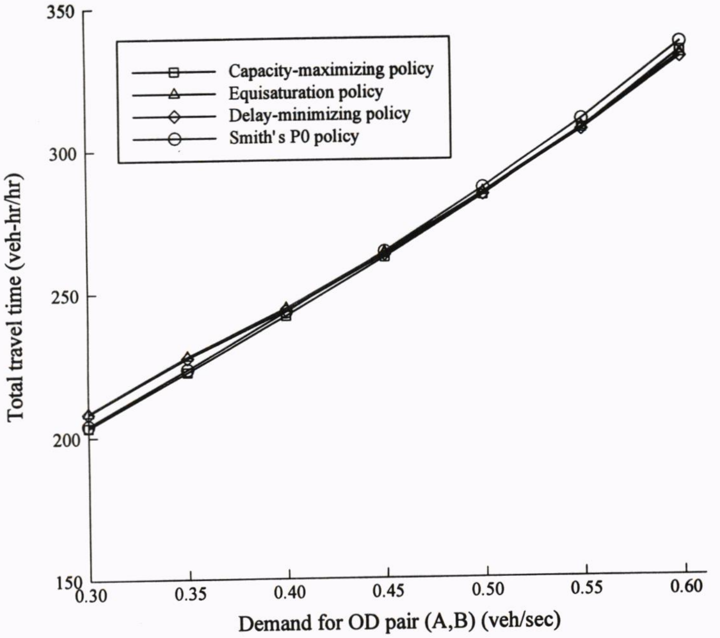


Figure 2.The Total Travel Times from the Four Signal-Controlled Policies

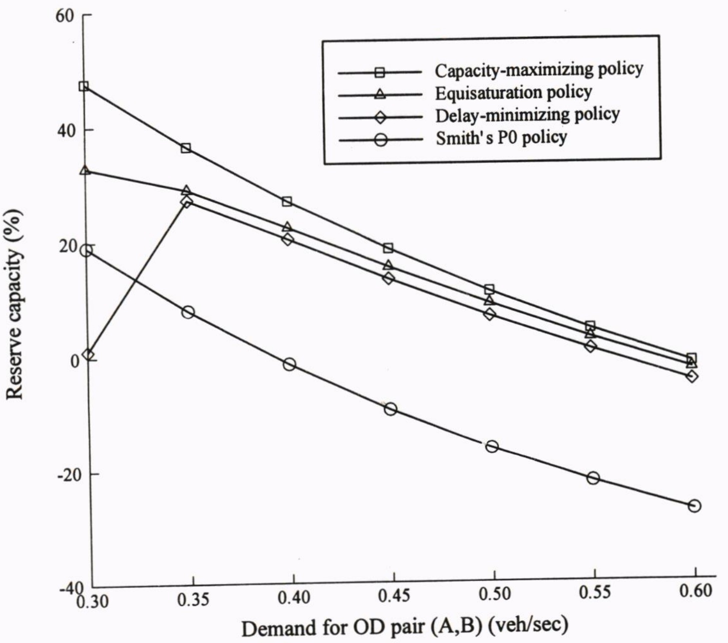


Figure 3. The Reserve Capacities from the Four Signal-Controlled Policies

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