THE RELATIONSHIP BETWEEN SUBJECTIVE TRAVELTIME AND TRAVELER'S EXPERIENCE

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Abstract: This paper focuses on the subjective travel time distribution. This study analyzes the shape of distribution, the formation process of the distribution and how it effects the fit on traffic behavior model. In order to examine these issues, I collect data from two kinds of computational experiment. Then, I get the result that the subjective standard deviation contributes to improve the fit of the route choice.

1. INTRODUCTION

Since its introduction by MacFadden(1974) and Ben-Akiva(1973), the disaggregated logit model using random utility theory is main stream of traffic demand modeling. The logit model is based on the hypothesis of maximizing utility such as "travelers make their decision to maximize their expected utility". Most of recent studies focus on the utility function specification to improve the goodness of fit of the logit model for actual traffic behavior. This study is adopts this approach. I focus on the uncertainty of travel time and attempt to insert the uncertainty into the utility function of traffic demand models to improve the prediction power.

According to some recent studies, it is clear that the travel time uncertainty produces an effect on travelers' behavior. When an uncertainty term is inserted into logit model, the expected utility function is obtained by the product of utility function and travel time distribution. So, if the utility function and the distribution of travel time are clear, it is possible to describe traveler's behavior. In existing models, the distribution is defined as actual distribution of travel time based on the rational expectation hypothesis, but it is unrealistic. Travelers make their decision by considering their subjective distribution of travel time. Therefore, using the subjective distribution for the expected utility function may improve the fit of demand model.

This paper focuses on the subjective distribution. I analyze the shape of distribution, the formation process of the distribution and how it effects the fit on traffic behavior model. In order to examine these issues, I collect data from two kinds of computational experiment. Then, I get the result that the subjective standard deviation contributes to improve the fit of the route choice.

Composition

This paper is composed of 6 chapters. In chapter 2, the motivation of this study will be demonstraited through the survey of existing studies and explanation of this study's argument. Chapter 3 will define travelers' utility functions and the optimal departure times in this paper.

Chapter 4 describes departure time choice models using the utility function. Then, a computational questionnaire will be designed to estimate endogenous parameters in the models. Finally, I will show that travelers subjective travel time is not same as actual travel time, and demonstrate traveler's learning process for travel time. In chapter 5, I will analyze the effectiveness of the subjective travel time for predicting route choice using the results of chapter 3 and 4. Chapter 6 gives conclusion and discussion.

2. SURVEY OF EXISTING MODEL AND APPROACH OF THIS MODEL

2.1. Survey of existing model

Golob(1970), Wong and Sussman(1973) and Prashker (1977) showed that the uncertainty of travel time had an effect on traffic demand. Following the studies, Brastow & Jucker(1977) defined travelers' expected utility, which depends on mean and variance of travel time, and analyzed the effect of the variance. Jackson & Jucker(1982) defined other linear utility functions, which were composed of various risk factors expressed as standard deviation. Then, they compared the models against each other. Because they used only linear expected utility functions, but the underlying travel response to risk appears to be nonlinear, the fit of their models was not satisfactory. In general, travelers have arrival time constraints. Therefore, it is difficult to express travelers' behavior by such simple linear utility functions. Yamashita & Kuroda(1996) defined travelers' route and departure time choice as a problem of decision making under uncertainty. To put it concretely, they defineded travelers' utility based on the relationship between their actual arrival time and their required arrival time. In addition, they estimated travelers' non-linear expected utility by combinating the utility and probability distribution of travel time. Then, they made a kind of logit model using the expected utility. They proved that this non-linear utility function was superior than linear mean-variance utility function for traffic demand prediction.

All previous studies have an unrealistic assumption that travelers know the actual distribution of travel time. I consider that the assumption causes reduced goodness of fit of traffic demand models.

2.2. Basic approach

According to the postulate of rationality, the best choice maximizes the expected utility function which is obtained by the product of the utility of the result and the probability distribution of the uncertain outcomes(Edwards, 1954 ; Coombs & Beadslee,1954). In this formulation, four types of model can be defined considering whether the utility function and distribution are subjective or actual.
In above-mentioned model (Yamashita & Kuroda 1996), the expected utility function is composed of the subjective utility function and the actual distribution of travel time. This approach holds good under the rational expectation postulate. However, it is hypothesized that travelers make their decisions using subjective utilities and subjective probabilities. Therefore, a demand model composed of subjective utility and subjective probability may improve the explanatory power of prediction.

This study focuses on the traveler's subjective probability distribution of travel time and examines the improvement from a demand model based on the subjective probability. It is assumed that travelers' subjective travel time distribution depends on their experience of the travel and especially the number of experiences. Next, I analyze the relationship between the travelers' experiences and the dynamics of the subjective distribution by laboratory computational experiments that repeatedly ask the participants to respond to make hypothetical departure time and route choice questions. For this analysis, a simple model of the learning process is built. This paper examines, in addition, the explanatory power of the discrete choice model that is based on the travelers' experience and the subjective distribution of travel time. Specifically, the choice model is a disaggregate logit model with non-linear utility terms expected that are obtained from the travel time subjective distribution made by the learning process model.

3. UTLITY FUNCTION AND OPTIMAL DEPARTURE TIME

3.1. Utility function by aversion of delay

This study assumes the following proposition when travelers make their decision of departure time choice.

1. A traveler's utility is increasing when the traveler postpones his or her departure time, because more time can be used productively at home (Cost of travel time)
2. The traveler's utility decreases by constant value when the traveler is late for their required arrival time (the constant value is called the penalty of delay).
3. When the traveler decides the departure time, the traveler cannot recognize whether he or she will be late or not. But the traveler can image the probability to be late.

Then, the traveler's expected utility function is defined as follow;

$$V_{ijk} = T_{ijk} - \gamma_{ijk} \Pr(T_{ijk}) \tag{1}$$

V : Traveler's expected utility

T_{ijk} : Departure time (<0, because the required arrival time T_d is defined as 0. The sum of actual travel time and safety margin is called as effective travel time. In this formula, $-T$ is the effective travel time.)

γ_{ijk} : Penalty of delay (>0)

$\Pr(T_{ijk})$: Probability of delay when traveler's departure time is T_{ijk}.

i : Index of traveler

j : Index of traffic mode or route

k : Index of frequency count ($k = 1,2,\cdots$) e.g., day the commute is made.

According to assumption 3, the traveler has his /her subjective distribution of travel time. I suppose that he/she estimates the probability of delay by the subjective distribution. Eq.(1) is transformed using $f_{ijk}(t)$ which is defined as density function of the subjective distribution of travel time.

$$V_{ijk} = T_{ijk} - \gamma_{ijk} \int_{-T_{ijk}}^{\infty} f_{ijk}(t)dt \tag{2}$$

In existing studies, the actual distribution of travel time is used for $f_{ijk}(t)$ based on the rational expectation hypothesis. I assume that $f_{ijk}(t)$ is subjective and depends on traveler's experience in this study. To the simplify model, $f_{ijk}(t)$ is defined from the based on the normal distribution N (μ_{ijk}, σ^2_{ijk}). Then, it assumed that μ_{ijk} and σ^2_{ijk} depends on the traveler's experience. Now, eq.(2) can be transformed as follows:

$$V_{ijk} = T_{ijk} - \gamma_{ijk} \int_{-T_{ijk}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_{ijk}} e^{-\frac{(t-\mu_{ijk})^2}{2\sigma^2_{ijk}}} dt \tag{3}$$

It seems that the assumption of normal distribution is too string. Therefore, I examined whether other distributions could improve the goodness of fit or not (Yamashita (1997)). According to the result of the examination, there is not much to choose between them.

In a more general formulation, the utility function will have costs and other factors. However, I omit every factor except the travel time factor in order to more closely examine the traveler's subjective travel time. In addition, according to eq.(3), the penalty of delay γ depends on traffic mode and traveler. However, it is not necessary for γ to depend on these parameters, as I will be explained it in more detail in the next chapter.

3.2. Optimal departure time

Travelers decide their departure time choice by maximizing the expected utility shown in eq.(3). I differentiate eq.(3) with respect to T.

$$\begin{aligned} \frac{\partial V_{ijk}}{\partial T_{ijk}} &= 1 - \frac{\partial}{\partial T_{ijk}} \gamma_{ijk} \left\{ 1 - \int_{-\infty}^{-T_{ijk}} \frac{1}{\sqrt{2\pi}\sigma_{ijk}} e^{-\frac{(t-\mu_{ijk})^2}{2\sigma^2_{ijk}}} dt \right\} \\ &= 1 - \gamma_{ijk} \frac{1}{\sqrt{2\pi}\sigma_{ijk}} e^{-\frac{(-T_{ijk}-\mu_{ijk})^2}{2\sigma^2_{ijk}}} \end{aligned} \tag{4}$$

Solving for T_{ijk} where $\frac{\partial V_{ijk}}{\partial T_{ijk}} = 0$ gives

$$T_{ijk} = -\mu_{ijk} \pm \sigma_{ijk} \sqrt{2\log\left(\frac{\gamma_{ijk}}{\sqrt{2\pi}\sigma_{ijk}}\right)} \tag{5}$$

Therefore, following equation minimize the utility function.

$$T^{*}_{ijk} = -\mu_{ijk} - \sigma_{ijk} \sqrt{2\log\left(\frac{\gamma_{ijk}}{\sqrt{2\pi}\sigma_{ijk}}\right)} \tag{6}$$

T^{*}_{ijk} is named the optimal departure time in this paper.

4. MODELS OF SUBJECTIVE DISTRIBUTION OF TRAVEL TIME

4.1. Estimation

In this chapter, travelers' subjective distribution of travel time is estimated using eq.(6). I assume that there is actual departure time data T_{ijk} that depends on travel mode and number of experience. The data is defined as
The parameter μ, σ and γ are determined by the Least Square Method as follows;

$$\min_{\mu,\sigma,\gamma} \sum_i \sum_j \sum_k (T_{ijk} - T^{*}_{ijk})^2 \tag{7}$$

There is diversity of models that can be used when the data are made by computational experiment. Potentially large number of endogenous parameters causes the diversity. In order to make the models robust, some parameters must be fixed when the others parameters are calculated. The diversity organized into two categories; exogenous parameters and endogenous parameters. In next secession, the model diversity is explained in detail.

4.2. Model diversity

4.2.1. Diversity about the mean of subjective travel time

In order to calculate parameters, there are two ways to define μ. One is to define μ by actual mean of travel time. The other is to define by subjective travel time which is predicted by travelers.

A. Actual mean of travel time (exogenous variable)

μ^{A}_{j} : Actual mean of travel time

B. Difference between departure time and traveler's expected arrival time (exogenous parameter)

Method-B is superior to method-A because travelers make their decision by their subjective parameter such as the mean of travel time defined by method-B. However, it is difficult to get a data set of travelers' subjective travel times. On the other hand, it is actually easy to get data of method-A. Therefore, method-A is effective for actual traffic demand prediction.

$\mu^{B}_{ijk} = (G_{ijk} - T_{ijk})$ $\qquad$ G_{ijk} : Traveler's expected arrival time

4.2.2. Diversity about standard deviation of travel time

In existing models, actual standard deviation of travel time is used as a factor of demand modeling. However, it is more difficult for travelers to get information about the standard deviation of travel time than that of the mean. So, it is unrealistic to assume that travelers make their decision based on the actual standard deviation. In this study, therefore, travelers' subjective standard deviation is defined as an endogenous parameter calculated by eq.(6).

Three methods can be considered for the estimation of σ. In the first method, σ is defined as constant value that depends on only traffic mode or route. The Maximum Likelihood Method or the Least Square Method can calculates this constant value. In the second method, σ is considered to depend on k which is the number of times of traveler's experience of the trip. The k σs are calculated. In the last method, σ depends on k and the relation between σ and k. The relation is obtained by a learning process model.

a. Constant standard deviation which depend on traffic mode or route j (endogenous parameter) :

$$\sigma_j^{a}$$

b. Standard deviation which depend on traffic mode j and number of times of experience k(endogenous parameter)

σ_{jk}^{b} $(k = 1,\cdots,K)$ K: Total number of iteration

c. Standard deviation which is defined by the learning process model (endogenous parameter)

In this study, the learning model is obtained by follows;

$$\frac{d\sigma_{jk}}{dk} = -c_j(\sigma_{jk} - a_j) \tag{8}$$

σ : Subjective standard deviation of travel time
j : Index of traffic mode or route
k : Index of frequency count $(k = 1,2,\cdots)$
a : Convergence value of the subjective standard deviation of travel time $(\sigma_{jk} > a)$
c : Velocity of convergence of the learning process

The boundary conditions of this equation are defined as follows;

$$\frac{d\sigma_{jk}}{dk} < 0, \qquad \lim_{\sigma_{jk} \to a} \frac{d\sigma_{jk}}{dk} = 0$$

The following model is obtained from solving the equation (8) under the boundary conditions;

$$\sigma_{jk} = a_j + b_j e^{-c_j k} \tag{9}$$

This model means that the subjective standard deviation is high for travelers' with low experience frequency and converges to a constant value a_j.

$\sigma_{jk}^{c} = a_j + b_j e^{-c_j k}$ (Endogenous parameters are a_j, b_j, c_j)

4.2.3. Diversity about the penalty of delay γ_{ijk}

The penalty of delay is shown to the participants in the experiment. However, the participants may not exactly appreciate the value of the penalty. Therefore, it may be necessary to calculate the parameter by the above-mentioned model. So, there are two definitions about the penalty of delay γ. First, γ is defined as the actual value which is shown to participants during the computational experiment. Then, γ is an endogenous parameter calculated by the Least Square Method.

I. Actual value of penalty of delay (exogenous parameter)

γ^{1} : Actual value of penalty of delay that is shown to travelers

II. Estimated parameter which is independent of traveler and times of frequency (endogenous parameter)

γ_j^{**} : Estimated from eq.(6)

4.3. Reference of model diversity

I defined 12 models (2*3*2) based on three kind of diversities in section 4.2. In this section, the mathematical formulas of the models are shown. The Least Square Method with eq. (6) is used to calculate the endogenous parameters of every model.
Because subjective mean of travel time μ_j^A and μ_{ijk}^B are exogenous parameter, I use one character μ_{ijk} instead of μ_j^A and μ_{ijk}^B.
A-a-II, B-a-II, A-b-II, B-b-II are indefinite model. Therefore, I don't explain these models in this paper. The proof that these models are indefinite is shown in Yamashita & Hagiyama(1996).

A-a-I, B-a-I

$$\min_{\sigma_j^a} \sum_i \sum_k \left(T_{ijk} + \mu_{ijk} + \sigma_j^a \sqrt{2\log\left(\frac{\gamma^{\mathrm{I}}}{\sqrt{2\pi}\sigma_j^a}\right)} \right)^2 \tag{10}$$

Endogenous parameter; σ_j^a

A-b-I, B-b-I

$$\min_{\sigma_{jk}^b} \sum_i \left(T_{ijk} + \mu_{ijk} + \sigma_{jk}^b \sqrt{2\log\left(\frac{\gamma^{\mathrm{I}}}{\sqrt{2\pi}\sigma_{jk}^b}\right)} \right)^2 \tag{11}$$

Endogenous parameter; $\sigma_{jk}^b \ (k = 1, \cdots, K)$

A-c-I, B-c-I

$$\min_{a_j, b_j, c_j} \sum_i \left(T_{ijk} + \mu_{ijk} + \sigma_{jk}^c \sqrt{2\log\left(\frac{\gamma^{\mathrm{I}}}{\sqrt{2\pi}\sigma_{jk}^c}\right)} \right)^2 \tag{12}$$

Subject to $\sigma_{jk}^c = a_j + b_j e^{-c_j k}$
Endogenous parameter; a_j, b_j, c_j

A-c-II, B-c-II

$$\min_{a_j, b_j, c_j} \sum_i \left(T_{ijk} + \mu_{ijk} + \sigma_{jk}^c \sqrt{2\log\left(\frac{\gamma^{\mathrm{II}}}{\sqrt{2\pi}\sigma_{jk}^c}\right)} \right)^2 \tag{13}$$

Subject to $\sigma_{jk}^c = a_j + b_j e^{-c_j k}$
Endogenous parameter; $a_j, b_j, c_j, \gamma_j^{\mathrm{II}}$

4.4.Condition of Experiment I

In order to calculate the endogenous parameters in the model, I conducted the following experiment on departure time choice behavior, which involves repeated hypothetical departure time choices for simulated commuter trips to the workplace. To simplify the experiment, let us take only one O-D pair connected by one route. All commuters are assumed to travel in one direction. To make sure the participants properly understand the experimental situation, the following instructions are given;

(a) the purpose of trip is going to your workplace,
(b) each iteration of the experiment corresponds to a day,
(c) required arrival time is at 9:00 am,
(d) penalty of delay is equal to 120 minutes.

Each iteration of the experiment consist of the following steps;

(a) the experimenter presents the last iteration's results to participants,
(b) the participants predict their arrival time (or travel time) for this iteration,
(c) the participants choose their departure time.

The participants were directed to predict the travel time based on the last iteration's result (actual travel time, actual arrival time and score) and previous driving experience (actual travel time, delays to required arrival time and total score). Then, they had to choose the departure time, which supposedly make them be in time for the required arrival time.

Actual travel times were calculated for each iteration of the experiment by random numbers according to the normal distribution $N(50, 10^2)$. The parameters of the distribution were hidden from the participants.
Then, this iteration's score is calculated as follows;

$$Score(i,k) = -(9{:}00\text{-departure time}) - 120*d(i,k)$$

d(i,k): if I-th participant's k-th iteration's arrival time is before 9:00 then d=0, if the arrival time is after 9:00 then d=1.
In order to encourage the participants to choose the departure time seriously, the experimenter gave the participants a money reward in proportion to their total score. All participants are college students in the information technology department of Ochanomizu University.

Table 1. The Condition of Experiment I

Experiment condition (Experiment I)	
Network	1OD,1 link
Actual parameter of travel time	$N(50,10^2)$
No. of participants	39
No. of iterations	100
Penalty of delay (given to participant)	120 (min)
Choice objective	To maximize score
Information provided to participants every iterations	Travel time, Arrival time, delay or not for last iteration, Total score
Respondents' tasks	Departure time Prediction of arrival time

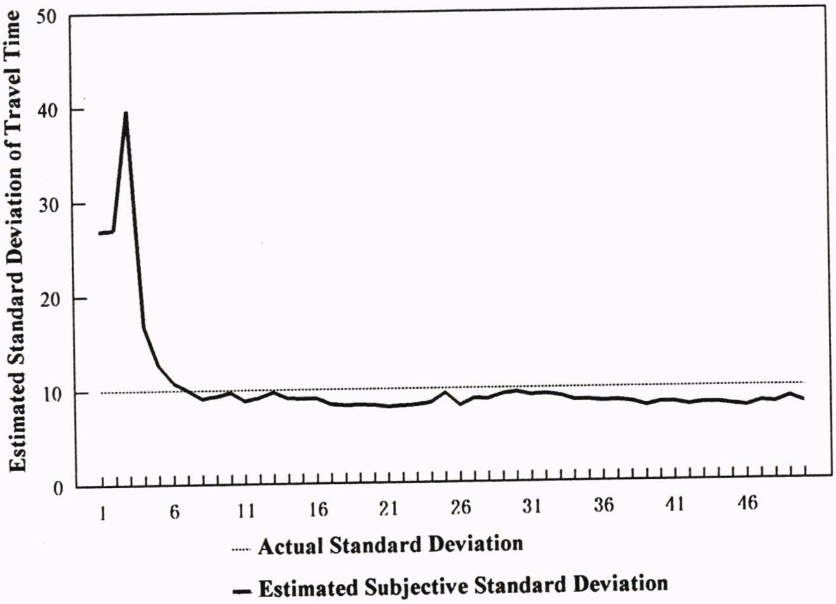

Figure 1. Learning Process in B-b-I

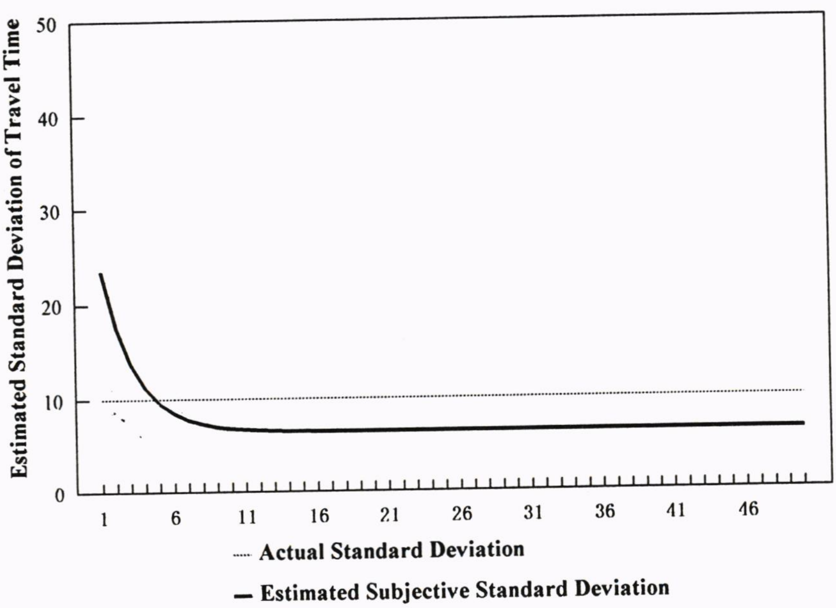

Figure 2. Learning Process in B-c-II

Table 2 The estimation results of the departure time choice models

Experiment I (No. of obs : 3900)						
Model	A-a-I	A-c-I	A-c-II	B-a-I	B-c-I	B-c-II
No. of parameters	1	3	4	1	3	4
Standard error	279500	236425	213459	154444	148868	148600
AIC	27730.6	27081.9	26685.4	25417.3	25277.8	25272.8
Estimated parameter						
Penalty of delay	(120)	(120)	1.0E+21	(120)	(120)	77.60
			3.6E+12			*0.11*
Standard deviation Of subjective travel time	9.06			5.76		
	0.06			*0.05*		
Learning model; a		8.45	1.63		5.53	6.53
		0.01	*67916.67*		*0.06*	*0.24*
b		32.60	8.19		7.33	26.09
		0.25	*1.1E+06*		*0.06*	*0.17*
c		0.44	0.75		0.25	0.43
		0.06	*1.1E+06*		*0.00*	*0.00*

Italic; standard error of estimated parameter
(parentheses); indicate constrained parameters

4.5. Results

Figure 1 shows the change of the subjective standard deviation of travel time that is calculated by the B-b-I model. This figure shows that the subjective standard deviation is high for travelers with low experience frequency. The lower standard deviation is, the higher experience frequency is. For high frequencies, the standard deviation seems to converge to a constant value. This result indicates that travelers without experience start early to avoid the risk of delay. I got similar results from the A-b-I model.

Table 2 shows the statistics and estimated parameters that are calculated by every model except A-b-I and B-b-I. Because each models has different number of estimated parameters, I must use AIC (Akaike Information Criterion) to compare the fit of the models. This study hypothesizes that the error distributions of these models are according to the normal distribution, because AIC requires the shape of error distribution.

$$\textbf{AIC} = -2*[\text{maximum likelihood}] - 2*[\text{No. of parameters}]$$

The model B-a-I, B-c-I and B-c-IIs AICs are lower than those of A-a-I, A-c-I and A-c-II. Therefore, I conclude that the travelers' subjective mean of travel time is effective for departure time choice models.

The optimal model is B-c-II, whose factors are subjective mean of travel time, the learning process model and the subjective penalty of delay. In the result of this model, the penalty of delay is estimated as 77.60, which is lower than the actual value 120. This means that the participants misunderstood the condition of this experiment regarding the penalty.

Figure 2 shows the curve of the subjective standard deviation of the learning process model (by B-c-II). In this result, it was confirmed that the standard deviation converges quickly on a constant value, and the value of convergence (= 6.53) is smaller than actual standard deviation of travel time (=10). This result shows that the travelers' decision making process is not based on the rational expectation hypothesis.

In additional, A-c-II model is almost indefinite. Better quality and quantity of data are required to evaluate the effect of this model.

5. ROUTE CHOICE MODEL USING THE SUBJECTIVE PARAMETERS

In this chapter, I examine the effect of the subjective distribution of travel time on traffic demand models. Some disaggregated logit models based on an expected utility function about the risk of delay are defined. Then, the parameters of the model are estimated using route choice data made by a computational experiment. Model goodness of fit -measured by AIC- are compared with that of the ordinal logit model.

5.1. Definition of the utility function

A simple network (1 OD pair 2 links) is used to analyze the route choice behavior. The travelers' expected utility function is defined as eq.(14) and (15). These functions include an inertia factor and route specific dummy, as well as the utility factors that were defined in chapter 4.

$$V_{i1x} = \theta_1\left\{T^{*}_{i1x} - \gamma\,\Pr(T^{*}_{i1x})\right\} + \theta_2 d_1(j_{i,k-1}) \tag{14}$$

$$V_{i2y} = \theta_0 + \theta_1\left\{T^{*}_{i2y} - \gamma\,\Pr(T^{*}_{i2y})\right\} + \theta_2 d_2(j_{i,k-1}) \tag{15}$$

V_{i1x} : Traveler's expected utility when the traveler choices the main route.

V_{i2x} : Traveler's expected utility when the traveler choices the side way.

i : Index of traveler

j : Index of route (1; main route, 2; side way)

k : Number of iteration $(x + y = k)$

x : Number of times that the traveler chose the main route

y : Number of times that the traveler chose the side way

θ_0 : Dummy parameter of the side way

θ_1 : Parameter of utility by departure time choice

T^{*} : The optimal departure time

γ : Penalty of delay

$\Pr(T^{*})$: Probability of delay when the traveler's departure time is T^{*}

d : Variables of inertia, the value is as follow;

$$d_1(j_{i,k-1}) = \begin{cases} 1 \;;\; j_{i,k-1} = 1 & (\textit{last coice is main route and present choice is main route}) \\ 0 \;;\; j_{i,k-1} = 2 & (\textit{last coice is side way and present choice is main route}) \end{cases}$$

$$d_2(j_{i,k-1}) = \begin{cases} 0 \;;\; j_{i,k-1} = 1 & (\textit{last choice is main route and present choice is side way}) \\ 1 \;;\; j_{i,k-1} = 2 & (\textit{last choice is side way and present choice is side way}) \end{cases}$$

θ_2 : Parameter of d

The probability of delay $\Pr(T^*)$ is as same as that in chapter 4. That probability is calculated from the subjective distribution of travel time which is assumed to be normally distributed $N(\mu_{ijk}, \sigma_{ijk}^2)$.

$$\Pr(T^*) = \int_{-T^*}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_{ijk}} e^{-\frac{(t-\mu_{ijk})^2}{2\sigma_{ijk}^2}} dt \tag{16}$$

Two methods are available to calculate the optimal departure time. One uses actual departure time choice data, the other uses the numerical solution of eq.(6) whose variables are mean of the subjective deviation of travel time , standard deviation and penalty of delay. Both methods will be explained in detail in section 5.3.

5.2. Parameter estimation of the logit model

Parameters are calculated using a binomial logit model that has the expected utility function defined as equation (14) and (15). In the binomial logit model, the probabilities of choice P_{i1k}, P_{i2k} are defined as follows:

$$P_{i1k} = \frac{e^{V_{i1x}}}{e^{V_{i1x}} + e^{V_{i2y}}} \tag{17}$$

$$P_{i2k} = \frac{e^{V_{i2y}}}{e^{V_{i1x}} + e^{V_{i2y}}} \tag{18}$$

The maximum likelihood method is used to be estimate parameters in the models.
The likelihood function is the joint probability of every travelers' every route choice;

$$L = \prod_{i=1}^{N} \prod_{k=1}^{K} P_{i1k}^{\delta_{i1k}} \cdot P_{i2k}^{\delta_{i2k}} \tag{19}$$

$\delta_{i1k} = 1,\ \delta_{i2k} = 0$; *i*-th traveler chooses the main route at k-th iteration.
$\delta_{i1k} = 0,\ \delta_{i2k} = 1$; *i*-th traveler chooses the side way at k-th iteration.
(Note: $\delta_{i1k} + \delta_{i2k} = 1 \quad \forall i,k$)
N : Number of travelers
K: Number of iterations

5.3. Model diversity

There is diversity about eq. (14) and (15) when the parameters are calculated using the data made by computational experiment. The diversity is caused by a high number of parameters. In order to make the model be robust, some parameters must be exogenous when the others are calculated. The diversity organizes into two categories of parameters; exogenous parameters and endogenous parameters.

5.3.1. Diversity about the optimal departure time

There are two methods to treat the optimal departure time, as an endogenous parameter or as an exogenous parameter.

A. Actual departure time (exogenous parameter)
This parameter can be taken from the actual traveler's choice data. This is an accurate way to describe the travelers' behavior. However, it is difficult to get the departure time data when the traveler chooses the other route. These data are needed to estimate models using above-mentioned formula.

B. Endogenous parameter estimated by departure time choice model
I show the optimal time choice formula as eq. (6) in chapter 3. I can eliminate T^* from eq. (14) and (15) by the substituting eq. (6) into these equations. According to this method, it is not necessary to get the special data regarding traveler's departure time choice.

Therefore, I analyze only method B in this study.

$$\text{B} \quad \Pr(T^*) = \int_{-T^*}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_{ijk}} e^{-\frac{(t-\mu_{ijk})^2}{2\sigma_{ijk}^2}} dt \qquad \text{(endogenous parameter)} \quad (20)$$

5.3.2. Diversity about standard deviation of travel time
In chapter 4, I concluded that the subjective standard deviation is effective to describe the travelers' behavior. Therefore, It may be better to calculate the subjective standard deviation by the logit model than to use the parameters used in laboratory experiment. I defined three methods to calculate this parameter.

a. Actual Standard deviation of travel time
This method is alternative hypothesis of this analysis. It is included for comparison since most conventional studies examine the actual standard deviation of travel time, rather than the subjective standard deviation.

σ_j^{a} (exogenous parameter)

b. Standard deviation that is defined by the learning process based on the number of times of travelers' experience (endogenous parameter)
This method is almost same as eq.(9) in chapter 4. But, the actual standard deviation is substituted for a_j in this equation as an exogenous parameter. b_j and c_j are endogenous parameters that are independent of route j. Therefore, the two parameters are estimated by the binomial logit model.

$\sigma_{jk}^{b*} = a_j + be^{-ck}$ (a is an exogenous parameter, b and c are endogenous parameters)

c. Standard deviation calculated exogenously
This way has a two-stage estimation method. The parameters of the learning process model are calculated by the same method as chapter 4. The learning model is exogenously substituted for the utility function of the binomial logit model.

$\sigma_{jk}^{\check{}}$ (exogenous parameter)

5.3.3. Diversity about the penalty of delay.
This diversity is same as that in chapter 4.

I Actual value of penalty of delay (exogenous parameter)

γ^{I} : Actual value of penalty of delay that is shown to travelers

II. Estimated parameter which is independent of traveler and experience

γ_j^{II} :(endogenous parameter)

5.3.4. Conditions about the others parameter

The other parameters are estimated according to the conditions as follows;

・ Dummy parameter of the side way θ_0 is defined as an endogenous parameter in the logit model.

・ The parameter for the optimal departure time choice θ_1 is defined as an endogenous parameter in the logit model.

・ The inertia variable d is obtained from the computational experiment. (Exogenous variable)

・ The parameter of the inertia variable θ_2 is defined as an endogenous parameter in the logit model.

Therefore, 6 models are defined by 3 definitions about the standard deviation and 2 definitions about the penalty of delay.

5.4. Conditions of experiment II

In order to estimate the endogenous parameters in the models, I conducted the following experiment on route choice and departure time choice behavior. To simplify the experiment, I considered only one O-D pair connected by 2 routes (main route and side way). The following instructions are given;

(a) The purpose of trip is going to your workplace,
(b) Each iteration of the experiment corresponds to a day,
(c) Required time is at 9:00 am and present time is 7:00 am
(d) The penalty of delay is equal to 120 minutes.
(e) The travel time of main route follows the normal distribution N(60, 13^2) and that of side way follows N(70, 5.2^2).

Each iteration of the experiment consists of the following steps;

(a) The experimenter presents the last iteration's results to participants,
(b) The participants predict arrival time (or travel time) for this iteration,
(c) The participants choose departure time and route.

Other conditions are same as the experiment I.

The analysis considers the case where a pool of travelers departs from a given origin to a single destination connected by two parallel alternative routs.

5.5.Results

Table 4 shows the result of estimation, AIC, logarithmic maximum likelihood and estimated parameters. The marks * and # in the table identify exogenous parameters.

According to the comparison between a-I and a-II, c-I and c-II, the maximum likelihood cannot be improved by the estimating the penalty of delay. Therefore, a-I is better than a-II, c-I is better than c-II by AIC.

Table 2. Condition of Experiment II

Experiment condition (Experiment II)	
Network	1OD,2 link
Actual parameter of travel time	N(60,13^2) (High risk, High return) N(70,5.2^2) (Low risk, Low return)
No. of participants	39
No. of iterations	100
The penalty of delay (given to participant)	120 (min)
Choice objective	To maximize score
Information provided to participants at every iterations	Travel time, Arrival time, delay or not for last iteration, Total score
Respondents' tasks	Departure time Prediction of arrival time Route choice

The AIC indicates that b-I is the best model. This model combines the logit route choice model with the learning process model.
a-I and a-II use the actual standard deviation of travel time. On other hand, c-I, c-II and b-I use traveler's subjective standard deviation. By comparing the AIC of these models, we see that the subjective standard deviation contributes to improve the fitness of the route choice model.

Table 4. The estimation results of the logit model's parameter using route choice data

#; Parameters given to participants during the experiment

*; Parameters of the learning model estimated by experiment I

Experiment II (No.of data: 3900)						
Model		a-I	a-II	c-I	c-II	b-I
No. of parameter		3	4	3	3	5
Log of Maximum likelihood		-843.53	-843.53	-821.03	-834.78	-796.38
AIC		1693.07	1695.07	1648.06	1675.56	1602.75
Dummy parameter of side way; $\theta 3$		0.11	0.25	3.78	2.58	0.11
Dummy parameter of inertia; $\theta 2$		2.70	2.70	2.47	2.64	2.34
Parameter for departure time choice; $\theta 1$		0.08	0.06	0.24	0.13	0.10
Penalty of delay; γ	(common parameter of both way)	#120.00	107.04	#120.00	-	#120.00
	(parameter of main)	-	-	-	* 62.58	-
	(parameter of side way)	-	-	-	*187.12	-
Standard deviation	a (parameter main)	# 13.00	# 13.00	* 6.00	* 8.13	* 13.00
of subjective travel time	a (parameter of side way)	# 5.20	# 5.20	* 4.90	* 4.37	* 5.20
	b (common parameter for both way)	-	-	-	-	12.28
Model cI, cII, bI use the	b (parameter of main road)	-	-	* 3.60	* 12.79	-
learning model,	b (parameter of side way)	-	-	* 3.40	* 2.79	-
B,c are parameter for	c (Common parameter for both way)	-	-	-	-	0.03
The learning model.	c (parameter of main road)	-	-	* 0.45	* 0.61	-
a+be-ck	c (parameter of side way)	-	-	* 0.11	* 0.11	-

6. DISCUSSION

This paper showed the formation process of travelers' subjective distribution of travel time through two types of computational experiments and the application to traffic demand modeling.

In chapter 4, I made it clear that travelers' subjective standard deviation of the travel time was high for travelers with low experience frequency, and the standard deviation was lower for travelers with higher experience frequency. For the high frequencies, the standard deviation converges to a constant value. The value of convergence is smaller than actual standard deviation of travel time. This result shows that the travelers' decision making process is not based on the rational expectation hypothesis.

In chapter 5, I compared the AIC of models that use the actual standard deviation and the models that use the subjective standard deviation. It is clear that the subjective standard deviation contributes to improve the fit of the route choice models.

Problems to be solved in future are shown as follows.

I defined the learning process in demand model in this study. This formula is based on the hypothesis that travelers don't forget their experience, they make their decision under the perfect memories about their past behaviors and results. This hypothesis may be unrealistic. Therefore, the effect of forgetfulness on route choice or departure time choice must be analyzed in future studies.

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