USING MULTI-OBJECTIVE FUZZY DE NOVO PROGRAMMING AND GIS IN HIGHWAY NETWORK INVESTMENT PLANNING

Cheng-Min FENG
Professor and Chairman
Institute of Traffic and Transportation
National Chiao Tung University
4F, 114 Chung-Hsiao W. Rd., Sec. 1,
Taipei 100, Taiwan
Tel: 886-2-23146515 Ext. 500
Fax:886-2-23822278
E-mail:cmfeng@sunwk1.tpe.nctu.edu.tw

Jennifer Y. J. WU
Ph.D. Student,
Institute of Traffic and Transportation
National Chiao Tung University
4F, 114 Chung-Hsiao W. Rd., Sec. 1,
Taipei 100, Taiwan
Tel: 886-2-23496799
Fax: 886-2-25450426
E-mail:jenny@iot.gov.tw

Abstract: This paper proposes the application of multi-objective fuzzy de Novo programming and geographic information system for highway network investment planning. The issues of efficiency and equity are considered in the objectives. The problem is formulated as the de Novo program to deal with the resource design of an optimal system. Then, it is solved by the fuzzy approach for achieving compromise solutions. The real highway network of Taiwan with its 1998's travel time survey data is built up in the geographic information system, which is used to derive the measurement of cities' accessibility/mobility. A real case is operated on the personal computer to demonstrate the effectiveness of investment planning through the proposed method.

1. INTRODUCTION

The problem of highway network investment is to select link improvement or link additions to an existing network such that social welfare is maximized. Some transportation planners like to use the network design problem (LeBlanc *et al,* 1986) to determine the appropriate investment alternatives under budget constraint. Basically, the network design problem is difficult to solve since two sets of decision makers with different objectives are inherently involved : the road users are engaged in "user optimization" and the government in "system optimization". Although some heuristic algorithms for solving network design problem have been proposed (Chang, 1992; Suwansirikul, 1987), actually we seldom use those approaches to deal with the real case because both the demand for O-D pairs and the travel-cost function for links are difficult to get.

This paper proposes a simple but practical method for multi-objective highway network investment planning to deal with the issues of the efficiency and equity. Equity implies a social or political consensus about the 'fairness' or 'justice' of the distribution of costs and benefits of a policy or program (Dear, 1978). For the equilibrium of national development, the equity in highway investment for areas and regions is very important. By maximizing the accessibility/mobility of main cities in each region and simultaneously minimizing the difference among them will be helpful to both requirements of efficiency and equity.

The accessibility/mobility of each city is represented by the travel time of its shortest path connecting to its regional center. For practical reason, the links of shortest paths are basically selected to be the candidates for improvement. The given links for the substitution of the existing shortest path are also allowed to be the candidates for improvement or addition. Those shortest paths can be easily generated in a geographic information system (GIS).

The highway investment often requires a considerable amount of resources (e.g. labor, land, facility, material and area for disposal). The concept of de Novo programming (Zeleny, 1986;

1995) is adopted in the proposed method, because it concerns not only the multi-objective optimization but also the resource design of an optimal system. For achieving the compromise solutions, the fuzzy programming approach (Zimmermann, 1978; Lee and Li, 1990) is operated to solve the problem.

The proposed method is tested for the case of west part of Taiwan, which covers 3 regions and 15 areas. The real highway network of Taiwan with its 1998's travel time survey data is built up in the geographic information system using TRANSCAD. By this method, the number of links in network has been downsized from 1929 to 120. The Pentium 586-300 has been used for running the real case and the proposed method has been proved to be effective.

2. PROBLEM DEFINITION AND FORMULATION

The problem is formulated as the de Novo program to deal with the resource design of an optimal system. Therefore, under the given total available budget, the resource (i.e. labor, land, facility, material and area for disposal) necessary to the highway network investment can be allocated rationally and prepared appropriately. In this chapter, we define the objectives of this problem first. Then, the de Novo program is introduced before we present the formulation of the highway investment problem.

2.1 Problem Definition

In the highway network investment planning, the government should consider the issues of efficiency and equity. The efficiency is usually measured by accessibility/mobility. Equity can be analyzed from a spatial viewpoint. In our opinion, all the main cities in the same region should have equal accessibility/mobility for the connection with the regional center. This aspect of 'intra-regional equilibrium' may support the concept of 'horizontal' equity (Truelove, 1993)— that persons in like circumstances should be treated identically. Also the 'inter-regional equilibrium' is one of government's goals for national development.

In our problem, the optimization of accessibility/mobility, the 'intra-regional equilibrium' and 'inter-regional equilibrium' are defined as objective (1)-(3) respectively.

Objective (1):

$$MIN \frac{1}{\sum N_i} \sum_i \sum_j \left(TT_{ij} - \sum_{l \in SP_{ij}} T_l \cdot x_l \right) \tag{1}$$

Objective (2):

$$MIN \frac{1}{\sum N_i} \sum_i \sum_j \left(TT_{ij} - \sum_{l \in SP_{ij}} T_l \cdot x_l - \frac{1}{N_i} \sum_j \left(TT_{ij} - \sum_{l \in SP_{ij}} T_l \cdot x_l \right) \right)^2 \tag{2}$$

Objective (3):

$$MIN \frac{1}{N} \sum_i \left[\frac{1}{N_i} \sum_j \left(TT_{ij} - \sum_{l \in SP_{ij}} T_l \cdot x_l \right) - \frac{1}{\sum N_i} \sum_i \sum_j \left(TT_{ij} - \sum_{l \in SP_{ij}} T_l \cdot x_l \right) \right]^2 \tag{3}$$

Notation:

SP_{ij}: in region i, the shortest path connecting the main city j to its regional center
TT_{ij}: current total travel time along SP_{ij}
T_l: saved travel time along link l while it is improved
$(T_l = T_l^c - T_l^f = T_l^c - D_l / S_l^f)$
T_l^c: current travel time along link l
T_l^f: forecast travel time along link l while it is improved
D_l: length of link l
S_l^f: forecast travel speed along link l while it is improved
x_l: decision variable for the improvement of link l
(x_l = 1, if link l is selected
= o, otherwise)
N_i: number of cities in region i
N: number of regions in the problem

The objective (1) is to minimize the average travel time from all the main cities to their regional center. The objective (2) is to minimize the difference of accessibility/mobility among cities in the same region. The objective (3) is to minimize the difference of accessibility/mobility among regions.

The accessibility/mobility of each city is represented by the travel time of its shortest path connecting to its regional center. Basically, the links of shortest paths are treated to be the candidates for improvement. Also, the given links for the substitution of the existing shortest path may become the candidates for improvement or addition.

For the estimation of the saved travel time by the improvement of a link, we need the forecast travel speed along the link to derive its forecast travel time. On account of deriving demand for highway improvement, we suggest use the speed value while the flow is near the capacity of the link.

2.2 De Novo Program

Multi-objective decision problems have been studied by many researchers. Most of these investigations concentrated on the optimization of a given system with resource constraints, while de Novo programming deals with the resource design of an optimal system. In general, a de Novo program with multiple objectives can be formulated as

(P1)
$$\max Z_k = \sum_{j=1}^{n} C_{kj} X_j,\ k = 1,2,\ldots,l$$
$$\min W_s = \sum_{j=1}^{n} C_{sj} X_j,\ s = 1,2,\ldots,r$$
subject to
$$\sum_{j=1}^{n} a_{ij} X_j - b_i \leqq 0,\quad i=1,2,\ldots,m$$
$$\sum_{i=1}^{m} p_i b_i = B$$
$$X_j \geqq 0,\quad j = 1,2,\ldots,n$$
where X_j, b_i are decision variables for projects and resources respectively, p_i represents the price of resource i, B is the total available budget.

Note that $\sum_{i=1}^{n} p_i a_{ij} = A_j$ represents the unit cost of project j. Using A_j, problem (P1) can be reformulated as

(P2) $\max Z_k = \sum_{j=1}^{n} C_{kj} X_j,\ k = 1,2,\ldots,l$

$\min W_s = \sum_{j=1}^{n} C_{sj} X_j,\ s = 1,2,\ldots,r$

subject to

$\sum_{j=1}^{n} A_j X_j = \mathrm{B}$

$X_j \geqq 0,\ j = 1,2,\ldots,n$

The ideal solution can be obtained by maximizing each Z and minimizing each W independently and subject to the given constraints. This set of ideal solutions can be represented by

$$I^* = (Z_1^*, Z_2^*, \ldots, Z_l^*, W_1^*, W_2^*, \ldots, W_r^*) \tag{4}$$

Although this ideal solution is generally infeasible, we still would like to achieve it if the constraints or resources could be adjusted. This is especially true in practical situations where the only real constraint is the total budget. Then, a metaoptimal problem can be constructed as follows:

(P3) $\mathrm{Min}\ B = \sum_{j=1}^{n} A_j X_j$

Subject to

$\sum_{j=1}^{n} C_{kj} X_j \geqq Z_k^*,\ k = 1,2,\ldots,l$

$\sum_{j=1}^{n} C_{sj} X_j \leqq W_s^*,\ s = 1,2,\ldots,r$

$X_j \geqq 0,\ j = 1,2,\ldots,n$

Zeleny (1986,1995) has considered this problem under the special case where the number of decision variables is equal to the number of objective functions. In this case the desired values of the decision variables, which are most probably infeasible and which give the optimum of all the objective functions simultaneously, can be obtained by solving the following (l+r) simultaneous equations

$$\sum_{j=1}^{n} C_{kj} X_j = Z_k^*,\ k = 1,2,\ldots,l \tag{5}$$

$$\sum_{j=1}^{n} C_{sj} X_j = W_s^*,\ s = 1,2,\ldots,r \tag{6}$$

where the equations were obtained from the constraints in Problem (P3).

Solving (5) and (6) yields B^*, each X_j^* and each b_i^*. The B* identifies the minimum budget to achieve each Z_k^* and W_s^* through each X_j^* and b_i^*. Since $B^* \geqq B$, the optimal-path ratio for achieving the ideal performance I* for a given budget level B is defined as $r^* = B / B^*$, and the optimal system design is :

$\forall i, j, k \quad X_j = r^* X_j^*,\ b_i = r^* b_i^*,\ Z_k = r^* Z_k^*$ and $W_s = r^* W_s^*$.

2.3 Problem Formulation In De Novo Program

Follow the formulation of (P1) and (P2), our highway network investment problem can be formed as:
(P4)

$$MIN \frac{1}{\sum N_i} \sum_i \sum_j \left(TT_{ij} - \sum_{l \in SP_{ij}} T_l \cdot x_l \right)$$

$$MIN \frac{1}{\sum N_i} \sum_i \sum_j \left(TT_{ij} - \sum_{l \in SP_{ij}} T_l \cdot x_l - \frac{1}{N_i} \sum_j \left(TT_{ij} - \sum_{l \in SP_{ij}} T_l \cdot x_l \right) \right)^2$$

$$MIN \frac{1}{N} \sum_i \left[\frac{1}{N_i} \sum_j \left(TT_{ij} - \sum_{l \in SP_{ij}} T_l \cdot x_l \right) - \frac{1}{\sum N_i} \sum_i \sum_j \left(TT_{ij} - \sum_{l \in SP_{ij}} T_l \cdot x_l \right) \right]^2$$

Subject to

$\Sigma_k A_l x_l \leqq B$ (i.e. $\Sigma P_k b_k \leqq B$ and $\Sigma_l a_{kl} x_l \leqq b_k$, for each k)
$x_l = 0$ or 1, for each l

B : total budget
A_l $(= \Sigma P_k a_{kl})$: cost needed for the improvement of link l
a_{kl} : amount of resource k needed for the improvement of link l
P_k : price of resource k
b_k : amount of resource k needed for optimal investment planning
(It is a decision variable.)

3. APPROACH TO SOLVING THE PROBLEM

For the purpose of resource design, we formulate the highway network investment problem as the form of a de Novo program. However, for the general case where number of variables is larger than number of objectives, no solution has been proposed in Zeleny's approach. Fortunately, based on Zimmermann's fuzzy programming approach(1978), Lee and Li (1990) proposed a methodology to deal with the general de Novo programming problem and it may generate compromise solutions.

In fuzzy programming approach, the above-mentioned ideal solution I^* in (4) and anti-ideal solution

$$I^- = (Z_1^-, Z_2^-, \ldots, Z_l^-, W_1^-, W_2^-, \ldots, W_r^-) \tag{7}$$

are used to be the reference points to define the membership functions which indicate the degree of satisfaction for each objective. In anti-ideal solution I^-, each element (Z_k^- or W_s^-) shows the worst possible performance under the total budget B for its objective independently and is considered as the tolerance limit for each objective respectively. The membership functions are defined as (8) and (9):

$\forall k = 1,2,\ldots,l$:

$$\mu_{\tilde{Z}_k}(Z_k) = \begin{cases} 0 & Z_k \leqq Z_k^- \\ (Z_k - Z_k^-)/(Z_k^* - Z_k^-) & Z_k^- < Z_k \leqq Z_k^* \\ 1 & Z_k > Z_k^* \end{cases} \tag{8}$$

$\forall s = 1,2,\ldots,r$:

$$\mu_{\tilde{W}_s}(W_s) = \begin{cases} 1 & W_s \leqq W_s^* \\ (W_s^- - W_s)/(W_s^- - W_s^*) & W_s^* < W_s \leqq W_s^- \\ 0 & W_s > W_s^- \end{cases} \tag{9}$$

The compromise-grade λ which represents the degree of overall satisfaction of the system design can be expressed as

$$\lambda = \min_{k,s} \{ \mu_{\tilde{Z}_k}(Z_k), \mu_{\tilde{W}_s}(W_s) \mid k=1,\ldots, l,\ s=1,\ \ldots,\ r \}. \tag{10}$$

We only need to maximize compromise-grade λ, so the problem (P2) is transformed to:

(P5) Max λ

Subject to

$$\lambda \leqq \left(\sum_{j=1}^{n} C_{kj}X_j - Z_k^-\right)/(Z_k^* - Z_k^-), \quad k = 1,2,\ldots,l$$

$$\lambda \leqq \left(W_s^- - \sum_{j=1}^{n} C_{sj}X_j\right)/(W_s^- - W_s^*), \quad s = 1,2,\ldots,r$$

$$\sum_{j=1}^{n} A_j X_j = \mathrm{B}$$

$$\lambda \in [0,1], \quad X_j \geqq 0, \quad j = 1,2,\ \ldots,\ n$$

Bit, Biswal and Alam (1992) concluded that the fuzzy programming algorithm is a good method to find a compromise solution for all types of multi-objective transportation problem. Bhattachacharya *et al* (1992) also used the approach to solve a multi-criteria facility location problem. However, there exists a risk while following Zimmermann's approach. The non-compensatory "min" operator used in (10) for the problem (P5) does not guarantee a non-dominated solution. Therefore, Lee and Li (1990) proposed a two-phase approach to solve this problem, where the non-compensatory operator "min" was used in the first phase to obtain the optimal compromise-grade λ, then a fully compensatory operator "averaging" was introduced in the second phase (P6) to find a non-dominated solution by restricting $\lambda_i \geqq \lambda$, . $\forall i = 1,2,\ldots,l+r$:

(P6) Max $\dfrac{1}{l+r} \sum_{i=1}^{l+r} \lambda_i$

Subject to

$$\lambda \leqq \lambda_k \leqq \left(\sum_{j=1}^{n} C_{kj}X_j - Z_k^-\right)/(Z_k^* - Z_k^-), \quad k = 1,2,\ldots,l$$

$$\lambda \leqq \lambda_s \leqq \left(W_s^- - \sum_{j=1}^{n} C_{sj}X_j\right)/(W_s^- - W_s^*), \quad s = 1,2,\ldots,r$$

$$\sum_{j=1}^{n} A_j X_j = \mathrm{B}$$

$$\lambda_k, \lambda_s \in [0,1], \quad X_j \geqq 0, \quad j = 1,2,\ \ldots,\ n$$

According to some papers (Li,1990; Bit,1992; Bhattacharya,1992) the fuzzy linear programming is easy to be solved. Also according to the other paper (Sasaki,1995), genetic algorithm has been proved to be able to solve fuzzy goal of nonlinear function with fuzzy nonlinear constraints. It seems that the fuzzy programming is a good alternative to deal with the multi-objective optimization design problem.

We transfer the form of our network investment problem for the adoption of two-phase fuzzy programming as (P7) and (P8).

(P7)

$$\begin{aligned}
&\text{Max } \lambda \\
&\text{Subject to} \\
&\lambda \leqq (W_1^- - \text{Objective}(1))/(W_1^- - W_1^*) \\
&\lambda \leqq (W_2^- - \text{Objective}(2))/(W_2^- - W_2^*) \\
&\lambda \leqq (W_3^- - \text{Objective}(3))/(W_3^- - W_3^*) \\
&\Sigma_k A_l x_l \leqq B \\
&x_l = 0 \text{ or } 1, \text{ for each } l \\
&\lambda \in [0,1]
\end{aligned}$$

(P8)

$$\begin{aligned}
&\text{Max } (\lambda_1 + \lambda_2 + \lambda_3)/3 \\
&\text{Subject to} \\
&\lambda \leqq \lambda_1 \leqq (W_1^- - \text{Objective}(1))/(W_1^- - W_1^*) \\
&\lambda \leqq \lambda_2 \leqq (W_2^- - \text{Objective}(2))/(W_2^- - W_2^*) \\
&\lambda \leqq \lambda_3 \leqq (W_3^- - \text{Objective}(3))/(W_3^- - W_3^*) \\
&\Sigma_k A_l x_l \leqq B \\
&x_l = 0 \text{ or } 1, \text{ for each } l \\
&\lambda, \lambda_1, \lambda_2, \lambda_3 \in [0,1]
\end{aligned}$$

where W_i^- : value of Objective(i) for the worst case
W_i^*: value of Objective (i) for the best case

The proposed method was applied for a real case that is described in next chapter.

4. TESTING CASE FOR THE PROPOSED METHOD

The proposed method is tested for the case of west part of Taiwan, which covers 3 regions and 15 areas with 15 main cities as Table 1. Three layers of polygon, link and point are built in the geographic information system using TRANSCAD for the representation of areas, highways and cities respectively. The real highway network of Taiwan with its 1998's travel time survey data is stored in the highway layer as Figure 1. There are four types of highway: National, provincial, county and connector highway. The shortest path connecting each main city to its regional center is generated as Figure 2. By the proposed method, the number of links in network has been downsized from 1929 to 120.

In the model, using the GIS is not necessary but is really helpful. The advantages of using the GIS are listed as follows:

1. It's easier to generate a shortest path in a network by giving any two locations.
2. It's easier to justify the rationality of the shortest path by using different colors to display the network in terms of the road type.
3. It's easier to check the accuracy of link length stored in attribute database by comparing it with the value generated automatically from geographic database.

Considering the cost and time in developing the GIS database, we suggest collect the geographic data of network by using GPS (Global Positioning System) simultaneously during the travel time survey.

Table 1. The Regions, Areas and Main Cities in the West Part of Taiwan

Region name	Regional center	Area name	Main city of area
North Region	Taipei City	Keelung	Keelung City
		Taipei	Taipei City
		Taoyuan	Taoyuan City
		Hsinchu	Hsinchu City
		I-lan	I-lan City
Middle Region	Taichung City	Miaoli	Miaoli City
		Taichung	Taichung City
		Nantou	Nantou City
		Changhwa	Changhwa City
		Yunlin	Douliou City
South Region	Tainan City	Chiayi	Chiayi City
		Shinying	Shinying City
		Tainan	Tainan City
	Kaohsiung city	Kaohsiung	Kaohsiung City
		Pingtung	Pingtung City

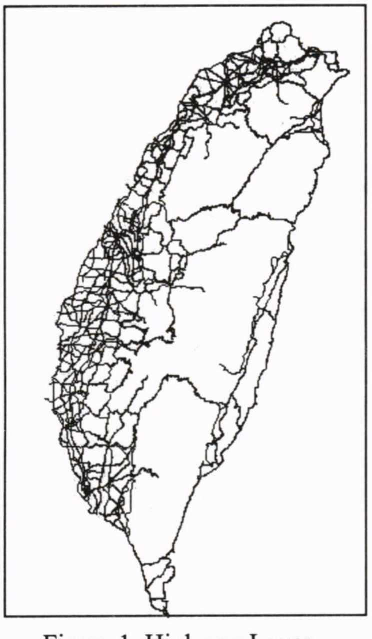

Figure 1. Highway Layer

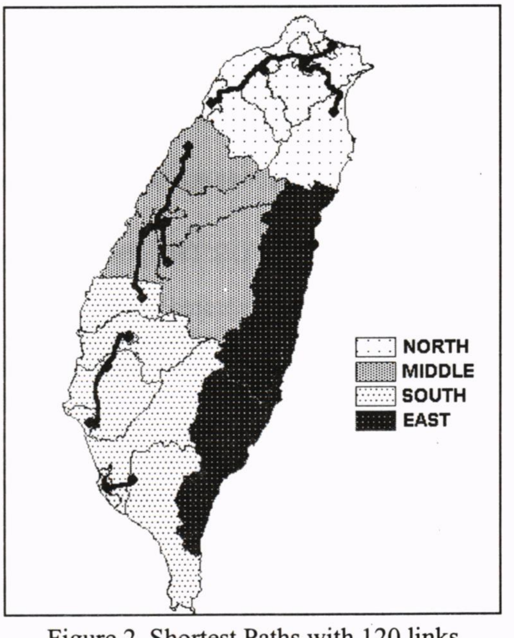

Figure 2. Shortest Paths with 120 links

The values of parameters in the problem formulation (P7) and (P8) are listed as follows:
A_l : cost needed for the improvement of link l
(=5/km * D_l, if link l is national highway or connector
=1/km * D_l, otherwise)

D_l : length of link l
S_l^f : forecast travel speed along link l while it is improved
(= 80km/hr, if link l is national highway
=45km/hr, otherwise)
B : total budget (=100)

For the purpose of testing the performance of fuzzy programming approach, we may ignore the value of a_{kl} (amount of resource k needed for the improvement of link l) and P_k (price of resource k) which are hidden in the model. When the a_{kl} and P_k are given, the b_k (amount of resource k needed for optimal investment planning) could be decided in the proposed method.

The Pentium 586-300 has been used for process the problem solving by fuzzy programming approach. To get the ideal solution I^* and anti-ideal solution I^-, three models for minimization of objective (1)- (3) separately under the total budget were processed. We got I^* = (50.38515, 277.9848, 20.08455) and I^- = (56.31894, 455.318,136.7277). Then, the first phase and second phase of two-phase fuzzy programming were processed to derive the compromise solutions. The summary of results is listed in Table 2.

Table 2. Summary of results in Testing Case

Model	Min objective (1)	Min objective (2)	Min objective (3)	Fuzzy programming	
				First phase	Second phase
Budget used	98.75	99.7	99.85	94.65	99.65
Time saved	104.0683	38.79667	62.02667	82.02667	82.32000
objective (1)	50.38515	56.31894	54.20712	52.38894	52.36227
objective (2)	438.4727	277.9848	455.318	337.4441	333.24934
objective (3)	136.7277	72.93536	20.08455	55.39943	52.138

In the case, each model was run by the SOLVER of EXCEL and the computing time is less than 8 minutes. The proposed method for the highway network investment planning has been proved to be effective and efficient.

REFERENCES

Bhattacharya, U., Rao, J.R. and Tiwari, R.N. (1992) Fuzzy Multi-criteria Facility Location Problem. **Fuzzy Sets and Systems 51**, 277-287.

Bit, A. K., Biswal, M.P. and Alam, S. S. (1992) Fuzzy Programming Approach to Multicriteria Decision Making Transportation Problem. **Fuzzy Sets and Systems 50**, 135-141.

Chang, C. J. and Chang S. H. (1992) A heuristic Algorithm for Solving the Discrete Network Design Problem. **Transportation Planning and Technology 16**, 39-50.

Dear M. J. (1978) Planning for mental Health care: a reconsideration of public Facility Location Theory. **International Regional Science Review 3**, 93-111.

LeBlanc L.J. and Boyce, D. E.(1986) A bilevel programming algorithm for each solution of the network design problem with user-optimal flows. **Transportation Research 21B**,259-265.

Li, R. J. and Lee, E.S. (1990) Fuzzy Approaches to Multicriteria de Novo Programs. **Journal of Mathematical Analysis and Applications 153**, 97-111.

Sasaki, M., Gen, M. and Yamashiro, M. (1995) A Method for Solving Fuzzy De Novo Programming Problem by Genetic Algorithms. **Computers Ind. Engng.** Vol.29, No. 1-4, 507-511.

Suwansirikul, C., Friesz, T. L. and Tobin, R.L. (1987) Equlibrium decomposed optimization: a heuristic for the continuous equilibrium network design problem. **Transportation Science 21**,254-263.

Truelove, M. (1993) Measurement of Spatial Equity. Environment and Planning C: **Government and Policy 11**,19-34.

Zeleny, M. (1995) Trade-off-Free Management via De Novo Programming. **International Journal of Operations and Quantitative Management 1(1)**, 3-13.

Zeleny, M. (1986) Optimal System Design with Multiple Criteria: De Novo Programming Approach. **Engineering Costs and Production Economics 10**, 89-94.

Zimmermann, H.J. (1978) Fuzzy Programming and Linear Programming with several Objective Functions. **Fuzzy Sets and Systems 1**, 45-55.