# INTEGRATED OPTIMIZATION MODELS FOR THE FEEDER/TRANSFER SYSTEMS IN A LINEAR HUB-AND-SPOKE INTER-CITY BUS NETWORK

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**Abstract:** This paper discusses the development and application of an integrated optimization feeder/transfer system model in a linear hub-and-spoke network. At first, genetic algorithms are employed to determine the feeder/transfer relationship among routes by clustering the routes into different partition sets. Then, an algorithm integrating feeder/transfer location selecting, routing, and scheduling is proposed to solve the optimal network and headway for each partition set. This integrated model was proved to be both efficient and effective by testing on an experimental example of 15 direct-service routes. The results showed that 3 feeder sets, a total of 10 routes altogether, were formed, while other 5 routes maintained direct service. As a whole, the total cost was lowered by 6.75% and the costs to both passenger and operator were brought down, illustrating that feeder/transfer systems can surely improve a direct-service system.

## 1.INTRODUCTION

Inter-city bus routes are conventionally conducted with direct service (non-stop service) to carry passengers from their origins to destinations quickly. However, because the buses on these routes are operated independently, they cannot be readily consolidated resulting in rather low load factors. For example, the load factors of three-fourth of the routes operated by Taiwan Motor Transport Company (TMTC), the largest bus company in Taiwan, are less than 50%. This fact indicates that buses on these routes are not fully utilized, which is a key factor in the financial deficit of TMTC. With the application of hub-and-spoke network techniques- setting up feeder stops at appropriate interchange or transfer stations at suitable spots such as rest areas- it could be feasible to consolidate bus operations to some extent to increase the load factor and thereby reduce the empty seat-mileage.

Because of the linearity of inter-city bus routes, after being redesigned as feeder/transfer systems, they will become linear hub-and-spoke networks. Then, not only the load factors can be enhanced but also the travel mileage for each passenger can be maintained at the same level without an adverse effect of detouring which is common in ordinary hub-and-spoke systems. However, a feeder/transfer system does increase the travel time and cause passengers inconvenience in making transfers. If there is no thorough planning and design process, the expected goal of feeder/transfer will not be achieved and the negative effect of service level deterioration might even occur. Therefore, how to develop a model for designing the inter-city feeder/transfer system to determine the optimal feeder/transfer location, routing, and scheduling is an issue worthy of study.

This study defines inter-city feeder/transfer system as having three levels. The first level is the location problem, which has to solve the optimal number of feeder stops/transfer stations and their locations. The second level is the routing problem, which has to solve the optimal network (direct, transfer or feeder service) based on the locations of feeder stops/transfer stations determined by the first level. The third level is the scheduling problem, which has to solve the optimal headway of each route based on the locations and networks determined by the first and second levels.

Most of the literature concerning the hub-and-spoke only discusses separately location selecting, routing or scheduling. Even that literature which considers the issues regarding these three levels is simplified to a large extent. For instance, Shih(1994) proposed a set of procedural structures on mass transportation network which considered transfer location selecting, routing and scheduling. However, since the structure was conducted with procedural modules, there was a lack of integration. Aykin(1995) designed a transfer system for the domestic air transportation network in the United States, including transfer location selecting and routing without addressing scheduling problem. Therefore, the expected benefit of transfer was given exogenously. Hsu *et al.*(1997) discussed the location selecting and routing problem for the inter-city bus transfer system, which employed a stepwise search algorithm to solve the optimal transfer location based on original timetable and given routing principles. But they did not consider the synchronization of bus scheduling.

Arisawa *et al.*(1977) in studies concerning routing and scheduling, solved the optimal scheduling of truck between hub-and-spoke to minimize the total cost of delayed cargo and empty-truck scheduling. Assad(1980) developed an optimal model for railway cargo scheduling and consolidation, transforming direct-service routes into transfer-service routes. Cargos are consolidated to increase compartment loading, thereby the operation cost is curtailed. Dobson *et al.*(1993) solved the optimal routing and scheduling for airline companies to maximize the profit under the hub-and-spoke system where there is only one single transfer station. Lan *et al.*(1997) and Lin *et al.*(1997) used analytical approaches to investigate the feeder/transfer systems of inter-city bus service under homogeneous (trip distribution and demand intensity do not vary as to time and space) and heterogeneous (trip distribution and demand intensity vary as to time and space) environments, respectively. Given the locations of transfer stations, they compared the benefits among four alternatives: direct-service, transfer, feeder, and feeder/transfer. In terms of synchronizing coordination, it was assumed that the average waiting time of passengers at origin ends, transfer stations, and feeder stops was in proportion to headway.

Relevant studies of scheduling (synchronizing coordination) are either designed as timed transfer or transfer optimization. The former designs the optimal slack time at transfer stations while the latter rearranges the optimal dispatching time of the first buses at origin ends in order to reduce waiting time for transfer passengers. As to minimizing waiting time at transfer stations, timed transfer is obviously better than transfer optimization. However, because timed transfer is to coordinate the buses of different routes to simultaneously arrive at transfer stations, each route has to be designed either common headway or integer ratio headway in which branch lines' headway is the integer ratio of trunk lines'. Transfer optimization is free of headway constraints. In addition, timed transfer works well in long headway routes in which headway exceeds 15 minutes, such as inter-city bus routes, but works poorly in short headway routes, such as city bus routes (e.g. Vuchic, 1981; Bookbinder *et al.*, 1992; Ting, 1997).

In terms of methodologies, the studies on timed transfer either employ analytical approaches to solve optimal headway and slack time (e.g. Salzborn, 1980; Vunchic, 1981; Hall, 1985; Lee *et al.,* 1991; Ting, 1997) or make system simulations to analyze the transfer cost for different transfer strategies (e.g. Abkowitz, 1987; Reynolds *et al.*, 1992). Because of the complexity of transfer optimization, most relevant studies employ procedural method or system simulation to determine the optimal timetable for the origin ends (e.g. Rapp *et al.*, 1976; Andreasson, 1977; Bookbinder *et al.*, 1992).

The relevant studies mentioned above, if they consider the issues of location selecting and routing, hardly get involved with the issue of scheduling. At the same time, they do not consider the fact that not all buses must be consolidated, even when stopping at the same transfer station. In order to make up for this defect, this paper employs genetic algorithms to determine feeder/transfer relationship among routes by clustering them into different partition sets. Then, an algorithm integrating feeder/transfer location selecting, routing and scheduling is proposed to solve the optimal feeder/transfer network and headway for each partition set. Finally, the efficiency and effectiveness of this model is tested on an experimental example.

## 2.FRAMEWORK

### 2.1 Assumptions

This paper divides inter-city bus service into three kinds. First, feeder service is defined as accommodating passengers who transfer from inter-city routes to local routes, or from local routes to inter-city routes at feeder stops. Second, transfer service is defined as accommodating passengers who transfer from one inter-city route to another inter-city route at transfer stations. Third, direct service or non-stop service is defined as accommodating passengers who are on the same bus from the origin end to the destination end without the maneuver of feeder or transfer.

We assume that the O/D distribution of passengers, the positions of interchange and rest area on the freeway are given. The demand of passengers is fixed and perfectly inelastic; that is, the demand will not change with different bus services-- direct, transfer or feeder. In addition, the demand has uniform distribution during a specific period of time and the travel speed of buses is deterministic. Each route consists of three parts: from the origin end to interchange, from interchange to interchange, and from interchange to the destination end. The transfer station is located at the rest area while the feeder stop is located near the interchange. Passengers make two transfers at most, however the number of feeders made by in-vehicle passengers has no limitation. Three kinds of bus service are illustrated in Figure 1. Figure 1a demonstrates the geometric layout of a linear freeway, Figure 1b shows four direct-service routes, Figure 1c shows the feeder network making use of interchanges I1, I2, I3, I7, I8, I9 as feeder stops. Figure 1d shows the transfer network making use of the rest area T2 and T4 as transfer stations.

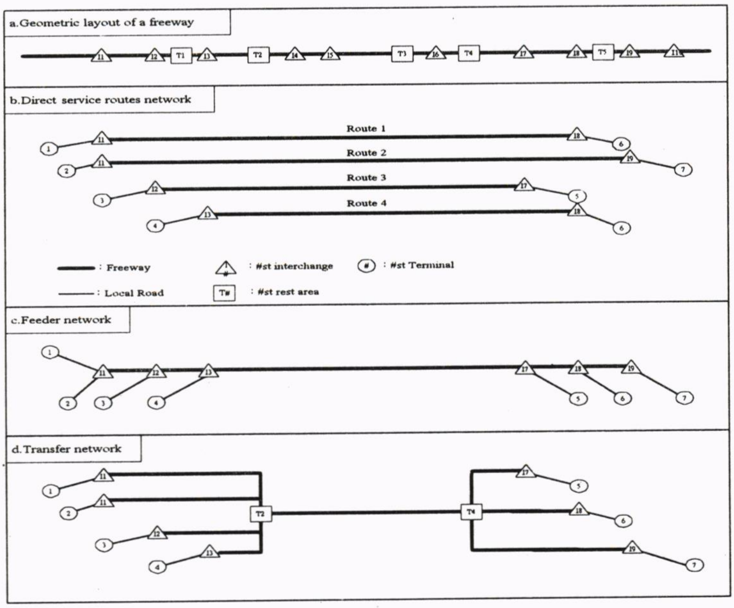


Figure 1. An example of feeder/transfer systems

## 2.2 Objective Function

The objective function is to minimize the total cost composed of three parts: passenger waiting cost incurring at origin ends, feeder stops, and transfer stations; passenger penalty cost incurring at transfer stations and feeder stops; and operator cost including operation and fleet. Each item of cost is elaborated as follows:

(1) Passenger waiting cost at origin ends

Let $WC_O$ denote the passenger cost of waiting time occurring at origin ends. It can be expressed as:

$$WC_O = \alpha \sum_{i=1}^{N} (\lambda_i + \lambda_i')W_i T \tag{1}$$

where $\alpha$ is the unit cost of waiting time at the origin station (dollar/hour). $\lambda_i$ and $\lambda_i'$ are the arrival rates of incoming and outgoing passengers of $i$th route (person/hour), respectively. $W_i$ is the average waiting time at origin ends (hour), which is a half of headway, assuming passengers do not know timetables in advance. $N$ is the number of routes. $T$ is a specific period of time (hour).

(2) Waiting cost at transfer stations or feeder stops

Let $WC_T$ denote the passenger cost of waiting time at transfer stations or feeder stops. It depends on the headway relationship between the original route and the route being transferred to. Without loss of generality, assume that the headway of original route and transferring route are $h_i$ and $h_j$, respectively. The average waiting time can then be expressed as (Ting, 1997):

$$W = 1/2[h_j - G(h_i, h_j)] \tag{2}$$

where $G(h_i, h_j)$ is the greatest common divisor of $h_i$ and $h_j$. Thus, in the case of integer ratio headway, the average waiting time from trunk line (headway $H$) to branch line (headway $nH$)

is $W=1/2[(n-1)H]$, while it is 0 from the branch line to trunk line. In the case of common headway, the average waiting time is 0, because all buses are coordinated to simultaneously arrive at feeder stops or transfer stations, feeder or transfer passengers do not need to wait for buses.

(3) Transfer penalty cost

Let $PC_T$ denote the transfer penalty cost, the inconvenience and discomfort that passengers suffer from changing buses at transfer stations or feeder stops. It can be expressed as:

$$PC_T = \gamma \sum_{i=1}^{N} \tau_i (\lambda_i + \lambda_i')T \tag{3}$$

where $\gamma$ is the passenger penalty cost of each transfer made (dollar/transfer). $\tau_i$ is the number of changing buses on $i$th route, $\tau_i=\{0,1,2\}$.

(4) In-vehicle passenger feeder penalty cost

Let $PC_F$ denote the feeder penalty cost, the inconvenience and discomfort to in-vehicle passengers who suffer from buses picking up or dropping off passengers at feeder stops. It can be expressed as:

$$PC_F = \delta \sum_{i=1}^{N} \pi_i (\lambda_i + \lambda_i')T \tag{4}$$

where $\delta$ is the passenger penalty cost of each feeder made (dollar/feeder). $\pi_i$ is the frequency of feeders on $i$th route, $\pi_i=\{0,1,2\ldots,m_i\}$. $m_i$ is the number of interchanges along the $i$th route.

(5) Operation Cost

Let $OC$ denote operation cost which is assumed proportional to bus-mileage, the product of route length, frequency of headway, and the period of time. It can be expressed as:

$$OC = \eta \sum_{i=1}^{N} \frac{2(l_i + L_i)T}{h_i} \tag{5}$$

where $\eta$ is the unit operation cost per bus-kilometer (dollar/bus-kilometer), and $l_i$ and $L_i$ are respectively the length on local road and freeway of $i$th route (kilometer).

(6) Fleet cost

Let $FC$ denote the fleet cost, depreciation of the minimum fleet size necessary to maintain the route headway. It can be expressed as:

$$FC = \theta \sum_{i=1}^{N} \left(\frac{2l_i}{vh_i} + \frac{2L_i}{Vh_i}\right) \tag{6}$$

where $\theta$ is the depreciation of a bus per day (dollar/bus-day). $v$ and $V$ are respectively the average travel speed of buses on local road and freeway (kilometer/hour).

Therefore, the objective function can be written as

$$\min TC = WC_O + WC_T + PC_T + PC_F + OC + FC.$$

### 2.3 Model Structure

In order to determine the feeder/transfer relationship among routes, this study divides all routes into several partition sets. The routes in the same partition set can be viewed as mutually feeder or transfer. Since clustering or set partitioning is of combinatorial character, which has been proved to be a NP-hard problem, thus rapidly becoming computationally intractable (Welch, 1983; Pinter *et al.*, 1991). However, genetic algorithms (GAs) are proved to perform efficiently and effectively in solving such combinatorial optimization problems (Goldberg, 1989; Lin *et al.*, 1993; Aytug *et al.*, 1994; Conway *et al.*, 1994). Thus, we first employ GAs to solve the set-partitioning problem. Then, a network optimization algorithm is proposed to solve the optimal network for each partition set. Finally, we utilize analytic approaches to solve the optimal headway for each network.

Figure 2 illustrates the integrated model of simultaneous clustering, while Figure 3 is the integrated model of stepwise clustering. Simultaneous clustering divides all routes into several partition sets simultaneously, then solves the optimal network and headway for each set. Stepwise clustering also chooses optimal partition set for each stage, then solves optimal network and headway for the chosen set, until the remaining routes are more cost saving for adopting direct service than for changing into feeder or transfer service. The following analysis compares the effectiveness and efficiency of these two models.

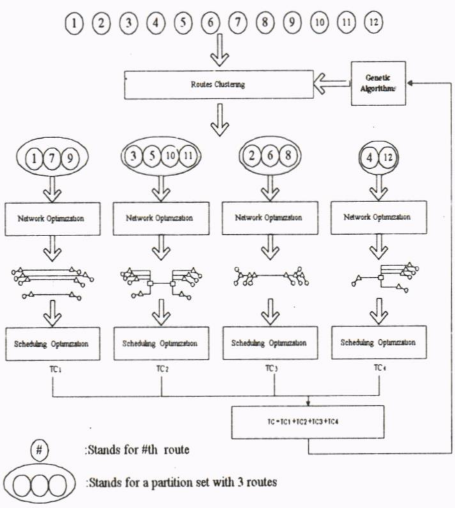


Figure 2. Framework of integrated model of simultaneous clustering

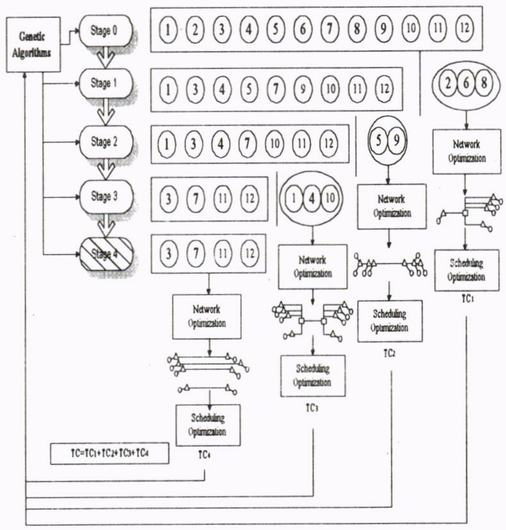


Figure 3. Framework of integrated model of stepwise clustering

## 3.SET PARTITIONING MODEL

### 3.1 Simultaneous Clustering Model

The mathematical programming model is

$$\min \quad TC = \sum_{k=1}^{K} TC_k \tag{7}$$

subject to

$$\sum_{k=1}^{K} X_{ik} = 1 \qquad i=1,2,\ldots,N \tag{8}$$

$$X_{ik} \in \{0,1\} \qquad i = 1,2,\ldots, N ; k = 1,2,\ldots, K \tag{9}$$

where $X_{ik}$=1 denotes that $i$th route is assigned to $k$th partition set, $X_{ik}$=0 otherwise . $N$ is the number of routes. $TC_k$ is the minimal cost of $k$th partition set. Whereas the routes belonging to the first partition set all employ direct service or else change to feeder or transfer service.

$K$ is the maximal number of partition sets, $K=[N/2]+1$ ($[]$ is Gauss sign). Adding 1 represents one more direct-service set is attached.

There are altogether $N \times K$ decision variables of the aforementioned model, as shown in Table 1. If there are more than two variables with value of 1 in the same column, it means that they mutually form a partition set. However, if $X_{ik}$ is encoded as a gene, it will cause the length of the chromosome too long (for instance, the number of decision variables for 10 routes is 60, for 20 routes is 220, and for 60 routes is 1860) and will result in the memory of computer being insufficient. In addition, it will be difficult to handle the constraints if one and only one variable equals 1 and else equals 0 in the same row.

Table 1. Relationship between routes and partition sets

| Routes | Partition sets | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | … | k | … | K |
| 1 | $X_{11}$ | $X_{12}$ | $X_{13}$ | | $X_{1k}$ | | $X_{1K}$ |
| 2 | $X_{21}$ | $X_{22}$ | $X_{23}$ | | $X_{2k}$ | | $X_{2K}$ |
| . . . | | | | | | | |
| I | $X_{i1}$ | $X_{i2}$ | $X_{i3}$ | | $X_{ik}$ | | $X_{Ik}$ |
| . . . | | | | | | | |
| N | $X_{N1}$ | $X_{N2}$ | $X_{N3}$ | | $X_{Nk}$ | | $X_{NK}$ |

In order to deal with the problem, this paper uses encoding/decoding techniques to replace each row of the decision variable matrix with a shorter gene string. Take 15 routes for example, there are at most 8 partition sets ($8=[15/2]+1$). Because each route is likely to be assigned to any set, these partition sets requires 3 genes to represent them, as shown in Table 2.

Table 2. Matching rules for gene strings, integers, and partition sets

| Gene strings | Integers | Partition sets |
|---|---|---|
| 000 | 0 | 1 |
| 001 | 1 | 2 |
| 010 | 2 | 3 |
| 011 | 3 | 4 |
| 100 | 4 | 5 |
| 101 | 5 | 6 |
| 110 | 6 | 7 |
| 111 | 7 | 8 |

The replacement of each row with these three genes not only curtails the length of chromosomes (4 genes can represent the problem of 31 routes, 5 genes can represent the problem of 62 routes) but also avoids the problem that a route might be assigned to several sets or be unassigned. Table 3 illustrates a feasible clustering result for these 15 routes. The chromosome of Table 3 is composed of 45 genes (000001011000101101010000000100000 0011011000000). Every 3 genes of the chromosome are then decoded into an integer of 0~7 sequentially, representing the partition set into which each route is assigned according to the matching rules stated in Table 2. After being decoded, the chromosome represents 2 partition sets with 7 direct-service routes (including routes 1, 4, 8, 10, 11, 14, 15 in partition set 1 and route 7 in partition set 3), 2 partition sets with 2 routes in each of them (including routes 2, 9 in partition set 2 and routes 5, 6 in partition set 6) and 1 partition set with 3 routes (including routes 3, 12, 13 in partition set 4).

Table 3. Relationship between routes and partition sets for an example of 15 routes

| Routes | Partition sets | | | | | | | | Encoding |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 000 |
| 2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 001 |
| 3 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 011 |
| 4 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 000 |
| 5 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 101 |
| 6 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 101 |
| 7 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 010 |
| 8 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 000 |
| 9 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 001 |
| 10 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 000 |
| 11 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 000 |
| 12 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 011 |
| 13 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 011 |
| 14 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 000 |
| 15 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 000 |
| Subtotal(routes) | 6 | 2 | 1 | 3 | 0 | 2 | 0 | 0 | |

As for the objective value of this feasible solution (the fitness of the chromosome), it is the total of the minimal costs of each partition set. For the first partition set and the partition set with only one route (partition set 3), their minimal cost is obtained by summing up the minimal cost of each route with direct service. For other partition sets (partition sets 2,4,6), their minimal cost is to calculate the cost under the optimal network and headway. If the partition set is impossible to form a feeder or transfer network, a big number M is assigned.

### 3.2 Stepwise Clustering Model

Stepwise clustering model divides all direct-service routes into two partition sets at every stage, one of which keeps direct service (called direct-service set), while the other set is turned to feeder or transfer (called feeder/transfer set). The mathematical model of each stage is formulated as follows:

Stage 1

$$MP_{(1)}: \quad \min \quad TC_{(1)} = TC_{(1)}^{D} + TC_{(1)}^{H} \tag{10}$$

subject to

$$X_i \in \{0,1\} \qquad i = 1,2,\ldots,|R_{(1)}| \tag{11}$$

where $X_i$=1 denotes that $i$th route belongs to the feeder/transfer set, while $X_i$=0 represents $i$th route belongs to the direct-service set. $R_{(1)}$ is the set of all direct-service routes at the first stage(that is, the total routes set: $R$). $TC_{(1)}$ is the total cost at the first stage. $TC_{(1)}^{D}$ and $TC_{(1)}^{H}$ are the total costs of direct-service set and feeder/transfer set at the first stage, respectively. While $TC_{(1)}$ is minimal, the corresponding clustering result can be expressed as $R_{(1)}^{\bullet} = \{r_i | X_i^{\bullet} = 1\}$ and $R_{(1)}^{*} = \{r_i | X_i^{*} = 0\}$. The former stands for the feeder/transfer set, the latter the direct-service set. Their optimal costs are $TC_{(1)}^{H*}$ and $TC_{(1)}^{D*}$, respectively. Moreover, $R_{(1)} = R_{(1)}^{*} \cup \overline{R_{(1)}^{*}}$ .

Stage 2

Let $R_{(2)} = \overline{R_{(1)}^{*}}$ and solve MP(2). While $TC_{(2)}$ is minimal, the corresponding clustering result can be expressed as: $R_{(2)}^{*}$ and $\overline{R_{(2)}^{*}}$. Their optimal costs are $TC_{(2)}^{H*}$ and $TC_{(2)}^{D*}$, respectively.

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Stage K

Continue to solve MP(K) accordingly until $R_{(K)}^{*} = \Phi$. Then, the cost of $\overline{R_{(K)}^{*}}$ is $TC_{(K)}{*}= TC_{(K)}^{D*}$.

Sum up the costs of feeder/transfer sets at each stage ($TC_{(k)}^{H*}$) as well as the cost of the direct-service set at the final stage ($TC_{(K)}^{D*}$). The minimal total cost can be expressed as $TC = \sum_{k=1}^{K-1} TC_{(k)}^{H*} + TC_{(K)}^{D*}$.

Compared with simultaneous clustering, both encoding/decoding for stepwise clustering are much simpler, where $X_i$ is directly encoded as gene, so the length of chromosome can be largely curtailed and can be further reduced in the evolutions of optimization stages. Consider $N$ routes for instance, let $|R| = N$ denote $N$ routes in set $R$, the length of the chromosome at the first stage is $N$. If $|R_{(1)}^{*}| = Q_{(1)}$, the length of the chromosome at the second stage is $N\text{-}Q_{(1)}$. If $|R_{(2)}^{*}| = Q_{(2)}$, the length of chromosome at the third stage can be further shortened as $N\text{-}Q_{(1)}\text{-}Q_{(2)}$, and so forth.

## 4.NETWORK OPTIMIZATION MODEL

### 4.1 Network Types

Based on the network features and passenger demands, each partition set can employ either transfer or feeder service. This study divides transfer network into three types: single-hub, double-hub, and mixed transfer networks, as shown in Table 4. Since the locations of origin/destination ends and interchanges of the freeway are given, the feeder network would be determined once the partition set was formed.

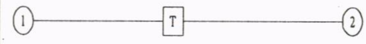
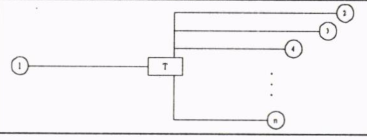
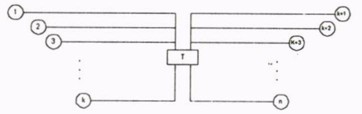
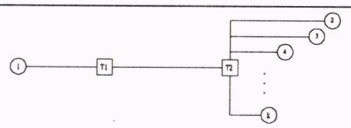
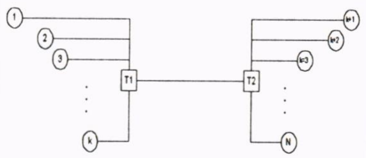
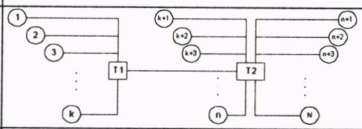
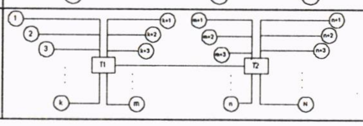
### 4.2 Network Optimization Algorithm

Network optimization algorithm is proposed as follows to determine the optimal network of a given partition set.

Step 1: If the partition set can form a feeder network, then calculate its minimal cost denoted as $TC_F$. If not, let $TC_F = TC_T = M$ and go to step 6.

Step 2: If the partition set can not form a transfer network, then let minimal cost of transfer network $TC_T = M$ and go to step 6.

Table 4. Types of transfer network and their characteristics

| Types | | Networks | Characteristics |
|---|---|---|---|
| Single-hub | One-to-one |  | • Passengers make exactly once transfer for each route.<br>• It is an inefficient network. |
| | One-to-many (Many-to-one) |  | • Passengers make exactly once transfer for each route.<br>• The closer to the most upstream destination end, the better the location of transfer station is. |
| | Many-to-many |  | • Passengers make exactly once transfer for each route.<br>• The optimal location of transfer station needs to be searched. |
| Double-hub | One-to-one |  | • Passengers make exactly twice transfers for each route.<br>• It is an inefficient network. |
| | One-to-many (Many-to-one) |  | • Passengers make exactly twice transfers for each route.<br>• It is an inefficient network. |
| | Many-to-many |  | • Passengers make exactly twice transfers for each route.<br>• The first transfer station is better located if closer to most downstream origin end ; the second transfer station is better located if closer to most upstream destination end. |
| Mixed | One part-to-two part |  | • Passengers make once or twice transfers depending on the route.<br>• The optimal location of transfer stations need to be searched. |
| | Two part-to-two part |  | • Passengers make once or twice transfers depending on the route.<br>• The optimal location of transfer stations need to be searched. |

Step 3: If all routes in the partition set have the same origin end, then one-to-many single-hub transfer network is employed. Calculate its minimal cost denoted as $TC_{so}$. Let $TC_T = TC_{so}$ and go to step 6.

Step 4: If all routes in the partition set have the same destination end, then many-to-one single-hub transfer network is employed. Calculate its minimal cost denoted as $TC_{sm}$. Let $TC_T = TC_{sm}$ and go to step 6.

Step 5:

(1) Let the partition set employ many-to-many single-hub transfer network. Calculate the minimal costs at different transfer stations, which are $TC_{s1}$, $TC_{s2}$, ..., $TC_{sn}$.

(2) Let the partition set employ many-to-many double-hub transfer network. Let the first transfer station be located at the rest area nearest to the most downstream origin end and the second transfer station be located at the rest area nearest to the

most upstream destination end. Calculate the minimal costs denoted as *TCd*.

(3) Let the partition set employ mixed transfer network. Arbitrarily select two rest areas as transfer stations. Calculate its minimal costs, which are *TCk1, TCk2 ,…,TCkm*.

(4) Choose the transfer network with the lowest cost and let $TC_T=min\{TCs1, TCs2, \ldots, TCsn, TCd, TCk1, TCk2, \ldots, TCkm\}$.

Step 6: Let $TC=min\{ TC_F, TC_T \}$; that is, if $TC_T > TC_F$, the partition set forms a feeder network; otherwise, a transfer network. Stop.

## 5.SCHEDULING OPTIMIZATION MODEL

Since the operation of buses as well as demand of passengers of one partition set is assumed independent of another set, the optimal headway for each partition set can be solved separately. This paper conducts synchronized coordination with timed transfer and adopts the integer ratio headway strategy (that is, the headway of branch line is integer ratio to the headway of trunk line). Such design allows the passengers transferring from branch line to trunk line without waiting, while passengers transferring from trunk line to branch line can also to a large extent be coordinated. Of course, if passenger demands on trunk line or branch line do not vary too much or if branch lines are rather short, the headway solved for trunk line and branch line might be the same; that is, it turns out to be a common headway strategy.

The decision variables of scheduling optimization model are the headway of trunk line (*H*) and the integer ratio of branch lines ($n_i$). The constraint is bus capacity(*C*). Take the feeder network as illustration, the optimal headway and optimal cost can be derived as follows and the mathematical formulations of cost items are shown in Table 5.

Table 5. Mathematical formulations of cost items for a feeder network

| Trunk/ Branch | Direction | $WC_O$ | $WC_T$ | $PC_T$ | $PC_F$ | $OC$ | $FC$ |
|---|---|---|---|---|---|---|---|
| Trunk line | Incoming | $\alpha\lambda T\frac{H}{2}$ | 0 | 0 | $\delta\lambda\tau T$ | $\eta\frac{2(L+L_1+L_2)T}{H}$ | $\theta\{\frac{2L}{VH}+\frac{2(L_1+L_2)}{vH}\}$ |
| | Outgoing | $\alpha\lambda' T\frac{H}{2}$ | 0 | 0 | $\delta\lambda'\tau T$ | | |
| Branch line | On-ramp | $\alpha\sum_i\lambda_i T\frac{h_i}{2}$ | $\beta\sum_i\lambda_i T(\frac{H}{2}-\frac{G(H,h_i)}{2})$ | $\gamma\sum_i\lambda_i T$ | $\delta\sum_k\lambda_i\tau_k T$ | $\eta\sum_i\frac{2l_i T}{h_i}$ | $\theta\sum_i\frac{2l_i}{vh_i}$ |
| | Off-ramp | 0 | $\beta\sum_i\lambda_i' T(\frac{h_i}{2}-\frac{G(H,h_i)}{2})$ | $\gamma\sum_i\lambda_i' T$ | $\delta\sum_k\lambda_i'\tau_k T$ | | |

Note: $\lambda$ and $\lambda'$ are the incoming and outgoing passenger arrival rates of trunk line, respectively. $H$ is the headway of trunk line. $\lambda_i$ and $\lambda_i'$ are the on-ramp and off-ramp passenger arrival rates of $i$th branch line, respectively. $h_i$ is the headway of $i$th branch line. $V$ and $v$ are average travel speed of freeway and local road, respectively. $L$ is the length of trunk line on freeway. $L_1$ and $L_2$ are the length of trunk line on local roads of two distinct ends, respectively. $l_i$ is the length of $i$th branch line.

Because the penalty cost is irrelevant to scheduling, we do not take it into account while deriving the optimal headway. Total cost function can be expressed as

$$TC=\alpha\{(\lambda+\lambda')\frac{H}{2}+\sum_i\lambda_i\frac{n_i H}{2}\}T+\beta\sum_i(\lambda_i'\frac{(n_i-1)H}{2})T+\eta\{\frac{2(L+L_1+L_2)}{H}+\sum_i\frac{2l_i}{n_i H}\}T$$

$$+\theta\{\frac{2L}{VH}+\frac{2(L_1+L_2)}{vH}+\sum_i\frac{2l_i}{vn_iH}\} \tag{12}$$

Taking the partial derivative of $TC$ with respect to $H$ and letting it equal to 0 yields the optimal headway of trunk line corresponding to arbitrary $n_i$

$$\tilde{H}=2\sqrt{\frac{\eta(L+L_1+L_2+\sum_i\frac{l_i}{n_i})T+\theta(\frac{L}{V}+\frac{L_1+L_2}{v}+\sum_i\frac{l_i}{vn_i})}{\alpha(\lambda+\lambda'+\sum_i\lambda_i n_i)T+\beta\sum_i\lambda_i'(n_i-1)T}} \tag{13}$$

Considering the constraint of bus capacity, then the optimal headway of trunk line corresponding to arbitrary $n_i$ is

$$\tilde{H}_{(C)}=min\{\ 2\sqrt{\frac{\eta(L+L_1+L_2+\sum_i\frac{l_i}{n_i})T+\theta(\frac{L}{V}+\frac{L_1+L_2}{v}+\sum_i\frac{l_i}{vn_i})}{\alpha(\lambda+\lambda'+\sum_i\lambda_i n_i)T+\beta\sum_i\lambda_i'(n_i-1)T}},\frac{C}{\lambda_{max}},\frac{C}{\lambda_{max}'}\} \tag{14}$$

where $\lambda_{max}$ and $\lambda_{max}'$ are maximal incoming and outgoing passenger arrival rate between two feeder stops along the freeway, respectively.

Because of integer constraint of $n_i$, it is impossible to derive the partial derivative of $TC$ with respect to $n_i$. Therefore, we use Largrangian relaxation to release the integer constraints. Taking the partial derivative of $TC$ with respect to each $n_i$ and letting it equal to 0 yields

$$\tilde{n}_i=\frac{2}{\tilde{H}}\sqrt{\frac{\eta l_i T+\theta\frac{l_i}{v}}{\alpha\lambda_i T+\beta\lambda_i'T}} \qquad for\quad i=1,\dots,I \tag{15}$$

Substituting all $\tilde{n}_i$ into Equation(14), the optimal headway of trunk line is

$$H^*=min\{\ 2\sqrt{\frac{\eta T(L+L_1+L_2)+\theta((\frac{L}{V}+\frac{L_1+L_2}{v})}{\alpha(\lambda+\lambda')T-\beta\sum_i\lambda_i T}},\frac{C}{\lambda_{max}},\frac{C}{\lambda_{max}'}\} \tag{16}$$

Substitute $H^*$ into Equation(15) and round each $\tilde{n}_i$ to yield the nearest integer $n_i^*$. Check the bus capacity constraint of each branch line. That is,

$$n_i^*\le\min\{\frac{C}{\lambda_i H^*},\frac{C}{\lambda_i'H^*}\} \qquad for\ all\ i \tag{17}$$

If all constraints of branch lines are satisfied, then $H^*$ and $n_i^*$ are the optimal result of scheduling. Otherwise, without loss of generality, if *i*th branch line is not satisfied, then set

$$n_i^*=\left[\min\{\frac{C}{\lambda_i H^*},\frac{C}{\lambda_i'H^*}\}\right] \qquad ([\,]\text{ is Gauss sign}) \tag{18}$$

Other variables remain the same. Taking all $n_i^*$ into Equation(14) yields $H^*$, and then solve all $n_i^*$ again. Repeat the procedures until the bus capacity constraints of trunk line and branch lines are satisfied. Similarly, the optimal headway of different types of transfer networks can be solved by the above procedures.

## 6.COMPUTATIONAL EXPERIMENTS

### 6.1 Experimental Design

The primary purpose of the computational experiments is to demonstrate and compare the efficiency and effectiveness of simultaneous clustering and stepwise clustering. An example of 15 routes is elaborated as follows:

(1) The geometric layout of the freeway: 6 interchanges are located at the mileage of 0, 30,

60, 90, 120, and 150 kilometer. 2 rest areas are located at 45 and 105 kilometer, which is shown in Figure 4.

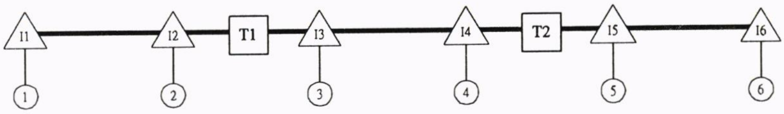



Figure 4. Geometric layout of the freeway of the experimental example

(2) Each interchange attaches only one end (origin or destination), which is 3 kilometers distant from the interchange. Every two distinct ends form a direct-service route, so there are totally 15 direct-service routes. The O/D distribution, route length, and passenger demand of each direct-service route are assumed as Table 6.

Table 6. Basic routes information of the experimental example

| Route | Origin | | Destination | | Route length (Kilometer) | | | Passenger demand (person/day) | |
|---|---|---|---|---|---|---|---|---|---|
| | O | IO | D | ID | Local at O | Freeway | Local at D | Incoming | Outgoing |
| 1 | 1 | 1 | 2 | 2 | 3 | 30 | 3 | 20 | 20 |
| 2 | 1 | 1 | 3 | 3 | 3 | 60 | 3 | 40 | 40 |
| 3 | 1 | 1 | 4 | 4 | 3 | 90 | 3 | 60 | 60 |
| 4 | 1 | 1 | 5 | 5 | 3 | 120 | 3 | 80 | 80 |
| 5 | 1 | 1 | 6 | 6 | 3 | 150 | 3 | 100 | 100 |
| 6 | 2 | 2 | 3 | 3 | 3 | 30 | 3 | 120 | 120 |
| 7 | 2 | 2 | 4 | 4 | 3 | 60 | 3 | 140 | 140 |
| 8 | 2 | 2 | 5 | 5 | 3 | 90 | 3 | 160 | 160 |
| 9 | 2 | 2 | 6 | 6 | 3 | 120 | 3 | 180 | 180 |
| 10 | 3 | 3 | 4 | 4 | 3 | 30 | 3 | 200 | 200 |
| 11 | 3 | 3 | 5 | 5 | 3 | 60 | 3 | 220 | 220 |
| 12 | 3 | 3 | 6 | 6 | 3 | 90 | 3 | 240 | 240 |
| 13 | 4 | 4 | 5 | 5 | 3 | 30 | 3 | 260 | 260 |
| 14 | 4 | 4 | 6 | 6 | 3 | 60 | 3 | 280 | 280 |
| 15 | 5 | 5 | 6 | 6 | 3 | 30 | 3 | 300 | 300 |

(3) The parameters of integrated optimization model are assumed as follows: $\alpha$=60 NT dollars per hour, $\beta$= 180 NT dollars per hour, $\gamma$=50 NT dollars per transfer, $\delta$=10 NT dollars per feeder, $\eta$=30 NT dollars per bus-kilometer, $\theta$=1000 NT dollars per bus-day, V= 90 kilometers per hour, v= 30 kilometers per hour, C=40 persons, T= 18 hours.

(4) The mechanism of genetic algorithms are designed as follows: population of each generation=100, roulette wheel selecting, two points crossover and gene mutation are being adopted.

### 6.2 Computational Results

Figure 5 shows the optimal cost solved by both simultaneous clustering and stepwise clustering under different mutation rates. As shown in Figure 5, different mutation rates have insignificant influence on the effectiveness of stepwise clustering. Except for the mutation rates of 0.006 and 0.009, stepwise clustering obtained the same solutions as that solved by totally enumerated (also under stepwise clustering concept), which indicates that GAs are rather effective. As for simultaneous clustering, its effectiveness is obviously inferior where the mutation rate is lower than 0.003 or higher than 0.1. However, it solved the same solution as that of stepwise clustering under mutation rates of 0.006, 0.01 and 0.03. Moreover, simultaneous clustering obtained even lower cost (NT 723,556 dollars) than the optimal value (NT 723,653 dollars) of stepwise clustering under mutation rate of 0.06 and 0.09. Of course, both optimization results of these two clustering models are lower than the cost of all routes with direct service (NT 775,919 dollars). Total cost is lowered by 6.75%.

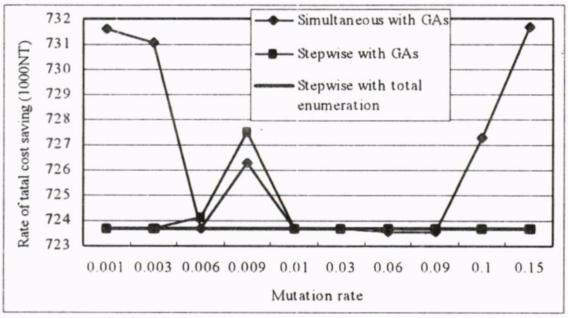



Figure 5. Optimal costs under various mutation rates

Figure 6 further shows the evolutions of simultaneous clustering and stepwise clustering under mutation rate of 0.06. In terms of effectiveness, simultaneous clustering is superior to stepwise clustering (97 dollars saved). In terms of efficiency, stepwise clustering is superior to simultaneous clustering, because most of the evolution curve of stepwise clustering falls below the curve of simultaneous clustering. For instance, the optimal solution is obtained at the 24$^{th}$ generation for stepwise clustering, while it is until 44$^{th}$ generation for simultaneous clustering to reach the same solution. The optimal solution of simultaneous clustering is obtained until 89$^{th}$ generation.

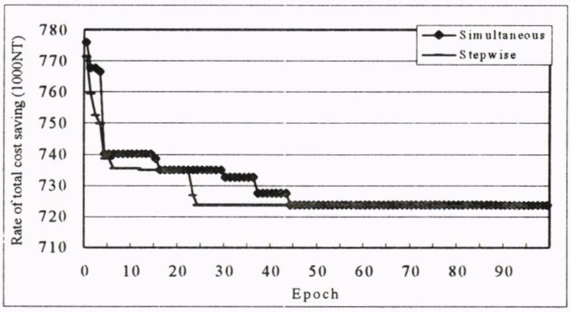



Figure 6. The evolutions of simultaneous clustering and stepwise clustering

The optimal results of simultaneous clustering are shown in Table 7. Notice that there are totally 4 partition sets. Except for one direct-service set (a total of 5 routes), the other 3 sets are formed as feeder networks (for a total of 10 routes all with common headway strategy). For each item of cost, $WC_O$ can be curtailed by NT 99,000 dollars; that is, the average waiting time for each passenger at the origin end changes from 81 minutes to 60 minutes after feeder service being provided. $PC_T$ increases by NT 80,000 dollars; that is, each passenger has to make an average of 0.33 transfers. $PC_F$ increases by NT 18,400 dollars; that is, each passenger has to make an average of 0.38 feeders. $OC$ drops by NT 51,626 dollars; that is, the overall bus-mileage is reduced by 1,721 kilometers. $FC$ drops by 138 dollars. In total, implementation of feeder/transfer systems can reduce total cost of passenger and operator by NT 600 dollars and by NT 51,764 dollars, respectively.

Table 7. The optimal results of simultaneous clustering (mutation rate=0.006)

| Partition sets | Routes | Network | Headway | $WC_O$ | $WC_T$ | $PC_T$ | $PC_F$ | $OC$ | $FC$ | $TC$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 10,12,13, 14,15 | D | — | 148300 | 0 | 0 | 0 | 143599 | 2704 | 294603 |
| 2 | 6,8,11 | F | 110 | 55000 | 0 | 34000 | 3200 | 58320 | 1418 | 151938 |
| 3 | 1,4,5,9 | F | 120 | 45600 | 0 | 28000 | 9200 | 87480 | 2067 | 172347 |
| 4 | 2,3,7 | F | 170 | 40800 | 0 | 18000 | 6000 | 38880 | 988 | 104668 |
| Cost with feeder/transfer service | | | (A) | 289700 | 0 | 80000 | 18400 | 328279 | 7177 | 723556 |
| Cost with direct service | | | (B) | 388700 | 0 | 0 | 0 | 379905 | 7315 | 775919 |
| Cost saving | | | (B-A) | 99000 | 0 | -80000 | -18400 | 51626 | 138 | 52364 |
| Rate of cost saving | | | (B-A)/B $\times$ 100% | 25.47% | 0 | - | - | 13.59% | 1.89% | 6.75% |

Note: 1.D stands for direct service set, F for feeder set.
2.To be realistic, each of headway is rounding to the nearest 5 minutes.

## 7.CONCLUDING REMARKS

This paper employs genetic algorithms (GAs) to determine the feeder/transfer relationship among routes by clustering the routes into different partition sets. To avoid lengthy discussion, readers are suggested to refer to Goldberg (1989) for the insights of GAs. An algorithm integrating feeder/transfer location selecting, routing, and scheduling is proposed to solve the optimal network and headway for each partition set. By testing on an experimental example of 15 direct-service routes, simultaneous clustering model is more effective but less efficient than stepwise clustering model. The results have shown that 3 feeder sets, a total of 10 routes altogether, were formed, while other 5 routes maintained direct service. As a whole, the total cost was lowered by 6.75% and the costs to both passenger and operator were brought down, illustrating that feeder/transfer systems can surely improve a direct service system.

This model can easily be applied in practice as long as the route characteristics and the geometric layout of a network are known as indicated in Table 6. However, this paper has, in order to simplify the problem, assumed that the bus speed is deterministic, passengers do not know timetables in advance, the demand is perfectly inelastic and without peak/off-peak variations. Perhaps future research can further release these assumptions and make the model more accommodating to the real world.

## ACKNOWLEDGEMENTS

The authors would like to thank the National Science Council of the Republic of China for financially supporting this research under Contract No. NSC 88-2211-E-009-019.

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