A STUDY OF GAME THEORETIC ANALYSIS OF FORMED AGREEMENT COST ALLOCATION MODEL

Hideki FUJIMURA
Graduate School of Science and Technology
Kumamoto Univ.
3-7-10,Takasukita,Kitakyusyu City, Fukuoka,
Japan 808-0147
Fax:+81- 93-582-2503
E-mail: fujimura @ interlink.or.jp

Shoshi MIZOKAMI
Professor
Dept. of Civil Environ. Eng.
Kumamoto Univ.

Ryuji KAKIMOTO
Assistant Professor
Kumamoto Univ.

Abstract: We proposed the Formed Agreement Cost-Allocation Model (F.A.C.A.)for the project composed several sections. This proposed allocation model has a special function that can accommodate the participant's opinion at the project. In this paper, we analysis the elemental factor and try to Game theoretic analysis to understand the quality of the solution on this proposed cost-allocation model. And to illustrate the property of this model, We picked up the case study ,the access to the Hibiki New Port in Kitakyushu City ,that is expected as an international leading port by the Ministry of Transport. And we want to construct the Infrastructure by use of this model on the point of P.F.I.(Public Finance Infrastructure).

1. INTRODUCTION

Recently, from the financial readjustment, both the Central Government and Local Government reexamine the allocation system of Infrastructure. P.F.I. is one of the most expected systems that can change the present infrastructure construction system as follows.1) This system based on the efficiency and profitability for the financial aspect. And 2) It based on the Public Private Partnership System. Now in Japan, when the Local Government wants to construct the bridge as a infrastructure, usually Local Government receive the half of the cost from the Ministry of Construction. In this system, we do not have any assessment system estimated the profit caused the infrastructure, and also, we do not have any allocation system including the using of the private's vitality. The lack of the P.F.I. system in Japan causes the infrastructure proceeding to difficult. Many of the businessmen want to make an allocation system recognized by the society that is "equity" and "fair". So, we proposed the new allocation system named "The Formed Agreement Cost Allocation Model"(F.A.C.A.) for to solve the difficult problem "How to allocate the project cost that is organized by several sections." This proposed allocation model has a special function, that can accommodate the participation's opinion. It is perfectly peculiar function, until now we can not find such a function in common allocation system for example "Remaining Saving Method". The next section deal with the outline of proposed allocation model, and can understand the characteristics of the solution of this model by use of simple model. The third section devote to the utility of this model, picked up the Second Wakato Bridge Project located in Kitakyushu city in the Kyushu Island in Japan, and organize a evaluating committee and calculate the weight of factor and vector at the A.H.P. method, and try to calculate the distribution in each sections. The penultimate section is devoted to the Game-theoretic analysis on the utility of this model and can illustrate the stability of distribution on this allocation model. At first we pick up the non-cooperate game and analysis the process of the formed agreement system. Next we pick up the cooperate game and compare the solution gained by the other allocation model for example Shaplay value and the Nucleus defined at the Game-theory on the point of Nash's equilibrium or not, and trace the process of the equilibrium point. And the final one is summary of this study.

2. THE CHARACTERISTICS OF THE SOLUTION BASED ON THE FORMED AGREEMENT COST ALLOCATION MODEL

2.1 The outline of the formed Agreement Cost Allocation Model

This allocation model based on to the each section's possible investment cost that can calculate the minimum of the substitute construction cost and the appropriate construction cost. And the Formed Agreement Cost Allocation Model has a characteristic function that can accommodate the evaluated committee's estimation to the possible investment cost. Up to date, the customary allocation model based on only the economic index but this proposed allocation model can measure the section's factor in each section, for example the emergency, the necessity, and the utility and so on. Those factor can calculate by A.H.P. method. Practically, we can calculate the solution of this model as follows. And the outline of proposed allocation model are shown in Figure 1.

Step-Ⅰ. At first we calculate the A_i ,it is the alternative construction cost in each section that participate the project. The suffix i means the number of section.

Step-Ⅱ. Calculate the appropriate investment cost B_i ,it is in the estimation of the benefit and the profit caused by the rise of the land-value or benefit caused by the increase of the traffic charge. Each benefit reduced by the interest at 6%, so in this section we exchange the appropriate investment cost to the current value.

Step-Ⅲ. Calculate the possible investment cost E_i as $min\{A_i, B_i\}$, the A_i means the alternative construction cost and the B_i means the appropriate investment cost.

Step-Ⅳ. Calculate the f_i that is the ratio of the share weight ,and we call it the share weight vector. The f_i is a total index determined on account of the emergency and the importance and the economics aspects at the evaluated committee. We use the A.H.P. method when we measure the weight of the factor at the evaluated committee.

Step-Ⅴ. Calculate the $F_i(=F*f_i)$,that means the share of the individual section evaluated by the committee. In this case, F means the total cost.

Step-Ⅵ. Calculate the remaining savings as $G_i = \max\{E_i - F_i, 0\}$

Step-Ⅶ .Calculate the ratio of the remaining savings as $H = D/\sum_{i \in N} G_i$ .In this formula $D = \sum_{i \in N} E_i - F$

Step-Ⅷ. Calculate the common cost $I_i = \{(E_i - F_i)*H\}$. It means that I_i can calculate $E_i - F_i$ (remaining savings) deliver to the H (the ratio of the remaining saving).

Step-Ⅸ. X_i (the allocation cost) can calculate as follows $E_i - F_i$. In this formula E_i means the possible investment cost, and I_i means common cost.

Step-Ⅹ. K_i (the pure benefit) can calculate as follows. $K_i = E_i - X_i$

2.2 The characteristics of the formed Agreement Cost Allocation Model

At the F.A.C.A(Formed Agreement Cost Allocation Model), The solves can calculate as follows.

$$X_i = \begin{cases} \left(1 - \frac{D}{G}\right) * E_i + \frac{D}{G} * F_i & (if \quad E_i \ge F_i) \\ E_i & (if \quad E_i \ge F_i) \end{cases} \quad (1)$$

Above the formula, i means the section, N means the number of section, E_i means the possible investment cost, and F_i means the share cost measured by the committee.

$G = \sum_{i \in N} \max\{E_i - F_i, 0\}$, $D = \sum_{i \in N} E_i - F$. D/G (the ratio of the remaining saving)

The solution is defined at the point between E_i and F_i distributed by the D/G. So, the

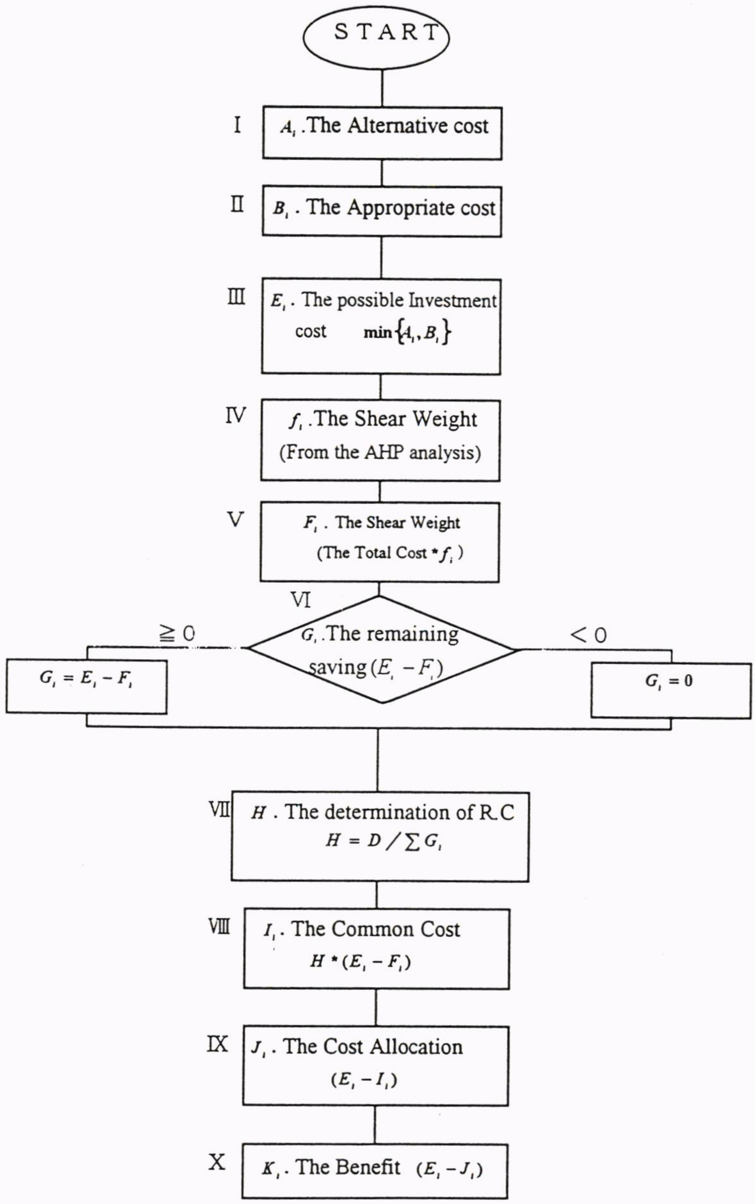

Figure 1. The Flowchart of the Formed Agreement Cost Allocation Model

allocation cost can calculate as formula (1). To understand the difference of the characteristic of the solution on the F.A.C.A, We set the three-persons cooperative game as Table 1.
Case1. All of the player's share-weights are within the individual rationality.
Case2. One player's share-weight is over than the individual rationality.
Case3. Two player's share-weight are out of the individual rationality.

Table 1. The Formula of the solution on the F.A.C.A. model

	x_1	x_2	x_3
Case 1 (X_1)	F_1	F_2	F_3
Case 2 (X_2)	$\left(1-D/G\right)*E_1+D/G*F_1$	$\left(1-D/G\right)*E_2+D/G*F_2$	E_3
Case 3 (X_3)	$F-E_2-E_3$	E_2	E_3

The characteristics of the solution are shown in Figure 2 .and we can understand as follows. At the case1 the solution points at share weight.(case 1. $E_i \le F_i, \forall\ i \in N$) At the case2, the solution slides to the line of the individual rationality, after then the solution is defined at the point between E_i and F_i distributed by the D/G (case 2). At the case3 the solution points the intersection of the individual rationality. (case 3)

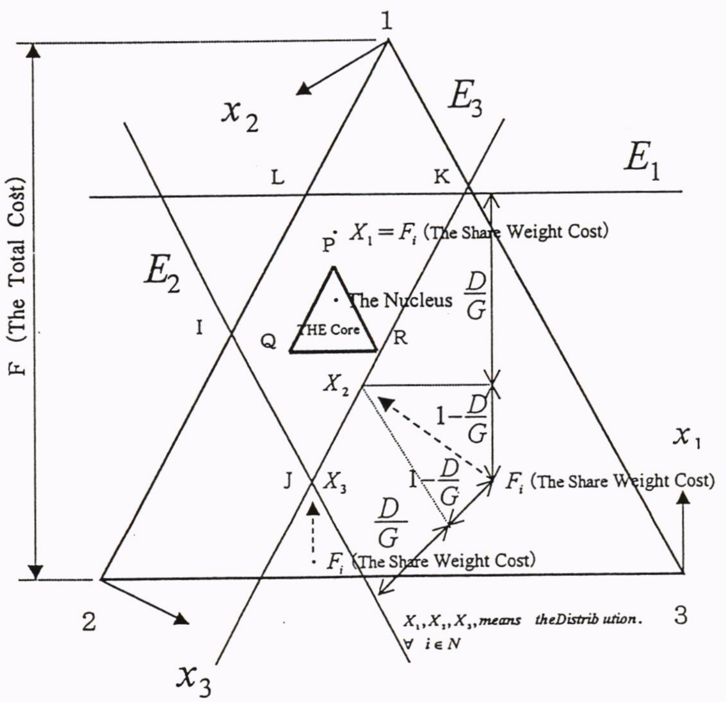

Figure 2. The Characteristics of the solution

2.3 The Game-theoretic Analysis of the distribution

If there is a much difference between F_i (the share weight) and E_i (the possible investment cost), and the evaluated committee do not want to sift the solution F_i to E_i .At the case, if we want to give a priority to the share weight, we can get the distribution on the condition as follows. At first, change the definition of the individual rationality to $v'(i) = max\{F_i, E_i\}$ and the group rationality to $v(s) = \alpha[v'(i) + v'(i+1)]$. α means the merit index that can get when formed the cooperation. We proposed the three distribution methods as follows and compare the each solution.

(A) M.C.R.S. method (Minimum Cost Remaining Saving method)

At first, we define the minimum cost $x_{i\,min}$ to $min\{E_i, F_i\}$, and the difference between total cost $v(N)$ and the sum of the minimum cost $\Sigma_{i \in N} x_{i\,min}$ define the remaining saving(RC). At this method, we distribute the (RC) in proportion to the difference the $x_{i\,max}$ and $x_{i\,min}$ to each participants as follows.

$X_i = x_{i\,min} + \beta_i * (RC)$

In this formula $\beta_i = (x_{i\,max} - x_{i\,min}) / \Sigma_{i \in N}(x_{i\,max} - x_{i\,min})$,RC= $v(N) - \Sigma_{i \in N} x_{i\,min}$, $x_{i\,max} = max\{F_i, E_i\}$, $x_{i\,min} = min\{F_i, E_i\}$

(B) The Solution based on the Nucleus

In this case ,at first we change the definition of individual rationality to $v'(i) = \max\{F_i, E_i\}$ and to $v(s) = \alpha[v'(i) + v'(i+1)]$, and can calculate the min max value as follows.

$X_i = \min_{x_i \in A} \max_{S} \{\Sigma_{i \in s} X_i - v'(s)\}$

Practically, we can get the solution by solve the LP(Linear Programming) as follows.

$min \;\; \mu, \; subject \;\; to \; x_i \le v'_{(i)} + \mu, \Sigma_{i \in s} x_i \le v'(s) + \mu, \quad \Sigma_{i \in N} x_i = v(N)$

Each condition means the individual rationality, group rationality, and total rationality.

(c) The Solution based on the Nash's Model of bargaining Problem

We can get the solution based on the Nash's Model of bargaining Problem as follows.

$X_i = \max_{x_i \in A^*} \{ \prod_{i \in N} (v'_{(i)} - x_i) : \;\; s.t. \;\; x_i \le v'_{(i)} \}$

In this formula, the solutions are defined the maximum of the product the difference of $v'(i)$ and solution x_i. Table2 shows the solution in each method when F_i (the share weight) has a priority to E_i (the possible investment cost) and on this table we add the shapley value used in the Game's theory calculated as follows. $X_i = \Sigma_{s \subset N} r(s)[v(s) - v(s-i)]$ $r(s) = \frac{1}{n!}(s-1)!*(n-s)!$.

The conditions are as follows. $E_1 = 80, E_2 = 60, E_3 = 30, F_1 = 11.0 F_2 = 11.0 F_3 = 88.0, \alpha = 0.6$

$\Sigma_{i \in N} F_i = 110.0, v(1,2) = 84.0 \;\; v(1,3) = 100.8 \;\; v(2,3) = 88.8 \;\; v(N) = 110.0$

The Figure3 shows the solution of table2.

Table 2. The solution in each method

The Methods of the solutions	x_1	x_2	x_3
Shapley Value	39.8	23.9	46.3
The F.A.C.A. method	44.9	35.1	30..0
(A)M.C.R.S Method	33.8	27.1	49.1
(B)The Nucleus	35.3	34.6	40.1
(C)The Nash bargaining problem	40.7	20.7	48.6
The individual rationarity $\max\{E_i, F_i\}$	80	60	88

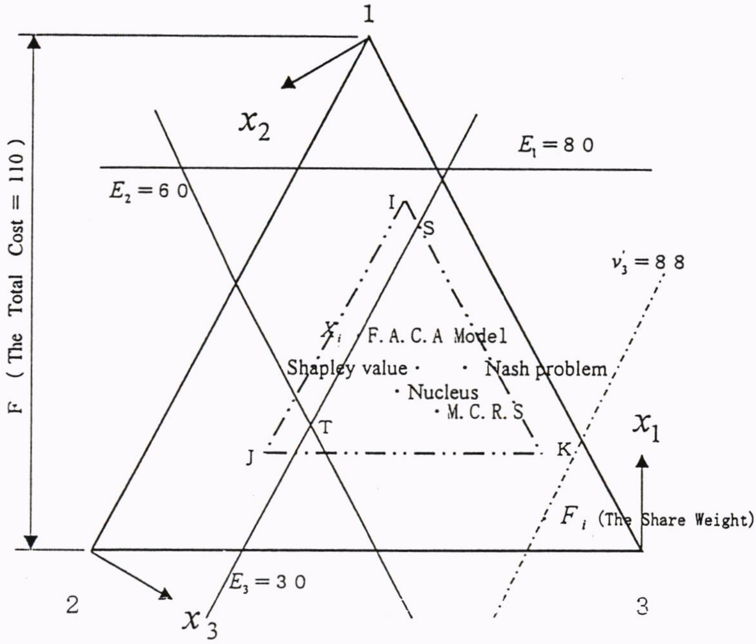

Figure 3. The solution in each method

2.4 The consideration of solutions

From the Table 2 and the Figure 3, we can conclude as follows. In the case of Figure 3, when the set surrounded by the E_i is compact and the difference of E_i and F_i is accurate, M.C.R.S., Nash's model, and Nucleus situate near the F_i (the share weight). It means that the order of the solutions of these methods are influenced by F_i .But we can not conclude this characteristics in each model are general, because these characteristics will change in proportion to E_i and F_i .From the comparison of the solution by F.A.C.A and the Shapley value, F.A.C.A, :Shapley value = (44.9,35.1,30.0) : (39.8,23.9,46.3), we can understand the characteristics of the solutions. At the Figure3, only F.A.C.A. has a direct connection between X_i and F_i . And $F_3 DOM_{\{1,2\}} x_3$ is formed, that means x_3 dominated by F_3 through the cooperate $\{1,2\}$ (the external controlling). If x_3 assert to reduce from E_3 ,the committee(F_3) criticize x_3 to hold-out gains. And X_i situated on the line of E_3 is defined as follows.

$X_i = \{x_1, x_2, E_3); \ x_1 + x_2 = F - E_3\}$ (The external stability) In this formula, if we can set $x_i = c \ (c \le E_i : \ i = 1,2)$ we can define the unique solution. At the F.A.C.A., this difficult problem is solved as to divide the differences of E_i and F_i in proportion to the ratio of the remaining saving. The solves of (x_1 and x_2) based on the E_3 , there is a cooperative characteristic, so the distribution solved by the F.A.C.A. has internal stability.

3.THE POSITIVE STUDY ON THE SECOND WAKATO BRIDGE

3.1 The outline of the Project

In December 1988, Kitakyushu city made a "Vision of Renaissance" that aimed at the city-image as "International technology city targeted the Asian countries" and this vision indicate the direction of the infrastructure for the 21 century. In this chapter, we picked up "The Second

Wakato Connecting Bridge" that connect the Hibiki new development area aim at the 2005. We already calculated the benefit and try to solve the allocation problem using the current data as follows. a) Make the traffic estimation model. b) Estimate the index of land-function .c) Calculate the benefit changing the traffic caused by the connecting bridge. d) Organize the evaluate committee .e) Define the distribution of each section. And we examine the property of the solution based on the F.A.C.A. The status of the project are shown in Figure 4.

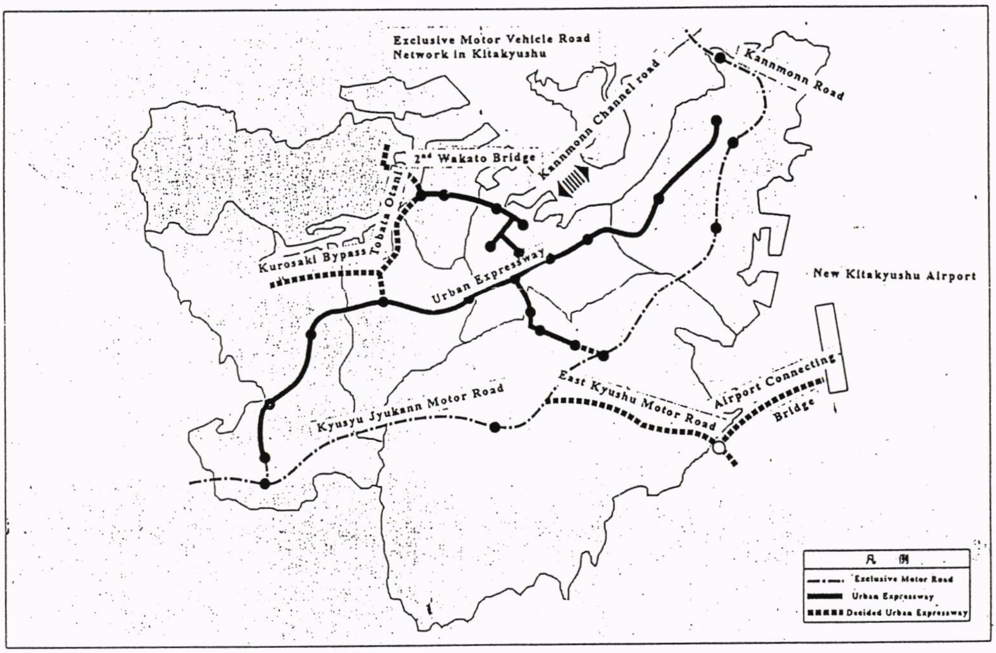

Figure 4. The situation of the project

3.2 The estimation of the share weight using the A.H.P. model

At the F.A.C.A., the share weight vector using the A.H.P. method at the committee has same weight as E_i (the possible investment cost). And the A.H.P. method is useful in evaluation of the emergency or the utility that is difficult to measure the benefit caused by the project. But the vector (by A.H.P.) was affected by the social condition at the time of the inquiries. Figure 5 shows the flow-chart of the survey from the organization of the evaluated committee. The committee was organized at the December in 1995, it was consisted of 10 members, Administrative members (3), the consultants (4), the real-estate judge (2), etc. and made the inquiries 3 times from June in 1996 to February in 1998. Figure 6 shows the hierarchy of the inquiry survey. We can understand from Figure6 as follows, in this case we picked up the 5 factor (ex. the emergency, the necessity, the economy, the finance, and the utility) that defined the priority of the allocation. And select the 4 sections (The Central Government, The Local Government, The Hibikinada Development Corporation, and The highway public corporation)

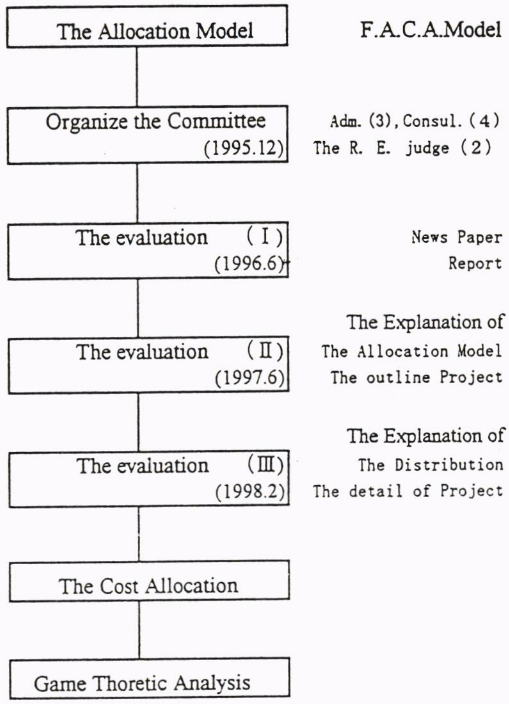

Figure 5. The flow -chart of the survey

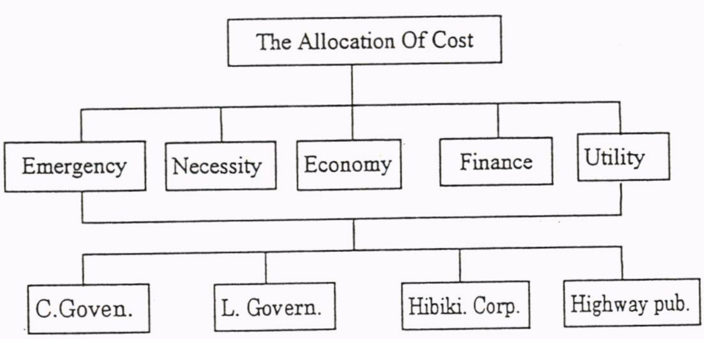

Figure 6. The hierarchy of the inquiry survey

3.3 The consideration of the factor and the share weight vector

Figure 7 shows the result of the change of the factor (the emergency, the necessity, the economy, the finance, and the utility) calculated by the inquiries at three times. We can understand in Figure7 as follows. The most changeable factor is "the emergency" (18.5%→20.4%→12.0%) ,secondary changeable factor is "the finance"(17.4%→12.5%→20.3%), the other three factor show stable score(the average of the change of the evaluated factor:3.8%).Next, we study the social condition for to examine the variant existed in the outcome of the inquiries. At the first inquiry (1996.6), the Cabinet authorized "The 9th National Port Construction Planning". From this report, we recognized the strengthen of the international competition, the necessity of the trade network system, and the rise the reliability. At the second inquiries(1997.6), the news that the Kobe International Port and the other ports in Japan are in drop down at the competition, so the necessity and emergency get the high evaluation. At the third inquiry (1998.2), there are financial readjustment of the Central Government and the falling down of the operation of the public cooperation. So the Economic and the Finance factor get the high score. On the other hand the Utility(21.4%→17.3%→21.2%) get a stable score through the inquiries.

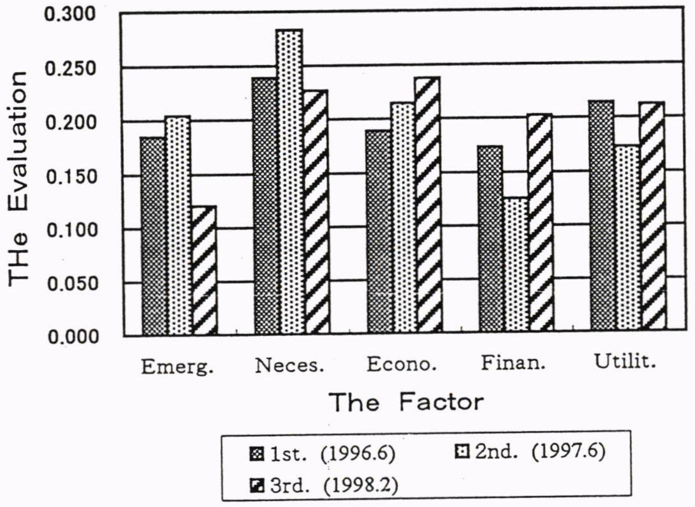

Figure 7. The evaluation of the factor

Figure 8 shows the weight vector in each section. We can understand the differences of the weight vector are small for the Local Government(29.1%~32.3%), the Central government(24.7% ~28.1%),the High way public corporation(20.4%~21.5%),The Hibikinada Development corporation(18.1%~25.9%).This is a reason that the section's evaluations based on the each factor are generally equal in each inquiry. The Hibikinada Development corporation(23.1%→25.9%→18.1%) that expected the development benefit has most difference evaluation in the sections. The other three sections get the low score at the second inquiries than that of the first inquiry, but at the third they get the high evaluation than that of the first. At the second inquiry, the Connecting Bridge Investigate budget are recognized by the Ministry of Transport, the committee recognized the development benefit as realistic. At the third inquiry, according to the reduction of the land value, and the falling down of the operation reduce the Hibikinada Development corporation's score. Through the all inquiries, The Local Government get a high evaluation. It was reflected the decentralization. And the Highway Public Corporation get the stable value, this is a reason that the member of the committee can understand the benefit caused by the increase of the tall fee easily.

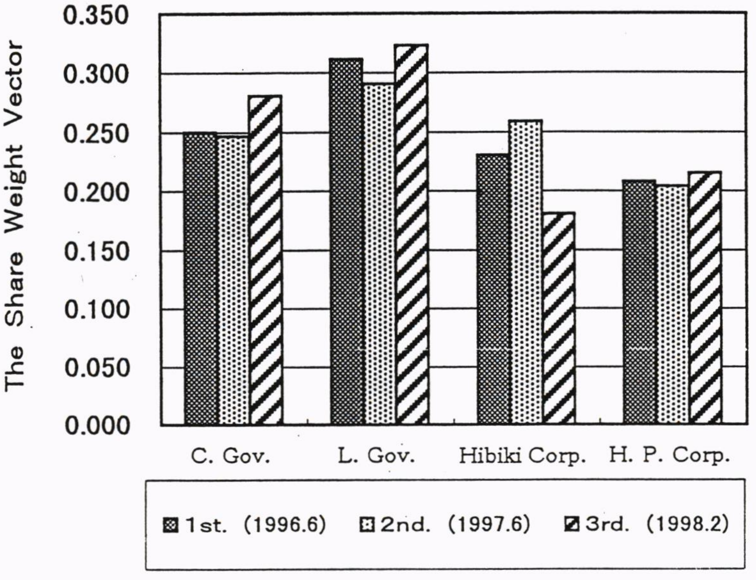

Figure 8. The change of the weight vector in each section

3.4 The Calculation for the Actual Model

In this section, we calculate the allocation correspond the change of the share weight. The condition and the result of the calculation are shown in Table 3. Now we assume that the Public(The Central and the Local Government) possible investigation cost can calculate as follows. (The total cost－The possible investment cost of The Highway Public Corporation) From the Table-3, we can understand as follows. The distribution of the Shapley value is 502.6(the ratio of the total cost is 71.8%) and the case of the Nucleus is 511.2(73.8%). In the case of the F.A.C.A. the distribution change from 409.6 to 453.5(64.6%). The distribution of the Hibiki Development Corporation is 128.9(18.4%) in the Shapley value, 137.5(19.6%)in the Nucleus, 143.7(20.5%)～ 171.5(26.5%) in the F.A.C.A. On the other hand, the Highway public corporation is 68.5(9.8%) by the Shapley value,51.3(7.3%) by the Nucleus, but equal in the F.A.C.A. This is the reason that the share weights (F_3) calculated from the weight vectors are over the possible investment cost, so the distribution is defined to the (E_3).

The share weights that evaluated by the committee is over the possible investment cost are shown in Figure 9, this is the same case picked up the 2.3. At the second inquiry, the solve leave from the nucleus and the Shapley value. After then at the third inquiry, the solve approach to the nucleus and the Shapley value than that of the first. Through the all inquiries, the evaluations of the committee indicate for the High way public corporation (x_3) over the possible investigation. But if the distribution for the High way public corporation (x_3) over the E_3, it means that the solve is out of the individuality, so the High way public corporation will assert to reduce the distribution to (E_3). So, we can understand the property of the distribution on the E_3 line based on the F.A.C.A. model.

Table 3 The condition and the result of the calculation

The design condition	$E_1 = 597.2$ $F_1 = 422.8$ $v(1,2) = 700$	$E_2 = 223.5$ $F_2 = 126.7$ $v(1,3) = 700$	$E_3 = 102.8$ $F_3 = 150.5$ $v(2,3) = 326.3$
Player	Public x_1	Third sector x_2	Highway corp. x_3
F.A.C.A. 1st	425.7	171.5	102.8
F.A.C.A. 2nd	409.6	187.6	102.8
F.A.C.A. 3rd	453.5	143.7	102.8
Shapley value	502.6	128.9	68.5
Nucleus	511.2	137.5	51.3
S.C.R.B. method	495.8	139.9	64.3

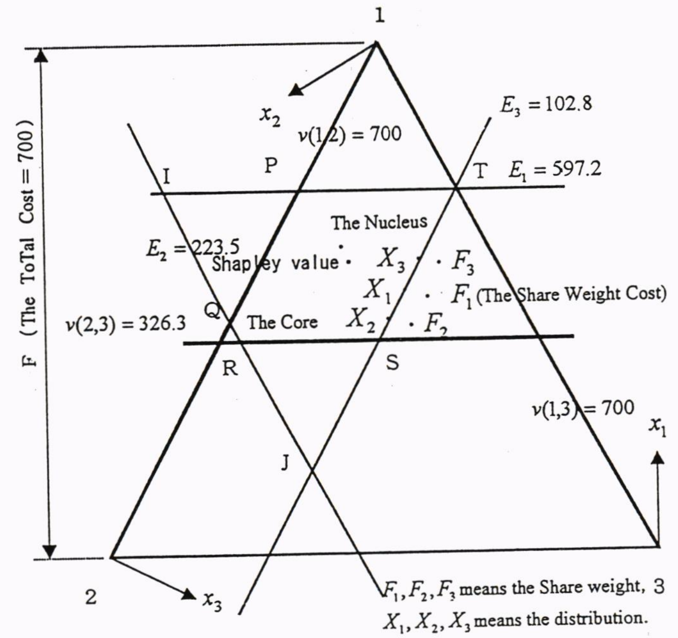

Figure 9. The result of the positive study

4.THE GAME-THEORETIC ANALYSIS ON THE F.A.C.A. MODEL

4.1 The study of the characteristics of the F.A.C.A. model

At the 2.2 we find the solution of the F.A.C.A.model can define in three cases(case1~case3). In this section, we study the stability on the point of the Game-theoretic analysis. At first, we calculate the pay-off table in each case. And we examine the stability of the solution on the point of the Nash's equilibrium or not. The result of the stability are shown in the table-4.In this table (1,1) field means that each section attend the project. And the (0,0) field means any section don't attend the project. From table-4 we can understand as follows. On the F.A.C.A. model, the (1,1) field get to the Nash's equilibrium, so we can understand the solution is stable. On the other hand at the case1 and case2 the (0,0) field does not get to the Nash's equilibrium, so the solutions of (0,0) at the case1 and case2 are instability. The (0,0) field means that the both sections do not want to attend the project. On the Shapley value both field of the solutions are instability, next on the Nucleus (1,1) field is stable and the (0,0) is instability. So we can conclude that these F.A.C.A. model is suite to the proceeding the project.

Table 4. The stability of F. A. C. A. model, Nucleus and Shapley value

	(1, 1)	(0, 0)
Case1 $F_i \le E_i (\forall i = N)$	○	×
Case2 $F_i \ge E_i$ $(i = 3)$	○	×
Case3 $F_i \ge E_i$ $(i = 2,3)$	○	○
Shapley value	×	×
Nucleus	○	×

注 (○ means the Nash's equilibrium.
× do not mean Nash's equili.)

The expectation of the player1(public sector) and player2 (private sector) can calculate as follows.

$$E_1(p,q) = \Sigma_{i\in 2}\ \Sigma_{j\in 2} p_i q_j B_1(p_i, q_j) \qquad (2)$$

$$E_2(p,q) = \Sigma_{i\in 2}\ \Sigma_{j\in 2} p_i q_j B_2(p_i, q_j) \qquad (3)$$

The Nash's equilibrium can calculate as follows.

$$E_1(p^*,q^*) = \max_p E_1(p,q^*) \qquad (4)$$

$$E_2(p^*,q^*) = \max_q E_2(p^*,q) \qquad (5)$$

The Nash's equilibrium of the mixed strategy can calculate as follows.

$$\frac{\partial E_1(p,q)}{\partial p} = 0 \qquad (6)$$

$$\frac{\partial E_2(p,q)}{\partial q} = 0 \qquad (7)$$

4.2 The Analysis of the expectation on the F.A.C.A. model

Figure10 shows the hierarchy of the co-operation at the Connecting Bridge Project in Kitakyushu-City in Japan. In Figure-10 E means the profit and V means the cost, suffix 1 means co-operative and suffix 2 means non-cooperative. The pay-off in the Connecting Bridge Project shows in table5. Above the formula we can conclude as follows $(p_1^*, q_1^*) = (0,0), (1,1)$, and the equilibrium of the mixed strategy on the F.A.C.A model is $(p_1^*, q_1^*) = (0.82, 0.42)$ and the equilibrium on the Shapley value is $(p_1^*, q_1^*) = (0.73, 0.52)$. The result of these equilibrium are shown in Figure11.

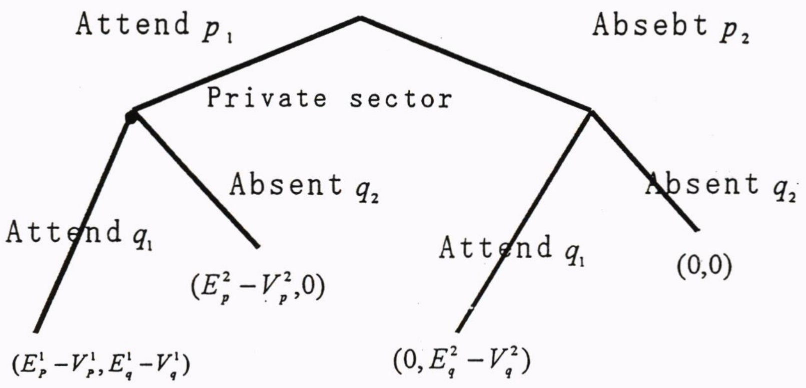

Figure10. The hierarchy of the co-operation at the Connecting Bridge Project

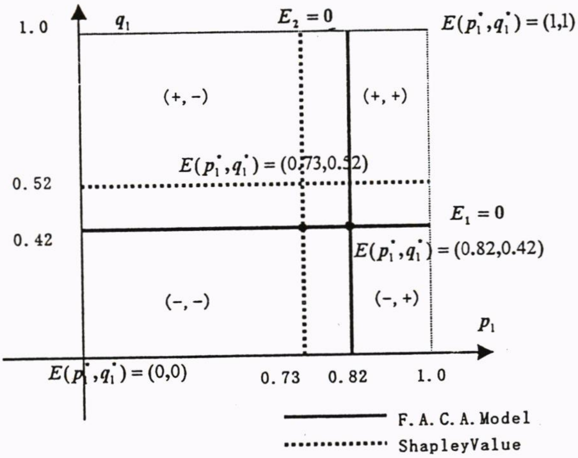

Figure11. The Equilibrium Process of The F.A.C.A model

Table5. The pay-off at the Connecting Bridge Project

Public sector (Central and Local government)	Private (The third Party and the highway corporation)	
	Attend q_j	Absent q_{j+1}
Attend p_i	143.7, 79.8 (94.6, 128.9)	-102.8, 0 (-102.8, 0)
Absent p_{i+1}	0, -373.7 (0, -373.7)	0, 0 (0, 0)

p_i means provability of the public sector. q_i means the provability of the private sector.

The above figure means F.A.C.A model and the below figure means Shapley value.

5.SUMMURY

In this study, we can conclude as follows.
(1) F.A.C.A. model has special function to the common allocation methods, that can evaluate not only the economy but also the emergency, the necessity, the utility, and the finance.
(2) The distribution based on the F.A.C.A. model always point the set surrounded the individual rationality, and can get the solution if the core exist or not.
(3) The evaluation of the factor changed according to the social condition, but the share weight vector was stabled through the inquiry.
(4) The Nucleus based on the Game's theory can calculate under the condition of the core, so there are much difficult for to use reality. And we can understand the solution based on the Shapley value does not have stability.
(5) The solution based on the F.A.C.A. model points the set surrounding the individual rationarity and it has a stability as Nash's equilibrium. So the F.A.C.A. model has a special utility under the condition that one player is anxious to proceed the project.

REFERENCES

a) Hideki Fujimura, Shoshi Mizokami, Ryuji Kakimoto, : A Study of Formed Agreement Cost Allocation Model, **Journal of the civil plannning**, No. 14, pp35-42, 1997.9

b) James P.Heaney and Robert E.Dickinson,Method for Apportioning the Cost of a Water Resource Project. **Water Resources Research**, Vol.18, No3, pp476-482,1982.6

c)Mituo Suzuki,Sigeo Muto,the theory of the cooperative game theory, **UP The Applied Mathmatical Selection** ,pp43-47