A NEW META-HEURISTIC APPROACH TO THE FLEET SIZE AND MIX VEHICLE ROUTING PROBLEM

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Abstract: The fleet size and mix vehicle routing problem (FSMVRP) decides the optimal vehicle fleet composition and routes under the consideration of a heterogeneous fleet. This paper presents a new meta-heuristic, generic intensification and diversification search (GIDS), for FSMVRP. A bank of twenty FSMVRP benchmark instances was utilized for evaluating the efficiency and the accuracy of GIDS. Preliminary results showed that the average deviation of the best known solution for the twenty instances was merely 0.698 %. Moreover, we had updated the best known solutions for six instances. Such results imply that GIDS should provide a powerful tool in solving FSMVRP.

1. INTRODUCTION

The fleet size and mix vehicle routing problem (FSMVRP) determines both the vehicle fleet compositions and the routes to serve a number of customers with respective demand from a central depot and under some side constraints. FSMVRP differs from the classical vehicle routing problem (VRP) in that FSMVRP deals with a heterogeneous fleet of vehicles. As an extension of VRP, FSMVRP considers not only variable link-traveled cost but also fixed vehicle-used cost. This consideration is important when firms make some tactical and operational decisions in order to minimize the cost of physical distributions. Effectively selecting, to lease or acquire, the number of vehicles in different type with a coherent routing policy for such vehicles to handle the system demand lies at the heart of the decision-making of transportation and logistic activities.

Due to the NP-hard complexity of FSMVRP, existing published methods for FSMVRP are almost all heuristics. Golden et al. (1984) adopted four modified savings methods which were derived from the conventional savings algorithm (Clark and Wright, 1964). In addition, Golden et al. (1984) proposed two giant tour partition methods, SGT and MGT. Gheysens et al. (1986) showed a generalized assignment based two-phase approach. Desrochers and Verhoog (1991) extended their matching based savings algorithm (MBSA) initially designed for the VRP to solve the FSMVRP. The perturbation procedure contributed by Salhi and Rand (1993) can be considered as a two-phased local search heuristic. Han and Chang (1995) presented two hybrid heuristics, MGSROR and MGORSR, which combined the multiple giant tour partition, the Or-opt exchange and the perturbation procedure.

The related vehicle routing research has trended toward artificial intelligence (AI) heuristics since the 1980s (Fisher, 1995). Various new concepts and approaches, currently named meta-heuristics, based on the traditional local search heuristics have achieved success in attacking a variety of practical and difficult combinatorial optimization problems. These meta-heuristics provide general frameworks that allow for creating new hybrids by combining various concepts derived from different areas such as classical heuristics, artificial intelligence, biological evolution, neural systems, physical science and statistical mechanics (Osman and Kelly, 1996). Many computational studies indicated that meta-heuristics gained results better than conventional heuristics to conquer hard combinatorial optimization problems.

Although the importance of FSMVRP has been recognized by researchers and practitioners, it has received relatively less attentions in literature than other vehicle routing and scheduling problems. In addition, few of the recently developed meta-heuristics are applied to solve FSMVRP. The main purpose of this research is to build a framework which combines the traditional local search heuristics and meta-heuristics for FSMVRP. This paper is organized as follows. Section 2 is devoted to a sketch about some basic ideas of meta-heuristics and the conceptual framework of our proposed meta-heuristic. Section 3 presents the implementation design of the new meta-heuristic for FSMVRP. Section 4 reports some computational results of this meta-heuristic which is tested on the twenty FSMVRP benchmark instances. Finally, conclusions and suggestions for future work are given in section 5.

2. THE META-HEURISTIC FRAMEWORK OF GIDS

This paper presents a new meta-heuristic, generic intensification and diversification search (GIDS), which combines the use of generic search methods with the concepts of intensification and diversification strategies for a more intelligent search. Before we present the framework of GIDS, it would be advantageous to briefly review some basic ideas of meta-heuristics and generic search methods at the outset.

2.1 Some Basic Ideas of Meta-heuristics

According to Osman and Kelly (1996), a meta-heuristic is one which guides subordinate (classical) heuristics combining concepts derived from artificial intelligence, and biological, mathematical, natural and physical sciences to improve their performance in the process of search. Though the myopic move from one incumbent solution to its neighborhood solution is basically the same as conventional local search methods, the overall search is guided by intelligent ideas or strategies to get away from trapped at some local optima.

In general, the basic ideas of some commonly used meta-heuristics and their related methods can be summarized as follows.

(1) Search by learning, e.g. neural network (NN) by Hopfield and Tank (1985), and ant colony optimization (ACO) by Dorigo et al. (1996);
(2) Keep good parts, e.g. genetic algorithm (GA) by Goldberg and Lingle (1985), and scatter search (SS) by Glover (1994);
(3) Allow bad solutions:
 – Tabu search methods, e.g. tabu search (TS) by Glover (1986), and tabu threshold

(TT) by Glover (1995),
- Generic search methods, e.g. simulated annealing (SA) by Kirkpatrick et al. (1983), threshold accepting (TA) by Dueck and Scheuer (1990), great deluge algorithm (GDA) by Dueck (1993), and record-to-record travel (RRT) by Dueck (1993);

(4) Change search space:
- By alternating neighborhoods, e.g. variable neighborhood search (VNS) by Mladenovic and Hansen (1997),
- By disturbing cost functions, e.g. noising method (NM) by Charon and Hudry (1993), and search space smoothing (SSS) by Gu and Huang (1994); and

(5) Start from different points, e.g. greedy random adaptive search procedure (GRASP) by Feo and Resende (1989), and jump search (JS) by Tsubakitani and Evans (1998).

Figure 1 gives a sketch of the basic strategies or ideas of some meta-heuristics. Note that this sketch is a general representation rather than a conclusive classification of the ideas or strategies embedded in different meta-heuristics. For example, tabu search consists of several comprehensive concepts, such as intensification and diversification strategies which can belong to the headings of keeping good parts or starting from different points. Details of some existing meta-heuristics could be referred to Glover and Laguna (1997), Osman and Kelly (1996), and Reeves (1993).

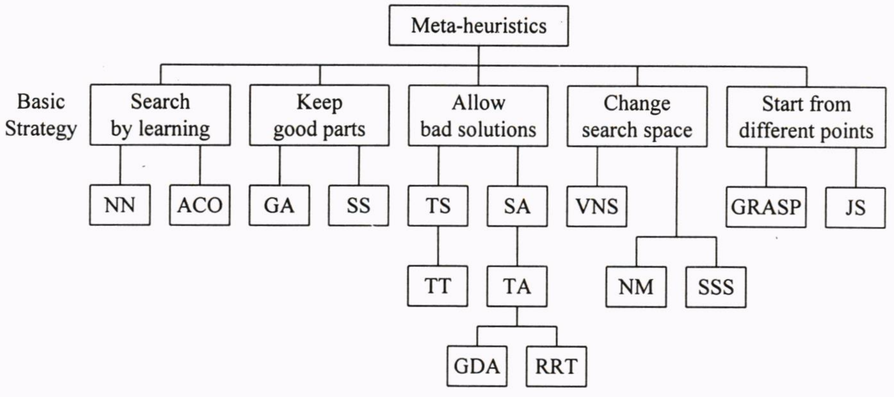

Figure 1. Some Basic Strategies of Meta-heuristics

2.2 Generic Search v.s. Local Search

Generic search (GS), the term we followed from Fisher (1995), is a meta-heuristic which adopts a more generous accepting rule than traditional local (or neighborhood) search. The traditional local search heuristic methods are subject to a rigid accepting rule which limits the move to an acceptable neighbor only when it is better than the current solution. Instead of such a rigid accepting rule, each different GS method has a more generous accepting rule in its own form which would allow the overall search move through some bad solutions when bypass the trap of a local optimum. Details of these GS methods are referred to Kirkpatrick et al. (1983), Dueck and Scheuer (1990), and Dueck (1993).

Figure 2 gives an implementation flowchart of a generic search method. As shown in Figure 2, while the guideline level of the meta-heuristic is governed by GS-related accepting rules, the basic search engines are still classical local search methods.

neighborhood which will be accepted by the associated accepting rule. The outcome of best-improvement of a local search heuristic certainly is more desirable than that of first-improvement. However, for GS-based meta-heuristic applications, the more efficient implementation of first-improvement may give more time for the overall search to explore more possible solutions. Accordingly, we included both first-improvement and best-improvement into our study.

2.3 Conceptual Framework of GIDS

GIDS, as mentioned earlier, combines generic search methods with the concepts of intensification and diversification strategies. Intensification and diversification strategies are two very important components of tabu search, which is one of the most notable meta-heuristic with successful applications to many complicated combinatorial optimization problems. Intensification strategies are based on adjusting choice rules to encourage historically found elite solutions and initiating a return to search them more thoroughly. Diversification strategies, on the other hand, encourage the search process to examine unvisited regions and to generate solutions that differ in various significant ways from those seen before. Note that although GIDS shares some ideas of intensification and diversification from tabu search, the implementation is very different to tabu search.

The conceptual framework of GIDS is shown in Figure 3. This framework consists of three subsystems: local solution constructor (LSC), generic search for intensification (GSI), and perturbation search for diversification (PSD). Referring to Figure 1 as shown earlier, GIDS respectively employs those ideas of allowing bad solutions, changing search space, and starting from different points in GSI and PSD. For intensification in GSI subsystem of GIDS, we applied generic search methods and the strategy of alternating neighborhoods to enhance the area of local search. For diversification in PSD subsystem of GIDS, we adopted the strategies of disturbing cost functions and starting from different points to diversify the sphere of local search. Details of the implementation design of GIDS for FSMVRP is described next.

3. IMPLEMENTATION DESIGN OF GIDS FOR FSMVRP

While the conceptual framework of GIDS is shown in Figure 3, its application requires more detailed design of specific heuristic methods and modules to implement. The flowchart of our implementation design of GIDS for FSMVRP is given in Figure 4. As showed in Figure 4, LSC subsystem includes two modules, initial solution (IS) and neighborhood search (NS); GSI subsystem includes two modules, generic search 1 (G1) and generic search 2 (G2); and PSD subsystem includes two modules, cost perturbation (CP) and weighted savings (WS).

Table 2 lists some available subordinate heuristics for each module of LSC, GSI and PSD subsystems. The related strategies of the modules are also given in Table 2. It is worthy of note that, based on these strategies, different modules can be designed for solving problems. In other words, GIDS, like TS, can be applied to solve varied combinatorial optimization problems.

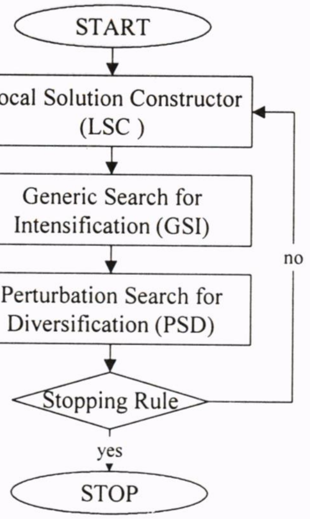

Figure 3. The Conceptual Framework of GIDS

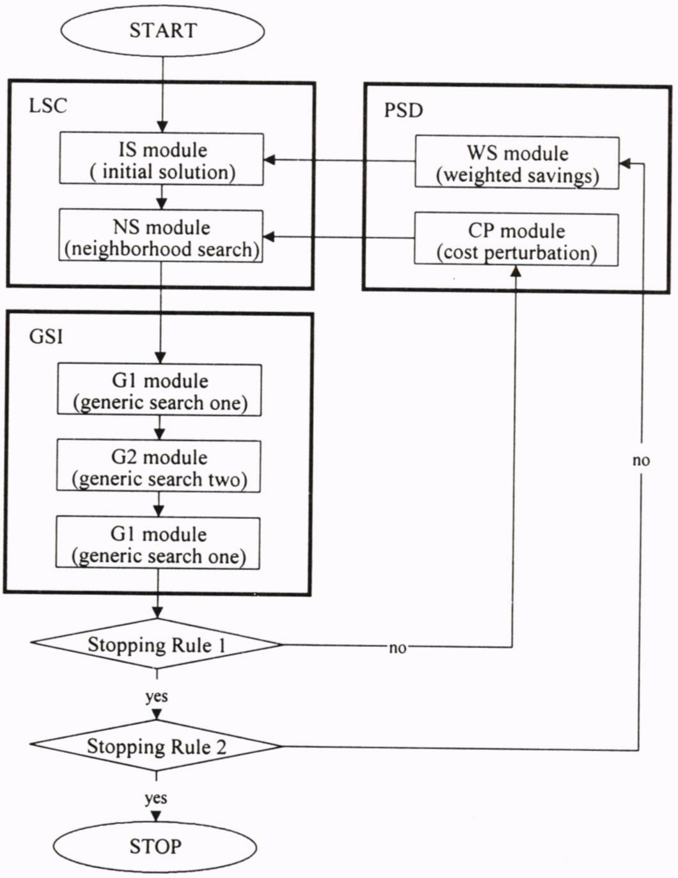

Figure 4. The Implementation design of GIDS for FSMVRP

Therefore, successful implementation of a generic search meta-heuristic also relies on efficient local search engines. The accepting rules, control parameters and related stopping rules for generic search methods such as SA, TA, GDA and RRT are listed in Table 1.

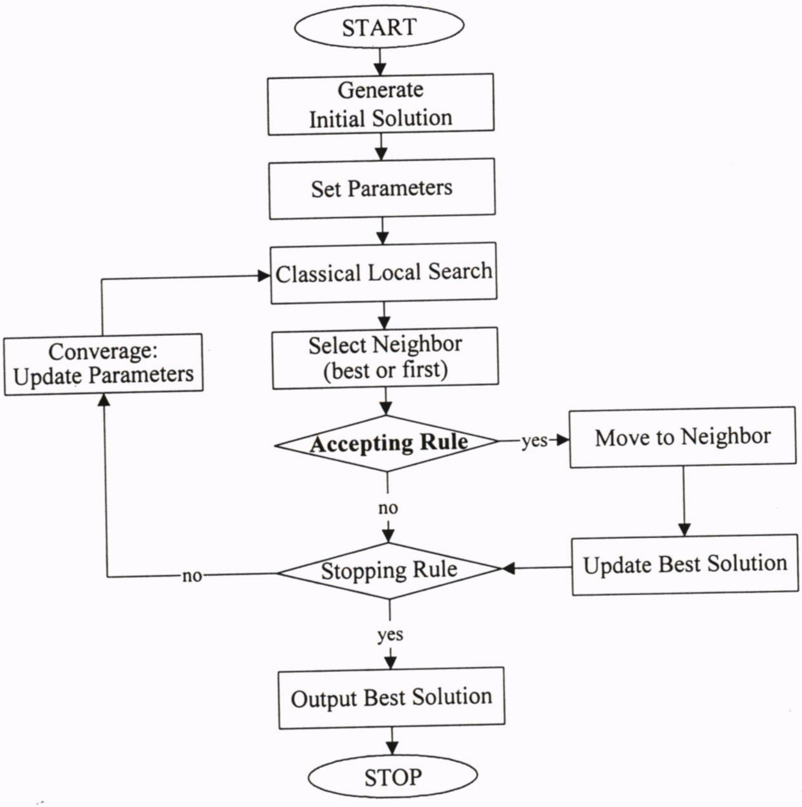

Figure 2. Flowchart of Generic Search Meta-heuristics

Table 1. Characteristics of Some Generic Search Methods

	SA	TA	GDA	RRT
Parameters	temperature (T) random value (r)	threshold (T)	water-level (L) rain speed (S)	deviation (D>0) record (R)
Accepting Rule	probabilistic: $r < e^{[-\Delta/T]}$	deterministic: $\Delta^{\dagger} < T$	deterministic: $C(X')^{\dagger} < L$	deterministic: $\Delta^{\dagger} < D$
Convergence	T decreases	T decreases	L = L - S	update R
Stopping Rule	K iterations	K iterations	all C(X') ≧ L	K iterations

† $\Delta = C(X') - C(X)$; C(X') is the objective value of neighbor solution; and C(X) is the objective value of current solution.

Note that there are two ways to implement a local search method within a meta-heuristic: the best-improvement or the first-improvement. Best-improvement is to find the best solution in the whole neighborhood of the incumbent solution that the heuristic could search for. First-improvement, on the other hand, is to find the first solution among the

Table 2. Strategies and Heuristics in GIDS Subsystems

Subsystem	Modules	Available Heuristics	Strategies
LSC	IS	modified savings methods	
	NS	perturbation procedure	best- or first-improvement
GSI	G1, G2	SA, TA, GDA, RRT	allow bad solutions change generic search alternate neighborhoods
PSD	CP	NM, SSS	disturb cost functions
	WS	weighted savings	start from different points

3.1 Design of LSC Subsystem

LSC subsystem first generates an initial solution, then produces a local optimal improvement by some neighborhood search heuristic. Therefore, LSC includes an initial solution (IS) module and a neighborhood search (NS) module.

(1) The IS module

The modified savings method is selected to build IS module. Table 3 summarizes five available modified savings formulas: combined savings (CS), optimistic opportunity savings (OOS), realistic opportunity savings (ROS), revised ROS (ROS-γ), and proportional usage savings (PUS). CS, OOS, ROS, and ROS-γ are adopted by Golden et al. (1982); PUS is our proposed savings method which consults the usage efficiency of vehicle. Notations in Table 3 are defined as follows:
C_{ij}: the travel cost of link (i, j),
$S_{ij} = C_{i0} + C_{0j} - C_{ij}$, is the savings value of travel cost by connecting link (i, j),
F(Z): the fixed cost of the smallest vehicle that can service a demand of Z,
P(Z): the capacity of the smallest vehicle that can service a demand of Z,
F'(Z): the fixed cost of the largest vehicle that has a capacity less than or equal to Z,
$w = P(Z_i + Z_j) - P(\max\{Z_i, Z_j\})$,
$\delta(w) = 0$, if $w = 0$; $\delta(w) = 1$, if $w > 0$,
$\gamma = 0.0$ to 0.3, is a route shape parameter, and
$U(Z) = F(Z)\times [Z\div P(Z)]$, is the proportional fixed cost of the smallest vehicle that can service a demand of Z.

Table 3. Summary of Savings Methods

Method	Savings Formula	
CS	$CS_{ij} = S_{ij} + F(Z_i) + F(Z_j) - F(Z_i + Z_j)$	(1)
OOS	$OS_{ij} = CS_{ij} + F[P(Z_i + Z_j) - Z_i - Z_j]$	(2)
ROS	$RS_{ij} = CS_{ij} + \delta(W)\times F'[P(Z_i + Z_j) - Z_i - Z_j]$	(3)
ROS-γ	$Rs_{ij}(\gamma) = RS_{ij} + (1 - \gamma)\times C_{ij}$	(4)
PUS	$US_{ij} = S_{ij} + U(Z_i) + U(Z_j) - U(Z_i + Z_j)$	(5)

For each modified savings method, both sequential and parallel constructing solution were considered. In our implementation, the sequential PUS was selected to generate initial solution.

(2) The NS module

The Salhi and Rand (1993) perturbation procedure (SRP) provided a neighborhood search heuristic for FSMVRP. The original perturbation procedure can be considered as a two-phased heuristic. First, given a fixed type of vehicle, FSMVRP is solved by using any classical VRP heuristics to generate an initial solution. Moreover, for each route of the initial solution, the matching process is used to determine the smallest vehicle which is large enough to serve it. Second, the matched routes are improved by systematically using seven refinement modules such as reduction, reallocation, combining, sharing, swapping, relax/comb and relax/share. These refinement modules can be regarded as complicated variants of traditional inter-route exchange methods.

In NS module, we selected the second phase of SRP to be the subordinate local search heuristic. Besides that the execution structure is simplified, three new refinement modules: relax/redu, relax/real, and relax/swap are added to the SRP procedure. Similar to original modules, these new modules allow replacing bigger vehicle while inserting or exchanging nodes. Procedure that employs three new refinement modules to replace original modules is called SRPX. Both SRP and SRPX were adopted in our implementation. Details of SRP and SRPX are referred to Han and Cho (1998).

3.2 Design of GSI Subsystem

GSI subsystem, based on generic search heuristic, is the core of GIDS. GSI subsystem considers two generic search heuristics, threshold accepting (TA) and great deluge algorithm (GDA), to guide the subordinate local search (i.e. SRP or SRPX) for extricating the bind of local optima. Two modules were designed for our GSI subsystem.

(1) The G1 Module

G1 module includes two loops to control the execution of generic search and local search, where GDA is the generic search method and SRP is the local search heuristic. In inner loop, first successively executes a GDA (based on first-improvement SRP) and a SRP with best-improvement; then a SRPX with first-improvement is optionally executed while unimproved.

(2) The G2 Module

G2 module performs a single loop which merely executes generic search at every iteration, then refines the final solution by local search once. TA (based on first-improvement SRP) is chosen as the generic search method and SRP with best-improvement as the refining local search heuristic.

Note that for GSI subsystem, G1 and G2 are implemented in the sequence of G1, G2, and G1. Such a design of GSI arises from the strategy of alternating neighborhoods which is similar to VNS. More details of G1 and G2 modules are given in Han and Cho (1998).

3.3 Design of PSD Subsystem

The purpose of PSD subsystem is to sequentially stretch the search range of GSI. Both cost perturbation (CP) module and weighted savings (WS) module are applied in PSD to achieve more drastic change of the territory to be searched.

(1) The CP Module

CP module is based on the strategy of changing search space by perturbing cost function. In CP module, we disturbed the cost function by a flip-flop (FF) method (Chen, 1996) and then improves current solution according disturbed cost by SRP with best-improvement. The flip-flop perturbation basically changes the sign of the cost function. Thus, the local maximum becomes local minimum and vice versa; the search accordingly would move to a location way from previously searched space.

(2) The WS module

Another diversification strategy is to create various start points and to re-search from LSC subsystem. Equation (6) shows a weighted savings (WS) method which regulates the weights of travel cost and vehicle cost in PUS savings formula.

$$US_{ij}(m) = w(m) \cdot S_{ij} + (1 - w(m)) \cdot U_{ij}, \qquad m = 1 \sim M \tag{6}$$

where, US_{ij} is the total savings; S_{ij} is the savings of travel cost; U_{ij} is the savings of vehicle cost; w(m) is the weight value, $0 \leqq w(m) \leqq 1$; and M, a predefined value, means the number of initial solutions. According to the weighted savings, IS module in LSC subsystem generate different initial solutions at each iteration.

Figure 5 gives an abstract presentation of the search process of GIDS. Each of the small circles denotes the neighborhood of a local (or neighborhood) search, and each of the big circles denotes the region explored by a specific generic search module. Dotted lines indicate jumps to diversified areas for CP or WS.

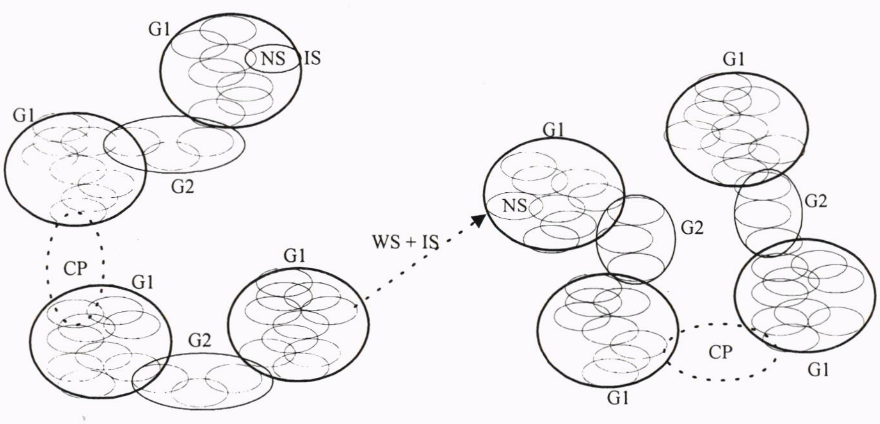

Figure 5. An Abstract Presentation of GIDS Search Process

3.4 Stopping Rules and Parameters setting

There are two stopping rules to control GIDS. Stopping rule 1 is the difference, d_k, of objective function values between the best solution found in LSC and that found in GSI. If d_k is more than zero, i.e. GSI gaining improvement, then executes CP module; otherwise, checks stopping rule 2. Stopping rule 2 is a counter related to the number of initial solutions, M. If the number of generated initial solutions is less than M, WS module must be executed; otherwise, stops the GIDS and prints out the best result found in the overall process.

The heuristics and parameters selected for application analysis of GIDS are listed in Table 4.

Table 4. Heuristics and Parameters of GIDS for FSMVRP

Module	Heuristics (parameters)
IS	PUS (sequential)
NS	SRP (best-improvement)
G1	GDA ($L = X_0 \times 1.2$, $S = X_0 \times 0.01$)* SRP (best-improvement) SRPX (first-improvement)
G2	TA ($T = X_0 \times 0.2$, $K = 10$)* SRP (first-improvement)
CP	FF SRP (first-improvement)
WS	PUS (M = 10)

* X_0 is the objective value of the initial solution.

4. APPLICATION ANALYSIS

4.1 Benchmark Test Instances of FSMVRP

Twenty FSMVRP test instances provided by Golden et al. (1984) are used to evaluate the performance of GIDS. The size of these instances vary from 12 to 100 customers except the depot. Table 5 summarizes the features and the best known solutions of the twenty instances. The best known solutions include total cost (objective value), applied method, and the source for each instance. These results provide the benchmark for evaluating the results of our GIDS applications.

4.2 Computational Results

The GIDS algorithm was coded in UNIX C language and ran under the SPARC 10 SUN workstation. The criteria selected to measure the performance of the proposed meta-heuristic are classified into two items: the quality of the solution and the computational effort. The quality of solution calculates the percentage of deviation between the best known solution and the best solution found by GIDS. To avoid the computational errors, all of the cost are previously rounded off to integers for a correspondent basis of comparison. The computational effort is measured by seconds of computer's CPU time.

Table 5. Summary of FSMVRP Benchmark Test Instances

No.	Size	Veh.‡	Best Known Solutions		
			Cost	Method	Source
1	12	3	602	ROM	Desrochers (1991)
2	12	3	722	SRP	Salhi and Rand (1993)
3	20	5	965	SGT	Golden et al. (1984)
4	20	3	6440	MGSROR[3]	Han and Chang (1995)
5	20	5	1006	MGSROR[3]	Han and Chang (1995)
6	20	3	6516	SRP	Salhi and Rand (1993)
7	30	5	7298	VRP solution†	Golden et al. (1984)
8	30	4	2349	VRP solution†	Golden et al. (1984)
9	30	5	2209	SRP	Salhi and Rand (1993)
10	30	4	2368	VRP solution†	Golden et al. (1984)
11	30	4	4763	MGT + OrOpt	Golden et al. (1984)
12	30	6	4092	SRP	Salhi and Rand (1993)
13	50	6	2438	MGT + OrOpt	Golden et al. (1984)
14	50	3	9132	MGT + OrOpt	Golden et al. (1984)
15	50	3	2615	MGORSR[3]	Han and Chang (1995)
16	50	3	2765	SRP	Salhi and Rand (1993)
17	75	5	1767	SRP	Salhi and Rand (1993)
18	75	6	2397	MGORSR[3]	Han and Chang (1995)
19	100	3	8700	ROM	Desrochers (1991)
20	100	3	4109	MGSROR[3]	Han and Chang (1995)

‡ the types of vehicle.
† the best known solution comes from the result of original VRP.

The computational results of GIDS can be compared with other well-performance heuristics. In Table 6, the computational results of six heuristics: MGT+Or, ROM, SRP, MGORSR, MGSROR, and GIDS, for all twenty instances of the FSMVRP are given. For each instance, the first column lists the number of instances, the second column gives the number of customers, the third column records the cost of the best known solution, and from the fourth to the ninth columns contain the results respectively for these heuristics expressed as a percentage difference from the best known solution. The computer CPU time spent in running GIDS program is recorded in the tenth column. Further, the final column updates the latest results of best known solution obtained by our proposed GIDS.

Table 6 shows that the average percentage difference among the twenty test instances of the proposed GIDS meta-heuristic is only 0.698 %, which yields the lowest average accuracy as compared with other five heuristics tested. Moreover, the standard deviation of the percentage error of GIDS is 1.143 %, and the worst deviation is less than 3.8 %. Overall, GIDS performed remarkably well than MGT+Or and ROM for the twenty benchmark instances, and presented almost similar quality to SRP, MGORSR and MGSROR for solving FSMVRP. It is worth noticing that the amount of instances which produced the best known solution by GIDS is eight, and that six of them update the best published results for instance 3, 7, 8, 10, 15, and 19. Detailed and the most updated results can be found on our research website of http://www.tem.nctu.edu.tw/~network.

Table 6. Comparison of Computational Results with Published Heuristics

No	Size	Best known solution	Difference (%)†							Updated best known solution
			MGT+Or	ROM	SRP	MGORSR	MGSROR	GIDS	(sec)*	
1	12	602	3.322	0.664	1.993	0.664	3.987	0.000	6.6	602
2	12	722	0.000	1.108	0.000	0.000	0.000	0.000	7.6	722
3	20	965	0.104	2.591	3.938	0.207	0.311	-0.415	27.5	961
4	20	6440	7.609	1.661	0.109	0.093	0.000	0.078	20.2	6440
5	20	1006	0.696	3.380	0.895	0.596	0.000	0.994	13.9	1006
6	20	6516	7.029	0.015	0.000	6.369	0.276	0.046	24.6	6516
7	30	7298	1.247	1.685	1.425	1.631	1.110	-0.069	106.4	7293
8	30	2349	0.766	1.618	0.766	0.639	0.639	-0.043	33.2	2348
9	30	2209	0.498	0.996	0.000	0.589	0.589	0.634	27.0	2209
10	30	2368	0.084	1.056	0.380	0.760	0.422	-0.042	21.5	2367
11	30	4763	0.000	2.079	1.176	2.037	0.189	0.336	72.7	4763
12	30	4092	1.075	3.960	0.000	0.122	0.073	0.367	58.2	4092
13	50	2438	0.000	3.568	2.256	0.451	0.451	2.912	173.1	2438
14	50	9132	0.000	0.252	0.230	0.909	0.821	0.329	144.7	9132
15	50	2615	0.956	0.268	0.306	0.000	0.956	-0.191	100.2	2610
16	50	2765	2.061	1.591	0.000	1.374	0.976	1.374	49.2	2765
17	75	1767	0.905	6.225	0.000	1.132	1.075	3.735	231.2	1767
18	75	2397	1.460	3.838	1.752	0.000	1.043	1.836	310.9	2397
19	100	8700	0.241	0.000	0.586	1.931	1.770	-0.138	772.7	8688
20	100	4109	2.093	3.383	1.898	0.170	0.000	2.215	656.5	4109
Average (%)			1.507	1.997	0.886	0.984	0.734	0.698	142.9	
Deviation (%)			2.170	1.634	1.052	1.417	0.906	1.143	212.4	
No. of best solutions			4	0	6	2	4	8		

† 'Difference' denotes the percentage of deviation between the best known solution (X*) and the best solution (X) found by specific heuristic. Difference (%) = $(X - X^*) \div X^* \times 100$.

* '(sec)' represents the CPU time in terms of second consumed while executing GIDS.

On the other hand, Table 6 shows the computer running time expanded on GIDS. Due to the diversity of computer equipment, the computational effort is considered as a reference. The average and standard deviation of CPU time for GIDS executing on the twenty instances are 142.9 and 212.4 seconds. The range of CPU time is from 6.6 to 772.7 seconds.

5. CONCLUSIONS

In this paper, we proposed a new meta-heuristic, generic intensification and diversification search (GIDS), which combined generic search methods and meta-strategies of intensification and diversification. GIDS was adapted for the fleet size and mixed vehicle routing problem (FSMVRP) by integrating two generic search methods, threshold accepting (TA) and great deluge algorithm (GDA), and flip-flop method (FF) into a special GIDS application.

The conceptual framework of GIDS mainly consists of three subsystems: local solution constructor (LSC), generic search for intensification (GSI), and perturbation search for diversification (PSD). Six modules, initial solution (IS), neighborhood search (NS), generic search one (G1), generic search two (G2), cost perturbation (CP), and weighted savings (WS), are designed to implement GIDS. Further, a modified saving formula, proportional usage savings (PUS) and a revised perturbation procedure (SRPX) were proposed. All of the modules were coded in UNIX C language and executed at a SPARC 10 SUN workstation. A bank of twenty FSMVRP instances is utilized to evaluate the performance of GIDS.

Computational results in terms of CPU time indicated that our proposed GIDS seems to perform more efficiently than most existing heuristic in solving FSMVRP. GIDS tested among the twenty instances produced an average percentage deviation of 0.698 % and updated best known solutions of six instances. The computer time ranged from 6.6 to 772.7 seconds. Thus, this results imply that GIDS seems to be an appropriate and powerful tool to solve FSMVRP.

Due to the well efficiency of computational test, the application of GIDS to practical logistics and transportation problems will become an vital research topic. In the future, several interesting directions are suggested as follows:

(1) Refinement of the general framework. Various strategies from other meta-heuristics can be properly added into the GSI and PSD subsystem to enhance the search function.
(2) Sensitivity analysis of GIDS. Experimental designs are necessary to validate the sensitivity of GIDS while altering the parameters, and adjusting the sequence or combination of distinct modules.
(3) Extension to other hard VRP related problem. The periodic vehicle routing problem (PVRP) and vehicle routing problem with time windows (VRPTW) are further considered.

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