RESEARCH ON PARAMETERS OF MECHANICAL MODELS FOR TRACK STRUCTURES OF HIGH SPEED RAILWAY

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Abstract : The results of research on the parameters of mechanical models for track structures of high speed railways, the computational formulas and the value ranges corresponding to these parameters are given in this paper. At the same time, the dynamic response characteristic parameters for the essential components of track structures are presented. These include wheel-rail interaction coefficients, nature frequencies and vibration accelerations measured in situ and their ranges as well as allowable maximum values in some key positions, which can be used to check the calculated results of the theoretical models.

1. INTRODUCTION

With the appearance of new problems in the development of railroad transportation resulting from high speeds, heavy haul, large traffic volume and traffic density, it is imperative to understand wheel-rail dynamic interaction. Numerical methods are used to investigate the wheel-rail system. Recently, a lot of theoretical models have been presented, in which the rolling stock, railroad and roadbed are analyzed as a whole system(Ahlbeck, D. R., Prause, R.H., and Meacham, H. C. 1973; Jenkins, H.H., et al 1974; Meacham, H. C. et al 1970; T.H.Tung 1989; Y.H. Huang, J.G. Rose 1988). By means of theoretical models it is possible to predict the dynamic responses and the stability of the track structures, and to assess new types of non-conventional railroads. In most mechanical models, track with complex dispersed parameters is generally transformed into a more simple one with lumped parameters of single or multiple degrees of freedom. But no matter what the models are, the essential parameters must be pre-determined. These parameters include the distributed masses per unit length of the various track components and their distributed spring rigidities per unit length, as well as wheel-rail interaction flexibility coefficients. If the parameters used do not coincide with the actual conditions, or do not reflect them at all, they will distort the results of the calculations, possibly with very serious errors. At present, such parameters are essentially based on past experience and the test results obtained, which are rather imperfect and incomplete in nature. Hence, there is much work to be done in this field of investigation. Studies on the parameters of mechanical models for track structures of high speed railways are discussed in this paper. Some analytical and empirical formulas used to calculate these parameters are presented. Meanwhile, the dynamic

response characteristic parameters for the main components of track structures, such as wheel-rail interaction coefficients, nature frequencies and vibration accelerations measured in situ and their ranges, as well as allowable maximum values in some key positions are presented. These can be used to check the calculated results of the theoretical models.
At present, the two types of track structures classified for high speed railways, are track with ballast and slab track. The track with ballast is normally used by European countries and it has the advantages of requiring lower capital investment, and having elastic properties for shock absorbing and cushioning. The disadvantage of this kind of track is that it is poor in resisting lateral force. The track with ballast is also employed by the Japanese National Railway except that in tunnels and high lever bridges where the slab track is substituted. This kind of track has been chosen for the Beijing-Shanghai high speed railway. Beijing-Shanghai high speed railway commences from Beijing and terminates at Shanghai, going through such cities as Tianjin, Jinan, Xuzhou, Bangbu, and Nanjing. It has a route length of 1300 *km* and 24 stations with an average distance between stations of 55 *km*. Beijing-Shanghai high speed railway is an entirely new high speed double track and electrified passenger dedicated railway line. In the initial period, high and medium speed trains will run on the same track with a maximum running speed of 300 *km/h*.
A section of track for Beijing-Shanghai high speed railway is shown in Fig.1 (Scientific Institute of Railway Minister, 1996). The track data are as follows

rail:	*60kg/m* continuously welded rail track
tie:	prestressed concrete tie III, *1680pcs/km, l=260cm*
fastening:	rail fastening with ω-shaped spring rods, gauge plate *60-10-17*
ballast:	special qualified crushed stone, *35cm* on deep for ballast, *20cm* or larger for sub-layer ballast

As an example of the parameter investigation, this paper focuses on the track used for the Beijing-Shanghai high speed railway. However the procedures are applicable to normal track with ballast of high speed railways.

2. PARAMETER DETERMINATION

The essential track parameters include the distributed spring rigidities per unit length of the various track components k_1, k_2, k_3 and their distributed masses per unit length m_1, m_2, m_3 in a model of continuous elastic foundations.

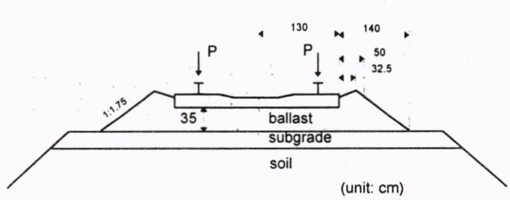

Figure 1. Section of the track for Beijing-Shanghai high speed railway

With respect to the distributed spring rigidities, it should be noted that the springs may be arranged either in a series or in a parallel formation. In the former case, springs with rigidities k_1, k_2 and k_3 are placed one above the other, and the lumped distributed spring rigidity of the system as a whole *k* will be

$$\frac{1}{k}=\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_3} \tag{1}$$

In the latter case, springs with rigidities k_1 and k_2 are placed side by side and the lumped distributed spring rigidity of the system as a whole, k ,will be

$$k=k_1+k_2 \tag{2}$$

In the calculation of the dynamic responses of the track, k_1 represents the distributed spring rigidity provided by the elasticity between the rail and tie, equal to the sum of the rigidities of the rail clips k_c and the rail pad k_p divided by the tie spacing *a*(since both are arranged in parallel). That is,

$$k_1=\frac{k_c+k_p}{a} \tag{3}$$

where k_2 and k_3 represent the distributed spring rigidities of the ballast and roadbed per unit length of the track underneath the half tie, respectively. They are more difficult to determine since they are closely related to the dimensions and spacings of the ties, depth and modules of elasticity of the ballast and the soil properties of the roadbed. In the calculation of k_2, we assume the load from tie pressure is uniformly distributed on top of the ballast and transmitted from the top of the ballast to the top of the roadbed with a spread angle φ obeying the law of linear spreading. Based on the above assumption, we can obtain (Arnold D.Kerr, 1978)

$$k_2=\frac{c(l-b)E_b}{a\ln\left[\frac{l(b+ch_b)}{b(l+ch_b)}\right]} \tag{4}$$

in which l is the effective supporting length at each end, b is the average width of the tie, h_b is the depth of ballast; $c=2tg\varphi$, φ is the spreading angle, and E_b stands for modules of elasticity of the ballast.

As to k_3, the following formulas can be proposed (X.Y.Lei, 1998)

$$k_3=\frac{E_s(l+ch_b)(b+ch_b)}{ah_s} \tag{5}$$

where h_s is the depth of soil concerned and E_s the module of the soil.

Since k_1, k_2 and k_3 are arranged in a series, the lumped distributed spring rigidity of the track as a whole, *k,* will then be

$$k=\frac{k_1k_2k_3}{k_1k_2+k_2k_3+k_3k_1} \tag{6}$$

where *k* is the module of elasticity of the rail foundations.

The distributed mass of rail per unit length of track m_1 is easy to obtain. For example, the distributed mass is *60.64kg/m* for *P60* rail and *74.414kg/m* for *P75* rail. The distributed mass of the tie per unit length of the track is equal to the mass of the half tie divided by the tie spacing.

$$m_2=\frac{m_t}{a} \tag{7}$$

where m_t is the mass of the half tie.

The distributed mass of ballast per unit length of track, m_3, is the mass of the ballast actually participating in the vibration of the track. It is comparatively difficult to determine. Employing the same assumption used to determine k_2, the following formula can be used to calculate m_3

$$m_3 = h_b \rho_b (b + h_b tg\varphi)(e + h_b tg\varphi) / a \tag{8}$$

where ρ_b is the density of the ballast.

3. LUMPED PARAMETER MODEL

The other way to calculate the dynamic track response is the lumped parameter model. In this model, the rail is considered to be a beam of infinite length which is laid on a continuous elastic foundation and on which there is a mass M_0 (equivalent to the unsprung mass of the car) excited by a cyclic force , $P_0 e^{j\omega t}$. The continuous elastic foundation is composed of rail fastenings, ties, ballast and roadbed. The properties of the rail fastenings, ties, ballast and roadbed are represented by a single set of springs and masses with distributed lumped rigidity, K_r, and mass, M_r, so as to form a vibrating system with a single degree of freedom.

The distributed lumped rigidity, K_r, can be expressed as (T.H.Tung 1989)

$$K_r = \frac{2k}{\beta} \tag{9}$$

where k is the elastic module of the rail foundation, shown in Eq.(6), and β is the "rigidity factor between rail foundation and rail"

$$\beta = \sqrt[4]{\frac{k}{4EI}} \tag{10}$$

in which EI is the vertical flexural rigidity of the rail.

The distributed lumped mass, M_r, can be obtained by means of the first natural frequency of elastic foundation

$$M_r = \frac{3}{2\beta}(m_1 + m_2 + m_3) \tag{11}$$

4. WHEEL-RAIL INTERACTION COEFFICIENT

The wheel-rail interaction coefficient depends not only on the contact force and relative contact displacement, but also on the properties of the contacted bodies and the configuration of the wheel tread and the rail surface.

According to the classical Hertz solution of contact stress, if the wheel and rail are assumed to be two elastic cylinders perpendicular to each other, we have

$$y = cp^{2/3} \tag{12}$$

where y is the relative contact displacement, c is the flexural coefficient, and p is pressure acting between two cylinders.

The contact spring coefficient K_0 can be calculated as

$$K_0 = \frac{dp}{dy} = \frac{3}{2c} p^{1/3} \tag{13}$$

When the wheel tread is conic in shape, the relationship between contact pressure and relative contact displacement can be written as

$$y = Gp^{2/3} \tag{14}$$

where

$$G = 4.57R^{-0.149} \times 10^{-8} \qquad (m/N^{2/3}) \tag{15}$$

and R is the wheel radius.
Similar to Eq.(13), the K_0 in this case can be expressed as

$$K_0 = \frac{3}{2G}p^{1/3} \tag{16}$$

When the wheel tread is worn and has concavity at the middle, the relationship between contact pressure and relative contact displacement is the same as Eq.(14) while the flexural coefficient is

$$G = 3.86R^{-0.115} \times 10^{-8} \qquad (m/N^{2/3}) \tag{17}$$

The experiment results show that variations in rail type and wheel load have little influence on K_0 whereas variations in diameter greater. Generally, K_0 is in the range of *12,250 ~ 14,210KN/cm.*

5. TRANSVERSE ELASTICITY OF THE TRACK

The transverse elasticity of the track is provided by two sets of springs. The first set is the spring for ties with spring rigidity per unit length $k_4\,(MN/m^2)$, which is provided by the local shearing deformation and creeping of the rail pads on concrete ties. This may be taken as approximately constant. The second set is the spring for ballast with spring rigidity per unit length $k_5\,(MN/m^2)$, which is provided by the elastic displacement of the ballast, and may also be taken as constant. The values of k_4 and k_5 can be determined from actual tests. Generally k_4 can be taken as $700MN/m^2$ and k_5 as $20MN/m^2$, both are assumed to be continuously and uniformly distributed.

6. DAMPING

Damping is also an important parameter used in calculation of dynamic track response . In a finite element model, the damping matrix of the system is usually expressed as (O.C.Zienkiewicz, 1977; O.C.Zienkiewicz and Morgan,1983)

$$[C] = \alpha[K] + \beta[M] \tag{18}$$

where *[K]* and *[M]* are the stiffness matrix and mass matrix of the system respectively; α and β are proportional coefficients which can be determined by

$$\alpha = \frac{2(\xi_i\omega_j - \xi_j\omega_i)}{(\omega_j^2 - \omega_i^2)}\omega_i\omega_j \qquad \beta = \frac{2(\xi_j\omega_j - \xi_i\omega_i)}{(\omega_j^2 - \omega_i^2)} \tag{19}$$

in which ω_i and ξ_i are the *i*th natural frequency and damping ratio , ω_j and ξ_j the *j*th natural frequency and damping ratio determined by experiments.
The essential track parameters used in theoretical models are proposed in Table 1.

7. SOME DYNAMIC RESPONSE MEASUREMENTS IN SITU AND ASSOCIATED ALLOWABLE VALUES

7.1 Wheel-rail Dynamic Factor

The stress and strain on track components produced by moving trains is far greater than those produced by stationary ones. This is because a dynamic wheel load is greater than a static wheel load. We call the ratio between the dynamic increments, or the difference between the dynamic and static wheel loads and the static wheel load, the dynamic load factor. Measurements for wheel-rail dynamic factors are listed in Table 2.

Table 1 Summary of Essential Track Parameters Used in Theoretical Models

parameters	range of value
rigidity between rail and tie k_1	$100 \sim 150\ MN/m^2$
rigidity of rail clips k_c/a	$5 \sim 10\ MN/m^2$
rigidity of rail pad k_p/a	$95 \sim 140\ MN/m^2$
rigidity of ballast k_2	$300 \sim 400\ MN/m^2$
rigidity of soil k_3	$70 \sim 240\ MN/m^2$
rigidity of combining ballast and soil k_{23}	$70 \sim 110\ MN/m^2$ for new line; $140 \sim 180\ MN/m^2$ for existing line
rigidity of single rail foundation k	$60 \sim 90\ MN/m^2$ for track with P.C. tie $70 \sim 110\ MN/m^2$ for track with P.C. broad tie
transverse rigidity of track k_4	$700\ MN/m^2$
transverse rigidity of track k_5	$20\ MN/m^2$
rigidity of wheel/rail interaction K_0	$1{,}225 \sim 1{,}421\ MN/m$
proportional coefficient α	$0.00005 \sim 0.0002$
proportional coefficient β	$0.00005 \sim 0.0002$

Table 2 Wheel-rail Dynamic Factor

speed (km/h)	$80 \sim 160$	$160 \sim 210$	>210
dynamic factor	$0.15 \sim 0.22$	$0.32 \sim 0.41$	$0.75 \sim 1.00$
test situation	Guang-Sheng quasi-high speed railline	Beijing circle railline	German ICE test

7.2 Ballast Resistance

Ballast resistance is the resistance against the longitudinal and transverse movement of the ties offered by ballast. It may be expressed as the resistance against each tie, *R(KN)*, or against each unit length of rail *p(KN/m)*.

Ballast resistance is closely related to the properties of the ballast particle, its grain size, the section of ballast, the quality of damping, the degree of fouling, and the weight of the track panel. Measurements of ballast resistance are given in Table 3.

Table 3 Ballast Resistance

ballast resistance*(KN/m)*		Jing-Shan railine	Jing-Guang railine
longitudinal resistance	ballast resistance by single tie	19.20	24.00
	ballast resistance per unit length of rail	33.68	42.11
transverse resistance	ballast resistance by single tie	14.45	11.20
	ballast resistance per unit length of rail	25.40	19.65

7.3 Natural Frequency

Natural frequencies include frequencies of the sprung and unsprung mass of the rolling stock, rail, tie and ballast, measurements of which are given in Table 4.

Table 4 Natural Frequencies *(Hz)*

sprung mass of the rolling stocks	unsprung mass of the rolling stocks	rail	tie	ballast
0 ~ 10	20 ~ 125	1400 ~ 2800	800 ~ 1200	120 ~ 250

7.4 Acceleration

Acceleration of the track components depends on the speed of the moving trains. Measurements of acceleration are shown in Table 5.

Table 5 Measurements of Acceleration *(g)*

speed of the moving train *(km/h)*	rail	tie	ballast
140	170	17	4
300	400	18	4

7.5 Specified Parameters

The allowable bearing stress on top of the ballast, as specified in China, is

for crushed stone ballast $[\sigma_b] = 0.49 MPa$

for screened gravel ballast $[\sigma_b] = 0.39 MPa$

The allowable bearing stress on top of the roadbed filled with ordinary sandy clay soils is

for new lines $[\sigma_s] = 0.13 MPa$

for existing lines $[\sigma_r] = 0.15 MPa$

Table 6 Allowable Accelerations *(g)*

type of railways	rail	ballast
for existing line	200	15
for new line	400	25

The vibrating acceleration in track components is one of the important parameters. As there are no specified accelerations in China, allowable values referred to the Japanese National Railway are proposed in Table 6.

8. DISCUSSION AND CONCLUTIONS

In designing components of track structure under rails, the following quasi-static formula is usually employed

$$R_d = \gamma(1+\alpha)P_0 \tag{20}$$

in which, R_d is dynamic rail pressure, γ and α are wheel load distributed factor and dynamic factor respectively, and P_0 is static wheel load.

Equation (20) is not only used to design standard railway lines, but also used to middle or high speed rail lines by some countries. In the paper, most observed values are obtained from the existing lines in China and some values are from computational results or from measurements of high speed lines in other countries. Since wheel loads of locomotives and cars for high speed railway are lower than these for standard rail line, the dynamic forces are nearly at the same level even though the dynamic factors in the former cases are higher than these in the later cases. Taking the dynamic rail pressure R_d as an example, results from different types of locomotives are shown in Table 7. In a sense, results or measurements from existing lines can be taken as an important references in designing track structure for high speed railway.

Table.7 Dynamic Rail Pressure R_d from Different Types of Locomotives

Types of locomotives	Max. speed (km/h)	axle arrangement	axle load (10^3kg)	γ	α	R_d (KN)
DF_{11}	170	3_0-3_0	23	0.46	1.5	132
SS_8	177	2_0-2_0	22	0.47	1.5	129
ICE	250	2_0-2_0	19	0.48	2.0	137
TGV	300	2_0-2_0	17	0.48	2.0	122

It should also be pointed out that even though there are some literatures concerning the parameters of mechanical models for track structures of railway in recent years, they are not systematic and comprehensive. In view of this fact, this paper may supplement the void.

Finally, the following conclusions can be drawn from the above analyses:

(1) The paper systematically and comprehensively discusses the parameters of mechanical model for track structures of railway.

(2) The essential track parameters are proposed in the paper, which can be used in numerical computation.

(3) Results or measurements presented in this paper can be taken as an important references in designing track structure for high speed railway.

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