EFFECTS OF STATE FACTORS ON ELASTIC PARAMETERS OF ANISOTROPIC SUBGRADE SOIL

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Abstract: Pavement materials are not only inelastic and nonlinear as commonly assumed but are also anisotropic. The latter means that their characteristics differ according to direction. The simplest form of material anisotropy is cross-anisotropy where the material has an axis of symmetry of rotation. Five elastic constants are required to describe cross-anisotropic material. Due to the complexity of the problems, there have been few attempts to measure the true properties of anisotropic pavement materials. A practical approach introduced by Graham and Houlsby enabled determination of the five elastic constants required to describe material anisotropy. The technique required the use of a repeated loading cubical-true-triaxial test apparatus. The technique was applied on a selected subgrade material. The experimental design allowed three levels of density, two levels of saturation, and two methods of compaction to be analysed statistically. The results allowed the identification of the effects of physical factors to the elastic parameters.

1. INTRODUCTION

The complexity of road pavements has necessitated the simplification of the mathematical models to describe the system. The models represent an idealization of the real pavement. Linear elastic theory, being the simplest approach, has been used extensively for pavement analysis. Current advancement in computers and numerical techniques have enabled increasingly realistic models of material behavior to be incorporated into the solutions of boundary value problems. Solutions, based on a simple linear isotropic elastic have been supplemented by solutions including inelasticity and nonlinearity.

In reality, however, pavement materials are not only inelastic and nonlinear but are also anisotropic. The latter means that their characteristics differ according to direction. The simplest form of material anisotropy is cross-anisotropy where the material has an axis of symmetry of rotation. The elastic properties of such a material are equivalent in all directions perpendicular to the axis of symmetry but differ, in general, from those in a direction parallel to the axis.

Australia probably is the first country in the world to adopt material anisotropy in its national road pavement design (NAASRA 1987). The policy was based on a fundamental assumption that all unbound materials are cross-anisotropic. This policy was stimulated by development of a computer program, CIRCLY, by the Commonwealth Scientific and Industrial Research Organisation of Australia (CSIRO) (Wardell 1976). The program enables the analysis of the response of multilayered cross-anisotropic media under multiple complex circular loads to be calculated.

Since it was firstly developed in 1987 many engineers became accustomed to assume that unbound materials are anisotropic. Nevertheless, there have been few attempts to measure the true properties of anisotropic pavement materials. Meanwhile, for an unclear reason, Australia seemed to confined to an assumption that all unbound materials to have a degree of anisotropy of two.

The work described in this paper was intended to address this related problem. It embraced a study on the effects of physical state conditions on the cross-anisotropic elastic behavior of typical subgrade material.

2. MATHEMATICAL MODEL AND LABORATORY PROCEDURE

The constitutive equation of an elastic cross-anisotropic material in a three-dimensional space may be written as:

$$\begin{bmatrix} \delta\sigma_1 \\ \delta\sigma_2 \\ \delta\sigma_3 \\ \delta\tau_{12} \\ \delta\tau_{13} \\ \delta\tau_{23} \end{bmatrix} = \begin{bmatrix} A & B & B & & & \\ B & C & D & & & \\ B & D & C & & & \\ & & & F & & \\ & & & & F & \\ & & & & & C-D \end{bmatrix} \begin{bmatrix} \delta\varepsilon_1 \\ \delta\varepsilon_2 \\ \delta\sigma\varepsilon_3 \\ \delta\gamma_{12} \\ \delta\gamma_{13} \\ \delta\gamma_{23} \end{bmatrix} \tag{1}$$

The elasticity stiffness matrix consists of five independent elastic constants (ie. A, B, C, D, and F). In order to be able to determine the five elastic constants, a laboratory test apparatus should have the capability to vary all three of the normal stresses as well at least two shear stresses. None of the currently existing apparatus is capable of fulfilling the requirements to fully characterised cross-anisotropic materials such as that given by Equation (1) unless assumptions are made regarding further relationships between the five elastic constants. This has been discussed in detail elsewhere (Zamhari 1998a).

Graham and Houlsby (1983) proposed a model that considers cross-anisotropy to be a special case of isotropy, by multiplying the stiffness coefficients in the horizontal direction with an anisotropy factor α. This is followed by multiplying the second and third columns by the same factor to fulfill the requirement of symmetrical matrix. This implies that the equation for isotropic material under a cubical true triaxial test (in which three normal stresses can be varied independently but no shear stresses applied) becomes:

$$\begin{bmatrix} \delta\sigma_1 \\ \delta\sigma_2 \\ \delta\sigma_3 \end{bmatrix} = \begin{bmatrix} A & \alpha B & \alpha B \\ \alpha B & \alpha^2 A & \alpha^2 B \\ \alpha B & \alpha^2 B & \alpha^2 A \end{bmatrix} \begin{bmatrix} \delta\varepsilon_1 \\ \delta\varepsilon_2 \\ \delta\varepsilon_3 \end{bmatrix} \tag{2}$$

An extension of Eq. (2) into Eq. (1) implies an interrelation between the five independent elastic constants,

$$[D/B]^2 = C/A = [(C-D)\ F]^2 = \alpha^2 \tag{3}$$

The value of α determines the material anisotropy. That is, the material is isotropic if $\alpha = 1$; it is stiffer horizontally if $\alpha < 1$; and it is stiffer vertically if $\alpha > 1$.

A substitution of the interrelationship (3) into the compliant matrix of the Generalised Hook's law for cross-anisotropic material in engineering terms established the following relationship:

$$
\begin{aligned}
E_v &= E^* \\
E_h &= \alpha^2 E^* \\
\upsilon_{vh} &= \upsilon^*/\alpha \\
\upsilon_{hh} &= \upsilon^* \\
2G_{vh} &= \alpha E^*/(1+\upsilon)
\end{aligned}
\tag{4}
$$

Eq. (4) shows that the five elastic constants, E_v, E_h, υ_{vh}, υ_{hh}, and G_{vh}, are now reduced to E^*, υ^*, and α.

For a cubical true-triaxial, the compliant matrix of Eq. (3) is,

$$
\begin{bmatrix} \delta\varepsilon_1 \\ \delta\varepsilon_2 \\ \delta\varepsilon_3 \end{bmatrix} = \frac{1}{E^*} \begin{bmatrix} 1 & -\frac{\upsilon^*}{\alpha} & -\frac{\upsilon^*}{\alpha} \\ -\frac{\upsilon^*}{\alpha} & \frac{1}{\alpha^2} & -\frac{\upsilon^*}{\alpha^2} \\ -\frac{\upsilon^*}{\alpha} & -\frac{\upsilon^*}{\alpha^2} & \frac{1}{\alpha^2} \end{bmatrix} \begin{bmatrix} \delta\sigma_1 \\ \delta\sigma_2 \\ \delta\sigma_3 \end{bmatrix}
\tag{5}
$$

This expression is a three-unknown-in-three-equation problem. It suggests that a cubical true-triaxial test is the ideal apparatus for determining the three unknown elastic parameters.

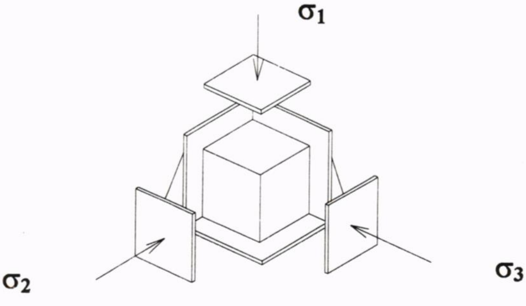

Figure 1. Arrangement of the Repeated Loading True Triaxial Test Loading Platens

A new repeated-loading-cubical-true-triaxial-test equipment was developed for this experiment. The system employed only three platens loaded by three mutually-perpendicular pneumatic jacks. Three reaction walls were placed in the opposite side of each loading platen. Figure 1 presents the basic principle of loading arrangement. The equipment consisted of an advanced pneumatic loading system which was controlled through a personal computer. The system provided a versatile control of forces in three perpendicular directions, enabling a good, if incomplete, simulation of the in-situ stress conditions in pavement. In brief the whole operation of the true-triaxial test equipment may be described as follows:

The required load histories were programmed into the computer which then generated the required electronic command signal for input to the Force Control Module. The loads were then applied to the test specimen by pneumatic actuators which were controlled precisely by a closed-loop servo system incorporating the electric servo-valves, the load cells, the Force Control Module and the Control and Data Acquisition System unit. As the loads were applied, the test specimen was deformed. The deformations of each surface of the test specimen were sensed by the displacement transducers. The electronic signals from the load cells and deformation transducers were amplified and then measured by the Control and Data Acquisition Unit. The data, in digital format, was collected and stored on the hard disc by the personal computer. During the operation, a real-time display of the selected measurements was generated to enable the operator to monitor the test progress. Details of the design and features of this equipment have been presented elsewhere (Zamhari 1998b).

3. DESIGN AND EXECUTION OF THE EXPERIMENTAL WORK

3.1 Material Selection

Subgrade soils play an important role in road pavement performance. The support that is provided by the subgrade determines the structure of the upper part of pavement. In a traditional method of pavement design, known as the CBR method, the thickness of the overlaying pavement structure is determined by the strength (i.e. the CBR value) of the subgrade soil. In many mechanistic design procedures, one of the critical responses that must be assessed for pavement design or evaluation is the compressive strain occurring at the top of the subgrade soil. Therefore, it was needed to study the characteristics of subgrade soils. Furthermore, it is generally accepted that over a large area of the world, subgrade soils exist in partially saturated conditions. For this reason the experimental work was restricted to specimens prepared to unsaturated conditions. As a consequence, the stress used in this experimental work is expressed in terms of the total stress, not effective stress.

The subgrade soil used in this study is a low plasticity, A-6 clay with Plastic Limit of 20% and Liquid Limit of 39%. The dry density/moisture content relationship for this soil at the standard and modified compaction is presented in Figure 2.

3.2 Independent Variables

The elastic response of subgrade soils is significantly affected by two main factors, namely, the stress state and the physical conditions. As a consequence, any description of the behavior of subgrade soils should refer to both conditions. This paper is restricted to the later. Three factors were studied. These were the saturation, the initial density and the method of compaction. The investigation was conducted for a single stress state (τ_{oct}= 60 kPa, k = 0.1768 and ζ = -0.50). Two methods of compaction were utilised in this experiment. The first was static compaction in which the soil was placed in the mould and subjected to a static pressure on the entire soil surface. The second was dynamic compaction similar to the laboratory procedure for standard and modified compaction (AASHTO T-99 and T-180).

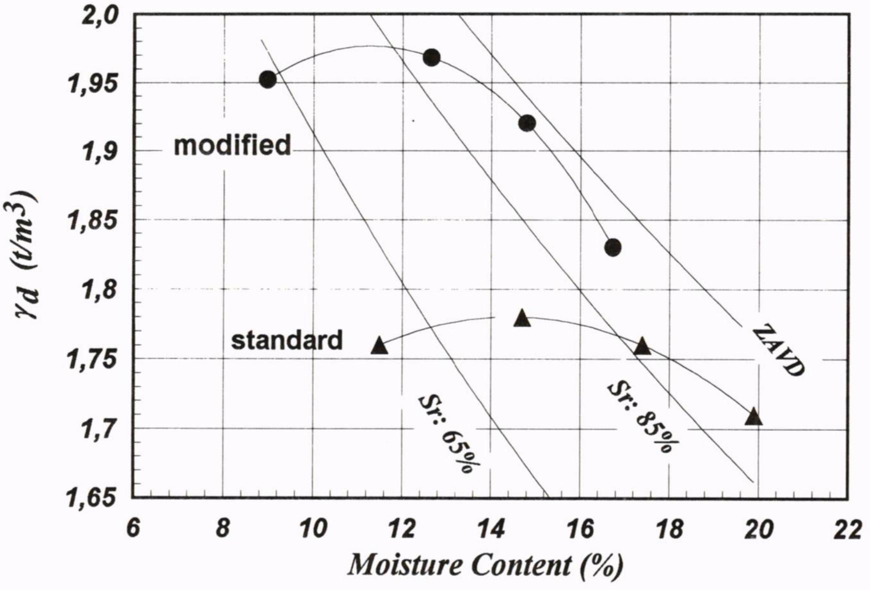

Figure 2. Moisture/density Relationship of Soil Sample

Initially, the design of the experiment for the effects of state and environmental conditions was a 3^2 factorial experiment. The factors to be studied were the methods of compaction, the densities and the saturations. There were two levels of density, related to the maximum densities at the standard and modified compactions respectively. These were densities, γ_d of 1.78 $^t/_m{}^3$ and 1.98 $^t/_m{}^3$. The two levels of the degree of saturation, Sr, were chosen to be 65% and 85%. For the standard compaction test specimens, the degree of saturation of 65% was related to the 12.50% moisture content. This degree of saturation was on the dry side of the optimum. The 85% degree of saturation was related to the dry-wet side optimum standard compaction and close to the optimum of the modified compaction.

Later, five more points were added. They were, Sr = 97% at γ_d = 1.98 $^t/_m{}^3$ (i.e. wet of optimum for modified density), Sr = 75% at γ_d = 1.78 $^t/_m{}^3$ (i.e. optimum for standard density), and γ_d = 1.88 $^t/_m{}^3$ at $Sr.$ = 65%, 80% and 85% respectively. The latter were the points lying midway between the standard and modified compaction.

3.3 Dependent Variables

Because the true-triaxial test apparatus used in this experiment was a stress-controlled type of equipment, the dependent variables were the three elastic principal strains, ε_1, ε_2 and ε_3.

From these basic variables, the five elastic parameters for cross-anisotropic material were determined by substitution of the applied principal stresses and the measured elastic principal strains into the equations described above.

3.4 Repeated Loading Testing Procedure

After curing for at least 24 hours, the test specimen, being wrapped in one layer of a thin plastic membrane, was placed in the specimen chamber of the cubical true-triaxial apparatus. The vertical axis of the test specimen was the same as the direction of the compaction during the moulding process. During the placement into the loading chamber, this vertical axis was arranged to coincide with the vertical loading plate. The vertical loading plate was set to impose the major principal stress. Thus the test specimen was subjected to major principal stress in its vertical position for the whole test series. It is believed that this arrangement is a reflection of in-situ stress condition.

Before the design stress state was set, the test specimen was subjected to 50 isotropic principal stresses applications of 50 kPa. The purpose of this conditioning load was twofold. Firstly, it functioned as the sample and equipment conditioning. Secondly, the elastic strains measured during this stage indicated the stiffness of the specimen in the three directions. The latter was important for determining which horizontal actuator was to act as the intermediate and which one as the minor principal stresses.

4. EXPERIMENTAL RESULTS AND DISCUSSION

The effects of the initial density and saturation on the elastic parameters are presented in Table 1. The design of experiment allowed three levels of density and two levels of saturation (Sr = 65% and 85%) to be analysed statistically. Typical analysis of variance of this data is summarised in Table 2. The effects of each variable are considered in detail in the following section.

Table 1. The Effects of Material State on Elastic Parameters at 10,000 Stress Applications

State Conditions		Static Compaction			Dynamic Compaction		
γ_d ($^t/_{m^3}$)	Sr (%)	n (Ev/Eh)	Ev (MPa)	υ_{vh}	n (Ev/Eh)	Ev (MPa)	υ_{vh}
1.78	65	1.39	135	0.444	1.36	174	0.428
	75	1.20	127	0.390	1.14	122	0.363
	85	1.36	112	0.446	1.09	115	0.348
1.88	65	1.16	215	0.334	1.33	182	0.451
	80	1.08	177	0.395	1.34	175	0.468
	85	1.12	180	0.412	1.24	164	0.448
1.98	65	1.23	286	0.392	1.34	270	0.426
	85	1.18	331	0.393	1.37	287	0.492
	97	1.01	210	0.39	1.33	215	0.506

4.1 Effects of the number of stress applications

The elastic strains were found to be relatively unaffected by an increase in the number of stress applications. A tendency for increase in the major principal strain was found at relatively high saturation, this is mostly related to static compaction. However, these variations were small. After about 2000 stress applications the strains were practically constant with a further increase in the number of stress applications. As a consequence, it may be postulated that the effects on the elastic parameters were not affected in the long term by increasing in the number of stress applications. This is confirmed by an analysis of variance, as summarised in Table 2. This table shows that the effects of the number of stress applications, either as a single factor or in terms of a two-way interaction between the states factors, were statistically insignificant. An evaluation of the results as a whole showed that, within the domain of the number of stress applications, the largest differences between the maximum and minimum values of E_v, υ_{vh}, and the degree of anisotropy, n, were 20%, 15% and 11% respectively. On average, the differences were not more than 7%. It was therefore considered reasonable to limit the observation to the condition related to the highest number of stress applications (i.e. N = 10,000 stress applications).

Figure 3 presents typical test result on the effects of material state on the elastic constants at 10,000 stress applications at the saturation, Sr, of 85%.

Table 2. Typical Analysis of Variance on Effects of Initial State and Number of Stress Applications on Elastic Parameters. *Dependent Variable: Degree of Anisotropy, n (= Ev/Eh)*

Source of Variation	Sum of Squares	DF	Mean Square	F	p(F)	Significant at (%)
Variables block	0.234	8	0.029	19.137	0.000	1
Compaction Method (C.M)	0.013	1	0.013	8.450	0.007	1
γ_d	0.184	2	0.092	60.152	0.000	1
Sr	0.036	1	0.036	23.261	0.000	1
N (Stress Rept.)	0.002	4	0.000	0.271	0.894	NS
2-way interactions	0.477	21	0.023	14.885	0.000	1
C.M - γ_d	0.313	2	0.157	102.520	0.000	1
C.M – Sr	0.005	1	0.005	3.422	0.074	10
C.M - N	0.002	4	0.001	0.389	0.815	NS
γ_d - Sr	0.135	2	0.068	44.235	0.000	1
γ_d - N	0.018	8	0.002	1.479	0.206	NS
Sr - N	0.003	4	0.001	0.566	0.689	NS
Explained	0.711	29	0.025	16.058	0.000	1
Residual	0.046	30	0.002			
Total	0.757	59	0.013			

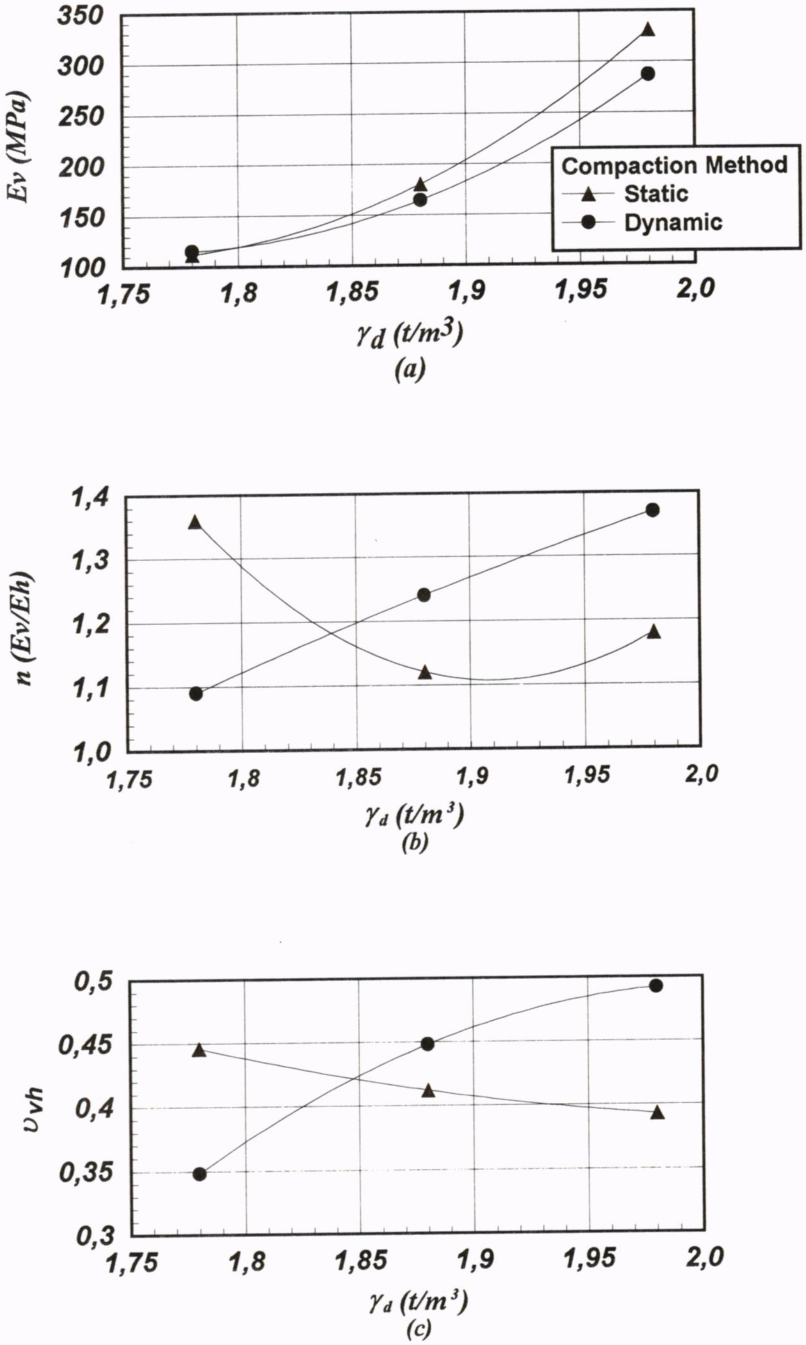

Figure 3 Effects of Material State on Elastic Parameters at Sr = 85%

4.2 Effects of Initial Density

Within the range of the experimental saturation, the stiffness in the vertical direction, E_v, tended to increase with an increase in the initial density irrespective of the methods of compaction. The effects of the initial density on the Poisson's ratio were dependent on both the methods of compaction and the degree of saturation.

The test specimen prepared under dynamic compaction and a relatively high degree of saturation had a Poisson's ratio which tended to increase with an increase in the initial density. By contrast, irrespective of the degree of saturation, those samples prepared under static compaction had a Poisson's ratio which decreased with an increase in the initial density.

The effects of the initial density on the degree of anisotropy, n, were similar to those of the Poisson's ratio, υ_{vh}. That is, n tended to decrease with an increase in the initial density for the sample prepared under static compaction, and tended to increase for the sample prepared under dynamic compaction at a higher degree of saturation. This phenomenon was probably reflected the interrelated effects of the density, the methods of compaction and the degree of saturation on the orientation of soil particles which, in turn, determined the degree of anisotropy.

4.3 Effects of Saturation

Regardless the method of compaction, the vertical stiffness, E_v, tended to decrease with an increase in the degree of saturation for almost all levels of initial density.

By contrast, the effects of saturation on the Poisson's ratio, υ_{vh}, were dependent on the method of compaction and the dry density. Within the saturation domain, the Poisson's ratio, υ_{vh}, of statically compacted specimens (to standard and intermediate densities) tended to increase with an increase in the degree of saturation. The test specimens prepared by dynamic compaction (to standard and intermediate densities) had Poisson's ratios, υ_{vh}, which decreased with an increase in the degree of saturation. However, at the maximum modified dry density, the Poisson's ratio remained constant with a variation of the saturation regardless of the methods of compaction. Also the degree of anisotropy, n, tended to decrease with an increase in saturation irrespective of the methods of compaction.

4.4 Effects of the Method of Compaction

The experimental results showed a very strong similarity for the variation of the vertical stiffness, E_v, with the density between the two methods of compaction. A closer observation suggested that it was possible to expect a higher vertical stiffness for statically compacted soil, as was reported by others. However, the variation was so small that it could not reflect in the analysis. Furthermore, the associated stress level required to yield such a condition was perhaps below the stress state which was used in this experimental work.

A distinction should be made between the test specimens prepared at different saturations, regarding the effects of the methods of compaction on the Poisson's ratio, υ_{vh}. At relatively low saturations, for a given density, the Poisson's ratio, υ_{vh}, approached similar values for both methods of compaction. By contrast, at a higher saturation, the specimens prepared by dynamic compaction had a Poisson's ratio, υ_{vh}, which increased with an increase in density, whilst the specimen prepared by static compaction had a Poisson's ratio, υ_{vh}, which increased with an increase in the initial density.

The effects of the methods of compaction on the degree of anisotropy, n, were similar to the effects on the Poisson's ratio, υ_{vh},. This meant, very small effects at low saturations and marked effects at high saturations. At high saturations, the static compaction tended to yield an increase in the degree of anisotropy with an increase in density. By contrast, the dynamic compaction tended to yield a decrease in the degree of anisotropy with an increase in density.

No direct observation of the orientation of the clay fabric was made in this experimental work. However, it is believed that the variation of the elastic response of test specimens for different methods of compaction may be attributed to differences in the clay fabric of the soil, caused by different methods of compaction.

5. CONCLUSIONS

The experimental work reported in this paper concerned the development of the characterisation procedure of anisotropic pavement materials. The work was focussed on cross-anisotropic subgrade soils. One of the aims of the experiment was to study the effects of the physical conditions on the elastic behavior of cross-anisotropic subgrade soils. The effects were studied under a constant stress state.

The Graham and Houlsby elastic-cross-anisotropic matrix equation was established by multiplying the stiffness coefficients of an isotropic material by the factor of anisotropy, α, to modify stiffness in one direction relative to the others. This approach reduced the number of the cross-anisotropic elastic parameters from five to three, i.e. by adding a new factor, α, to the other two isotropic elastic parameters. The 3x3 matrix equation of principal stresses allowed the cubical true-triaxial test the ideal apparatus for determining the elastic cross-anisotropic parameters. The proposed matrix equation had been proven to yield admissible cross-anisotropic elastic parameters.

Because more than one factor was studied, the work was designed to be a factorial experiment. An analysis of variance was used to examine the results. The analysis was applied, essentially, as a guide to the reliability of the conclusions derived from the experiments. It was demonstrated that this approach permits the examination of the relative significance of the effects of various physical state factors on the elastic parameters required to describe cross-anisotropic material.

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LIST OF PRINCIPAL NOTATIONS

E^* : modified Young's Modulus

E_h : modulus of elasticity in the horizontal direction

E_v : modulus of elasticity in the vertical direction

G_{vh} : shear modulus of cross-anisotropic material

G^* : modified shear modulus

k : octahedral stress ratio, $k = \dfrac{\delta\tau_{oct}}{\delta\sigma_{oct}}$

n : degree of anisotropy, $n = \dfrac{E_v}{E_h}$

$\delta\varepsilon_i$: incremental recoverable principal strain; $i = 1, 2, 3$

$\delta\sigma_i$: incremental principal stress, $i = 1, 2, 3$

$\delta\sigma_{oct}$: incremental octahedral normal stress,

$$\delta\sigma_{oct} = \frac{(\delta\sigma_1 + \delta\sigma_2 + \delta\sigma_3)}{3}$$

$\delta\tau_{oct}$: incremental octahedral shear stress,

$$\delta\tau_{oct} = \sqrt{\frac{(\delta\sigma_1 - \delta\sigma_2)^2 + (\delta\sigma_2 - \delta\sigma_3)^2 + (\delta\sigma_3 - \delta\sigma)^2}{3}}$$

υ^* : modified Poisson's ratio

υ_{hh} : Poisson's ratio, effect of horizontal strain on complimentary horizontal strain

υ_{vh} : Poisson's ratio, effect of vertical strain on horizontal strain

υ_{vh} : Poisson's ratio, effect of vertical strain on horizontal strain

ζ : Nadai-Lode stress parameter (in terms of incremental principal stresses)

$\zeta = (2\delta\sigma_2 - \delta\sigma_1 - \delta\sigma_3) / (\delta\sigma_1 - \delta\sigma_3)$