BUS ROUTING SCHEMES FOR A DISTANT DEMAND

Seung-Young KHO
Associate Professor
Department of Transportation Engineering
Myongji University
San 38-2, Nam-Dong, Yongin-Si
Kyunggi-Do, 449-728 Korea
Fax: +82-335-336-2885
E-mail: sykho@wh.myongji.ac.kr

Chang-Ho PARK
Professor
Department of Civil Engineering
Seoul National University
San 56-1, Shinlim-Dong, Kwanak-Ku
Seoul, 151-742 Korea
Fax: +82-2-889-0032
E-mail: parkch@gong.snu.ac.kr

Abstract — This paper is to analyze transit routing schemes for a distant demand, which is laterally apart from a major demand corridor. Analytical cost models for 2 routing schemes, no-zigzagging and zigzagging bus routes, are developed, solutions for zigzagging conditions are derived, and some important concepts, practical implications and useful results about zigzagging are suggested through a practical analysis for several access modes.

Keywords — Zigzagging of bus routes, distant demand, corridor demand, zigzag ratio, demand ratio, cost ratio

1. INTRODUCTION

Zigzagging of bus routes is defined as a route configuration in which buses deviate from their shortest paths to their final destinations such as CBDs (central business districts) to serve scattered demand, return to the shortest paths and repeat this kind of operation. Many transit routes have a zigzagging network configuration. The zigzagging of bus routes is falsely known to be inefficient because it causes increases of route distances. However, when it is not economical to run an additional transit route to serve a demand distantly located from major demand corridors, zigzagging of the nearest bus route to the distant demand can be an effective way to provide transit service and passengers located distantly can reduce their access distances.

On the contrary, zigzagging of bus routes can cause several negative aspects against passengers, transit companies and society. When a bus route detours, the passengers in the vehicles, who boarded before the route starts to detour ("on-board passengers"), have to detour also causing increases in their travel times. In addition, zigzagging increases route length causing an increase in vehicle operating cost. In some cases, despite that few distant passengers can access the nearest bus route without zigzagging, a bus route zigzags for some other reasons such as a political reason. It is helpful to know whether zigzagging is necessary or not, and in what conditions zigzagging of a bus route is more efficient than the access of distant passengers.

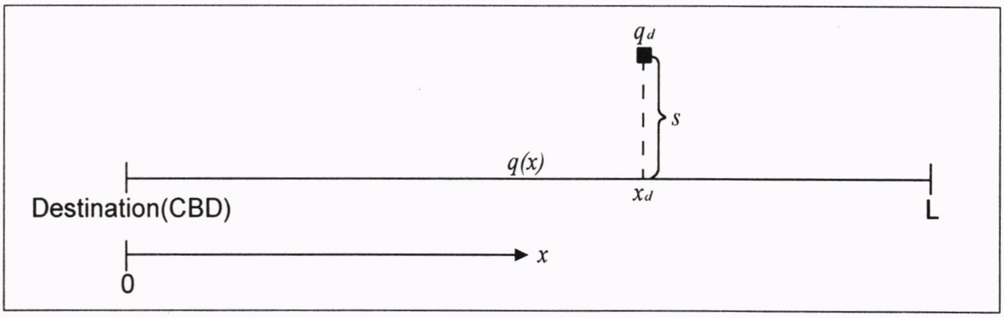
The purpose of this paper is to analyze the economy of zigzagging of transit routes for a certain amount of the transit demand, which is laterally apart from a major demand corridor. This paper attempts to develop analytical cost models for no-zigzagging and zigzagging bus routes, derive analytical solutions for zigzagging conditions, find important variables which determine whether or not to zigzag, and suggest some important concepts, practical implications and useful results about zigzagging through some simplifications.

It is assumed that a bus route serves a many-to-one demand located along a corridor toward a CBD ("corridor demand") and some demand laterally apart from the corridor ("distant demand") toward the same CBD. Also, it is assumed that the transit demand is transit-captive, that is, constant. This implies that the demand does not vary depending upon the route configuration.

2. COST MODELS

2.1 Total Transportation Costs for No-Zigzagging and Zigzagging

In order to provide transit service for the demand pattern as shown Figure 1, which is observed in many cities, two kinds of bus routing schemes can be considered.



Where S = Length of bus route (km)
$q(x)$ = Corridor demand at location x (persons/hr-km)
s = Lateral distance between route and the distant demand (km)
x_d = Longitudinal location of the distant demand (km)
q_d = Distant demand (persons/hr)

Figure 1. Demand Distribution of Cost Model

One is "No-Zigzagging" route, in which the route run along the corridor without detouring (zigzagging) to pick up the distant demand. In this route configuration passengers of the distant demand have to access to the nearest bus stop along the corridor by using an access mode. The access mode can be one of walking, bicycle, park-and-ride, kiss-and-ride, taxi, etc. The other is "Zigzagging" route, in which the route detour to pick up the distant demand starting at x_d, returns to the same point, and continues to serve the demand on the corridor.

The total transportation cost in this paper consists of the bus operating cost, passenger waiting time cost, passenger travel time cost, and passenger access cost. Assuming that only the lateral access distance of the corridor demand is considered (longitudinal access distances are assumed to be the same) and the average waiting time is a half of the headway, the total costs can be modeled simply as shown in Table 1.

Table 1. Cost Models for Two Routing Schemes

Cost Item	(1) No-Zigzagging	(2) Zigzagging
Bus Operating Cost(C_o)	$\alpha_o \cdot \dfrac{L}{v_b h_1}$	$\alpha_o \cdot \dfrac{L+2s}{v_b h_2}$
Passenger Waiting Cost(C_w)	$\alpha_w \cdot \dfrac{h_1}{2} \cdot \{ \int_0^L q(x)dx + q_d$	$\alpha_w \cdot \dfrac{h_2}{2} \cdot \{ \int_0^L q(x)dx + q_d \}$
Passenger Travel time Cost(C_t)	$\dfrac{\alpha_t}{v_b} \{ \int_0^L xq(x)dx + x_d q_d \}$	$\dfrac{\alpha_t}{v_b} \{ \int_0^{x_d} xq(x)dx + \int_{x_d}^L (x+2s)q(x)dx + (x_d+s)q_d \}$
Passenger Access Cost(C_a)	$\dfrac{\alpha_a}{v_a} \{ \int_0^L a(x)q(x)dx + (a_d+s)q_d \}$	$\dfrac{\alpha_a}{v_a} \{ \int_0^L a(x)q(x)dx + a_d q_d \}$

Where α_o = Unit bus operating cost (\$/vehicle-hr)
α_w = Value of passenger waiting time (\$/person-hr)
α_t = Value of passenger travel time (\$/person-hr)
α_a = Value of passenger access time (\$/person-hr)
h_1, h_2 = Headways
v_b = Bus operating speed
v_a = Access speed
$a(x)$ = Average access distance at point x
a_d = Average access distance of the distant demand

From the cost models, headways are included in bus operating and passenger waiting cost terms in both cases. They can be optimized for minimizing the total costs as below.

$$h_1^* = \sqrt{\frac{2\alpha_o L}{\alpha_w v_b Q}}, \quad h_2^* = \sqrt{\frac{2\alpha_o (L+2s)}{\alpha_w v_b Q}} \tag{1}$$

where $Q = \int_0^L q(x)dx + q_d$ = total demand per hour

From the above equations, we can see that the optimal headway is proportional to the square root of the route length. Also h_2 is always larger than h_1 because the route length of zigzagging bus route increases as much as $2s$, compared to no-zigzagging, in order to pick up the distant demand.

By substituting the headways in Table 1 by the optimal headways, the bus operating cost (C_o) and the passenger waiting cost (C_w) become to be equal and the sum of these two cost terms collapses into

$$C_o + C_w = \begin{cases} \sqrt{\dfrac{2\alpha_o \alpha_w L Q}{v_b}} & \textit{for no-zigzagging,} \\ \sqrt{\dfrac{2\alpha_o \alpha_w (L+2s) Q}{v_b}} & \textit{for zigzagging.} \end{cases} \tag{2}$$

2.2 Differences of Costs between Two Routing Schemes

Bus Operating and Passenger Waiting Costs

The bus operating and passenger waiting costs increase by zigzagging. The ratio of the cost for zigzagging to that for no-zigzagging is $\sqrt{1+2s/L}=\sqrt{\rho}$, in which the value $1+2s/L$ can be defined as ρ= "zigzag ratio" ($\rho \geq 1$), that is, the ratio of the route distance for zigzaging to the route length for no-zigzagging.

From this, it can be observed that the bus operating and passenger waiting costs do not increase as proportional as the increase in the route length by detouring. For example, if the detour distance is 20% of the original route distance, the bus operating and passsenger waiting costs increase less than 10%.

Passenger Travel Time Cost

By zigzagging, the passenger travel time cost increases by $2s\alpha_t\int_{x_d}^{L}q(x)dx$ for the detour of the on-board passengers ($\int_{x_d}^{L}q(x)dx$) and by $\alpha_t s q_d$ for the additional travel time of the distant demand. Consequently, in addition to the increases in the bus operating and passenger waiting costs, zigzagging causes an increase in the passenger travel time and travel time cost.

Passenger Access Cost

Buses zigzag because more people can ride buses with less access distances by zigzagging. Zigzagging saves the passenger access cost of $\alpha_a s q_d / v_a$ for the distant demand, which is the only benefit for zigzagging. It was shown in the above that zigzagging of buses causes increases in the bus operating, passenger waiting and travel time costs. Therefore, the amount of the passenger access cost saving due to zigzagging determines whether or not a bus route to zigzag. The amount of the passenger access cost saving should be compared to the increases in the bus operating, passenger waiting and travel time costs.

3. CONDITIONS FOR ZIGZAGGING

3.1 General Condition

In order for a bus route to zigzag, the saving in the passenger access cost by zigzagging should be greater than the sum of the increases in the bus operating, passenger waiting and travel time costs by zigzagging. Therefore, the conditions for zigzagging of bus routes is

$$\frac{\alpha_a s q_d}{v_a} > \sqrt{\frac{2\alpha_o\alpha_w LQ}{v_b}}(\sqrt{\rho}-1)+\frac{\alpha_t}{v_b}\left\{\int_{x_d}^{L}2sq(x)dx+sq_d\right\} \tag{3}$$

In the above equation, the term in the left-hand side is the amount of decrease in the passenger access cost for the distant demand by zigzagging. The first term of the right-hand side represents the sum of the increases in the bus operating and passenger waiting costs and the second term of the right-hand side is the increase in the passenger travel time cost by zigzagging.

Unless the above condition is satisfied, zigzagging should not be adopted. The above condition depends on various aspects of the situation. These include almost all the variables in the cost models above.

The concept of the above condition for zigzagging can be applied to other demand patterns than many-to-one demand. The above equation can be rearranged as

$$\left(\frac{\alpha_a}{v_a}-\frac{\alpha_t}{v_b}\right)sq_d > \sqrt{\frac{2\alpha_o\alpha_w LQ}{v_b}}\left(\sqrt{\rho}-1\right)+\frac{2s\alpha_t}{v_b}\int_{x_d}^{L} q(x)dx \tag{4}$$

The left-hand side represents the benefit of the distant demand by zigzagging. The right-hand side represents the increase in the bus operating cost by zigzagging and the increase in cost (inconvenience) of the on-board passengers.

The general condition for bus route zigzagging can be described as

> "Zigzagging of buses to pick up a distant demand can be more efficient than the access of the distant demand to the bus route, provided that the benefit of the distant demand by zigzagging is larger than the sum of increases in the bus operating and passenger waiting time costs and travel time cost of the on-board passengers."

Therefore, whether or not to zigzag is mainly determined by the size of the distant demand compared to the number of in-vehicle passengers, costs of available access modes if the distant demand have to access to the corridor, and the distance to detour.

3.2 Condition for the Same Headways

The condition for zigzagging in the previous section is very complicated. In this section, in order to simplify the condition, the headway of the zigzagging route is assumed to be the same as the optimal headway of the no-zigzagging route, i.e.,

$$h_2 = h_1^* = \sqrt{\frac{2\alpha_o L}{\alpha_w v_b Q}} < h_2^* = \sqrt{\frac{2\alpha_o(L+2s)}{\alpha_w v_b Q}}. \tag{5}$$

This assumption implies that even though the optimal headway of the zigzagging route should increase as the route length increases by zigzagging, the same headway is maintained as before.

This simplification causes an increase in the bus operating cost and a decrease in the passenger waiting time cost compared to using the optimal headway for zigzagging. The passenger travel time and access costs do not change by changes in headways. The changes in bus operating and passenger waiting time costs are shown in Table 2.

How the sum of the bus operating and passenger waiting time costs changes depending upon the routing scheme and the zigzag ratio is shown in Figure 2.

As shown in Figure 2, even though the sum of the bus operating and passenger waiting time costs increases slightly by not using the optimal headway for the zigzagging route, the amount of the increase is not large especially for a reasonable range of zigzag ratio. For example, the sum of the bus operating and passenger waiting time costs for not-using the optimal headway for the zigzagging route is about 1.06 times that of for using the optimal headway for $\rho = 2$.

Table 2. Changes of Cost for the Same Headways

Item	(1) No-Zigzagging	(2) Zigzagging	
		(2-1) $h_2 = h_2^*$	(2-2) $h_2 = h_1^*$
Bus Operating Cost (C_o)	$\sqrt{\frac{\alpha_o \alpha_w L Q}{2 v_b}}$	$\sqrt{\frac{\alpha_o \alpha_w (L+2s) Q}{2 v_b}}$	$\sqrt{\frac{\alpha_o \alpha_w (L+2s)^2 Q}{2 v_b L}}$
Passenger Waiting Time Cost (C_w)	$\sqrt{\frac{\alpha_o \alpha_w L Q}{2 v_b}}$	$\sqrt{\frac{\alpha_o \alpha_w (L+2s) Q}{2 v_b}}$	$\sqrt{\frac{\alpha_o \alpha_w L Q}{2 v_b}}$
$C_o + C_w$	$\sqrt{\frac{2\alpha_o \alpha_w L Q}{v_b}} = K$	$\sqrt{\frac{2\alpha_o \alpha_w (L+2s) Q}{v_b}} = K\sqrt{\rho}$	$\sqrt{\frac{2\alpha_o \alpha_w (L+s)^2 Q}{v_b L}} = K\frac{\rho+1}{2}$

Note *: $K = \sqrt{\frac{2\alpha_o \alpha_w L Q}{v_b}}$

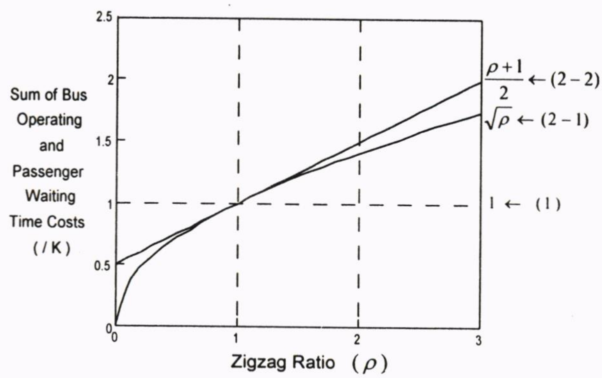


Figure 2. Changes in Sum of Bus Operating and Passenger Waiting Time Costs

The increase in the bus operating cost by zigzagging compared to that by no-zigzagging is $\frac{2\alpha_o s}{v_b h_1^*} = \sqrt{\frac{2\alpha_o \alpha_w Q}{v_b L}} \cdot s$. Therefore, the condition for zigzagging becomes slightly simplified as

$$(\frac{\alpha_a}{v_a} - \frac{\alpha_t}{v_b}) s q_d > \sqrt{\frac{2\alpha_o \alpha_w Q}{v_b L}} \cdot s + \frac{2s\alpha_t}{v_b} \int_{x_d}^{L} q(x)dx \tag{6}$$

The above condition is the same as the general condition in the previous section except for the first term of the right-hand side, which represents the increase in the bus operating cost by zigzagging. This condition can be described as

> "Zigzagging of buses to pick up a distant demand can be more efficient than the access of the distant demand to the bus route, provided that the benefit of the distant demand by zigzagging is, at least, larger than the sum of increases in the bus operating cost and travel time cost of the on-board passengers."

3.3 Condition for the Same Operating Costs

The condition for zigzagging in the previous section can be simplified further by neglecting the increase in the bus operating cost for a small value of ρ as shown in Figure 2.

By this simplification, the condition for zigzagging is simplified into

$$\left(\frac{\alpha_a}{v_a}-\frac{\alpha_t}{v_b}\right)sq_d > \frac{2s\alpha_t}{v_b}\int_{x_d}^{L} q(x)dx \quad \text{(minimum condition for zigzagging)} \qquad (7)$$

The left-hand side represents the benefit of the distant demand by zigzagging, that is, the decrease in access cost of the distant demand. The right-hand side represents the increase in the travel time cost of the on-board passengers. Therefore, buses should not zigzag unless the decrease in the access cost of the distant demand is greater than the increase in the travel time cost of the in-vehicle passengers at least.

When neglecting the increases in the bus operating and passenger waiting time costs, whether or not to zigzag should be determined by comparing the decrease in the access cost of the distant demand against the increase in the travel time cost of the in-vehicle passengers.

The above minimum condition for zigzagging can be rephrased as

$$P > 2\big/(R-1) \quad or \quad R > \frac{2}{P}+1 \qquad (8)$$

where $P = q_d \Big/ \int_{x_d}^{L} q(x)dx =$ "Demand Ratio",

$R = \dfrac{\alpha_a/v_a}{\alpha_t/v_b} =$ "Cost Ratio".

P represents the ratio of the distant demand to the number of on-board passengers. R represents the ratio of the access cost to the travel time cost for unit distance, which is determined by the access mode of the distant demand. Figure 3 show the minimum condition for zigzagging in terms of P and R.

In Figure 3, zigzagging of buses to pickup a distant demand can not be efficient at all for combinations of P and R in the region below the curve. Zigzagging may be efficient for combinations of P and R in the region above the curve. In the region above the curve, the saving of access cost of the distant demand is greater than the increase in the travel time cost of on-board passengers for zigzagging. However, because zigzagging causes increases in the bus operating and passenger waiting costs in the region above the curve, these increases should be considered further for zigzagging.

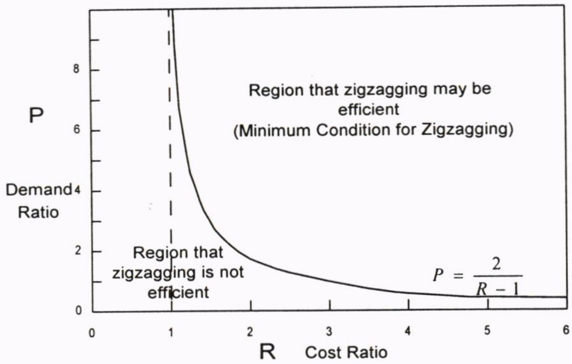


Figure 3. Condition for Zigzagging

For example, for $R = 3$ (the access cost per unit person distance is 3 times of the travel time cost per unit person distance), zigzagging can be considered at least for $P \geq 1$ (the number of the distant demand is equal to or larger than the number of the on-board passengers).

4. PRACTICAL CONSIDERATIONS

In the previous section, it is known that the efficiency of zigzagging is mainly determined by the values of P and R. Also, the value of R is mainly influenced by the access mode of the distant demand. In this section, values of R are examined for various access modes for a major transit route in reality. Here, typical values of R for walking, kiss-and-ride, park-and-ride, taxi, feeder bus, and bicycle are calculated.

The travel time cost of passengers on bus and an access mode per hour, α_t, is defined as the amount of money that they are willing-to-pay for travel time saving. In this paper, the travel time cost of \$1.71/person-hour is assumed which is 40% of the average hourly wage of typical commuters in urban areas in the country in 1996. When the average bus operating speed of 21.6 Km/h is assumed, the bus travel time cost for unit distance, α_t/v_b is \$0.079/person-km. Typical vaules of the variables necessary to obtain the access costs of various access modes for unit distance and R are obtained and shown in Table 3.

Among various access modes, the largest value of P, 25, is required for bicycle. Relatively small values of P, 0.45 and 0.8, are required for walking and taxi. This imples that if there exists an efficient access mode such as bicycle, operating straightly along a major demand corridor without zigzagging is an efficient routing scheme. On the other hand, if access is quite inefficient causing a lot of access cost such as walking and taxi, buses have to zigzag to pick up the distant demand frequently.

5. CONCLUSIONS

This paper attempted to analyze the two transit routing schemes for a distant demand, zigzagging and no-zigzagging. Through the transportation cost modeling and subsequent

Table 3. Typical Values of Various Access Modes

Item / Mode	Fixed Cost ($/person-hr)	Variable Cost		Access Time Cost, α_a ($/person-hr)	Operating Speed, v_a (Km/hr)	Access Cost per Unit Distance, α_a/v_a ($/person-Km)	R	Min. P
		Wage ($/person-hr)	Others ($/person-hr)					
Walking	0	0	0	1.71	4	512	5.40	0.45
Bicycle	0	0	0	1.71	20	102	1.08	25
Kiss-and-Ride	1.78	0	0.65	5.84[1)]	27	260	2.74	1.15
Park-and-Ride[2)]	1.78	0	0.65	4.14	27	184	1.94	2.13
Taxi	3.05	2.15	0.56	7.46	27	332	3.50	0.8
Feeder Bus	0.20	0.13	0.04	2.08	18.8	133	1.40	5

Notes 1): Driver's travel time cost of a half of the passenger's travel cost is added
2): Parking cost is ignored

analysis, it was found that it may be better to zigzag to pickup the distant demand when the demand ratio P is large, that is, the distant demand is comparatively larger than the number of on-board passengers for a given cost ratio R. This situation, in which zigzagging is more efficient than the distant demand's access, happens potentially at the beginning of a transit route.

Consequently, at the beginning of a bus route, when the number of on-board passengers is very small, buses better off to zigzag to pickup passengers. Whereas, in the middle of a transit route, where the number of on-board passengers is quite substantial relative to the distant demand, it is better to go straight to the final destination without zigzagging. Whether or not to zigzag is mainly determined by the cost and demand ratios.

Zigzagging should not be evaluated as an inefficient routing scheme. Depening uopn the location and the amount of the distant demand, zigzagging can be an efficient routing scheme. The results obtained in this paper can be applied to any other demand patterns, as long as the saving in the access cost of the distant demand is compared to the increases in other costs by zigzagging, such as the increase in the travel time cost of on-board passengers.

In this paper, idealized situations about demand pattern and bus route configuration were assumed. In reality, demand pattern and route configuration can be far more complicated. A further research has to be performed for complicated circumstances.

REFERENCES

Kho, Seung-Young (1989), Design of Bus Routes for a Many-to-Few Travel Demand. Ph.D. Dissertation, Department of Civil Engineering, University of California at Berkeley, U.S.A.

S. C. Wirasinghe and N. S. Ghoneim (1981), Analysis of a Radial Bus System for CBD Commuters Using Auto Access Modes. **Journal of Advanced Transportation**, Vol. 16, No. 2, Summer.