MODELING AIRLINE COMPETITION WITH ALLIANCES AS COOPERATIVE GAMES

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ABSTRACT: This paper analyzes the airline competition behavior with various types of alliances, such as code sharing and acquisition, by applying cooperative game theory. The study first reviews the definition and theorem of the core, the stable sets, and the bargaining sets, then formulates the airline's payoff as functions of air fares and service frequencies. The payoff function is composed of air travel demand model and cost model. Next, mathematical programs are developed to solve for the core, the stable sets, and the bargaining sets. A solution approach is then presented to find the payoff configuration and the coalition structure under market equilibrium. Finally, a case study based on data of the domestic airline market in Taiwan is illustrated as a model application.

1. INTRODUCTION

Taiwan's Domestic air travel market has become very competitive due to the deregulation in the last decade. Currently, the cutthroat competition in airfares among airlines has led to the increase in accident rate and the degrading in the level of service. To prevent the further deterioration in safety and quality of service, the Bureau of Civil Aviation in Taiwan encourages airlines to be merged or to form alliance.

Meanwhile, the ongoing processes of merging and cooperation among airlines were never ended. In practice, alliance was often viewed as a strategy for airlines to reduce costs and enhance competitiveness. Currently, the joint venture of three Taiwan's domestic airlines became Uni Air [Central Daily News, July, 26, 1998]; two of the domestic airlines, Far Eastern Airlines and Formosa Airlines, had joint flights in several market segments since 1996. In addition, there are numerous airline alliances around the world; the following are some of the new developments:

- The joint service between EVA Air and All Nippon Airways in Taipei-Osaka market [Central Daily News, Apr. 2, 1998];
- The alliance between Northwest Airlines and KLM [Commercial Times, Oct. 13, 1997];
- The alliance between British Airways and Air Austral [Commercial Times, Aug. 11, 1997];

- The Star Alliance: Air Canada International, Lufthansa, SAS, Trans Brazil, Thai Airways, and United Airlines. It has a network that covers 108 countries and 654 cities with the average frequency of one take-off per 15 seconds [Commercial Times, Oct. 13, 1997];
- The One-World Alliance: British Airways, Cathy Pacific Airline, Air Canada, American Airlines, and Air Austral. It has a network that covers 138 countries and 632 cities with the average frequency of one take-off per 14 seconds [Commercial Times, Sep. 22, 1998].
- The alliance of Singapore Airline, Air New Zealand, Ansett Australia, and Ansett International. It has a network that covers more than 40 countries and more than 200 cities.

The goal of the paper is to apply cooperative game theory to explore the interactive relationship among airlines in the process of alliance, and to develop an analytical model to predict the formulation of airline alliances.

The content of the paper is as follows: first, a brief review on cooperative games theory, i.e., the solution concepts of core, stable sets, Shapley value, and the bargaining sets; second, the formulation of airlines' payoff functions; third, a proposed solution approach; finally, the conclusion and future directions of study.

2. LITERATURE REVIEW

2.1 Review of Cooperative Game Theory

The fundamental theorem of cooperative games, based on Owen [1982] and Curiel [1997] are as follows:

2.1.1 Basic Notations and Assumptions

A cooperative game is often noted as $<N, V>$, where $N = \{1, 2, 3, \ldots\ldots, n\}$, is the set of all players, V is the characteristic function, S is the set of coalition, and $V(S)$ represents the payoff in the set S.

1. The Assumption of Super-additive

$V(S)+V(T) \le V(S \cup T) \quad \forall S, T \in 2^N$, such that $S \cap T \neq \emptyset$.

This assumption states that the profit produced by the union of S and T is no less that the sum of the profits produced by S and T separately.

Where $2^N = \{S \mid S \subset N\}$, the set of all subsets S, $\emptyset$ = the empty set.

2. The Assumption of Imputations

Let x_i be the profit allocated to player i, then the following conditions must be satisfied:

(i) $\sum_{i \in N} x_i = V(N)$

(ii) $x_i \geq V(\{i\}) \quad \forall i \in N$

Condition (i) states that the sum of profits from all players must be equal to the profit of union of all players. In other words, with the super-additive assumption, *V(N)* is the maximum of profit that could be produced by all players, therefore, the imputation of profit should be based on *V(N)*. Condition (ii), on the other hand, describes the assumption that the profit allocated to player *i* through the process of cooperation should be no less than the profit produced by player *i* alone.

3. Imputation **x** dominates **y** through coalition *S*

Let **x** and **y** be two imputations, if the following conditions are satisfied, it is referred as **x** superior to **y** through coalition *S* (notation: $\mathbf{x} \in=_s \mathbf{y}$) if

(i) $x_i \geq y_i \quad \forall i \in S$

(ii) $\sum_{i \in S} x_i \leq V(S)$

4. Carrier

If the condition $V(S) = V(S \cap T)$ holds, then T is called the carrier, i.e., coalition *S* has dummy members.

2.1.2 Solution Concepts of Cooperative Games

The following solution concepts describe the approaches to find the coalition structure and the allocation of payoffs among allied members.

1. Core

If the following conditions hold, then the core, noted as C(*V*), of the cooperative game is found. It should be noted that each player could not find any other imputation better than the solution of the core.

(i) $\sum_{i \in S} x_i \geq V(S) \quad \forall S \subset N$

(ii) $\sum_{i \in N} x_i \geq V(N)$ (1)

2. Stable Set

If the core does not exist, the other solution concept is called the stable set *U*. The set is defined as follows:

(i) If $\mathbf{x}, \mathbf{y} \in W$, then $\mathbf{x} \in\neq_s \mathbf{y}$
(ii) If $\mathbf{x} \notin W$, then there exists $\mathbf{y} \in W$, such that $\mathbf{y} \in=_s \mathbf{x}$ (2)

In other words, condition (i) states that a stable set satisfies the two conditions of internal stability (no imputation in *W* dominates another), and condition (ii) describes the external stability (any imputation outsides *W* is dominated by some imputation in *W*).

3. Shapley Value

Define $\varphi_i[V]$ to be the Shapley value in the game *V* for player i, the value is an index of power for each player. The following conditions must be satisfied:

(i) If *S* is any carrier of *S*, then $\sum_S \varphi_i[V] = V(S)$.
(ii) For any imputation π and $i \in N$, $\varphi_\pi(i)[\pi V] = \varphi_i[V]$
(iii) If *U* and *V* are any games, then $\varphi_i[U+V] = \varphi_i[U] + \varphi_i[V]$.

These are the Shapley's axioms. The Shapley value can be proven to become:

$$\varphi_i[V] = \sum_{\substack{T \subset N \\ i \subset T}} \frac{(t-1)!(n-t)!}{n!}[V(T) - V(T-\{i\})] \quad (3)$$

4. Multi-linear Extensions (MLE)

One of the difficulties with the Shapley value is that its computation requires the sum of a very large number of terms. Therefore, the multi-linear extension (MLE) of the game is a helpful tool to solve the problem. The MLE of *V* is a function of *f* defined by:

$$f(x_1, x_2, \ldots\ldots x_n) = \sum_{S \subset N} \left\{ \prod_{i \in s} x_i \prod_{i \notin s} (1 - x_i) \right\} V(S) \quad (4)$$

For $0 \le x_i \le 1$, $i = 1, 2, \ldots, n$.

The MLE approach has two advantages:

(i) Limit theorems of probability theory, such as central limit theorem, can be applied.
(ii) The behavior of MLE under the composition of games can be dealt with.

5. The Bargaining Set

To address the concept of the bargaining set, we need to define some of the terminology as follows:

Definition: A payoff configuration is a pair

$$(\mathbf{x},\mathbf{T}) = (x_1,\cdots,x_n,T_1,\cdots T_m),$$

where **T** is a coalition structure of an n-person game, i.e., a partition of N, and **x** is an imputation satisfying

(i) $\sum_{i\in T_k} x_i = V(T_k)$, for $k = 1, \ldots, m$.

(ii) $x_i \geq V(\{i\})$

(iii) $\sum_{i\in S} x_i \geq V(S)$ (5)

If the payoff configuration satisfies condition (ii), it is said to be individually rational; if the payoff configuration satisfies condition (iii), it is said to be coalitionally rational. This paper will deal with the case of coalitionally rational payoff configuration (c.r.p.c.).

Definition: Let $\mathbf{T} = (T_1,\cdots T_m)$ be a coalition structure, and let K be a coalition. Then the partner of K in **T** means that

$$P(K,\mathbf{T}) = (i \mid i \in T_k, T_k \cap K \neq \emptyset)$$

Definition: Let (**x**, **T**) be a c.r.p.c. for a game V. Let K and L be nonempty disjoint subsets of some $T_k \in \mathbf{T}$. Then an objection of K against L is a c.r.p.c. (**y**, **U**) satisfying

(i) $P(K,\mathbf{U}) \cap L = \emptyset)$

(ii) $y_i > x_i \quad \forall i \in K$

(iii) $y_i \geq x_i \quad \forall i \in P(K,\mathbf{U})$ (6)

Then a counter-objection of L against K is a c.r.p.c. (**z**, **V**) satisfying

(iv) $K \not\subset P(L,\mathbf{V})$

(v) $z_i \geq x_i \quad \forall i \in P(L,\mathbf{V})$

(vi) $z_i \geq y_i \quad \forall i \in P(L,\mathbf{V}) \cap P(K,\mathbf{U})$ (7)

Thus, a c.r.p.c. (**x**, **T**) is called stable if for every objection of a K against L, L has a counter-objection. The bargaining set **M** is the set of all stable c.r.p.c.s. Based on the concept of bargaining set, the solution approaches such as the Kernel and the Nucleolus can be developed.

6. The Nucleolus

One of the most commonly used solution concepts is the nucleolus. Let V be an n-person game, and let **X** be a set of n-vectors (payoff vectors). Define $\theta(\mathbf{x})$ as the excess of S, i.e.,

$$\theta(\mathbf{x}) = V(S) - \sum_{i\in S} x_i \qquad (8)$$

Let $\Theta(\mathbf{x})$ be the 2^N vector whose components are $\theta(\mathbf{x})$'s, the excesses of the 2^N subsets of $S \subset N$, arranged in descending order. Define the relation $\theta(\mathbf{x}) \leq_L \theta(\mathbf{y})$ as the lexicographic ordering of **x** is no less than **y**, then the nucleolus of V over the set **X** is the set N(**X**) defined by

$$\mathbf{N(X)} = \left\{ \mathbf{x} \middle| \begin{array}{c} \mathbf{x} \in \mathbf{X} \\ \text{If } \mathbf{y} \in \mathbf{X}, \text{then } \mathbf{x} \preceq \mathbf{y} \end{array} \right\} \quad (9)$$

Where, $\mathbf{x} \preceq \mathbf{y}$ if and only if $\theta(\mathbf{x}) \leq_L \theta(\mathbf{y})$;

$\mathbf{x} \prec \mathbf{y}$ if and only if $\theta(\mathbf{x}) <_L \theta(\mathbf{y})$.

Based on the theorems proven in the bargaining set, the nucleolus can be solved by the following linear program:

Min α
Subject to

$$\sum_{i \in S} x_i + \alpha \geq V(S), \quad S \subset N, \quad x \in \mathbf{x} \quad (10)$$

2.2 Review of Related Papers

Currently there are only a few papers that deal with the Taiwan's airline alliances or analyze the airline's competition behavior by applying game theory. The followings are the brief summary of these researches:

Ko [1997] described various forms of cooperation and alliances among airlines. Detailed insights regarding the motivation and the achievement of these alliances are presented in the paper. Li and Tang [1997] presented some quantitative measures to evaluate the effectiveness of alliances. The before-and-after ridership data of these allied airlines are examined by statistical hypothesis test procedures. Li [1997] first applied game theory to analyze the competition behavior of the domestic airline market, then Shyr [1998] and Wu [1998] extended Li's work to the hub-and-spoke network environment. The basic framework used by Li [1997], Shyr, and Wu [1998] are as follows:

(i) The airline's payoff function is consists of an O-D demand model, a market share model, and a cost model;
(ii) The O-D demand model is a function of average airfares, total flight frequency, and related attributes of other competing modes;
(iii) The market share model is calibrated by using Logit model based on stated preference data and revealed preference data. The variables include two fare classes and frequency;
(iv) The cost model is calibrated by using Hansen's equation [1990].
(v) Solution approaches in four separate models, i.e., the static and dynamic airfare competition models, and the static and dynamic frequency competition model, are proposed to solve for the market equilibrium.

In addition, Wu [1998] applied the market leader-and-follower relationship into the dynamic game of the airfare competition model. The basic framework of airlines' payoff functions is also used in the paper.

3. MODEL FORMULATION

The basic assumptions of the model are as follows:

1) The sole objective of each airline is profit maximization;
2) Airfares are subject to change in the short term, but flight frequency remains constant;
3) The demands of various O-D pairs are independent;
4) The demands of business and leisure travelers are independent;
5) The factors affecting payoffs are limited to airfares and frequency;
6) All coalitions satisfy the property of superadditive;
7) All airlines have perfect information of payoffs.

The first assumption states the fact that each airline would seek for the maximum profit in the cooperative games. The second one is reasonable because currently Taiwan's aerospace is very crowded and there is no room for the increase of flight frequency. The third and the fourth assumptions are often adopted by most studies and will allow us to solve the optimization problem based on one fare class of an O-D pair. The fifth one assumes that the demand of domestic air travel is very sensitive to airfare and frequency but less sensitive to other service attributes. The assumption is acceptable in Taiwan's domestic market because all airlines possess similar service quality. The last two assumptions are essential to the cooperative games. The sixth assumption means the bigger the alliance, the better for everyone in the coalition.

3.1 The Airline's Payoff Functions

The airline's payoff function consists of the estimated profits and the costs associated with the coalition k. The demands and costs of coalition k are to be estimated by the models described in the following sections.

$$\pi_{ijk} = p_{ijk} \times q_{ijk} - Cost_{ijk} \times F_{ijk} \tag{11}$$

Where

π_{ijk} = the payoff of airline coalition k for the (i, j) O-D pair;

p_{ijk} = the airfare of airline coalition k for the (i, j) O-D pair;

q_{ijk} = the revenue passengers of airline coalition k for the (i, j) O-D pair;

$Cost_{ijk}$ = the air travel cost per flight of airline coalition k for the (i, j) O-D pair;

F_{ijk} = the frequency of airline coalition k for the (i, j) O-D pair.

3.2 The Airline's Demand and the Loading Factor Functions

One of the indices used by airlines to record revenues generated by air services is called revenue passenger mileage (RPM). Although the potential air travel demand may not be bounded by airline's capacity if airlines offer attractive low prices, the revenue passengers per flight, on the other hand, should be no more than the number of seats provided by the airline on each flight. As a result, this study formulates the loading factor R_{ijk} to be a logistic function of airfares and frequency such that the revenue passengers would not exceed the capacity.

$$q_{ijk} = seat_{ijk} \times F_{ijk} \times R_{ijk} \tag{12}$$

$$R_{ijk} = 1 \Big/ 1 + \exp[a_0 + a_1 \cdot p_{ijk} + a_2 \cdot p_{ijl} + a_3 \cdot \ln F_{ijk} + a_4 \cdot \ln F_{ijl}] \tag{13}$$

Where,

R_{ijk} = the loading factor of airline coalition k for the (i, j) O-D pair;

p_{ijl} = the airfare of the rivalry coalition l for the (i, j) O-D pair;

F_{ijl} = the frequency of the rivalry coalition l for the (i, j) O-D pair;

a_0, a_1, a_2, a_3, a_4 = the model parameters of the loading factor functions.

The logistic function can be further transformed into a linear form as shown in equation (14) such that general linear regressions can be applied to estimate model parameters.

$$\ln\left(1 \Big/ R_{ijk} - 1\right) = a_0 + a_1 \cdot p_{ijk} + a_2 \cdot p_{ijl} + a_3 \cdot \ln F_{ijk} + a_4 \cdot \ln F_{ijl} \tag{14}$$

Meanwhile, the logic of considering two alliances in the model would be explored in section 4.2.

3.3 The Cost Function

Due to fact that cost data are very confidential in airline industry, it is very difficult to acquire accurate cost data from Taiwan's airlines. However, the estimated rate of return in the domestic airline industry is known to be no more than 5%. Equation (15) shows the suggested cost model. The cost model consists of the discount factor δ and the cost function calibrated by Hansen [1990]. It should be noted that only the variable costs are taken into account in the model.

$$Cost_{ijk} = \delta_{ij} \times (\beta_0 + \beta_1 \cdot seats_{ijk} + \beta_2 \cdot miles_{ij} + \beta_3 \cdot seats_{ijk} \cdot miles_{ij}) \tag{15}$$

$Cost_{ijk}$: costs per flight of airline coalition k for the (i, j) O-D pair in US dollars, including operating costs, capital costs, depreciation costs, and maintenance costs;
δ_{ij}: the discount factor for the (i, j) O-D pair;
$seats_{ijk}$: the average number of seats per flight of airline coalition k for the (i, j) O-D pair;
$miles_{ij}$: the flight distance for the (i, j) O-D pair.
β_0, β_1, β_2, β_3: the model parameters estimated by Hansen [1990].

4. SOLUTION APPROACH

This section describes the solution approach with two parts, the first step is to calibrate the model parameters with data generation, and the second step is to solve for the core, the Shapley value and the nucleolus of the cooperative games.

4.1 Data Generation and Model Calibration

Due to the fact that the data of loading factors for all coalitions are not available, a stated preference survey was conducted to derive the potential demand if airline alliances are formed. The procedures for deriving these loading factors are as follows:

1) Calibrate the market share models S_{ijk} described in the following section for all airline alliances by using stated preference data;
2) Calibrate the average loading factor model described in the following section, then estimate the revenue passengers for the (i, j) O-D pair with the following relationship:

$$Q_{ij} = \overline{R}_{ij} \times TF_{ij} \times \overline{Seats}_{ij} \tag{16}$$

Where,
Q_{ij} = the estimated revenue passengers for the (i, j) O-D pair;
$\overline{R}_{ij}$ = the average loading factor calibrated to the (i, j) O-D pair;
TF_{ij} = the total flight frequency provided by all airlines for the (i, j) O-D pair;
$\overline{Seats}_{ij}$ = the average number of seats per flight for the (i, j) O-D pair.

3) Predict the loading factors of all airline alliances under various scenarios with the following equation:

$$R_{ijk} = \frac{Q_{ij} \cdot S_{ijk}}{F_{ijk} \cdot seats_{ijk}} \tag{17}$$

4) Use the predicted values of loading factors as the input data to the regression model shown in equation (13).

4.1.1 The Airline Market Share Functions

Given the assumption that air travelers have no strong preference over one specific airline to another in Taiwan's domestic market, this model describes how travelers choose the airline alliances. The market share model includes all the attributes related to the quality of service and airfares. The formulation of the model is shown below:

$$S_{ijk} = \frac{e^{V_{ijk}}}{\sum_{k=1}^{K} e^{V_{ijk}}} \tag{18}$$

Equation (18) shows that the market share model are formulated as a multinomial logit model [Ben-Akiva, 1985]. The variables of these utility functions usually consist of the airfares and frequency of other competing alliances and the alternative-specific constants. The functional forms of these utility functions are as follows:

$$V_{ijk} = \alpha_0 + \alpha_1 \cdot p_{ijk} + \alpha_2 \cdot F_{ijk} \tag{19}$$

Where,

$\alpha_0, \alpha_1, \alpha_2$: the model parameters in the utility functions;

4.1.2 The Function of Average Loading Factor for an O-D Pair

In addition, the travel time and fares of the other competing modes should be included in the model. Wu [1990] had calibrated the domestic air travel demands, but the model parameters need to be estimated with updated data.

$$\overline{R}_{ij} = \frac{1}{1+\exp[U_{ijk}]}$$

$$U_{ijk} = b_0 + b_1\overline{P}_{ij} + b_2 \ln TF_{ij} + b_3 Income_{ij} + b_4 \overline{Seats_{ij}} + b_5 Rail_{ij} + b_6 D_{1ij} + b_7 D_{2ij} \tag{20}$$

Where,

$\overline{P}_{ij}$ = the average airfare weighted by airline's capacity for the (i, j) O-D pair;

$Income_{ij}$ = the average household income for the (i, j) O-D pair;

$Rail_{ij}$ = the price of express railway for the (i, j) O-D pair;

D_{1ij} = 1 if it was the peak seasons, i.e., from June to August; 0, otherwise, i.e., the seasonal factor for the (i, j) O-D pair;

D_{2ij} = 1 for a period of three months followed by an accident; 0, otherwise, i.e., the safety factor for the (i, j) O-D pair;

4.2 Solution Approach to Cooperative Games

Given the assumption that flight frequency remains constant in the short term, airfare becomes the only decision variable in the solution process. With the assumption that the demands of various O-D pairs are independent, the payoff of each O-D pair can be solved separately. The proposed approach is as follows:

Step I: List all possible coalition structures, then find the minimum payoffs among all coalition structures.

For example, in the case of four airlines, the coalition structures could be:

1) No coalition is formed, i.e., $\mathbf{T} = \{T_1=\{1\}, T_2=\{2\}, T_3=\{3\}, T_4=\{4\}\}$;
2) A coalition of two, e.g., $\mathbf{T} = \{T_1=\{1,2\}, T_2=\{3\}, T_3=\{4\}\}$;
3) A coalition of three, e.g., $\mathbf{T} = \{T_1=\{1,2,3\}, T_2=\{4\}\}$;
4) Two Coalitions of two airlines, e.g., $\mathbf{T} = \{T_1=\{1,2\}, T_2=\{3,4\}\}$;
5) A unified coalition, i.e., $\mathbf{T} = \{T_1=\{1,2,3,4\}\}$.

It should be noted that there are: 6 combinations of case 2, 4 combinations of case 3, and 6 combinations of case 4. Because there are so many coalition structures, we need a criterion to set up the payoff values under various coalition structures. For example, the payoff of airline 4 would vary from case 1 to case 3, then which one should be the appropriate payoff value for airline 4?

In this case, the criterion is to choose the minimum payoffs among all coalition structures. With the assumption of superadditive, i.e., the bigger the alliance, the better for everyone in the union, the worst scenario for an individual airline would be to compete with a coalition of three airlines. In other words, the minimum payoffs for all individual airlines should be derived from case 3. Similarly, the payoffs for all coalitions of two should be derived from case 4. Therefore, we can focus on the calibration of payoff functions in case 3, case 4, and case 5 for the cooperative games.

Step II: Calibrate the parameters of the payoff functions, π_{ijk}'s.

Step III: Solve the following system of equations to find the optimal airfares F_{ijk}'s;

$$\frac{\partial \pi_{ijk}}{\partial F_{ijk}} = 0, \quad \forall k, \quad \forall (i,j) \tag{21}$$

It should be noted that the system of nonlinear equations shown in Equation (21) could be solved by Mathematica [Varian, 1993].

Step IV: Calculate the payoffs with F_{ijk}'s derived from Equation (21).

Step V: Apply Equation (10) to solve for the Nucleolus.

The problem could also be solved by applying Mathematica [Varian, 1993].

5. CASE STUDY

Since the solution approach can be duplicated in each O-D pair with each fare class, to simplify the task, we would focus on one fare class in one O-D market. Table 1 shows the estimated costs per flight in the market. The flight distance between Taipei and Tainan is 195.6 miles. The discount factor δ is approximately 0.6.

Table 1: The Estimated Costs in the Taipei-Tainan Market

Airlines	Average number of seats/flight	Flight distance (mile)	Costs/flight (NTD)
1	163	195.6	173654
2	183	195.6	197172
3	151	195.6	159543

NTD: New Taiwan Dollar, 1 USD = 32 NTD.

Table 2: Parameters of Market Share Models (t values in parentheses)

Variables \ Models	{1, 2} versus {3}	{1, 3} versus {2}	{2, 3} versus {1}
Constant	1.2458 (4.509)	0.8247 (3.180)	1.1629 (4.155)
Airfare (NTD)	-0.0016 (-3.166)	-0.0024 (-5.206)	-0.0019 (-4.084)
Flights/day	0.0168 (0.956)	0.0345 (1.423)	0.0251 (0.962)
ln L(β)	-188.61	-225.64	-203.80
ln L(0)	-268.25	-286.96	-280.72
ρ^2	0.30	0.21	0.27
No. of sample	387	414	405

Table 3: Parameters of the Average Loading Factors (t values in parentheses)

Variables \ Models	Loading Factor	Loading Factor (with significant parameters)
Constant	8.204(4.903)	8.373(5.185)
ln Frequency	-0.807(-2.728)	-0.841(-2.976)
Airfare	-0.0029(-2.163)	-0.0032(-2.376)
Income	0.00018(4.817)	0.00017(4.681)
Seats	-0.011(-4.153)	-0.010(-4.030)
Rail	-0.00067(-1.171)	NA
D1	0.0035(0.066)	NA
D2	-0.054(-0.952)	NA
R^2	0.754	0.747
Adjusted R_a^2	0.731	0.734

Table 4: Loading Factor for All Coalitions (t values in parentheses)

Variables \ Models	Loading Factors: {1, 2} versus {3}		Loading Factors: {1, 3} versus {2}		Loading Factors: {2, 3} versus {1}	
	{3}	{1, 2}	{2}	{1, 3}	{1}	{2, 3}
Constant	4.177 (9.208)	12.082 (6.578)	11.943 (6.731)	10.603 (4.759)	5.031 (11.167)	11.965 (6.461)
F_{12} (flights/day)	0.178 (1.836)	-2.356 (-5.987)	-1.290 (-3.394)	-2.082 (-4.363)	-0.157 (-1.630)	-2.230 (-5.621)
F_3 (flights/day)	-1.331 (-11.162)	1.648 (3.412)	-2.422 (-5.191)	1.693 (2.891)	-1.260 (-10.639)	1.563 (3.209)
Airfare P_{12} (NTD)	0.00094 (8.494)	-0.0054 (-12.086)	0.0033 (7.581)	-0.0061 (-11.248)	0.0014 (12.806)	-0.0057 (-12.654)
Airfare P_3 (NTD)	-0.0026 (-21.849)	0.00043 (0.091)	-0.0052 (-11.249)	0.001 (1.742)	-0.0030 (-26.043)	0.0003 (0.627)
R^2	0.969	0.943	0.892	0.921	0.975	0.944
R_a^2	0.962	0.932	0.869	0.904	0.970	0.932

Table 2 shows the calibration results of the market share model in the Taipei-Tainan market with the stated preference demand data, and Table 3 is the estimated parameters of the average loading factor model described in equation (20). Then, applying the procedure in section 4.1 produces Table 4. It should be noted that Table 2, Table 3 and Table 4 are all consistent with the a priori. Namely, the market shares decrease as airfares increase, and more flight frequency leads to higher utility; on the other hand, higher airfares and more frequency produce the lower loading factors.

Table 5: Payoffs and Optimal Airfares for All Coalition Structures

Coalition S	V(S) (NTD/day)	Optimal Airfare (NTD)	Revenue Passengers per Day	Loading Factors	Frequency (flights/day)
12	2958150	1241	6328	0.85	22
3	-628528	744	1471	0.48	9
13	2929160	1230	5862	0.87	20
2	225053	953	2967	0.80	11
23	3783460	1333	5837	0.86	20
1	-47287	937	2396	0.64	11
123	14464100	1895	8752	0.84	31

Table 6: Shapley Values and the Nucleolus

Airlines	Shapley Values	Ratio	Nucleolus	Ratio
1	4592915	0.32	4924334	0.34
2	5456235	0.36	5196674	0.36
3	4714950	0.33	4343093	0.30

By the application of Mathematica, the payoffs of these airlines under different coalition structures are calculated and shown in Table 5 and the Shapley value and the Nucleolus are described in Table 6.

It should be noted that the property of superadditive is satisfied in Table 5. Table 5 also shows that the airfares increase as the discrepancies between two alliances increase. Namely, when a bigger coalition is formed in the market, it will enjoy more monopolistic power over the airfare.

By examining Table 6, we find that the discrepancy between the Shapley values and the Nucleolus is insignificant. As a result, the imputation of payoff can be based on the Shapley values or the Nucleolus. Meanwhile, airline 2 has the biggest share of the payoff because it has better safety record and better quality of service in this market.

In addition, the nucleolus shows the final settlement among theses three players if they could all form a unified coalition. However, with the application of anti-trust law, penalty associated with monopoly should be taken into account in the case of one unified alliance. If the penalty is so severe such that V{1,2,3} is no longer greater than V{2,3}, then the coalition {2,3} could be the outcome of the market equilibrium.

6. CONCLUSION

The major achievements of the paper are as follows:

- The paper demonstrates how to apply cooperative game theory to interpret the cooperation and competition behavior among airlines.

- The proposed payoff function reveals the complexity of transportation behavior. The function consists of an O-D demand model to be calibrated by the aggregate demand data, a market share model to be calibrated by logit model, and a cost model to be calibrated by Hansen's formulation and to be adjusted by a discount factor.

- The solution approach describes the procedures of solving the nucleolus of the cooperative game.

- The case study presents the estimated parameters of the payoff functions based on data collected from one of Taiwan's major domestic markets. The payoff values of all coalitions are calculated, and the solution of nucleolus is interpreted.

However, there are some research directions needed to be followed in the future. These directions are as follows:

- A comprehensive survey of cost functions for major airlines on domestic markets.

- Costs reduction due to coalition as well as penalty applied to monopoly should be taken into account in the future studies.

- Incorporating decisions regarding flight frequency into the cooperative games.

- Extension to various levels of alliance in the formulation of coalitional payoff functions.

REFERENCES

Ben-Akiva, M. and Lerman, S. (1985) **Discrete Choice Analysis**. MIT Press, Cambridge, Massachusetts.

Commercial Times. (1998) **Cathy Pacific Joins the Star Alliance**, Sep. 22, pp. 3, Taipei, Taiwan.

Commercial Times. (1997) **The Impact of Star Alliance on Taiwan's Airlines**, Oct. 13, pp. 9, Taipei, Taiwan.

Commercial Times. (1997) **The Alliance of Singapore Airlines, Air New Zealand, Ansett Australia, and Ansett International**, Aug. 11, pp. 9, Taipei, Taiwan.

Central Daily News. (1998) **The Trend of Alliance in Airline Business**, July 26, pp. 5, Taipei, Taiwan.

Central Daily News. (1998) **EVA's Joint Services with ANA in Taipei-Osaka Route**, Apr. 2, pp.5, Taipei, Taiwan.

Curiel, I. (1997) **Cooperative Game Theory and Applications**, Kluwer Academic Press, Boston.

Hansen, M. (1990) Airline competition in a hub-dominated environment: an application of non-cooperative game theory, **Transportation Research B**, Vol. 24B, No. 1, 27-43.

Ko, C. (1997) **A Study of the Motivation and Types of Alliances among Domestic Airlines**. Master thesis, National Cheng-Kung University, Tainan, Taiwan.

Li, J. J. and Tang, M. (1997) **A Study of the Alliances Among Domestic Airlines**. Research report submitted to Development Funds for Chinese Airline Industry, Department of International Business, National Taiwan University, Taipei, Taiwan.

Li, C. (1997) **A Study of the Airline Oligopolistic Competition Behavior in the Domestic Market**. Master thesis, Tamkang University, Taipei, Taiwan.

Shyr, F. and Li, C. (1997) Modeling airline competition with two fare classes under static and dynamic games. **Proceedings 8th World Conference on Transport Research**, UFSIA, Antwerp, Belgium, 12-17, July 1997.

Owen, G. (1982) **Game Theory - second edition**, Academic Press Inc., Orlando, Florida.

Varian, H. (1993) **Economic and Financial Modeling with Mathematica**, Springer-Verlag Publishers, The Electronic Library of Science, Santa Clara, California.

Wu, Y. (1998) **Modeling Airline Competition under Hub-and-Spoke Networks**. Master thesis, Tamkang University, Taipei, Taiwan.

Wu, C. H. (1990) **An Analysis of Domestic Airline Passenger Demand in Taiwan**. Master thesis, National Taiwan University, Taipei, Taiwan.