

## A SPATIAL ANALYSIS OF EXTERNAL ECONOMIES AND ITS ASSOCIATION WITH TRANSPORTATION INFRASTRUCTURE: THE CASE OF SENDAI CITY

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**Abstract:** The existence of external economies in land markets is an well-acknowledged fact. Empirical studies, especially in a spatial context, show that spillover effects act as determinants of land prices. The purpose of this study is to apply the technique of Geographically Weighted Regression (GWR), used to obtain spatially localized estimates of model parameters, to a spatial econometric model of land prices. The objective is to compare the spatial distribution of spillovers and other variable parameters to that of public transportation infrastructure provision, in particular rail systems. A case study is conducted with the city of Sendai in Japan. It is shown in the case study that considerable parameter variation over space exists, and that spatial spillover effects are, in average, more favorable where transportation infrastructure exists. Parameter distribution for other variables also reveals some interesting aspects relating to the structure of the city.

### 1. INTRODUCTION.

It is well known that the concentration of activities in urban spaces gives rise to a number of external economies. In particular, urban development and transportation activities are characterized for producing externalities. Negative externalities include various emissions from land use activities and congestion. But there are also beneficial effects, as could be the increase in land prices that transportation projects presumably entail, especially when well coordinated with land development.

Spillover effects in the land market have been previously shown to exist in empirical analyses of land prices, usually based in regression techniques that estimate the price effects of different variables. More recent studies measure the magnitude of spatial externalities by means of an association term that accounts for spatial external effects. However, a limitation to this framework is that the spatial term, and the model parameters in general, are obtained at a regional scale. To the present, this has prevented the advancement of geographically detailed studies of external effects to compare with the local configuration of transportation services.

The purpose of the present study is to apply a set of spatial analysis techniques to examine, among other variables, the spatial external economies in land markets. This will be done at a geographically detailed level to be able to assess the relation of external effects to transportation infrastructure provision. In particular, our objective is to reveal the spatial variation of external economies affecting land prices, something that can be accomplished applying the Geographically Weighted Regression method (Brundson et al., 1996; Fotheringham et al., 1998) to a spatially autoregressive model. The GWR method shifts the

emphasis from global estimation of parameters to a local scale, while a spatially autoregressive model gives the potential of analyzing spillover effects. Combination of these analytical techniques is expected to produce local, as the opposite of regional estimates of the parameters, in particular the spatial association parameter, which can then be inspected for the search of spatial patterns.

Although it is commonly acknowledged that spatial externalities exist, by comparing the local variation of the external effect with the spatial pattern of transportation infrastructure provision, it can not only be shown whether a link exists between them, but also its strength can be assessed. Presumably, the areas that receive the greatest benefits from external effects would be among the most attractive locations for development. If such locations can be shown to coincide with transportation facilities, an argument for coordinated transportation and land use development and a subsequent internalization of effects would arise, since value capture of the projects would be easier to achieve.

The structure of the paper is as follows. In section 2, a general formulation of spatial models is described and the interpretation of the spatial association effects is given. Section 3 describes the GWR method and a general formulation with spatial components of the model is derived, as the basis of the model finally used for the case study, and section 4 covers a technical aspect of GWR estimation. Application of the above techniques is exemplified in sections 5 and 6 with the case of Sendai City, a major city in northeastern Japan, for which it is shown that considerable spatial variation of the parameters exists. Attention is set on the rail systems of public transportation (train and subway) and the distribution of spatial externalities, but the results for the rest of the parameters reveal some interesting aspects relating to the strongly monocentric structure of urban land prices.

## 2. SPATIAL ECONOMETRIC MODELS OF URBAN LAND PRICES.

A common practice in analyzing land prices is to estimate land price functions, in what is known as the hedonic approach method. In this well-known approach, the price of land (or more generally of real estate) is considered to be a function of a number of variables, which in every case include some measure of accessibility (for instance, to city center: see *inter alia* Asabere and Huffman, 1991; Hansen and Kristensen, 1991), and in some cases a measure of proximity or accessibility to transportation infrastructure (Hoch and Wadell, 1993; Tsutsumi et al, 1998). The interest in obtaining a functional form for urban land prices through a statistical or econometric methodology, is to assess the influence and significance that individual variables (distance to city center, land uses, distance to train stations, etc.) have in determining the price of land. In transportation studies, the obvious interest lies in searching for the effects that transportation infrastructure might have in this process.

There is in addition an interesting aspect of econometric models of real estate prices, intimately related to the fact that they usually involve working with spatial data. A well-known characteristic of spatial data is that it often shows a form of serial arrangement in space that invalidates many standard econometrics methods (Anselin, 1988), a results that in turn forces the analyst to resort to specialized methods of spatial analysis. Although such methods are often more complex than the simple OLS regression approach, they hold a legitimate value. In the particular context of analyzing the prices of real estate, they allow for the spatial effects present in the process to be controlled under the form of models with spatial error autocorrelation, which is a way of spatially explaining what is not an explicit part of the model. Another option, and one of particular interest for the purpose of our

study, is to model those effects as spatial spillovers, in fact a spatial form of external economies. Examples of both kind of approaches appear in the works of Can (1992), Dubin (1992) and Chica-Olmo (1995).

A general specification for a spatial econometric model is given by the following equations (Anselin, 1988) that cover a spatially autoregressive scheme and spatial error autocorrelation, by means of spatial interactions matrix  $W$  and parameters  $\rho$  and  $\lambda$ :

$$Y = \rho WY + X\beta + \varepsilon \quad (1)$$

$$\varepsilon = \lambda W\varepsilon + \mu \quad (2)$$

Here,  $Y$  is a vector ( $n \times 1$ ) of observations,  $X$  a matrix ( $n \times k$ ) of explanatory variables,  $\beta$  a vector ( $k \times 1$ ) of  $k$  estimation parameters that include the usual constant, and  $\varepsilon$  and  $\mu$  are vectors ( $n \times 1$ ) of stochastic error terms. In this kind of models, the spatial structure of regional interactions is determined by the spatial interactions matrix  $W$ , which is a  $n \times n$  square matrix (corresponding to a sample of  $n$  locations-observations) that will have non-zero entries for interacting location pairs, and 0's in the diagonal and elsewhere. In practice, and for ease of interpretation, matrix  $W$  is usually row-standardized, which means that every entry is divided by its row's total sum so that each row is adjusted to add up to 1:

$$w_{ij}^{st} = w_{ij} / \sum_j w_{ij} \quad (3)$$

$$\sum_j w_{ij}^{st} = 1 \quad (4)$$

with  $w_{ij}^{st}$  as the row  $i$  and column  $j$  value of the row-standardized matrix. Setting  $\lambda = 0$  in the above regression specification, that is assuming no spatial autocorrelation in the error terms, the spatially autoregressive (SAR) form of the model is obtained. The remaining element of the spatial econometric specification is parameter  $\rho$  (the spatial association term) in equation (1). This parameter measures the intensity of spatial interactions and is interpreted in the present context as the magnitude of spatial spillovers. Since the objective variable  $Y$  is the price of land, when a row-standardized interactions matrix is used, the term  $\rho WY$  will represent the proportion of the average price at neighboring locations that will be reflected in the price of a given plot of land, with 'neighborhood' being defined by the structure of  $W$ . It is clear that the spatial association term has a legitimate interpretation, and constitutes, under the definition of externalities, a spatial external economy.

### 3. SPATIAL MODELS AND THE GWR METHOD.

The model specified by equations (1) and (2) represents a clear improvement over conventional regression analysis, not only for being a formally correct way of analyzing spatial data, but also because it permits the explicit modeling of *spatial interactions*. However, in spite of these advantages, it produces limited results in terms of analyzing *spatial variation*. Once such a model has been estimated, the parameters  $\beta$  and  $\rho$  obtained are fixed over space (only one set of parameters is obtained and it is assumed to hold for all the region) and give no hint at possible variations in localized sectors of the study region. This sort of models are termed global models, because their parameters represent averages over the whole area, a characteristic that in many cases may hide interesting sub-regional (local) variations. Application of the Geographically Weighted Regression method, on the other hand, appears as a tool to measure spatial variations which would otherwise remain hidden. Its application has helped to show before that, at times, the parameter stationarity assumption of the global model will not uncover information important to a better understanding of the problem (Fotheringham et al., 1998).

Thinking about transportation planning, a global model of land prices constitutes a useful tool to study the general structure of the city. However, it is likely that a GWR model will give deeper insight into the characteristics and behavior of land price controls at different points of the city. Since the objective here is not only to measure the magnitude of external effects, but to study their spatial relation with the provision of transportation infrastructure, the application of a spatial model under a scheme of geographical weights to obtain localized parameter estimators seems more appropriate. In this way, it should be possible to compare spatial variation of the parameters (variation that can be displayed in the form of maps) to the spatial configuration of public transportation service.

The concept of applying geographical weights, although simple, yields a powerful tool to investigate local functional relationships. It consists basically in the introduction of a set of values, that will assign larger weights to observations more relevant to a particular estimation, and smaller weights to less important observations. The net effect is that less relevant observations are penalized and will accordingly have less influence in the outcome of the estimation. The key feature of the GWR method is that the weights are a function of location in space, and more specifically of distance from a given location in space. In accordance with the geographic principle that close locations are more related than distant locations, larger weights will be assigned to observations in close proximity, with weights decreasing at larger distances, until the effect of the most distant observations becomes negligible.

Now, a way of introducing a weighting scheme in the spatial econometric specification presented before is as follows (this particular case can be found in Brundson et al, 1998):

$$Y = \rho_i WY + X\beta_i + \varepsilon \quad (5)$$

$$\varepsilon = D_i \mu \quad (6)$$

where  $D_i$  is a diagonal matrix ( $n \times n$ ) specific to location  $i$ , that will become the instrument of the model's geographical weighting scheme. Please notice that once a geographical weighting instrument is introduced, any number of estimations can be carried out just by varying the values of the geographical weights, something that may be done changing the location. The value of the weights assigned to observations will depend on their distance from  $i$ , and further detail should not be needed in order to notice that parameters  $\rho_i$  and  $\beta_i$  will no longer be 'global', but will correspond to estimation for a specific set of weights  $D_i$  centered at location  $i$ . For this reason, the parameters of the GWR model are distinguished from those in the global model by the addition of the locational subindex  $i$ .

To derive the likelihood function of the model, on which estimation is based, it is assumed for the above specification that the vector of random errors is normally distributed with a covariance structure matrix  $\Omega$ :

$$\mu \sim N(0, \Omega)$$

with the diagonal elements of  $\Omega$  taking positive and non-zero values.

Now, since the error covariance matrix  $E[\mu\mu'] = \Omega$  is diagonal, a well-behaved vector of homoskedastic errors, say  $v$ , exists:

$$v = \Omega^{-1/2} \mu \quad (7)$$

that may be alternatively be presented as:

$$\mu = \Omega^{1/2} v \quad (8)$$

Defining  $A_i = I - \rho_i W$ , the following simplification results with equation (5) now as:

$$A_i Y = X\beta_i + \varepsilon \quad (9)$$

Substituting equation (8) into (6) results in:

$$\varepsilon = D_i \Omega^{1/2} v \quad (10)$$

which in turn can be introduced into (9) to yield:

$$A_i Y - X\beta_i = D_i \Omega^{1/2} v \quad \text{or} \quad v = \Omega^{-1/2} K_i (A_i Y - X\beta_i) \quad (11)$$

where  $K_i D_i = I$ .

The error terms  $v$  in equation (8) above, although homoskedastic and well-behaved, can not be observed. Since the likelihood function has to be based on observations vector  $Y$ , they have to be transformed through the use of the Jacobian function (see Anselin, 1988):

$$J = \det\left(\frac{\delta v}{\delta Y}\right) = \det\left[\frac{\delta}{\delta Y}\left\{\Omega^{-1/2} K_i (A_i Y - X\beta_i)\right\}\right] = \left|\Omega^{-1/2}\right| |K_i| |A_i| \quad (12)$$

The log-likelihood for this model, assuming a multivariate normal distribution for error terms  $v$ , can then be obtained as:

$$L = -\frac{n}{2} \ln \pi - \frac{1}{2} \ln |\Omega| + \ln |K_i| + \ln |A_i| - \frac{1}{2} v' v \quad (13)$$

with  $v'v$  as the sum of squares of the appropriately transformed regression residuals.

For the purpose of estimation, the first order conditions are obtained taking the partial derivatives of the log-likelihood function with respect to the parameters. Before that, it is further assumed that the covariance matrix can be expressed in terms of a reduced number of parameters, and in the simplest case of homogeneity will depend of a single parameter corresponding to the variance term:

$$E[\mu\mu'] = \Omega = \sigma^2 I \quad (14)$$

The parameters of the model turn out to be  $\rho$ ,  $\beta$  and  $\sigma^2$ . Taking derivatives, the following first order conditions result:

$$\begin{aligned} \frac{\delta L}{\delta \beta_i} &= \frac{\delta}{\delta \beta_i} \left( -\frac{n}{2} \ln \pi - \frac{1}{2} \ln |\Omega| + \ln |K_i| + \ln |A_i| - \frac{1}{2} v' v \right) = -v' \frac{\delta v}{\delta \beta_i} \\ &= -v' \frac{\delta}{\delta \beta_i} \left[ \Omega^{-1/2} K_i (A_i Y - X\beta_i) \right] = v' \Omega^{-1/2} K_i X \end{aligned} \quad (15)$$

and:

$$\begin{aligned} \frac{\delta L}{\delta \sigma^2} &= \frac{\delta}{\delta \sigma^2} \left( -\frac{n}{2} \ln \pi - \frac{1}{2} \ln |\Omega| + \ln |K_i| + \ln |A_i| - \frac{1}{2} v' v \right) \\ &= -\frac{1}{2} \frac{\delta}{\delta \sigma^2} \ln |\Omega| - \frac{1}{2} (A_i Y - X\beta_i)' K_i' \frac{\delta}{\delta \sigma^2} \Omega^{-1} K_i (A_i Y - X\beta_i) \\ &= -\frac{1}{2} \text{tr}(\Omega^{-1}) + \frac{1}{2} v' \Omega^{-3/2} K_i (A_i Y - X\beta_i) = -\frac{n}{2\sigma^2} + \frac{1}{2} v' \Omega^{-3/2} K_i (A_i Y - X\beta_i) \end{aligned} \quad (16)$$

The log-likelihood function will attain its maximum value when the partial derivatives are

equal to 0, which is the condition used to derive the ML estimators of the parameters. For the case of parameter vector  $\beta_i$ , the ML estimators are obtained from equation (15) as

$$\beta_i = (X'K_i'K_iX)^{-1}X'K_i'K_iA_iY \quad (17)$$

while the variance can be obtained from equation (16) as:

$$\sigma^2 = \frac{1}{n}(A_iY - X\beta_i)'K_i'K_i(A_iY - X\beta_i) \quad (18)$$

Substituting equations (17) and (18) in the log-likelihood given by (13), and doing some simplification, it can be shown that a concentrated log-likelihood of the following form results:

$$L_c = C - \frac{n}{2} \ln \left[ \frac{1}{n} (e_o - \rho_i e_L)' K_i' K_i (e_o - \rho_i e_L) \right] + \ln |A_i| \quad (19)$$

with

$$e_o = Y - Xb_o \quad (20)$$

$$e_L = WY - Xb_L \quad (21)$$

and

$$b_o = (X'K_i'K_iX)^{-1}X'K_i'K_iY \quad (22)$$

$$b_L = (X'K_i'K_iX)^{-1}X'K_i'K_iWY \quad (23)$$

It is clear that  $e_o$  and  $e_L$  above represent the error terms ensuing from the regression of  $X$  on  $Y$  and  $WY$  respectively, and are nothing else than ordinary and lagged errors. Its introduction in the concentrated log-likelihood function allows it to be expressed as a function of only one parameter ( $\rho$ ), that can be optimized using numerical techniques. Further noting that the determinant of  $A_i$  can be expressed as (Ord, 1975):

$$|A_i| = |I - \rho_i W| = \prod_n (1 - \rho_i \omega_n) \quad (24)$$

with  $\omega_n$  as the  $n$ th eigenvalue of  $W$ , it is clear that the maximum likelihood will be attained for the value of parameter  $\rho_i$  that maximizes the following expression (compare with Brundson et al., 1998):

$$-\frac{n}{2} \ln \left[ \frac{1}{n} (e_o - \rho_i e_L)' K_i' K_i (e_o - \rho_i e_L) \right] + \sum_n \ln (1 - \rho_i \omega_n) \quad (25)$$

Once  $\rho_i$  has been obtained, the parameter estimators  $\beta_i$  and the variance can be calculated introducing its value in (17) and (18), as the rest of the elements there are known.

#### 4. GEOGRAPHICAL WEIGHTS AND CALIBRATION OF THE KERNEL FUNCTION.

In the previous section, the geographical weighting method was introduced and adapted for the case of a spatial model specification. However, little was said about the weights themselves ( $K_i$  in eq. 11), except to note that they depend on location and should conform to the geographical principle of strong interaction with proximity. The problem is now to select a method or function to obtain quantitative values for the weights. Although a very simple solution would be to assign a weight of 1 to observations within a distance  $d$  of location  $i$  and 0 to those beyond this critical distance (the underlying assumption in global

models is unitary weights for all observations), this problem is best solved in GWR by means of a continuous distance-decay or kernel function. Recalling equation (11):

$$v = \Omega^{-1/2} K_i (A_i Y - X\beta_i)$$

it can be seen that the purpose of weights  $k_{ij}$  in the diagonal matrix  $K_i$  is to adjust the error corresponding to observation  $j$ , according to distance  $d_{ij}$  from location  $i$  for which parameters are being estimated.

A choice of kernel function is a Gaussian curve, that does not present the discontinuity problem of a binary weighting scheme, and has the desirable property of having a maximum value of 1 at  $d_{ii} = 0$  (that is, at location  $i$  itself). A function such as:

$$k_{ij}(d) = \exp(-d_{ij}^2 / \phi^2) \quad (26)$$

will emphasize observations around the location of the estimation, by down-weighting distant observations and making the most remote virtually negligible for estimation centered at  $i$ . There is however an extra issue raised by the kernel function given by equation (26) that must be solved before the model is made operational. As it stands, the value of weight  $k_{ij}$  depends, in addition to distance  $d_{ij}$  between locations, on a distance-decay parameter  $\phi$  (the kernel bandwidth) that controls the steepness of the Gaussian curve. It is critical to select an appropriate value of  $\phi$ , since let it to increase, the weights for distant observations will increase tending in the limit to the global model, and let it to decrease, only the closest observations will be of any estimation relevance (and as a consequence the variance will increase).

The problem of calibrating the kernel function, which is to say of obtaining a value for parameter  $\phi$ , is to find a balance between these two extremes. This is done in GWR by means of a least squares criterion using a procedure known as *cross-validation*. In cross-validation, the objective is to minimize the sum of squared errors that results from the difference between the observed value  $y_i$  and its estimated value  $y_{*i}(\phi)$ , fitted for kernel bandwidth  $\phi$  without considering observations at  $i$  during estimation. Taking out observations at  $i$  is done to prevent the kernel bandwidth from tending to 0, a trivial case in which the only observation taken into account would be  $i$  itself and would therefore have a perfect 'fit' with an error of 0. The form of the cross-validation score is:

$$CVS = \sum_i [y_i - y_{*i}(\phi)]^2 \quad (27)$$

Estimating the CVS for different kernel bandwidths and then plotting it against the values of  $\phi$  is a guideline to select an appropriated bandwidth. In the case of spatial models, it is important to notice that the estimation procedure will involve reducing and re-standardizing  $W$  matrix  $n$  times to eliminate the row and column corresponding to  $i$ . Although the calculation of the CVS can become quite cumbersome because new eigenvalues are needed after each reduction, an alternative is to use for calibration of the kernel not all observations, but a representative or randomly selected sample of locations.

A last point worth noticing about the weighting scheme is that, although distance  $d_{ij}$  is measured to observation  $j$  (the observation whose error term will be adjusted), a set of weights  $K_i$  can be defined for *any* location  $i$ , regardless of whether or not an observation is recorded there. This feature is an useful precursor to mapping, as estimation can be carried out for any number of points over a plane just by moving around point  $i$ , and then the value of the parameters obtained for those locations can be contoured or mapped to produce graphical output depicting their spatial variation.

## 5. CASE STUDY: GLOBAL MODELS.

Before proceeding to the application of the GWR model, introduction of the study area and application of spatial models of the form given by equations (1) and (2) is in order. The case study is Sendai City, capital of Miyagi Prefecture in northeastern Japan, and one of the country's 10 largest cities with a population close to 1 million. Sendai has a strongly monocentric structural shape, served by radial rail transportation in the form of a subway line in the south-north axis, and train lines in the south-north and east-west axes. The south-north railway line runs parallel to the subway line, starting from the commercial subcenter at Nagamachi, passing Sendai Station in the center, and up to Kita Sendai. From there, the train line goes to the west, while the subway line continues in direction north up to Izumi Chuo, the second commercial subcenter of city. Two other train lines part from Sendai Station and run in the east direction (fig. 1).

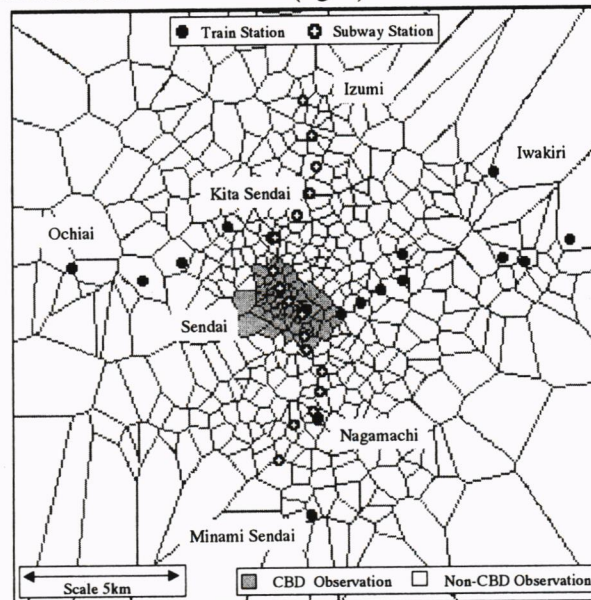


Figure 1. Sendai City. Transportation Infrastructure and Configuration of Spatial Interactions

Variable selection is based on urban economics theory, that shows how urban land prices decrease with distance from the city's central business district, where most services locate (monocentric distance-decay model). Another factor affecting land prices is the use of land, and here we consider the intensity of commercial land uses, both as a measure of land use activity and as a measure of accessibility to services. The third factor under consideration is accessibility to transportation infrastructure. The variables thus proposed for the models are: land price (LPR; Sendai City Information Office, 1996); distance to CBD, (DIST; in this case distance to Sendai Station calculated using a GIS); percentage by area of commercial land use by zone (CommPct; Basic Planning Survey for Sendai Metropolitan Area); and distance to nearest subway or train station, calculated using a GIS (DISTN).

Two model formulations are attempted. The first is a simple (single region) spatially autoregressive model (equations 1 and 2), and the second is an alternative form of this model, namely a spatially switching regression or SSR, to account for possible spatial heterogeneity. A switching regression is used when the data may be separated into a small number of congruent regimes in terms of some affinity. In the case of a spatially switching



regression, the data is separated in sub-regions (spatial regimes) that share a common characteristic. For the models below, two regimes turn out to be enough to represent the spatial structure of Sendai City, the first corresponding to the CBD area and the second covering the rest of the city (Paez, 1998). Formally, the two-regime spatially switching specification of a spatially autoregressive model is as follows:

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \rho W \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} + \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \quad (28)$$

In this form of the model, the observation vectors  $Y$  and  $X$  are rearranged so that they can be separated in two vectors according to their spatial regime, while the spatial interactions matrix  $W$ , and the parameter and error vectors  $\beta$  and  $\varepsilon$  are rearranged to match their corresponding observations. What the switching regression actually does, is estimate a different set of parameters for each spatial regime. Estimation of this model can be carried out in a way very similar to that of the (single region) spatially autoregressive model, noting that, just by redefining the variables, equation (28) may be expressed in the following form, which is identical to the SAR model in equation (1):

$$Y^* = \rho WY^* + X^*\beta^* + \varepsilon^* \quad (29)$$

A basic difference of this model with the simple, single region SAR, is that in order to account for spatial heterogeneity, the structure of the error's covariance is given in terms of two parameters instead of one: there will be two (potentially different) variance parameters for two spatial regimes. This is expressed as follows:

$$E[\varepsilon^* \varepsilon^{*'}] = \Omega = \begin{bmatrix} \sigma_1^2 I_1 & 0 \\ 0 & \sigma_2^2 I_2 \end{bmatrix} \quad (30)$$

where  $\sigma_1^2$  and  $\sigma_2^2$  are the variance terms for regimes 1 and 2 respectively, and  $I_1$  and  $I_2$  are identity matrices of dimension  $n_1$  and  $n_2$  (the sample size of regimes 1 and 2). Estimation of parameters, inference, and statistical tests against misspecification of a two-regime spatially switching SAR (SS-SAR) model are introduced in Paez (1998).

The structure of spatial interactions was defined based on the observed land prices ( $n=479$ ) using a nearest neighbor criterion. To do this, a binary weight matrix was obtained (1's for interacting location pairs, 0's in the diagonal and elsewhere), that was then row-standardized to obtain spatial interactions matrix  $W$ . The spatial structure of the region can be observed in figure 1, where physical contiguity between areas (i.e. shared zone borders) is the criterion used to define spatial interaction.

Estimation results of the global models are presented in table 1. The first model is a simple spatially autoregressive specification, that produced a statistically significant estimator for the spatial association term  $\rho$ , confirming the existence of spatial spillovers, as well as expected signs of the parameters: negative for distance from CBD and positive for the land use variable. The model fails, however, to account for spatial error autocorrelation, giving a significant value of the Lagrange Multipliers statistic against the null hypothesis of  $\lambda=0$  in equation (2), and therefore is not thought to be well-specified. The second and third models are spatially switching SAR, with a different selection of variables. Again, the parameter for spatial spillovers is significant in both, but only SS-SAR #1 appears to be a correctly specified model that accounts for all systematic variation in the data, spatial and else, as SS-SAR #2 presents spatial error autocorrelation.

Although theoretical results to test the specification of a spatially autoregressive GWR are

as yet not available, the global models serve as a guideline for selecting the functional form of the GWR model. Of the global models, two of them, the single region SAR and the spatially switching SAR with the DISTN variable, fail to account for spatial error autocorrelation as indicated by significant values of the LM statistic. Consequently, these model specifications and variable selections are discarded as forms of the GWR, since the possibility exists that the misspecification of the global model is either a consequence of, or will be reflected in, misspecification of the local models.

Also, note in passing that adding the DISTN variable to SS-SAR #1 to obtain SS-SAR #2, had the interesting consequence of making a correctly specified model become afflicted by spatial error autocorrelation. This result stands in contrast to the somewhat extended belief that an effective countermeasure against spatial autocorrelation is the addition of locational variables. In our case, addition of a locational variable with a markedly non-random spatial distribution (normalized Moran's *I* statistic for DISTN is  $Z(I)=32.06$ ) adds little to the explanation but has the side effect of passing down the autocorrelation effect to the model.

**Table 1.** Global Land Price Models. Spatially Autoregressive (SAR) and Spatially Switching SAR

Variable	Spatially Autoregressive (SAR)		Spatially Switching SAR #1		Spatially Switching SAR #2	
	Parameter	t-value	Parameter	t-value	Parameter	t-value
$\rho$	0.773	22.317 **	0.464	16.85 **	0.444	15.895 **
CONST1 ( regime 1 ) <sup>1</sup>	60.329	1.216	1348.052	3.31 **	1801.204	4.008 **
DIST	-0.014	-1.801 *	-0.731	-3.01 **	-0.575	-2.295 *
CommPct	6.209	4.128 **	8.919	1.18	5.007	0.653
DISTN	-	-	-	-	-1.023	-1.997 *
CONST2 ( regime 2 )	-	-	86.758	6.93 **	94.432	7.546 **
DIST	-	-	-0.007	-4.32 **	-0.005	-2.910 **
CommPct	-	-	2.473	5.63 **	2.349	5.323 **
DISTN	-	-	-	-	-0.009	-2.949 **
LOG LIKELIHOOD	-3334.18		-2694.62		-2688.46	
$\sigma_1^2$	112598.64		802015.94		755116.3	
$\sigma_2^2$	-		4512.99		4440.68	
$n_1$	479		60		60	
$n_2$	-		419		419	
<i>&lt;&lt; Lagrange Multipliers test for spatial error autocorrelation &gt;&gt;</i>						
LM Statistic ( $\chi^2 - 1$ D.F.)	34.59**		1.69		10.84**	

<sup>1</sup> Regime 1 in the switching regressions corresponds to the CBD

\* Significance > 0.05      \*\* Significance > 0.01

**6. CASE STUDY: GWR MODEL.**

Adopting previous section's SS-SAR #1 as the form of the model to apply the weights, estimation of the local models can be now conducted. This is done in a similar fashion to the method described above, only that now the corresponding term of the log-likelihood function to be maximized will take the following form:

$$-\frac{n_1}{2} \ln(\sigma_1^2) - \frac{n_2}{2} \ln(\sigma_2^2) - \frac{1}{2} (A_i^+ Y^+ - X^+ \beta_i^+) K_i^+ \Omega^{-1} K_i^+ (A_i^+ Y^+ - X^+ \beta_i^+) + \sum_n (1 - \rho_i \omega_n) \quad (31)$$

with  $\beta_i^+$  estimators:

$$\beta_i^+ = (X^{++} K_i^{++} \Omega^{-1} K_i^+ X^+)^{-1} X^{++} K_i^{++} \Omega^{-1} K_i^+ A_i^+ Y^+ \quad (32)$$

and variance terms for regions 1 and 2:

$$\sigma_1^2 = -\frac{n_1}{2} (A_i^+ Y^+ - X^+ \beta_i^+) K_i^+ \begin{bmatrix} I_1 & 0 \\ 0 & 0 \end{bmatrix} K_i^+ (A_i^+ Y^+ - X^+ \beta_i^+) \quad (33)$$

$$\sigma_2^2 = -\frac{n_2}{2} (A_i^+ Y^+ - X^+ \beta_i^+) K_i^+ \begin{bmatrix} 0 & 0 \\ 0 & I_2 \end{bmatrix} K_i^+ (A_i^+ Y^+ - X^+ \beta_i^+) \quad (34)$$

Equations (31) to (34) are fairly straightforward extensions of the concepts presented in sections 3 (spatial econometric models) and 5 (spatial models and geographical weights), and estimation of the model, with only minor modifications, follows the general guideline of the previous models.

Now, the first step towards estimating a GWR model is calibration of the kernel function (eq. 26) by minimization of the Cross Validation Score (eq. 27). Given the large size of the sample ( $n = 479$ ), calibration using it in full would have been extremely intensive in terms of computational resources. To avoid computational overhead, and following a suggestion by Brundson et al (1998), estimation was carried out using a sub-sample of observations, consisting of 100 randomly drawn observations ( $n_s = 100$ ), or roughly 21% of the total. Results of the calibration are shown below, where figure 2 plots the CV score against the value of kernel bandwidth  $\phi$  in the range of 2500 to 4000 meters. The score was found to be minimized with reasonable precision, given the dimensions of the study area, by a value of 3043 m of the kernel bandwidth. This value was therefore used for the kernel function to estimate the GWR model.

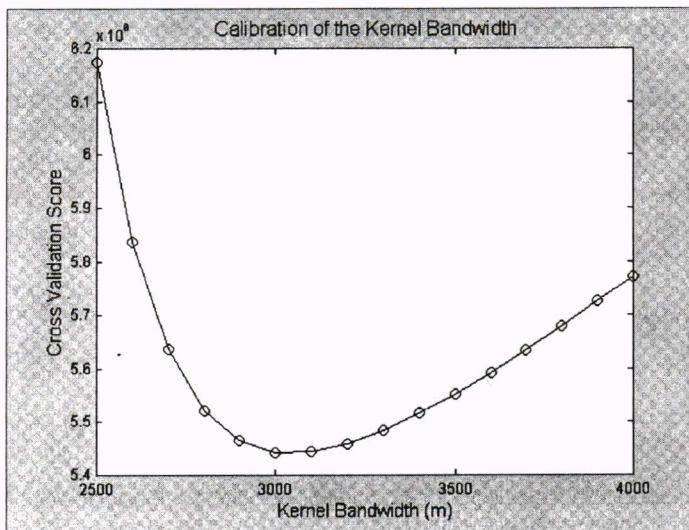


Figure 2. Calibration of the Kernel Function. CV Score and Kernel Bandwidth ( $\phi$ )

Estimation of the model's parameters was done using expressions (31)-(34) for an irregular grid made of 488 locations that, in general, did not correspond to recorded observations, but that were distributed to reflect the actual distribution of observations in space, in general more dense in the commercial business district area. Obtaining parameters over a grid was done to take advantage of it to produce smoother maps. Consequently, estimation was done for  $i = 1, \dots, 488$  local models for which parameters were obtained and then mapped to produce the graphical output given by figures (3) through (6). It is important to notice at this point that a switching regression will produce parameter vector  $\beta^* = [\beta_1, \beta_2]'$  consisting of separate estimators for each spatial regime. However, although all parameters are used to calibrate the kernel function, for the purpose of reporting the results only those parameters corresponding to the spatial regime to which the location belongs are used.

The first example of graphical output (figure 3) corresponds to the spatial distribution of the  $\rho_i$  parameter of spatial association. This parameter represents the magnitude of spatial spillovers, and had a value of 0.464 in the underlying global model s-r SAR #1. However,

it can be seen from the figure that a global level is at best a rough average of the actual distribution as given by the GWR model. The maximum value for a local  $\rho_i$  parameter was found to be of 0.68 (that is, a 68% of the average of neighboring prices will be reflected in the price at the location), and the average over the region was found to be 0.29, well below the value of the global model. Some areas were found to have negative spatial external effects, and a minimum of  $-1.00$  was obtained, but in a limited border area to the west of the city. The variation among locations is large and does not appear to be random, as the normalized version of Moran's  $I$  statistic, calculated using a quadratic distance decay interactions matrix, produced a significant value of  $Z(I) = 40.88$  (meaning positive spatial autocorrelation).

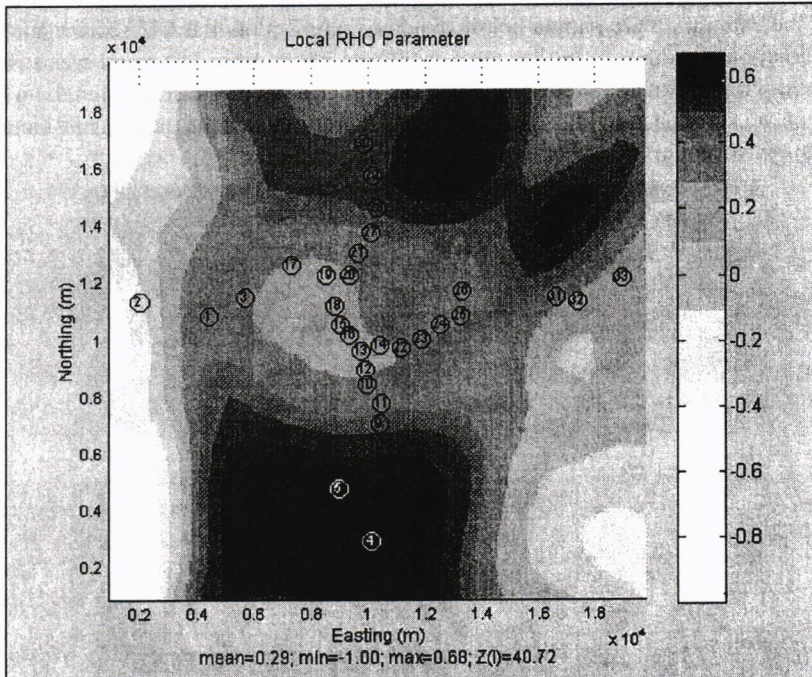


Figure 3. Spatial Distribution of the Local RHO Parameter ( $\rho_i$ ) and Statistics.

Positive external effects were obtained for almost all the region, but with the highest values concentrating around three easily spotted sub-regions: the area south of Minami Sendai, the surroundings of Iwakiri station, and between this and Izumi station. A somewhat unexpected result is that the size of externalities in the area around the CBD is not as large as in these three spots, but this might be a consequence of the level of development already achieved by the CBD, which is relatively high and possibly reaching saturation. An interpretation could be as a moderately high benefit of locating in a developed area, but where agglomeration economies are operating close to their limits.

Although conclusive evidence of an association pattern with transportation infrastructure was not found, it is still clear that high values of spatial spillovers occur in regions well serviced by public transportation, as the regions with the lowest values are locations in the fringes and distant from any kind of rail transportation. An interpretation of negative externalities might be offered in terms of competition, since the price of lots in less desirable regions of the city are reduced in an amount proportional to the average of neighboring prices, apparently to maintain their relative attractiveness. In general (see table

2), relatively high values of the spatial association parameter were obtained for the locations of the stations, whose average (0.359) is higher than the regional average.

Worth of mention appears the fact that among the stations with the higher values of the spatial spillover parameter, we find those between Nagamachi and Minami Sendai to the south, and the stations in the neighborhood of Izumi district to the north, where the commercial sub-centers of the city locate. These areas are experiencing rapid growth, but do not approximate yet to the level of development observed in the CBD.

**Table 2.** Parameter Estimation Results for the Locations of Train/Subway Stations

Station #	RHO	CONST	DIST	CommPct	Station #	RHO	CONST	DIST	CommPct
1	0.366	121.5	-0.014	4.33	18	0.248	2076.8	-0.982	9.06
2	-0.177	132.5	-0.007	2.76	19	0.255	120.1	-0.016	6.99
3	0.299	158.6	-0.020	4.51	20	0.261	114.3	-0.014	6.97
4	0.664	61.9	-0.007	3.06	21	0.323	92.9	-0.010	6.03
5	0.645	42.5	-0.007	5.71	22	0.282	149.1	-0.029	4.69
6	0.563	46.0	-0.007	6.79	23	0.294	151.5	-0.030	4.04
7	0.543	52.2	-0.008	6.38	24	0.317	145.3	-0.025	3.12
8	0.521	53.4	-0.008	6.60	25	0.345	138.8	-0.020	2.08
9	0.456	68.3	-0.009	6.41	26	0.372	119.4	-0.015	1.74
10	0.297	1555.3	-1.019	16.37	27	0.414	76.6	-0.006	4.07
11	0.375	92.6	-0.013	6.23	28	0.520	53.6	-0.001	2.01
12	0.270	1653.2	-1.017	15.96	29	0.547	38.2	0.003	1.45
13	0.267	1749.6	-0.987	13.99	30	0.518	65.1	-0.001	2.02
14	0.271	1745.6	-0.980	13.65	31	0.341	78.4	-0.001	0.45
15	0.256	1948.2	-0.978	11.14	32	0.225	110.8	-0.003	0.34
16	0.263	1854.4	-0.974	12.38	33	0.189	123.4	-0.005	0.69
17	0.268	144.1	-0.019	5.88	34	0.603	33.9	-0.001	1.14

Figures (4) to (6) are graphic depictions of the rest of the parameter's spatial variation as measured by the estimation of the GWR model. It is evident that significant variation exists, and the normalized  $Z(I)$  values of the spatial autocorrelation statistic, large and statistically significant, point at the non-randomness of it. The maps are for the constant term of the regression (figure 4), and for parameters corresponding to independent variables DIST and CommPct (figures 5 and 6 respectively).

Inspection of the above figures supports the assertion that the structure of the city is strongly monocentric, something that was hinted by the global model in its switching regression form, but that appears to be confirmed by the GWR model. Especially informative is the map of the local constant term, as it measures, once the effect of all the relevant variables has been considered, the spatial variation of land prices. From figure (3) it is clear that the highest values of land, all else being equal, are in the CBD area, but with an important element of spatial non-stationarity. There, the parameter ranges between 600-800 to a maximum of roughly 2100, in contrast with the single global model's value of 1348 for the same area. In addition, an inverse relationship can be observed between the variation of the constant and the DIST parameter.

The DIST parameter also confirms the importance of locating close to the central station, as the regional average of the DIST parameter is negative. And, although variation is smooth for the most part of the city, the relevance of distance becomes ever more important as a location moves toward Sendai Station. Another evidence is given by the CommPct parameter, which measures the importance of commercial land use intensity. Variation of this parameter declines almost concentrically from a maximum at the CBD, showing that the element of commercial presence is more beneficial the closer locations are to central city. It is interesting to notice that a similar effect is not observed around the commercial sub-centers.

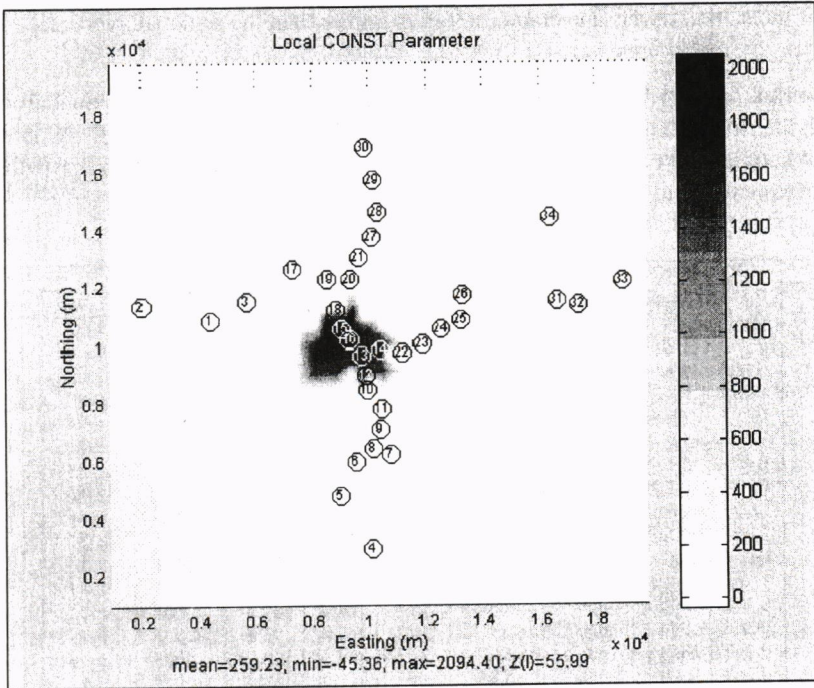


Figure 4. Spatial Distribution of the Local CONST Parameter and Statistics.

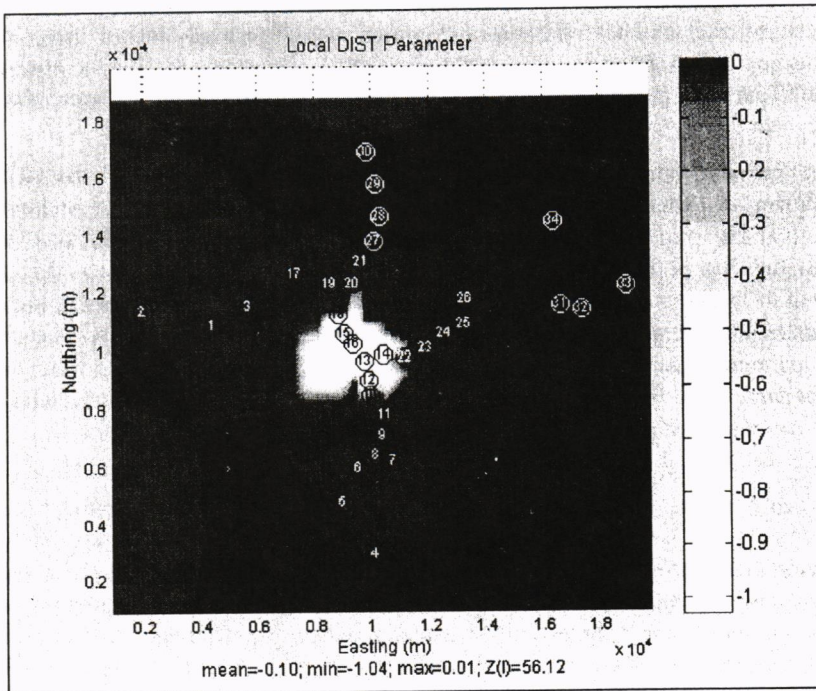


Figure 5. Spatial Distribution of the Local DIST Parameter and Statistics.

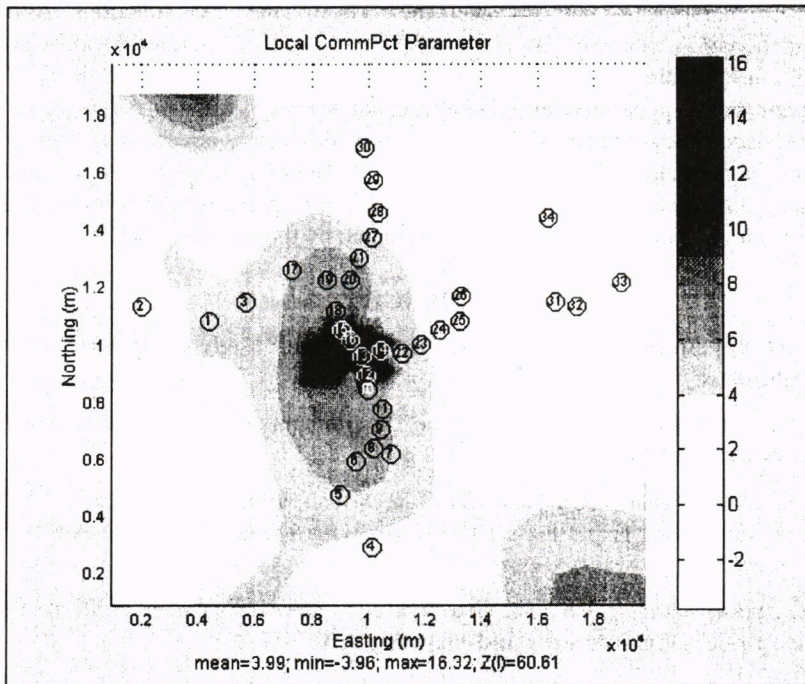


Figure 6. Spatial Distribution of the Local CommPct Parameter and Statistics.

## 7. SUMMARY AND CONCLUSIONS.

The main goal of this paper has been to present a spatially detailed study of land prices, to compare the results with the spatial configuration of transportation service provision. An important objective of the analysis has been the study of spatial external effects in the land markets, but at a local level to reveal any potential association with transportation infrastructure. Two spatial analysis methodologies have been used to that end, the first drawing from a spatial econometric approach to measuring interactions, and the second being the GWR method to measure spatial parametric variation.

Specifically, this paper presented an empirical application of the GWR methodology (used to obtain local estimates of the model's parameters), based on a spatial econometric specification to model spatial external effects. Application of GWR techniques to spatially specified models stems from the work of Brundson et al (1998), but extends the theoretical results to obtain a particular model, namely a geographically weighted, spatially switching SAR. This form was needed to represent the characteristics of the case study without giving way to the possibility of model's misspecification.

As for the results of the model, it was found that spatial external effects exist, and that there is indeed considerable spatial variation of the parameters, a result that was masked by the global model approach. Besides visual inspection of the results, selection of a kernel bandwidth revealed that the GWR model represents an improvement over the global model, as a definite kernel bandwidth could be found with a smaller sum of squared errors than the larger values to which the global model tends. In addition, spatial parametric variation appears to be non-random, as statistically significant values obtained for the spatial autocorrelation statistic  $I$  suggest. In this sense, the GWR helped as a tool to explore more

in depth, at a geographically detailed level, the form of land price functions. Although decisive evidence of correlation between external economic effects and the provision of transportation infrastructure was not found, it could still be noticed that, in general, high values of parameter  $\rho_i$  appear in relation with train and subway stations. It is suggested that the results obtained here could be used to further study the factors affecting the variation of this parameter, in a would-be expansion of the model. Thinking about the spatial variation of the constant, distance and commercial land use parameters, the results seems to confirm the strongly monocentric structure of land prices hinted by the global model.

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