The Model of Optimal Non-Queuing Pricing for Port Container Yard

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Abstract: As there are limited space and hardware equipments of Port Container Yard, export containers have to queue outside for entry if the container yard reaches its capacity saturation. Therefore, some containers probably could not be completed the loading preparation on time, which would cause great damage to the shipper. In order to solve this problem, this research establishes a queuing pricing model for port container yard. After implementation of this toll scheme, it will be effective in dispersing containers' arriving time, and so as to eliminate the queuing phenomenon outside the entrance of container yard.

Keywords: Port Container Yard, Capacity, Queuing Pricing, Arrival Time

1. INTRODUCTION

In the international trade, sea and air transportation would be the major ways of shipping. However, sea transportation has accounted for more than 90 percent of total transportation amount, which has played an important role in transnational commerce. In the history of ocean shipping, container transportation has been used since from 1957 due to its particular characteristics. For example, using standardized containers could not only be beneficial to transshipping, which possesses a characteristic of intermodal transportation, but also offer a door-to-door service with low unit transportation cost. Therefore, container transportation has been playing an critical role in marine transit.

During the process of container transportation, the sellers will load the containers with cargos in factory, haul to the dock, hang over those filled containers to the ship owners, and then transport to the port of destination. Afterwards, that will be transported to the buyers through inland transportation. Port container yard is located on the rear of berth, and its purpose is used for containers' transshipment between land and sea transportation. With regard to operating full container yard, for the sake of convenient and time-saving loading, harbor managers will request the consignors to pile up those about to transport containers in the designated yard in advance before ship anchoring.

Due to the restricted land resources of container yard, however, under the global trend of developing of large-scale container ships, there must be more and more containers that have to stack up in the yard prior to ships' approaching. For this reason, there is a increasing requirements for the handling efficiency and the wider available space of yard. In order to hold competitiveness, except for strengthening the operational efficiency for each port to accelerate the handling efficiency, it still has to make the best use of the limited space to stack more containers. Nevertheless, owing to the high density of stacking, it has caused the difficulty in retrieving containers.

The current operational ways for port container yard can be divided into two modes. One is operating by the bureaus own, and the other is running by ship owners' operating lease. In order to attract more shippers to come, ship owners will often offer few days of free storage time for containers, which may save time for handling cargos activities while it strikingly cause the jam-packed yard and the growing costs of shipping companies. This phenomenon will be happened especially on the eve of major holidays, e.g. Christmas. The dramatically increasing amounts of export and import containers would lead to the crowded yard, so the operating time would be lengthen, and more seriously, it even results in the container yard which has becoming saturated. Therefore, arrived containers should have to queue outside for any available space to enter. In terms of import containers, if the port container yard has been saturated, ship owners can haul their cargos to stack at the inland container yard, while this would definitely raise the cost of land transportation. On the other hand, in relation to export containers, if the capacity of port container yard is getting saturated, the containers could not be piled up prior to the scheduled loading time, so it would have to arrange for transshipment or wait for the next sailing date. This result will make the shippers as well as ship owners have suffered losses. Consequently, if there is no any opportunity for expanding insufficient instruments and space, the only solution is expected from implementing the toll scheme to disperse containers' arrival time and to relieve the queuing phenomenon in port container yard.

In order to successfully arrange container loading, the bureau will urge that export containers have to be piled up in port container yard in advance before shipping. Firstly, shippers have to make a decision for the exporting ways, e.g. FCL (Full Container Load) or LCL (Less Container Load). However, this research assumed that all shippers would like to use FCL way of export. The next procedure is to book shipping space through carrier or the agent, and then is to draw the shipping order (S/O). Afterwards, the process will be to the designated container yard to receive the empty containers and return to the factory (warehouse) for loading. And then the containers would be dragged to the port container yard, in compliance with the rules of the shipping order. The final step is that containers have to be entrusted customs broker to complete the pre-shipment formalities, and after that, they can be ready for loading.

The research is aimed at investigating port container yard's queuing phenomenon that shipper may face when the container yard has reached its saturation limit. Additionally, shippers have to calculate the relevant costs, which was generated from the queuing problems, yet the cost is without taking into consideration of free storage period provided by ship owners. During the certain periods, if there are a great number of containers likely to enter the container yard in a short time, it will cause the yard to be crowded and lower the operational efficiency. Furthermore, if container yard is unable to dispose of the containers immediately, it would easily make the capacity become saturated. This way, shippers have to complete the inspection outside the control station and wait for available space to enter, following the order of first come first served. This research only considers the containers' queuing problem, which may occur from control station to yard, so when containers can be stacked at the yard prior to the scheduled loading time, they could be viewed as successful loading. Therefore, this research would not investigate how does container yard arrange the loading procedures.

When the capacity of container yard has been saturated, even containers have been already arriving at the control station, they still have to queue outside for available space to enter. This research classifies containers' scheduled loading time into three different modes, i.e. early, on-time, and late arrival time, and then the cost function and equilibrium cost could be constructed for shippers. Afterwards, according to the principle of "conservation of equilibrium cost", it could be developed the optimal non-queuing toll scheme for port container yard to radically disperse the queuing phenomenon outside the control station.

2. LITERATURE REVIEW

Literature review can be divided into two portions, which are examining the relevant literatures regarding to operating port container yard and queuing model.

2.1 Container yard operation

Taleb-Ibrahimi, *et al.* (1993) discussed the impact on export container's handling work and storage strategies, with regard to the space of container yard. Based on the queuing theory to establish a mathematical model, the concept of this strategy was to allocate arrived containers to be placed on the temporary area, and using dynamic models assigned containers to enter the main container yard for stacking in order to reduce the handling moves. This paper also described how to decrease the operating frequency and how to predict the minimum number of operations, which would be beneficial for analysing the long-term management of running container yard.

Kim, *et al.* (1999) probed into examining the optimal routing algorithm for transfer cranes in container yard. According to the inclusion of space cost, equipment fixed costs, variable cost, and time-consuming cost for trailer, these total costs could be investigated into forecasting the space requirements of import containers, planning the optimal space for making better use of container yard, and computing the optimal number of instruments. Within the three highlighted random patterns, i.e. fixed arrival, weekly-cycle arrival, and the random arrival, these could help to build up the optimized solution by developing the mixed integer program. Additionally, these also could be contributive to minimize the total container handling time of a transfer crane and determine the optimal routing algorithm.

Steenken, *et al.* (2004) organized and classified a great amount of literatures related to container terminal operations. Owing to the last four decades of highly development of containerization, there was an ever increasing demand for container freight terminals. Therefore, how to manage it in a more efficient way has been becoming a remarkable issue. For the others important literatures about operation and handling work regarding to container freight terminals were described and classified in this paper. Moreover, Stahlbock and Voβ (2008) have expanded and updated some critical views, in compliance with previous literatures about operation research.

Kim, *et al.* (2007) proposed an online search algorithm in order to enhance container yard's operational productivity, and this model could be effective in dealing with the stacking and the retrieving problems. Utilising a simulation approach automatically assigned the cranes in container yard; additionally, in accordance with the types and levels of containers and the crane's moving distance, containers could be piled up at the optimal position within the container yard through allocating under the dynamic approach. Consequently, the result of examination showed an online search algorithm could be conductive to step up the operating efficiency.

2.2 Queuing model

Sen (1980) solve two types of constrained optimization problems to demonstrate the potential gains associated with a priority structure. A single server system with Poisson arrival and departure (M/M/I) is studied in the sequel although many of the arguments are applicable to a general queuing system.

A model of tolling a queuing bottleneck was initially introduced by Vickrey (1969). This model was built up through contemplating every commuter's total time costs of journey and the costs of time-delay, so commuters could depend on the basis of minimizing the total costs to decide their departure time. Furthermore, Braid (1989) and Arnott, *et al.* (1990) concluded the non-queuing optimal toll scheme for road bottleneck to radically eliminate all commuters' total queuing time spent at bottleneck entrance. The concept of non-queuing optimal toll collection was according to different entry time spot to charge its corresponding toll, which could be viewed as continuously variable charges. Even though this model could extremely eliminate queuing time spent, in fact it is not feasible in practice.

Based on the model derived from Arnott, *et al.* (1990), Laih (1994) had developed a set of step toll scheme to be an alternative plan for substituting the optimal non-queuing toll scheme, which would be apt to administer and fulfil the purpose of flexible charging function. Although the step toll scheme cannot fundamentally eliminate all commuters' queuing time spent, its best advantage is to help decision makers reduce the time spent through assessing variable charging fees corresponding to different time interval. Consequently, the government or the affiliated authorities could have more flexibility to regulate the road bottleneck tolling.

Laih (2004) expanded the *n*-steps toll scheme (n=1,2,3...) for the queuing pricing model of road bottleneck, and Laih figured out that when the charging steps increased one by one, the changes between each step of tolling amount and charging time spot, the related equilibrium cost, the equilibrium departure rates and moving tracks of departure time of auto-commuters would vary regularly. The conclusion had been summarised as follows: (1) Prior to implementation of the optimal *n*-step tolling scheme, it could conduce to forecast how many numbers of users who were willing to pay or not. (2) It would be possible to forecast the varying behaviour of departure time for all commuters. (3) Those commuters who were unwilling to pay the tolls that their departure time would not be changed, while commuters who were willing to pay would like to adjust their departure time. The above conclusion could be a useful reference for decision makers who would like to execute the *n*-step tolling pricing model.

Pettitt (2007) investigated whether the transfer of regulation from the States to the Commonwealth will serve to alleviate congestion problems in Australian ports. The transfer of power to the Commonwealth may lead to the Commonwealth being able to take further powers from the states, not only in relation to the ports but in relation to other areas such as industrial relations. Industry groups claim they have been pushing for a larger role for the Commonwealth in the running of the nation's ports, but question as to what extent the Commonwealth should be involved.

Laih, *et al.* (2007 and 2008) established the optimal non-queuing toll scheme and the optimal *n*-step toll scheme for container ships queuing at the anchorage of a busy port. As the optimal non-queuing tolling scheme has to keep varying its amount of fees, it would have some difficulties in executing indeed. Accordingly, the optimal *n*-step tolling scheme has been claimed to be a suitable alternative. This research was performed in a way of dynamic analysis to compute the dispersing result of container ships' arrival rate and the change of varying arrival time, compared the difference between before and after implementing the optimal *n*-step toll scheme. Furthermore, it also indicated that the arrival time for those ships which had paid the tolls would be backward extended, compared with not implementation of tolling. However, the results of arrival time would still remain steadfast for those ships without paying any tolls. As the result, the behaviour of ships' arrival time would be changed according to the execution of toll collection, and the tolling administration would definitely relieve the queuing situation.

3. THE OPTIMAL QUEUING PRICING MODEL

The purpose of establishing the queuing pricing model for port container yard is to disperse containers' arrival distribution at control station by using tolling scheme. Firstly, this chapter has classified three different arrival modes, i.e. early arrival, on-time arrival, and late arrival, according to export container's scheduled loading time. Additionally, these three modes would be applied to develop cost function for shippers and calculate the optimal non-queuing pricing scheme for port container yards, which could radically eliminate containers' total time spent for queuing outside the control station.

3.1 Non-toll equilibrium

The research supposed that there will be a great amount of import and export containers during the shipping season, and the limited space of container yard will cause a queuing phenomenon outside the container yard. With regard to import containers, consignors can drag their containers to be piled up at the inland yard, so it can effectively relieve the jam-packed circumstance. On the other hand, regarding to export containers, the bureau managers will often request shippers to stack their containers at the yard before loading in order to speed up the following preparation. However, if containers cannot enter the container yard at the scheduled time, it would make them incapable of loading on-time, so they have to tranship or wait for the next sailing date. Therefore, so as to facilitate all containers could enter the container yard at their scheduled time; this research will discuss the influence of queuing pricing model for port container yard, which could be conductive to disperse container's arrival time distribution and reduce their queuing time spent outside control station.

For the purpose of simplicity, the model has been developed under the following assumptions:

a. Supposed that the capacity of container yard has been saturated, the first arrived container must start to queue outside the control station for entry.
b. Containers which have been queuing outside the control station can make use of the waiting time to complete the inspection in advance. The reason is that containers can enter immediately and do not have to waste another time to check, if the yard has available space.
c. Presumed that the time spent and the costs are fixed for the clearance of containers and its loading working. Customs clearance includes many procedures, e.g. stacking containers at the designated position, clearance of goods, and customs inspection, while loading working contains transit from the container yard to alongside the ship and loading operations.
d. The costs in this model will only count the direct cost and its derived cost which are related to queuing. Other costs will not be considered if there is no direct relation to queuing.
e. According to the arrival order, containers will be handled following the principle of first come first served.

The model assumed the container yard has been saturated, so arrived containers have to wait for another available space for entry. In accordance with the sailing schedule issued by shipping company, consignors have to conform to it to reserve the shipping space and obtain the S/O from carriers or the agency. Afterwards, containers need to get into the container yard prior to the scheduled loading time, recorded on the shipping order. Therefore, according to the scheduled loading time, it will divide the arrived containers into three different arrival patterns, i.e. early arrival, on-time arrival, and late arrival. The time relationships among the three different arrival patterns for containers can be listed as follows:

Early Arrival:

$$\begin{aligned} & t + T_Q(t) + T_Y + T_E(t) = t^* \\ & T_E(t) = t^* - t - T_Q(t) - T_Y \end{aligned} \tag{1}$$

On-Time Arrival:

$$\tilde{t} + T_Q(\tilde{t}) + T_Y = t^* \tag{2}$$

Late Arrival:

$$\begin{aligned} & t + T_Q(t) + T_Y - T_L(t) = t^* \\ & T_L(t) = t + T_Q(t) + T_Y - t^* \end{aligned} \tag{3}$$

In equations (1)~(3), t is defined as the arrival time spot for containers, which have arrived at the control station of port container yard. Since the capacity of port container yard has been saturated, the arriving containers have to queue outside for available space, and $T_Q(t)$ is defined as the length of queuing time period for containers. After the length of queuing $T_Q(t)$, containers then can get into the container yard. Once containers enter the yard, managers would assign containers to be piled up at the specific place and request containers have to complete the customs procedures and inspections. This time length would be regarded as T_Y. After fulfilment all of the prescribed working within the container yard, then start loading the container ship. If the start loading time is exactly equal to the scheduled loading time spot, t^*, which means that containers arrive the control station at the time spot, $\tilde{t}$, then these containers could complete the loading preparation on-time. In other words, after processing two periods of $T_Q(\tilde{t})$ and T_Y, containers could complete the loading preparation just on the scheduled loading time, so equation (2) could be regarded as on-time arrival. If containers have finished the loading preparation prior to the scheduled loading time, t^*, it could be defined as early arrival, i.e., the time-span for early arrival could be expressed as $T_E(t)$. On the contrary, if containers have not yet to complete the loading preparation later than the scheduled loading time, t^*, it could be indicated as late arrival, i.e., $T_L(t)$ as the symbol of time interval of late arriving. The early and late arrival cases are shown as equations (1) and (3), respectively.

Under the three different arrival modes for containers, the total cost function related to queuing can be listed as below:

$$TC(t) = \alpha \cdot T_Q(t) + \beta \cdot T_E(t) + \gamma \cdot T_L(t), \quad t_q \le t \le t_{q'} \tag{4}$$

In terms of the equation (4), α is defined as the unit time cost of containers queuing outside the control station, including the related cost for using container trailer, e.g. driver salaries, management fees, fuel costs, maintenance and component costs, depreciation of container trailers, insurance, etc. β is regarded as the additional costs which would be burdened by shippers, such as the space rent because of earlier entry the container yard. On the other hand, γ is defined as the late cost, generated from the expenses of transhipment or the cost of waiting for the next sailing date if containers are unable to loading as scheduled due to late entering the yard. The cost functions for three different arrival patterns of containers are shown as follows:

Early Arrival:

$$TC(t) = \alpha \cdot T_Q(t) + \beta \cdot T_E(t)$$
$$= \alpha \cdot T_Q(t) + \beta \cdot \left(t^* - t - T_Q(t) - T_Y\right) \tag{5}$$
$$T_L(t) = 0$$

On-Time Arrival:

$$TC(\tilde{t}) = \alpha \cdot T_Q(\tilde{t})$$
$$= \alpha \cdot T_Q(\tilde{t}) \tag{6}$$
$$T_E(\tilde{t}) = T_L(\tilde{t}) = 0$$

Late Arrival:

$$TC(t) = \alpha \cdot T_Q(t) + \gamma \cdot T_L(t)$$
$$= \alpha \cdot T_Q(t) + \gamma \cdot \left(t + T_Q(t) + T_Y - t^*\right)$$
$$T_E(t) = 0$$
(7)

As each shipper is pursuing for spending the minimum cost of using container yard, all shippers' arrival time cost would be equivalent which could achieve equilibrium, i.e., $\frac{dTC(t)}{dt} = 0$. Consequently, the conditions of equilibrium from equations (5) to (7) are expressed as below:

Early Arrival:

$$\frac{dTC(t)}{dt} = \alpha \cdot \frac{dT_Q(t)}{dt} + \beta \cdot \left(-1 - \frac{dT_Q(t)}{dt}\right) = 0$$
$$(\alpha - \beta) \cdot \frac{dT_Q(t)}{dt} = \beta \tag{8}$$
$$\frac{dT_Q(t)}{dt} = \frac{\beta}{\alpha - \beta}$$

On-time Arrival:

$TC(\tilde{t}) = \alpha \cdot T_Q(\tilde{t})$ is a time spot which corresponds to a *TC* value, so it cannot be differential.

Late Arrival:

$$\frac{dTC(t)}{dt} = \alpha \cdot \frac{dT_Q(t)}{dt} + \gamma + \gamma \cdot \frac{dT_Q(t)}{dt} = 0$$
$$(\alpha + \gamma) \cdot \frac{dT_Q(t)}{dt} = -\gamma \tag{9}$$
$$\frac{dT_Q(t)}{dt} = \frac{-\gamma}{\alpha + \gamma}$$

In light of equations (8) and (9) could be obtained the slope relations of arrival time (t) of container and the length of queuing time spent $T_Q(t)$. In reality, the arrival time of containers from a shipper would not be determined only by the length of queuing time spent. However, the cost functions of equations (5)~(7) developed in our model are only related to queuing for the purpose of deriving the optimal queuing pricing. This is the reason why the results of equations (8) and (9) exclude other possible factors that have nothing to do with queuing.

Generally speaking, the queuing cost (α) is greater than the cost of early arrival (β), and the cost of late arrival (γ) is the highest among these three values. Accordingly, equation (10) shows the three values in the order.

$$\gamma > \alpha > \beta > 0 \tag{10}$$

It is clear to recognise from equation (10) that equations (8) and (9) are positive and negative slopes, respectively. On the basis of the slope relations of equations (8) and (9), both could help to draw the diagram to express the whole course of queuing time spent, from the starting point (t_q) to the ending point ($t_{q'}$), as shown in Figure 1.

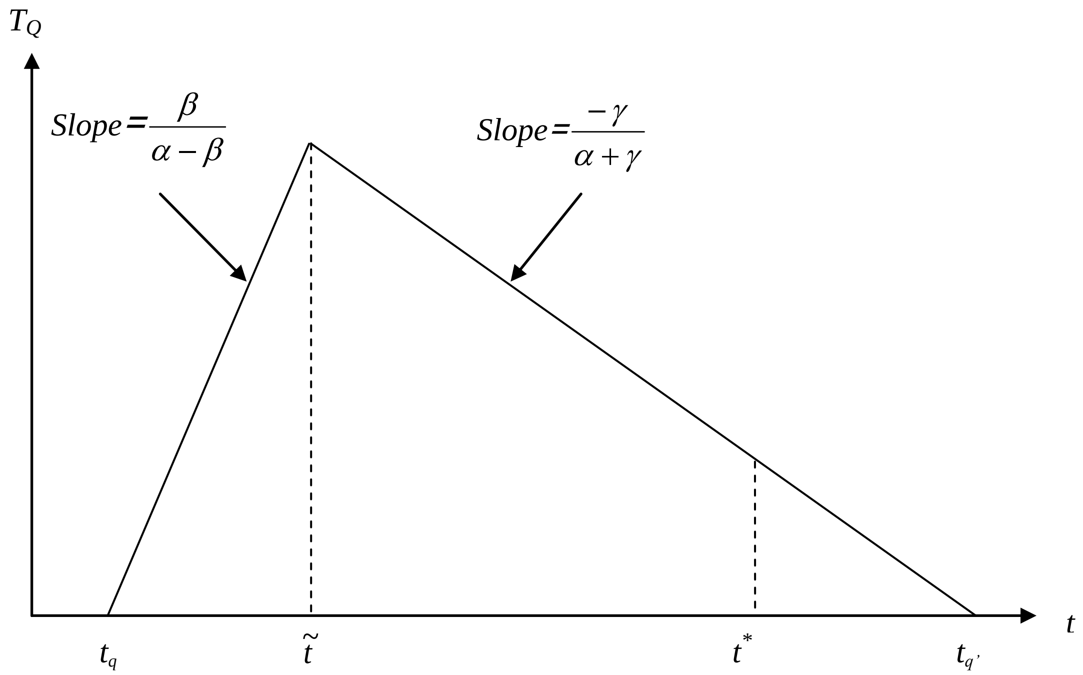

Figure 1. The Relationship of Container's Arrival Time and the Length of Queuing Time

The queuing phenomenon will be arisen from the saturated container yard, and the total length of queuing time spent would be affected by the rate of retrieving containers and their leaving speed. In general, the portal crane would be adopted in the container yard. It could only retrieve a container once a time and also shows a poor mobility. Hence, in order to save time and the fuel costs, it would be suitable for working in sequence from near to far, starting to do alongside the trailer parking area. For the sake of making better use of the capacity, the stacking place in the container yard would be classified from sailing route or sailing date; in addition, the similar weight of containers would be piled up together as well. The container yard will usually forecast how many containers would like to loading and plan the appropriate space for them to use in advance.

As containers which would be transported by the same ship will be stacked together, so while containers are preparation for retrieving, if a stack of containers will be transported by the same ship, it would be impossible to stack another new container right after retrieving one container, yet it should wait for all stacks of container retrieval finished. However, if the top of target container will be transported by different ship, compared with the below containers, the new container could be stacked instantly after retrieving. Only the numbers of retrieval containers are greater than the numbers of queuing containers, the queuing phenomenon could be radically eliminated.

Supposed after passing through control station, the averagely handling time spent (retrieving or stacking) for each container would be *T*. Additionally, the ship-loading sequence is starting from around trailer area by near to far. If there are the numbers of *N* containers queuing outside the control station, the total queuing time spent for the *N*-th container would be equal to the time spent for retrieving the number of *Q* containers plus the time spent for stacking the number of (*N*-1) containers. Therefore, the total spending time for retrieving is $T \cdot \sum_{i=1}^{N} Q_i$, and the total spending time for entering container yard is $T \cdot (N-1)$, so the total queuing time spent is $T \cdot \sum_{i=1}^{N} Q_i + T \cdot (N-1) = T \cdot \left(\sum_{i=1}^{N} Q_i + N - 1 \right)$. Where Q_i is defined as the total number of *i*-th row of containers which require retrieving.

During the span of queuing time from the starting point (t_q) to the ending point ($t_{q'}$), there are the number of *N* containers queuing outside control station because of waiting for entry the container yard. Assumed the total queuing time spent would be θ, which could be calculated form the following formula, shown on equation (11).

$$t_{q'} - t_q = T \cdot \left(\sum_{i=1}^{N} Q_i + N - 1 \right) = \theta \tag{11}$$

According to the formula of on-time arrival mode: $\tilde{t} + T_Q(\tilde{t}) + T_Y = t^*$, the following two equations could be obtained from Figure 1.

$$\tilde{t} + \frac{\beta}{\alpha - \beta} \cdot (\tilde{t} - t_q) + T_Y = t^* \tag{12}$$

$$\tilde{t} + \frac{-\gamma}{\alpha + \gamma} \cdot (\tilde{t} - t_{q'}) + T_Y = t^* \tag{13}$$

With regard to the equations from (11) to (13), they could help to calculate the three respective time spots, i.e. $\tilde{t}$, t_q and $t_{q'}$, under the equilibrium situation prior to execution of toll collection. These three time values are shown as below:

$$\tilde{t} = t^* - \frac{\beta\gamma}{\alpha(\beta + \gamma)} \cdot \theta - T_Y \tag{14}$$

$$t_q = t^* - \frac{\gamma}{\beta + \gamma} \cdot \theta - T_Y \tag{15}$$

$$t_{q'} = t^* + \frac{\beta}{\beta + \gamma} \cdot \theta - T_Y \tag{16}$$

Under the equilibrium status, the total cost for all shippers are equivalent. Substituting the obtained values from the equations (14) to (16) into equations (5) to (7), the equilibrium cost (TC^e) for all shippers can be obtained as $\frac{\beta\gamma}{\beta+\gamma} \cdot \theta$. Figure 2 shows the relation between containers' (shippers') arrival time and their costs. The red thin lines of $(t_q, \tilde{t})$ and $(\tilde{t}, t_{q'})$ represents the queuing time cost ($\alpha \cdot T_Q(t)$) for the early and late arrivals, respectively, while the blue thick line signifies the time cost of early arrival ($\beta \cdot T_E(t)$), and the green thick line symbolises the time cost of late arrival ($\gamma \cdot T_L(t)$). In the light of early arrival mode ($t_q \le t < \tilde{t}$), the sum of red line and blue line costs represent the equilibrium cost (TC^e). On the contrary, in the late arrival mode ($\tilde{t} < t \le t_{q'}$), the equilibrium cost (TC^e) could be calculated by the sum of red line and green line. Moreover, the conditions of on-time arrival mode do not have to count the early arrival cost as well as late arrival cost, yet it has to burden the maximum queuing time spent.

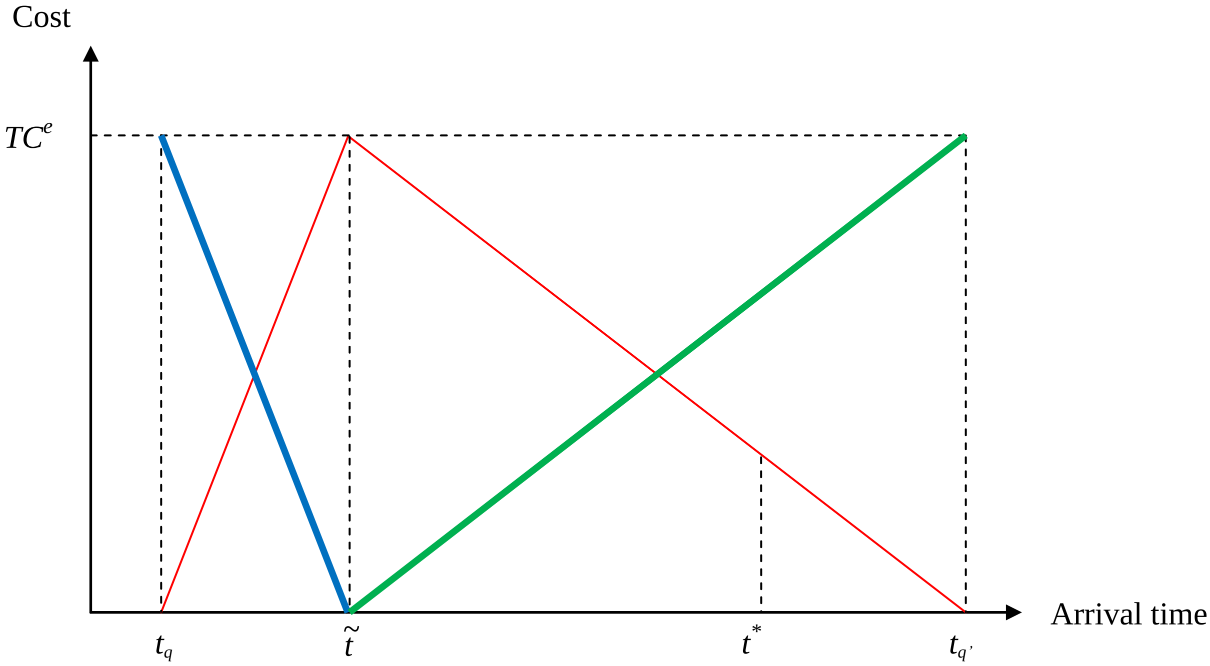

Figure 2. The Queuing Time, Early and Late Costs through the Queuing Period

3.2 The optimal non-queuing toll scheme and arrival rates

The non-queuing pricing model is established under the concept of charging the variable tolls in compliance with the different arrival time so as to completely substitute the queuing time cost of containers and further eliminate all the queuing time spent. Based on the rule of "conservation of equilibrium cost", the equilibrium cost (TC^e) is supposed to remain the equivalent amount whether administering toll collection or not. Therefore, the two important prerequisites of $T_Q(t) = 0$ and $TC(t) = TC^e$ have to be fulfilled after execution of non-queuing toll collection. In view of the above, the equations from (5) to (7) could be reworded, and the non-queuing toll collection ($\Omega(t)$) could also be obtained, which are presented as follows:

Early Arrival:

$$
\begin{aligned}
& TC(t) = \beta \cdot T_E(t) + \Omega(t) = TC^e \\
& \Omega(t) = TC^e - \beta \cdot (t^* - t - T_Y) \\
& t_q \le t < t^*
\end{aligned}
\tag{17}
$$

On-Time Arrival:

$$
\begin{aligned}
& TC(t) = \Omega(t) = TC^e \\
& \Omega(t) = TC^e \\
& t = t^*
\end{aligned}
\tag{18}
$$

Late Arrival:

$$
\begin{aligned}
& TC(t) = \gamma \cdot T_L(t) + \Omega(t) = TC^e \\
& \Omega(t) = TC^e - \gamma \cdot (t + T_Y - t^*) \\
& t^* < t \le t_{q'}
\end{aligned}
\tag{19}
$$

As seen in Figure 3, $\triangle t_q a t_{q'}$ would be the queuing time cost generated before tolling, while $\triangle t_q b t_{q'}$ would be the optimal non-queuing toll scheme, which would be continuously variable corresponding to different arrival time (t). As the two triangles are congruent, it could be speculated that the optimal non-queuing toll collection could wholly substitute the total queuing time cost before tolling execution. Under the non-queuing toll scheme, the range before point t^* is defined as the early arrival interval after toll collection, and the slope of $\overline{t_q b}$ could be indicated as β from equation (17). Furthermore, the range after t^* could be represented as the late arrival interval after toll collection, and the slope of $\overline{b t_{q'}}$ could be viewed as $-\gamma$ in terms of equation (19). Therefore, t^* could be regarded as on-time arrival, even though the sum costs of early arrival and late arrival could be excluded, the tolling amount required to pay would be the highest among three different modes.

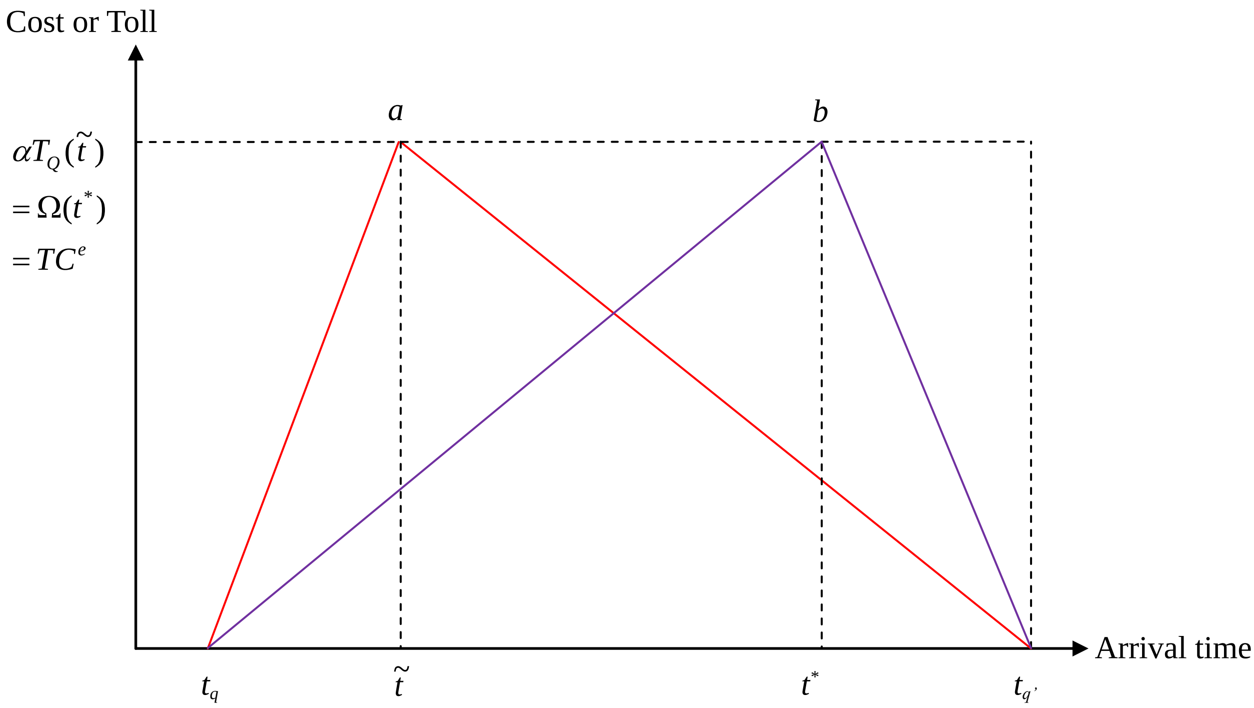

Figure 3. The Queuing Time Cost and Optimal Non-Queuing Toll Scheme

Supposed that during the queuing time periods, the average hourly arrival rate of containers is s, then the arrival rate of variable containers could be expressed as the marginal arrival rate $\frac{d(s \cdot T_Q(t))}{dt}$. Thus, before toll collection, the marginal arrival rate for early arrival periods ($t_q \le t < \tilde{t}$) would be $\frac{\beta \cdot s}{\alpha - \beta}$, and its total arrival rate would be indicated as $\frac{\alpha \cdot s}{\alpha - \beta}$ $(= s + \frac{\beta \cdot s}{\alpha - \beta})$. On the other hand, the marginal arrival rate for late arrival periods ($\tilde{t} < t \le t_{q'}$) before tolling is $\frac{-\gamma \cdot s}{\alpha + \gamma}$, so its total arrival rate could be calculated as $\frac{\alpha \cdot s}{\alpha + \gamma}$ $(= s + \frac{-\gamma \cdot s}{\alpha + \gamma})$. Finally, since $T_Q(t)$ is equal to zero through the queuing period ($t_q \le t \le t_{q'}$), the total arrival rate under the optimal non-queuing toll scheme would always be s.

4. NUMERICAL EXAMPLE

Assumed that there have 75 containers (N) intended to enter the container yard to stack during the same loading periods, while they have to queue outside the control station because of the limited capacity of yard. All of the queuing containers will be handled under the principle of “first come first served”. As the capacity of container yard has been saturated, the total number of containers that need to be retrieved is $\sum_{i=1}^{N} Q_i = 75$. Further assumed that the average handling time spent (T) for per container in the container yard would be 15 minutes, so the total queuing time spent would be equal to the time spent for retrieving 75 containers plus the time spent for stacking 74 containers. Therefore, the sum of queuing time spent would be $T \cdot \sum_{i=1}^{N} Q_i + T \cdot (N-1) = 15 \cdot 75 + 15 \cdot 74 = 2235$ minutes, i.e., $t_{q'} - t_q = 37.25$ hours.

Take Keelung port container yard, located in Taiwan, for the investigation example, the unit queuing time cost (α) of containers which wait at the outside of control station have included driver salaries, management fees for container car, fuel costs, maintenance and components costs, depreciation of container car, insurances, and taxes upon license and fuel; accordingly, the total estimated amount would be around NT$ 371.97461 per hour (NT$:US$=30:1). If a container has arrived earlier and has earlier entered the container yard for stacking, it would further add some additional fees, involving rental for using yard and insurances against stacking containers in the yard, so its unit time cost (β) of early arrival is estimated about NT$ 65.55 per hour. On the other hand, if containers will be late to the yard which will also lead to delay for entry, the loading cannot be successfully. Therefore, it will need to arrange for transhipment or wait for the next scheduled departure, so its unit time cost of late arrival (γ) have to include freight forwarding for export containers, container demurrage, and the added rental for using container, which total fees could be computed to NT$ 630.44 per hour. In order to facilitate the calculation, we hypothesised that the containers would start to queue at 00:00, and the queuing would be finished at 37:15 after the 75 containers had totally entered the yard. In practice, the actual loading time could be backward adjusted, while the results would not be affected. The queuing pricing model for the port container yard in this research is based on the above values to calculate the following outcomes:

$t_{q'} - t_q = \theta = 37.25$ hours, $t_q = 0 = 00:00$, $t_{q'} = 37.25 = 37:15$.

$$\tilde{t} = \frac{TC^e}{\frac{\alpha\beta}{\alpha-\beta}} = \frac{\frac{\beta\gamma}{\beta+\gamma} \cdot \theta}{\frac{\alpha\beta}{\alpha-\beta}} = 27.7957 \text{ hours.}$$

$$t^* = 37.25 - \frac{TC^e}{\gamma} = 37.25 - \frac{\frac{\beta\gamma}{\beta+\gamma} \cdot \theta}{\gamma} = 33.7417 \text{ hours.}$$

In terms of the above computation, containers would start to queue outside the control station for entry at 00:00 (the first day), and the queuing would be ended at 37:15 (the second day) after all containers entering the container yard. Take the upper half of Figure 4 for example, the blue triangle (with the base from $t_q = 0$ to $t_{q'} = 37.25$, and the apex $TC^e = 2211.7688$ located on $\tilde{t} = 27.7957$) shows the equilibrium queuing time cost before toll execution. The blue lines of $(t_q, \tilde{t})$ and $(\tilde{t}, t_{q'})$ represents the queuing time cost for the early and late arrivals, respectively. On the other hand, the green triangle (with the same base as the blue triangle, and the apex $TC^e = 2211.7688$ located on $t^* = 33.7417$) represents the optimal non-queuing toll scheme. The green lines of (t_q, t^*) and $(t^*, t_{q'})$ represents the tolls that should be paid by the early and late arrivals, respectively. After executing the optimal non-queuing toll scheme, all of the containers' arrival time will be effectively dispersed at the entrance of port container yard, so the phenomenon of queuing will no longer exist. Furthermore, take the lower half of Figure 4 for example, the two blue solid lines indicate the container's arrival rates ($\frac{s\alpha}{\alpha-\beta}$=2.44410 and $\frac{s\alpha}{\alpha+\gamma}$=0.74713 for the early and late arrivals, respectively) before tolling. While the green horizontal solid line represents the ship's arrival rate (s=2.0134) after implementing the optimal non-queuing toll scheme. Therefore, the numbers of the total early arrivals and total late arrivals of containers are equal to 67.9355 and 7.0645, respectively.

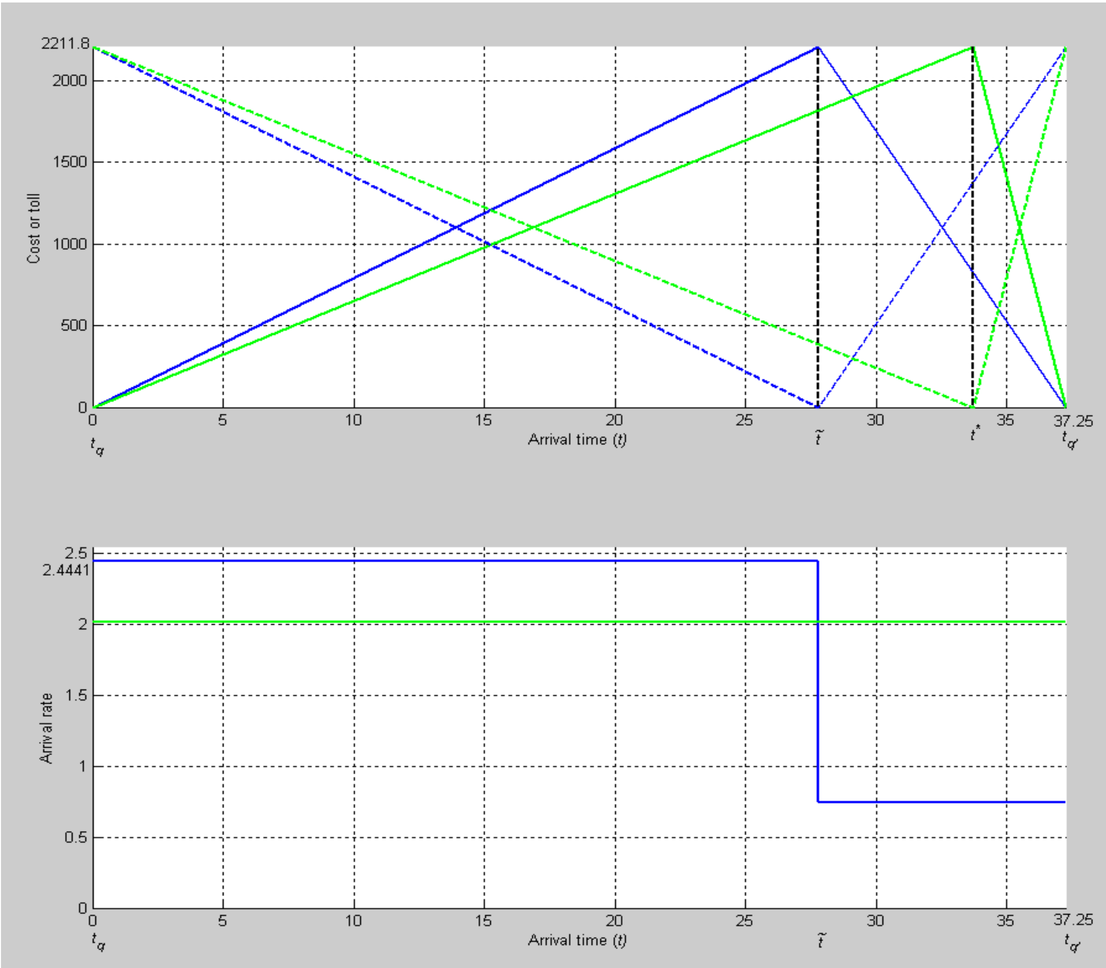

Figure 4. All Results in the Numerical Example

5. CONCLUSION

As the capacity of port container yard has been saturated, there has no any empty space inside the yard to use. Containers arrived the control station at this saturated moment should queue outside in sequence for waiting available space. Such queuing phenomenon would be reasonably eliminated by using the queuing pricing model to disperse containers' arrival time. This research is on the basis of "conservation of equilibrium cost" to compute the optimal non-queuing toll scheme, in compliance with the different arrival time charging for the relative tolls. This model could radically eliminate the queuing phenomenon and facilitate the using efficiency of container yard. With regard to shippers, it could be effective in saving the time cost of transportation; in relation to managers of container yard, it could enhance the operational efficiency toward the container yard in order to attract more shippers who are willing to make use of it. Consequently, the non-queuing toll scheme could help to achieve win-win objective no matter for consignors or for managers. In practice, we suggest that the policy of queuing pricing model would be suitable for using on shipping season. As the overall volume of containers on the peak period will be greater than the dull season, so if the scale of container yard is not large enough, it could easily bring about the long queuing phenomenon outside the yard for export containers to enter due to its restricted capacity.

The numerical example has computed the respective unit time cost of queuing periods, early and late arrival periods. These three significant parameter values have determined each container's equilibrium cost, the optimal non-queuing toll scheme, arrival rates, and the consequence of dispersing containers' arrival time after toll collection. Consequently, the results of numerical example could provide a useful reference for the port container yard manager to consider the implementation of queuing pricing.

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