

MULTI-OBJECTIVE PROGRAMMING FOR PAVEMENT MANAGEMENT USING GENETIC ALGORITHMS

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Abstract

The need for actual multi-objective optimization in Pavement Management Systems (PMS) has been recognized by many practitioners. Due to the difficulty in formulation and lack of appropriate analytical tool, researchers and practitioners has been relying on either traditional single-objective optimization or some form of ad hoc techniques to handle multiple objective functions. Genetic Algorithms (GA), an artificial intelligence technique, has been found to be a robust and suitable algorithm for multi-objective optimization. This paper presents detailed description of a GA multi-objective optimization algorithm developed at the National University of Singapore. A rank-based fitness assignment technique and the concept of Pareto optimality were used in the formulation. The proposed methodology has been demonstrated with an example pavement maintenance problem. Various sets of multi-objectives including two-objective, three-objective and four-objective formulations are studied.

1. INTRODUCTION

In Pavement Management Systems (PMS), maintenance and rehabilitation programs are traditionally scheduled based on single-objective optimization. The conventional single-objective optimization techniques such as Linear Programming (Lytton 1985), Dynamic Programming (Feighan *et al.* 1987, Li *et al.* 1995), Integer Programming (Fwa *et al.* 1988) have been used. The difficulty in modeling and formulation, and lengthy computation time are the main reasons that limit the usage of such models. This situation becomes worse when of multiple objectives are involved.

Though the existence of various conflicting objectives in PMS programming is recognized, there is no significant research reported in the literature to address the implications and procedures of considering these objectives simultaneously. Some of the common objectives are to attain minimum overall maintenance costs, highest level-of-service, minimum safety hazards, maximum available resource utilization and minimum disruption to normal traffic flows, etc. Any maintenance policy established based on a particular

objective often overlooks or downgrades the importance of other objectives. For example, the objective of minimizing maintenance costs would find a policy that requires the least cost compared to other policies, but most likely this least-cost policy would not provide the highest level-of-service. Similarly, the highest level of service policy would cost more than the least-cost policy. To deal with such conflicting objectives, simultaneous multiple-objective optimization is necessary.

A robust search technique known as Genetic Algorithms (GA), a branch of artificial intelligence, formulated based on natural genetics (Holland 1975), has recently been found as a suitable algorithm for single-objective optimization of PMS problems (Fwa *et al.* 1994a, Fwa *et al.* 1994b, Fwa *et al.* 1996). GA is also capable to accommodate multiple objectives (Goldberg 1989) of any form of linear, non-linear or any other complex expressions. The authors have successfully developed a multi-objective optimization GA scheme to solve a pavement management programming problem (Fwa *et al.* 1999). This paper provides the detailed background to the development work, and gives illustrative examples to demonstrate its application. The methodology is tested with an example PMS problem. The study covers the case of two-objective, three-objective and four-objective optimization.

2. APPLICATION OF GA TO MULTI-OBJECTIVE OPTIMIZATION

2.1 Handling of Multiple Objective Functions

Conventional optimization techniques cannot be easily extended to true multi-objective optimization mainly because they were not designed to handle multiple solutions (Fonseca and Fleming 1995). On the other hand, genetic algorithms have been identified as one with good application potential for multi-objective optimization (Fonseca and Fleming 1995). There are four GA based approaches generally adopted for multi-objective optimization: (a) Plain aggregate approach, (b) Population-based non-Pareto approach, (c) Pareto-based approach, and (d) Niche induction technique.

(a) Plain aggregate approach: This approach is similar to other scalar fitness function, in which objectives are weighted and combined to form a scalar function. The advantage of this system is that it produces a compromised solution which requires no further interaction from the decision maker. In the event that the compromised solution is not acceptable to the decision maker, the program has to be re-run to find another compromised solution. In addition to this, there is no unique guideline for the combination.

(b) Population-based non-Pareto approach: This was first introduced by Schaffer (1985) where he termed the technique as Vector Evaluated Genetic Algorithm (VEGA). In VEGA, GA solutions are divided equally and assigned to each objective separately for evaluation to select fitter solutions. Next, these all groups of fitter solutions are combined, crossover and mutation operator are applied to produce new solutions. VEGA performs implicit combination of objectives which leads to split the population into species, a phenomenon known as speciation (Schaffer 1985). It has noted by Fleming and Pashkevich (1985) that the points of a trade-off surface in concave surface cannot be found by optimizing the linear combination of objectives.

Although this approach does not use the concept of Pareto-optimality and also not well-suited for problems with concave trade-off surface, VEGA has been found to produce better results than conventional methods.

- (c) **Pareto-based fitness assignment technique:** This technique was first proposed by Goldberg (1989) where non-dominated solutions are identified and an equal probability of reproduction is applied to each non-dominated solution. Fonseca and Fleming (1993) proposed a rank-based fitness assignment technique for defining Pareto optimality. The rank-based fitness assignment technique has two advantages (i) Pareto-ranking is blind to the convexity or the non-convexity of the trade-off surface and (ii) it rewards solutions with good performances with respect to all objectives (Fonseca and Fleming 1995). This is a promising technique for multi-objective optimization and therefore, this approach is adopted in this paper. The concept of Pareto-optimality and fitness assignment is discussed in later part of this paper.
- (d) **Niche induction technique:** In the Pareto-based approach all non-dominated solutions receive equal probability of sampling but that does not guarantee that they may be uniformly sampled. Fonseca and Fleming (1995) noted that finite populations with the multiple equivalent optima tend to converge to only one of them due to what is known as genetic drift. To overcome the genetic drift, Goldberg (1989) proposed fitness sharing scheme known as Niche Induction Technique. The goal of fitness sharing is to distribute the solutions along the Pareto optimal frontier.

2.2 Concept of Pareto-optimality

Unlike single-objective optimization, the solution to the multi-objective problem is not a single point but a family of points of non-dominated solutions. The set of non-dominated solutions is known as the Pareto set. For a two-objective problem, this Pareto set can be on a curve which is known as Pareto front while in the case of three-objective problem, it would be a surface. The whole purpose of multi-objective programming is to find the global Pareto front from where decision maker would select a maintenance policy. The solutions in the Pareto set provide an insight into the characteristics of the problem before a final decision is made. In the GA process, the optimization process continues until the globally non-dominated solutions are obtained. This global Pareto set is known as the Pareto optimal set. In this study, by the introduction of improved algorithm (discussed later) and avoiding any duplication of solutions, no genetic drift is observed. The improved algorithm works well enough to prevent any genetic drift for the optimization process.

3. EXAMPLE PROBLEM

3.1 Description of Problem

A network level pavement maintenance scheduling problem solved by Fwa *et al.* (1998) using the plain-aggregate approach is considered for the development and analysis of multi-objective programming. This classic pavement maintenance problem was initially solved by Fwa *et al.* (1988) using integer programming. Later on, GA was employed (Fwa *et al.* 1994a) to solve the problem. It includes resource constraints, rehabilitation constraints, highway classes, severity distress levels, priority to each repair activity and

more importantly, several objectives can be derived for the problem. Interested readers are referred to the literature (Fwa *et al.* 1988) for in-depth knowledge of the problem. Only a few aspects of the problem are discussed here.

There are five constraints, namely budget constraint, manpower availability, equipment availability, production constraints and rehabilitation constraints. The example problem considers four highway classes (I, II, III and IV), four repair activities (A, B, C and D) for defects and three need-urgency levels (high, medium and low). The necessary input data are recorded in Table 1 to Table 3.

Table 1 Resource Requirements

(a) Required Workdays for Repair

Highway Class	Repair Activity			
	A {Need Urgency: high, medium, low}	B {Need Urgency: high, medium, low}	C {Need Urgency: high, medium, low}	D {Need Urgency: high, medium, low}
I	{4, 6, 3}	{6, 4, 25}	{8, 2, 13}	{2, 3, 18}
II	{2, 2, 4}	{6, 10, 20}	{9, 8, 15}	{2, 8, 15}
III	{5, 5, 5}	{8, 2, 15}	{6, 10, 15}	{5, 10, 10}
IV	{3, 4, 15}	{4, 16, 15}	{8, 12, 18}	{4, 20, 15}

(b) Required Man-days

Repair Activity	Required Manpower (Man-days/Production Day)			
	Supervisor	Driver	Labourer	Equipment
A	0	2	4	0
B	1	1	5	1
C	1	3	5	2
D	1	2	2	4

(b) Required Man-days

Repair Activity	Required Equipment (Equipment-days/Production Day)					
	Dump Truck	Pickup Truck	Crew Cab	Distributors	Loaders	Rollers
A	1	0	1	0	0	0
B	1	1	0	0	0	1
C	3	1	1	1	0	1
D	2	1	0	1	0	0

Table 2 Repair Priority and Rehabilitation Constraints

Highway Class & Other Parameters		Repair Activity			
		A {Need Urgency: high, medium, low}	B {Need Urgency: high, medium, low}	C {Need Urgency: high, medium, low}	D {Need Urgency: high, medium, low}
I	Priority	{90, 63, 54}	{100, 90, 60}	{70, 63, 42}	{50, 45, 30}
	Rehabilitation Factors	{0.82, 0.7, 1.0}	{0.83, 0.9, 1.0}	{1.0, 0.9, 1.0}	{0.8, 1.0, 1.0}
II	Priority	{72, 54, 45}	{80, 70, 50}	{56, 49, 35}	{40, 35, 25}
	Rehabilitation Factors	{0.93, 0.84, 0.81}	{1.0, 1.0, 1.0}	{1.0, 1.0, 1.0}	{0.92, 0.96, 0.9}
III	Priority	{76.5, 58.5, 40.5}	{85, 75, 65}	{59.5, 52.5, 31.5}	{42.5, 37.5, 22.5}
	Rehabilitation Factors	{0.92, 0.78, 0.8}	{1.0, 1.0, 1.0}	{1.0, 1.0, 1.0}	{0.83, 0.91, 0.96}
IV	Priority	{70.5, 36, 18}	{65, 40, 20}	{45.5, 28, 14}	{32.5, 20, 10}
	Rehabilitation Factors	{1.0, 1.0, 1.0}	{1.0, 1.0, 1.0}	{1.0, 1.0, 1.0}	{1.0, 1.0, 1.0}

Table 3 Work Production, Costs and Resource Information**(a) Work Production and Costs**

Maintenance Activity	Production Rate			Unit Cost		
	Need Urgency Level			Need Urgency Level		
	Low	Medium	High	Low	Medium	High
Shallow Patching (kg/day)	6,537.6	3,813.6	2,542.4	0.0938	0.1311	0.1751
Deep Patching (kg/day)	17,978.4	9,443.2	6,174.4	0.0852	0.1333	0.1817
Premix Leveling (kg/day)	10,896.0	80,448.8	49,940.0	0.0403	0.0420	0.0467
Crack Sealing (km/day)	10.1	13.5	16.4	81.37	70.19	63.98

(b) Resource Availability

Parameter	Value
Analysis Period	45 working days
Budget Allocation	\$18,000 (shallow patching), \$20,000 (deep patching), \$13,000 (premix leveling), \$9,000 (crack sealing)
Manpower Availability	90 man-days (supervisors), 135 man-days (drivers), 270 man-days (laborers), 90 man-days (operators)
Equipment Availability	135 days (dump trucks), 45 days (pickup trucks), 45 days (crew cabs), 45 days (distributors), 45 days (loaders), 45 days (rollers)

3.2 GA Coding and Operation

Coding of problem parameters is a key element in GA optimization because GA does not work on the problem itself directly but on the coded parameters. There are altogether 48 decision variables, i.e., maintenance decisions in equivalent workdays unit. The decision variables are integer numbers and therefore, integer coding is adopted for the example

problem. Each decision variable is located at the gene and all the genes constitute a chromosome, and the number of genes is represented by the string length or length of chromosome.

GA analysis starts with generating a pool of initial solutions randomly. These solutions are evaluated against the predefined objective functions and based on their evaluation values artificial fitness are assigned. Next, generation of solutions (offspring) is created from the best of the last generation through reproduction and mutation operations. The creation of offspring and evaluation is continued till the stopping condition is met.

The efficiency of genetic algorithm, however, depends largely on how constraints are modeled. GAs tend to produce a number of invalid offspring during offspring generation process. There are several techniques to handle various constraints. One of them is Decode and Repair Algorithm. In this method, initial GA solutions are generated such a way that all of the constraints are satisfied and offspring are checked against each of the constraints and if any constraint violation is identified, the invalid solution is repaired by a repair algorithm. This study employed an decode and repair algorithm for the example problem.

3.3 PMS Problem with Two Objectives

Two-objective problem is the simplest form of multi-objective optimization problem because the problem parameters can be easily analyzed and visualized graphically. The problem with more than two objectives is difficult to analyze due to the involvement of too many attributes and dimensions. In order to explain GA operations for multi-objective optimization, the example problem is solved for two objectives initially. The two objectives are (a) to maximize the maintenance work production in workday units and (b) to minimize the total maintenance cost. The maintenance work production is calculated as the sum of the number of equivalent workdays for each maintenance multiplying with associated priority weights. The total maintenance cost is computed by the unit price listed in Table 1 and the number of workdays for each maintenance activity.

3.3.1 Fitness Assignment

When the two objectives are optimized simultaneously, the solutions in the pool spread all over the two-dimensional space as shown in Fig. 1. It is obvious from the figure that none of the solutions from the pool may be chosen without having an artificial fitness for each solution. This is where multi-objectives optimization differ from single-objective optimization. In single-objective optimization, fitness are computed by the objective function values of that objective only while for multi-objective optimization, all the objective function values are to be accounted for in fitness assignment.

Goldberg (1989) proposed the first technique for fitness assignment. This method works by assigning rank 1 to all the non-dominated solutions and removing them from the contention, and then finding a new set of non-dominated solutions and ranking them as rank 2, and so forth. Fonseca and Fleming (1993) proposed a slightly different approach where the rank of each solution is determined by the number of solutions in the pool that dominates the subject solution. Those solutions that are not dominated by any other

solutions have a rank of 1. The solutions with rank 1 is the non-dominated solutions. Equation (1) can be used for the rank calculation.

$$R(S_i) = 1 + S_n \quad (1)$$

where, $R(S_i)$ = rank of solution S_i in any particular pool;
 S_n = number of solutions that dominate S_i in that pool.

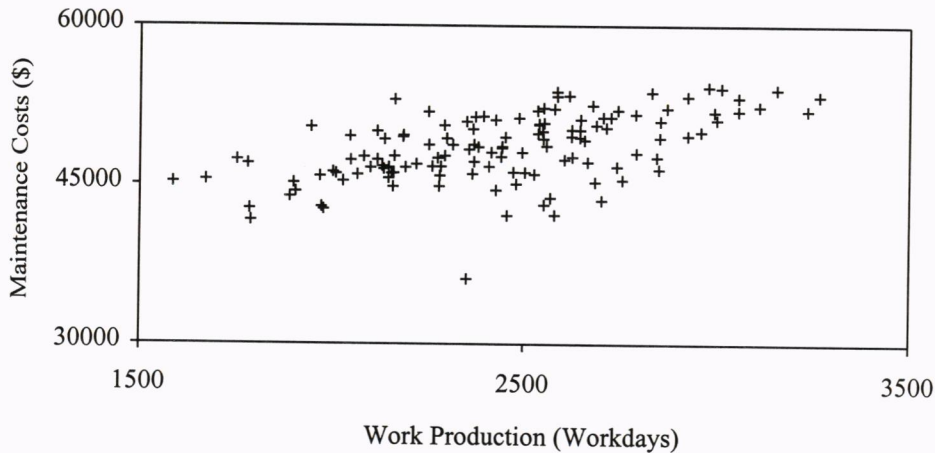


Fig. 1 Distribution of GA solutions on two-dimensional plane

Fig. 2 plots the solutions with rank 1 to rank 10. According to the direction of objective functions, the solutions that require less costs but produce better work production are preferable. It should be noted that this rank-based fitness changes after every GA iteration because once new solutions come in the fresh fitness of each solution has to be determined.

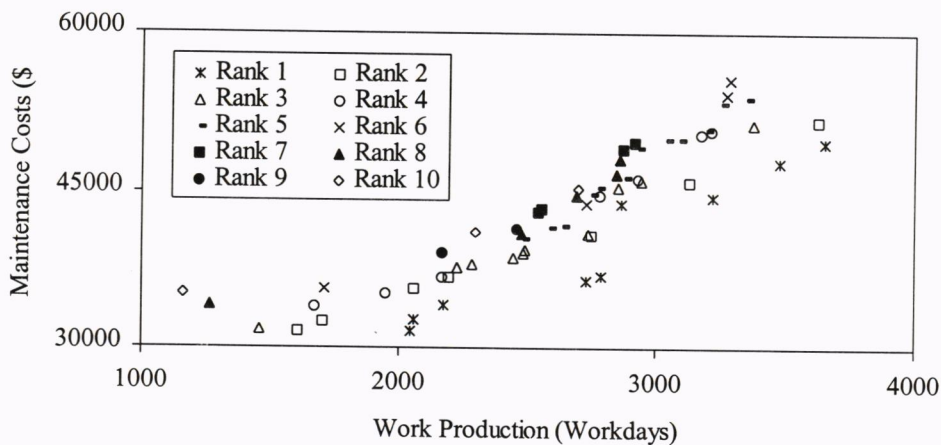


Fig. 2 GA solutions with rank 1 to rank 10 after 1st iteration

3.3.2 Optimization by Simple GA (SGA)

The GA procedures for multi-objective programming is similar to that of single-objective optimization as shown in a flow chart (Fig. 3). The main difference is in the evaluation and fitness assignment technique as explained earlier. Another important factor to consider is the optimization direction. GA performs the optimization in one direction only, i.e., either minimization or maximization. For multi-objective programming the inherent objective is to find global non-dominated solutions (solution of rank 1). Thus, the direction of GA optimization would be to find solutions with rank 1. In other words, GA would work to minimize the ranks of the solutions.

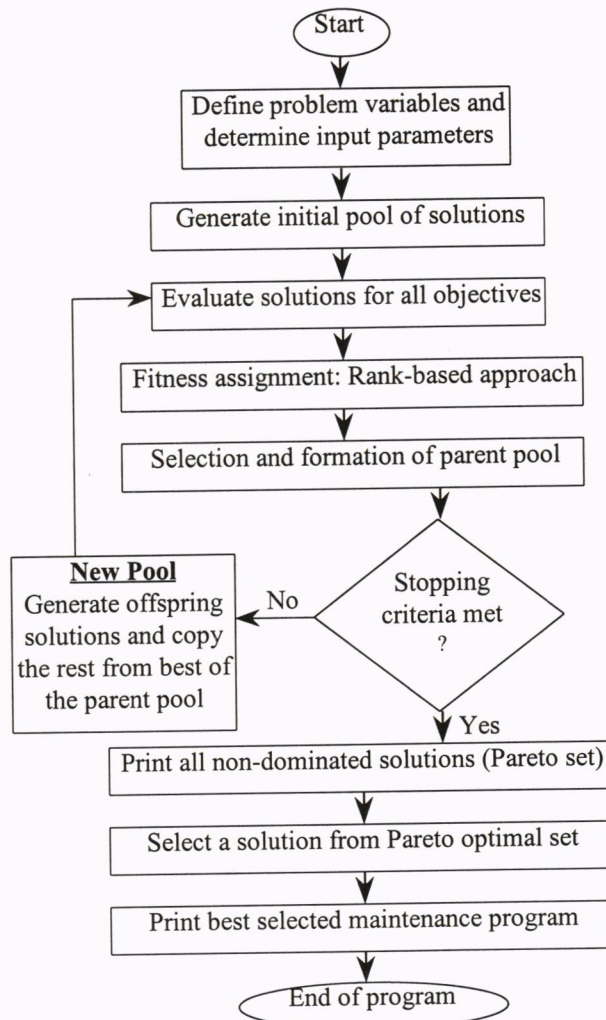


Fig. 3 Multi-objective optimization procedure by SGA

Initially, it was assumed that Simple GA operation procedure would be able to perform reasonably. However, the results revealed that GA convergence may not occur after a large

number of iterations. Fig. 4 plots the convergence trends with the number of GA iterations. A pool of 200 solutions with 80% offspring at each iteration could not reach convergence even after 500 iterations. This could happen because of two possible reasons. First, in each iteration the GA solution pool maintains only 20% best solutions from the parent pool and 80% new solutions are generated. There is no guarantee that the newly generated 80% solutions would have better fitness compared with their parents. A number of inferior solutions are generated in each iterations while some of the fitter solutions in the parent pool are not maintained in the new pool and this leads GA operation to lose many fitter solutions.

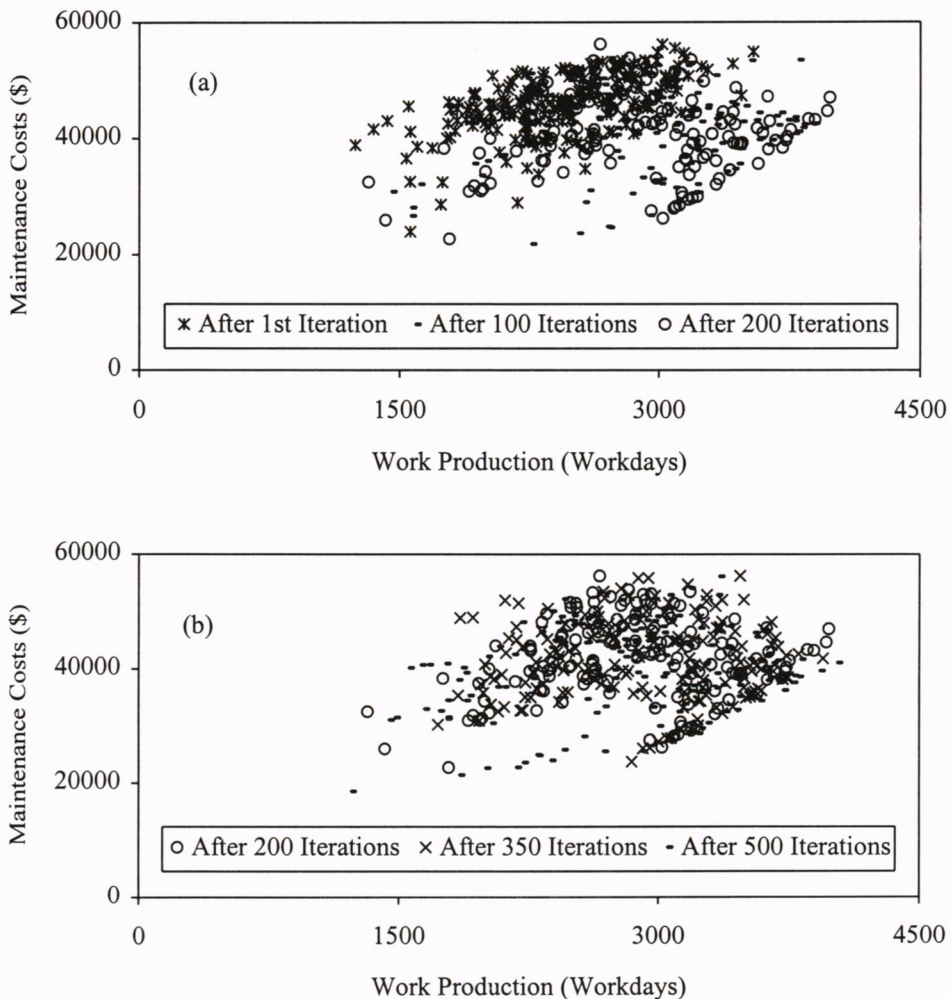


Fig. 4 Convergence of Pareto frontier by SGA optimization

Second, unlike single-objective optimization, multiple objective programming cannot concentrate on a single point or solution. In single objective programming it may be beneficial to have many duplicates of the best solution because this increases their

possibility of being chosen for the next generation. However, if it is allowed to produce duplicates of the best solution for multiple objective programming, there is a high possibility that the solution pool will consist of only a few individual solutions. When plotted on a graph all the duplicate solutions merged into a point and they will not appear as a Pareto Frontier (Fig. 4). Thus the purpose of finding a Pareto Frontier may not be achieved.

3.3.3. Optimization by Improved GA (IGA)

The aforementioned two reasons suggest that if it is possible to prevent losing better solutions from the parent pool and not to allow any duplication, the convergence problem could be solved. This study proposes an improvement algorithm that is capable of serving the purpose.

In the simple GA approach, offspring pool is created by copying certain number of the best solutions from the parent pool and the rest of the offspring are generated by crossover and mutation operation (Fig. 3). The improved algorithm works slight differently from the above approach. The stepwise procedures are shown in Fig. 5.

Offspring solutions are generated first from the best solutions and their fitness are assessed against parent solutions. By comparing the ranks of all solutions in the parent pool and all newly created solutions, a new pool is obtained with the fitter solutions. This approach guarantees that no fitter solutions are left in the parent pool.

The above approach coupled with the "no-duplication" strategy has worked well. Fig 6 plots convergence curves. The difference is more evident when Fig 6 is compared with Fig 4. The improved GA, after 50 iterations, was able to produce a well-defined Pareto frontier. Though the program was executed for 500 iterations, it is clear that the IGA could converge after 250. It is to note that the both approaches took almost same computation time of less than 1.0 CPU second, while running on Silicon Graphics Workstation (IMPACT R10000). The superiority of the improved algorithm (IGA) is obvious.

3.4 PMS Problem With More Than Two Objectives

For illustration purpose, the example problem is also solved for three and four objectives respectively. The three-objective problem assumes an additional objective to maximize pavement conditions for the network together with the previous two objectives. The inclusion of the third objective, i.e., maximizing pavement condition, helps achieving better pavement conditions (Fig. 7(a)) while compromising on the other objectives. This means, better pavement condition is achieved at the expense of more costs (Fig. 7(b)). It is noticeable that better pavement conditions can be obtained by the three-objective optimization.

The fourth objective was to minimize the total manpower (in mandays) requirements. This objective conflicts with that of maximum work production and highest pavement condition. However, due to manpower shortage many authorities may prefer to have a maintenance program that would require minimum manpower while other vital objectives are not overlooked. The inclusion of the fourth objective produces results that are not

easily explainable. Certainly, the program produces a pool of non-dominated solutions that requires less manpower compared to that of 2-objective and 3-objective (Fig. 7(d)). Interestingly, the final pool of solutions require more costs than the other two cases (Fig. 7(b)). This could due to be the complexity of maintaining all the four objectives simultaneously. It is also interesting to note that while the program spends so much budget, it fails to produce better pavement conditions and higher work productions. Thus, the inclusion of the fourth objective only ensures minimum manpower requirement but all other objectives are compromised.

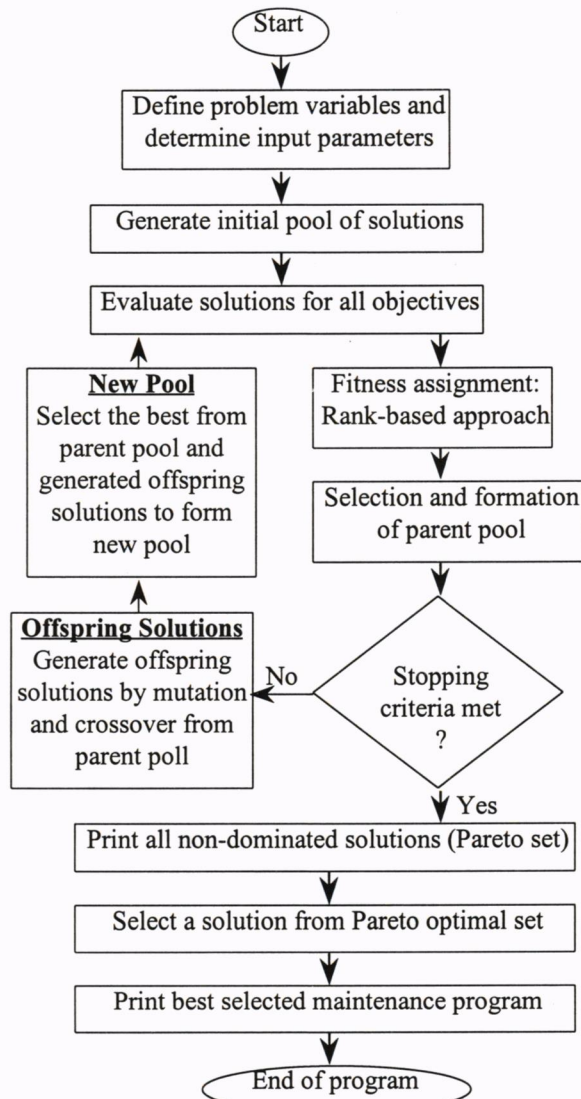


Fig. 5 Multi-objective optimization procedure by IGA

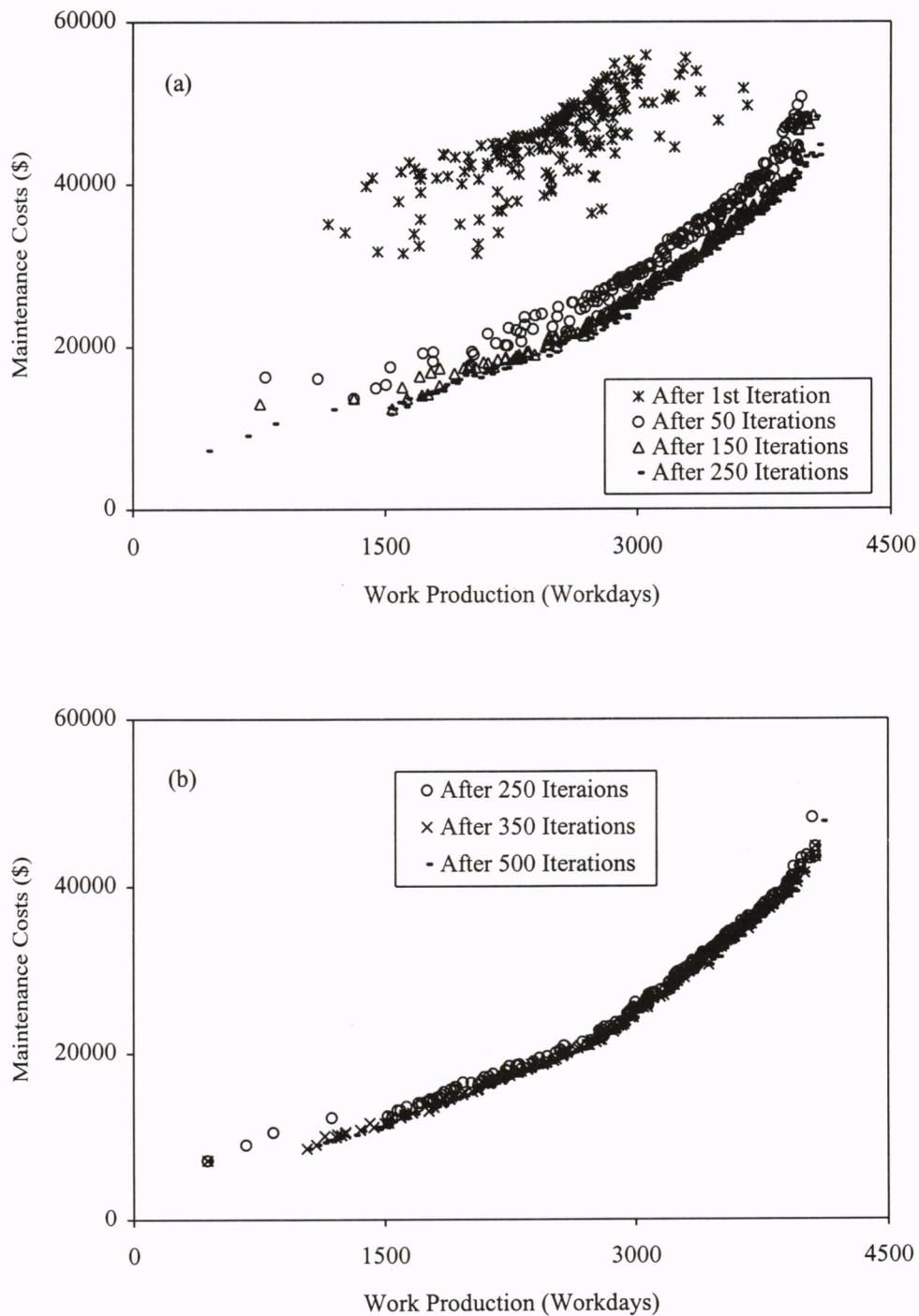
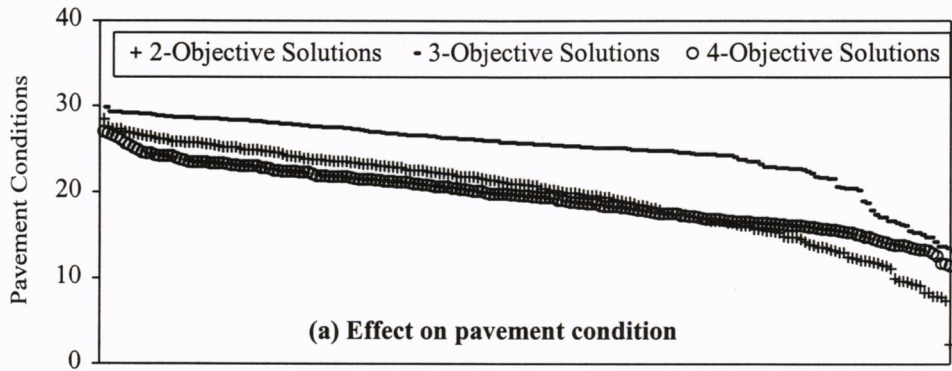
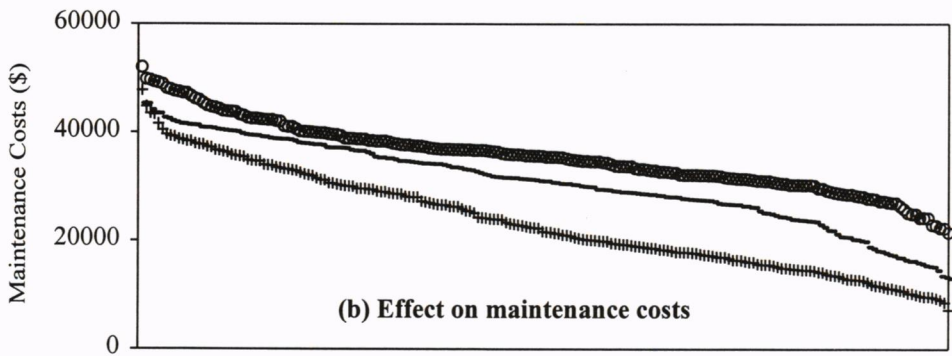


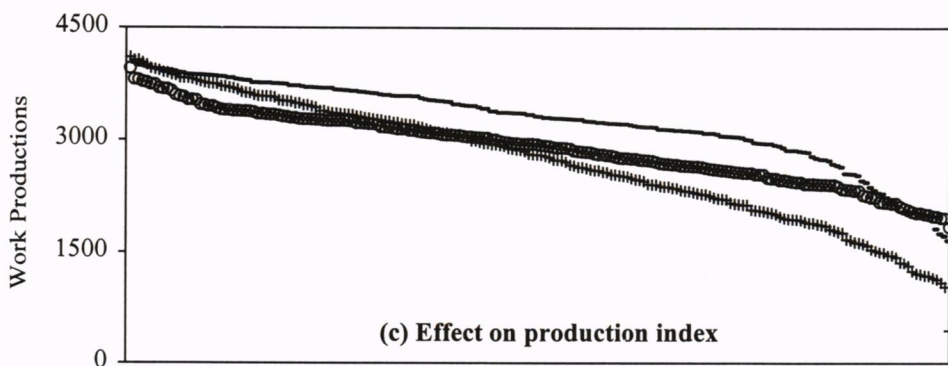
Fig. 6 Convergence of Pareto frontier by IGA optimization



Solutions arranged in descending order of pavement condition

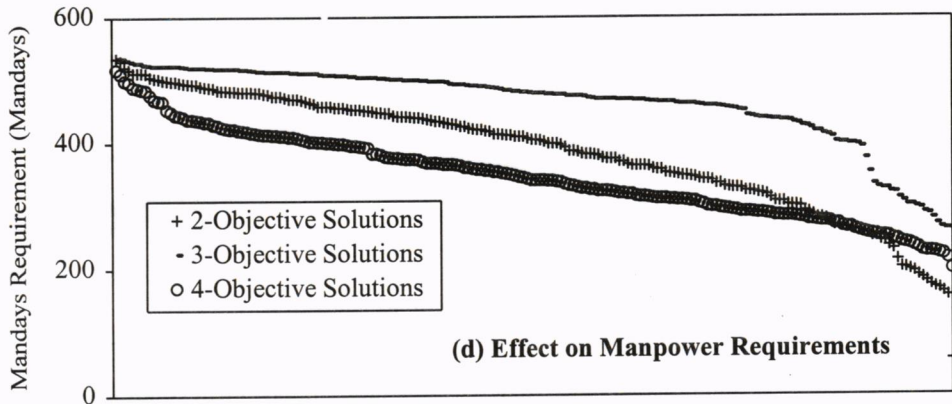


Solutions arranged in descending order of maintenance costs



Solutions arranged in descending order of work productions

Fig. 7 Effect of third and fourth objectives on various parameters



Solutions arranged in descending order of manpower requirements

Fig. 7 Effect of third and fourth objectives on various parameters (continued)

4. SELECTION OF MAINTENANCE PROGRAM

The multi-objective optimization program, whether two or more objectives are considered, produces a pool of non-dominated solutions as explained earlier. All these non-inferior maintenance programs bear the same level of preference. Now the problem arises as to how a non-dominated solution from the Pareto set should be selected. In other fields of engineering, there are many ways to select one where an individual can exercise his or her judgement in selection. One of such techniques would be to select a maintenance program that would require a maintenance budget close to the expected goal (target of maintenance budget set after the analysis). Considering the 2-objective analysis for the example problem, if the authority is likely to fix a budget of \$35,000, the optimum maintenance program would be the one that requires a total maintenance cost of \$34,984. Similarly, an optimal maintenance program for the 3-objective and 4-objective analysis can be selected accordingly.

5. CONCLUSION

This paper presents the methodology and procedure of multi-objective optimization of PMS problems using rank-based fitness assignment technique and genetic algorithms. The methodology was developed to find the Pareto optimal set of solutions (i.e., maintenance programs) and was tested with an example maintenance planning problem. Two sets of procedures (optimization by Simple GA (SGA) and Improved GA (IGA)) have been examined to study the various aspects of multi-objective optimization. It was found that SGA did not perform satisfactorily while IGA's performance was excellent. The idea for the development of IGA was not to generate any duplicate solution and select the new pool of solution such that fitter solutions are not left out in the parent pool. This procedure worked very successfully for the analysis of the example problem.

In addition to the 2-objective problem, this paper also examined the implications and effects of inclusion of third and fourth objectives to the optimization. It was revealed that as more objectives were considered the problem became more complex and it was very difficult to explain a problem physically. However, it was obvious that the inclusion of any particular objective led to better value of that objective while compromising on other objectives. The selection of a maintenance program from the Pareto set is also important. One of the selection methods could be to fix the budget as a goal after the analysis is done and find a solution close to that goal. This method can be applied to any analysis that comprises two or more objectives.

REFERENCE

- Feighan K. J., Shahin M. Y. and Sinha K. C. (1987) A dynamic programming approach to optimization for pavement management systems. **Proceedings 2nd North American Conference on Managing Pavement**, Ontario Ministry of Transportation and communications, and U.S. Federal Highway Administration, pp. 2.195-2.206.
- Fleming P. J. and Pashkevich (1985) Computer aided control system design using a multi-objective optimization approach. **Proceedings of the IEE Control'85 Conference**, Cambridge, U. K., pp. 174-179.
- Fonseca C. M. and Fleming P. J. (1993) Genetic algorithm for multi-objective optimization: formulation, discussion and generalization. In Forrest S. (eds), **Genetic Algorithms: Proceedings of the Fifth International Conference**, Morgan Kaufmann, San Mateo, C.A. pp. 416-423.
- Fonseca C. M. and Fleming P. J. (1995) An overview of evolutionary algorithms in multi-objective optimization. **Evolutionary Computation**, 3(1), pp. 1-16.
- Fwa T. F., Sinha K. C. and Riverson J. D. N. (1988) Highway routine maintenance programming at network level. **Journal of Transportation Engineering, ASCE**, 114(5), pp. 539-554.
- Fwa T. F., Chan W. T. and Tan C. Y. (1994a) Road maintenance planning using genetic algorithms. II: Analysis. **Journal of Transportation Engineering, ASCE**, 120(5), pp. 710-722.
- Fwa T. F., Chan W. T. and Tan C. Y. (1994b) Optimal programming by genetic algorithms for pavement management. **Transportation Research Record**, No. 1455, Transportation Research Board, pp. 31-41.
- Fwa T. F., Chan W. T. and Tan C. Y. (1996) Genetic-algorithm programming of road maintenance and rehabilitation. **Journal of Transportation Engineering, ASCE**, 122(3), pp. 246-253.
- Fwa T. F., Chan W. T. and Hoque K. Z. (1998). Analysis of Pavement Management activities programming by genetic algorithms. **Transportation Research Record**, No. 1643, pp. 1-7.

Fwa T. F., Chan W. T. and Hoque K. Z. (1999). Multi-Objective Optimization for Pavement Management Programming. Submitted for Publication in **Journal of Transportation Engineering, ASCE**.

Goldberg D. E. (1989) **Genetic Algorithms in search, optimization and machine learning**. Addison-Wesley Publishing Company, Reading, Mass.

Holland J. H. (1975) **Adaptation in natural and artificial systems**. The University of Michigan Press, Ann Arbor.

Horn J., Nafpliotis N. and Goldberg D. E. (1993) Multi-objective optimization using Niched Pareto Genetic Algorithms. **Proceedings 1st IEEE Conference on Evolutionary Computation**, Vol. 1, Piscataway, NJ, pp. 82-87.

Li N., Xie W. and Haas R. (1995) A new application of Markov modelling and dynamic programming in pavement management. **Proceedings 2nd International Conference on Road and Airfield Pavement Technology**, Pavement Engineering Society, Singapore, Vol. 2, pp. 683-691.

Lytton R. L. (1985) From ranking to true optimization. **Proceedings North American Conference on Managing Pavement**, Ontario Ministry of Transportation and communications, and U.S. Federal Highway Administration, pp . 5.3-5.18.

Schaffer J. D. (1985) Multiple objective optimization with vector evaluated genetic algorithms. In Grefenstette, J. J. (eds), **Proceedings of the First International Conference on Genetic Algorithms**, pp 93-100.