A Ladder-type Mechanism of Option Pricing for Truck-only Toll Lanes

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Abstract: Truck-only toll (TOT) can help reduce traffic congestion and improve traffic flow on highways. Except implementing congestion price on roads, this study focuses on establishing a pricing model with TOT lanes, which adopt a call option from the perspective of transportation option. In the model, we particularly consider a guaranteed mechanism in contract, which provides compensation by different boundaries for the road users who bought a call option from government. Stakeholders in road can apply this model to solve their problems. Freight industries can also acquire hedge benefit from the risk of congestion. Also, this model can be used to control traffic flow and capacity on highways for government.

Keywords: Truck-only Toll, Traffic Congestion, Call Option, Transportation Option, Guaranteed Mechanism

1. INTRODUCTION

The problems of traffic congestion occurred on highways have become a major transportation-managing challenge for both government and many companies. Although each highway generally has speed limit and many lanes, small cars mix with large vehicles (such as heavy-duty trucks, or tractor-trailers, etc.). Under such congesting situation, the uncertainty of traveling time for road users may be resulted (van Lint et al., 2010); while the transportation efficiency for large vehicles may be affected. In addition, the oppressed feeling caused by large vehicles can lead other small-vehicle drivers to psychological stress which will endanger and put the road users at high risk. Further, it can make a serious traffic congestion which impedes the rapid transportation and the saving of travel time if a traffic accident involving a collision with a large vehicle happens.

To solve the problem caused by the mixing traffic flows, and to enhance the road fluency on the highways, Forkenbrok and March (2005) proposed an idea related to the arrangement of truck-only toll (TOT). They argued that specific truck traveling lane should be implemented to significantly reduce the risk of vital car accident as well as traffic congestion. Other researches supported the establishment of TOT and indicated several advantages of TOT for freight transportation which include: providing a more economical driving scheme, improving the service quality of delivery (Forkenbrok and Hanley, 2005), and reducing the emission of carbon dioxide (CO2) on the highways (Chu and Meyer, 2009), in spite of an unsolved question, that is how to establish a reasonable and acceptable pricing mechanism for TOT lanes. And this will become an important issue if the government decides to implement the policy of TOT lanes. In reality, the governments often ignore the good-pricing practice when they set up the TOT lanes.

Cherry and Adelakun (2012) interviewed about 500 truck drivers for their willingness to pay for the use of TOT lanes. The results showed that most truckers were willing to pay in order to save: 10 minutes in average. Two implications can be indicated: firstly, TOT-lane policy still failed to improve transportation efficiency; and secondly, TOT-lane policy was unable to provide strong evidence to show cost-effectiveness. The cost of using TOT lanes might be higher than the using of general lanes. When truck drivers used TOT lanes, they still needed to pay extra costs for the avoiding and reducing of traffic congestion.

Tibben-Lembke and Rogers (2006) offered a transportation option which indicated that traffic congestion can be measured by putting a monetary value. This option can be regarding as one of the transportation resources in the future. One of the advantages of such option is that it can help the freight transporters to reduce the uncertain losses with economic and financial measures (Yao et al., 2010). Friesz et al. (2008) also used the call options in the pricing procedure. They suggested that traffic congestion should be securitized and doing so will, allow dynamic user equilibrium and further solve the risk of road congestion.

This study adopts the method of real options to solve the problems resulted from the policy TOT lanes. We designed a call option of pricing model to fulfill the goal of cost-effectiveness by purchasing TOT lanes options. In the process of designing the model, a guaranteed mechanism can help increase potential buyers' motivation of purchasing the TOT lane options. The option period of such mechanism can last for one year and be practiced at any of the four quarters of the year. Furthermore, the design of such device takes the existing demand of dynamic uncertainty resulted from industrial competition into consideration. The pricing model we design is so flexible and practical to freight transporters because it reflects the intrinsic and time value embedded in the option.

2. THE TOT LANE CONTRACT

2.1 TOT Lane Contract Specification

TOT lane options allow the contract buyers to efficiently execute efficiently the management of delivery freight transportation. Compared to users without pre-purchasing TOT lane options, the buyers can have a more favorable price and have guaranteed rights and interests to practice these options. The operators can practice the contract on a specific date in the future, wherein represents the due date of the contract.

S indicates traffic flow, and it is divided into three speed levels, represented by K_1, K_2, K_3 respectively. The government provides a guaranteed mechanism for purchasing TOT lane options. Before the due date, the holders can execute the contract during the period of the TOT lane options. The government will offer road users a rebate if the average speed on the highway is lower than a specific speed margin listed in the contract. On the other hand, drivers who do not purchase this option cannot get any compensation when they are late. However, maximizing the capacity of the highway, we convert the traffic flow into the number of vehicles served by the dedicated TOT lanes based on traffic flow. And there are three specified level of services provided by the dedicated TOT lanes, indicated by k_1, k_2, k_3.

In the contract, $C(S,T)$ represents the specific prices for TOT lane prior to the due date of the contract T. Three different speed levels of the payoff for the options before the due date are shown as follows:

$$(S,T)=\begin{cases}(S-K_3)*k_3, & if\ S \geq K_3 \\ (S-K_2)*k_2, & if\ K_3 > S \geq K_2 \\ (S-K_1)*k_1, & if\ K_2 > S \geq K_1 \\ 0, & if\ otherwise\end{cases} \tag{1}$$

2.2 Guaranteed Mechanism

The different levels of speed suggested above would stimulate companies of transportation industry to purchase TOT lane option. Further, the government based on originally condition in contract also provides R_b as a guarantee rebate. Two trade-off questions will exist if the policy is put into practice: how to maintain a more flexible space for the implementation of TOT lanes options for holders and how to effectively manage traffic flow on highway.

Demand variability will affect the usage for TOT lane option. We consider demand dynamic in different quarter for practitioners. Another perspective from government will be its possibility to provide reasonable fee for holders phrase TOT lanes option. However, building the same threshold of traffic flow for each quarter does not match real situation. For example, companies in freight industry have different delivery schedule on different seasonal demand.

We assume that a holder purchases a TOT lane option at the 1st quarter for one year. This gives the holder a right to exercise an option from the 1st date of the next quarter until maturity date at the 31st of the fourth quarter among any of the quarters. Figure 1 shows the relationship between the dates of a contract and an operation time.

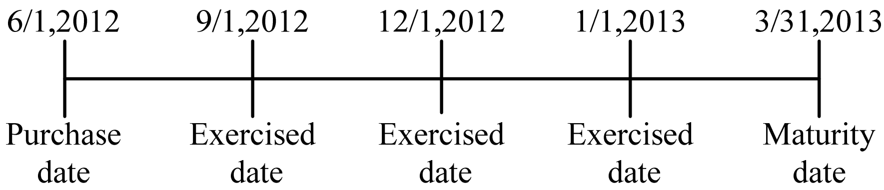

Figure 1. The relationship between time and dates on the contract

With different service levels, the pricing model decides the setting of four boundaries for each quarter, wherein, B_1, B_2, B_3, B_4 represents the upper boundaries of the traffic flow respectively, and $R_{b1}, R_{b2}, R_{b3}, R_{b4}$ indicates the rebate of the four boundaries respectively. Figure 2 illustrates the four boundaries that display the relationship between time and exercised traffic flow, t_0 indicates purchase time, $t_1 - t_4$ indicates the implementation time, and T indicates maturity date.

The companies in freight industry has purchased TOT lanes on the date t_0 during the period of implementation, $(t_1, t_4]$. When the average speed on the highways is lower than a specific speed margin listed in the contract; in other words, when the traffic flow exceeded one of the boundaries B_1, B_2, B_3, B_4, the rebate guaranteed mechanism will be activated immediately. When a driver finds that an average speed on the highways is lower than a specific speed listed in the contract, he may implement the option at a specific time t_2. The holder not only preserves a rebate R_{b1} provided at a specific time t_2, but also obtains the optional payoff C_2^* next time.

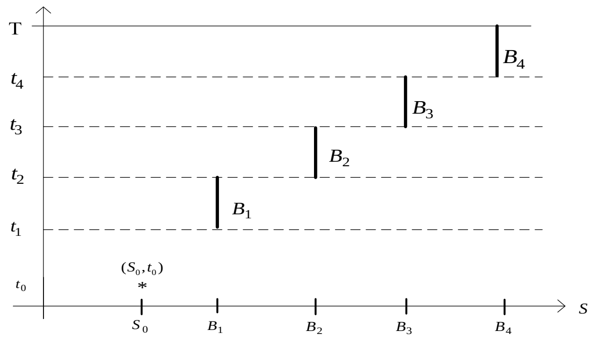

Figure 2. The initial relationship between boundaries and time

$C_i(B_i,t)$ represents the given rebate R_{bi}, by the time when the traffic flow has exceeded the boundary during the period of implementation $(t_1, t_4]$. For every stage of upper boundary, a rebate plus an additional value of next TOT lane option, C_i^* will be provided.

Given the maturity date, T, upon the implementation date, t_4, the payoff will be the rebate R_{b4} plus a value of general TOT lanes option C^*. Thus, when encountering the upper boundaries, the values of general TOT lanes options are as follows:

$$\begin{cases} C_1(B_1,t) = R_{b1} + C_2^*, & if\ t_1 < t \le t_2 \\ C_2(B_2,t) = R_{b2} + C_3^*, & if\ t_2 < t \le t_3 \\ C_3(B_3,t) = R_{b3} + C_4^*, & if\ t_3 < t \le t_4 \\ C_4(B_4,t) = R_{b4} + C^*, & if\ t_4 < t \le T \end{cases} \tag{2}$$

3. THE TOT LANES PRICING MODELS

We supposed that traffic flows S satisfies the lognormal distribution presented and the stochastic process flows Geometric Brownian Motion (GBM) as $dS = \mu S dt + \sigma S dz$, where dz is the stand normal distribution and $dz \sim N(0, dt)$.

According to the assumption of Black-Scholes model, the buyer can purchase a TOT lane option at any time during a given purchase date. There will be no transaction cost and taxes for buying a TOT option without arbitrage opportunity. For this case, we can further establish a no-arbitrage hedge portfolio.

Because we use financial derivation to measure the price in TOT lanes, we could establish a hedge portfolio that employs the number of traffic flow instead of the financial index. We consider that there is a relationship between freights and stockholders in freight market, which results in a negative relationship between traffic flow on the TOT lane and stock price. Hence, we could set up the no-arbitrage hedge portfolio $F(S,t)$:

$$F(S,t) = C(S,t) - \Delta S \tag{3}$$

where $C(S,t)$ is the value of the TOT lane option at time t. The proportion Δ is the hedging ratio. However, we should use portfolio to derive a Black-Scholes partial differential equation (PDE):

$$\frac{\sigma^2}{2}S^2C_{SS}+RSC_S+C_t=RC \tag{4}$$

where R is the expected rate of return from the investment. C is the TOT lane option price. S is the traffic flow of the TOT lane option. t is the time to maturize the contract. σ is the average volatility of traffic flow.

3.1 Variable Transformation

In this section, we use variable transformation and boundary integral method, which calculate two portions, including the condition of maturity and the situation of exceeding the boundary during exercise date.

Now, we use three new equations (5) to transform the variables:

$$\begin{cases} \tau = T-t \\ x=\ln S+(R-\frac{\sigma^2}{2})\tau \\ u(x,\tau)=C(S,t)e^{R\tau} \end{cases} \tag{5}$$

In equation (5), τ denotes remaining time and x expresses the traffic flow after transformation. We use $\ln S$ and $(R-\frac{\sigma^2}{2})\tau$ to demonstrate the distance of drift movement, the intercept and the drift of volatility, respectively. The third item means $u(x,\tau)$, which equates the future value of $C(S,t)$. $e^{R\tau}$ is the continuous future value interest factor that implies how much a TOT lane option can cost at maturity.

Consequently, take equation (5) into equation (1), (2), (4); then it produces a new PDE equation (6), and this formula is called the homogenous equation:

$$\frac{\sigma^2}{2}u_{xx}(x,\tau)=u_\tau(x,\tau) \tag{6}$$

Then, the new equation (7) is a payoff based on different traffic flow at maturity indicated in equation (5). It's an initial condition when remaining time $\tau=0$, r_1,r_2,r_3 are the conversion rate of three grades and $u_0(x,0)$ indicates a value at maturity:

$$u_0(x,0)=\begin{cases} (e^x-K_3)*r_3, & if\ e^x\geq K_3 \\ (e^x-K_2)*r_2, & if\ K_3>e^x\geq K_2 \\ (e^x-K_1)*r_1, & if\ K_2>e^x\geq K_1 \\ 0, & if\ otherwise \end{cases} \tag{7}$$

Final, we have a new equation (8) when the boundary exceeded by traffic flow at anytime during exercised date:

$$\begin{cases} b_1(\tau) = \ln B_1(T-\tau) + \left(R_1 - \dfrac{\sigma_1^{\ 2}}{2} \right)\tau \\ b_2(\tau) = \ln B_2(T-\tau) + \left(R_2 - \dfrac{\sigma_2^{\ 2}}{2} \right)\tau \\ b_3(\tau) = \ln B_3(T-\tau) + \left(R_3 - \dfrac{\sigma_3^{\ 2}}{2} \right)\tau \\ b_4(\tau) = \ln B_4(T-\tau) + \left(R_4 - \dfrac{\sigma_4^{\ 2}}{2} \right)\tau \end{cases} \tag{8}$$

In equation (8), $B_i(T-\tau), i = 1-4$ are on the upper boundaries of traffic flow that government guaranteed, $\left(R_1 - \dfrac{\sigma_1^{\ 2}}{2} \right)\tau$ is a distance of drift.

Figure 3 shows the relationship between the remaining time τ and traffic flow x after transformation. (x_0, τ_0) means the purchase time, and $b_i(\tau), i = 1-4$ are the value of threshold for the four boundaries.

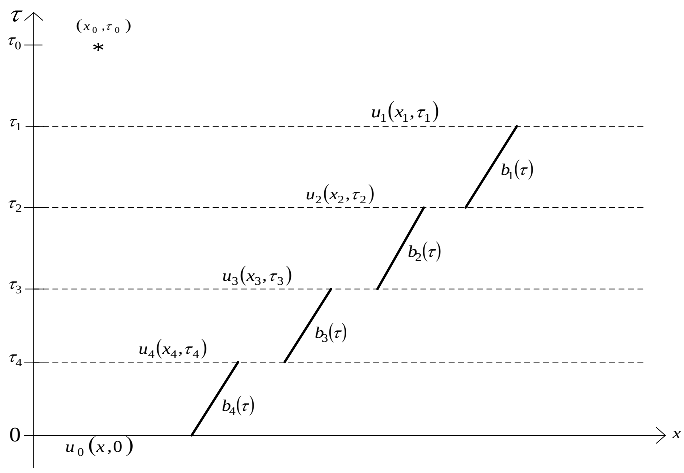

Figure 3. The transformed relationship between traffic flow and time

Equation (9) shows the status of boundary condition in transformation.

$$u_{bi}(b_i(\tau), \tau) = R_{bi}^{\ *}(\tau) * e^{R\tau} + u_i^* \tag{9}$$

Next, we calculate the prices of the TOT lane options between a purchase date and an exercise date based on the conception of expected value, and then use integral representation to evaluate the value from exercised time to maturity with the four boundaries $\tau_i, i = 1-4$.

3.2 The Value of TOT Lane Option

We use Green's function $G^*(\cdot)$, which is obtained via combining the lognormal distribution at any time $\left(\bar{x},\bar{\tau}\right)$ and follows the condition $G^{*}\left(b(\tau),\tau;x_0,\tau_0\right)=0$.

$$G^{*}\left(x,\tau;\bar{x},\bar{\tau}\right)=\frac{1}{\sqrt{2\pi\sigma^2\left(\bar{\tau}-\tau\right)}}\left[\exp\left(-\frac{\left(x-\bar{x}\right)^2}{2\sigma^2\left(\bar{\tau}-\tau\right)}\right)-\exp\left(-\frac{\left(x-2b\left(\bar{\tau}\right)+\bar{x}\right)^2}{2\sigma^2\left(\bar{\tau}-\tau\right)}+\alpha\right)\right] \tag{10}$$

where $0\le\bar{\tau}\le\tau_i, i=1-4$, and α is a constant variant, $\alpha=\frac{2\left(R-\frac{\sigma^2}{2}\right)\left(b\left(\tau_0\right)-x_0\right)}{\sigma^2}$. After using the boundary integral method, we obtain the integral representation as equation (11)-(14), in which the first item is effective for the first boundary condition and the second item is effective for the next boundary condition where we have $\tau_1-\tau_4$.

$$u_1\left(x_1,\tau_1\right)=\int_{-\infty}^{b_1(\tau_2)}u_2\left(x_2,\tau_2\right)G^{*}\left(x,0;x_1,\tau_1\right)dx+\int_{\tau_2}^{\tau_1}u_{b1}\left(b_1(\tau),\tau\right)\left(-\frac{\sigma^2}{2}G_1^{*}\left(b_1(\tau),\tau;x_0,\tau_0\right)\right)d\tau \tag{11}$$

$$u_2\left(x_2,\tau_2\right)=\int_{-\infty}^{b_2(\tau_3)}u_3\left(x_3,\tau_3\right)G^{*}\left(x,0;x_2,\tau_2\right)dx+\int_{\tau_3}^{\tau_2}u_{b2}\left(b_2(\tau),\tau\right)\left(-\frac{\sigma^2}{2}G_2^{*}\left(b_2(\tau),\tau;x_1,\tau_1\right)\right)d\tau \tag{12}$$

$$u_3\left(x_3,\tau_3\right)=\int_{-\infty}^{b_4(\tau_4)}u_4\left(x_4,\tau_4\right)G^{*}\left(x,0;x_3,\tau_3\right)dx+\int_{\tau_4}^{\tau_3}u_{b3}\left(b_3(\tau),\tau\right)\left(-\frac{\sigma^2}{2}G_3^{*}\left(b_3(\tau),\tau;x_2,\tau_2\right)\right)d\tau \tag{13}$$

$$u_4\left(x_4,\tau_4\right)=\int_{-\infty}^{b_4(0)}u_0\left(x,0\right)G^{*}\left(x,0;x_4,\tau_4\right)dx+\int_{0}^{\tau_4}u_{b4}\left(b_4(\tau),\tau\right)\left(-\frac{\sigma^2}{2}G_4^{*}\left(b_4(\tau),\tau;x_3,\tau_3\right)\right)d\tau \tag{14}$$

The accumulation for TOT option could validly continue when the traffic flow exceeds the boundary level. The boundary condition appears as follows.

$$u_b\left(b(\tau),\tau\right)=\int_{-\infty}^{\infty}u_0\left(x,0\right)G^{*}\left(x,0;b(\tau),\tau\right)dx \tag{15}$$

The function $G^{*}(\cdot)$ is the lognormal probability density function as follows.

$$G\left(x,\tau;\bar{x},\bar{\tau}\right)=\frac{1}{\sqrt{2\pi\sigma^2\left(\bar{\tau}-\tau\right)}}\exp\left(\frac{\left(x-\bar{x}\right)^2}{2\sigma^2\left(\bar{\tau}-\tau\right)}\right) \tag{16}$$

According to equation (15), (16), the value of the TOT lane option from the exercised date to the maturity date is as follow:

$$u_1\left(x_1,\tau_1\right)=\begin{cases}\int_{-\infty}^{b_1(\tau_2)}u_2\left(x_2,\tau_2\right)G^{*}\left(x,0;x_1,\tau_1\right)dx+\int_{\tau_2}^{\tau_1}u_{b1}\left(b_1(\tau),\tau\right)\left(-\frac{\sigma^2}{2}G_1^{*}\left(b_1(\tau),\tau;x_0,\tau_0\right)\right)d\tau & ,if\ x_1<b_1(t_1)\\ R_{b1}e^{R\tau_1}+C_2^{*}e^{\mathrm{R}\tau_1} & ,if\ x_1\ge b_1(t_1)\end{cases} \tag{17}$$

$$u_2(x_2,\tau_2)=\begin{cases}\int_{-\infty}^{b_2(\tau_3)}u_3(x_3,\tau_3)G^*(x,0;x_2,\tau_2)dx+\int_{\tau_3}^{\tau_2}u_{b2}(b_2(\tau),\tau)\left(-\dfrac{\sigma^2}{2}G_2^*(b_2(\tau),\tau;x_1,\tau_1)\right)d\tau\ ,if\ x_2<b_2(t_2)\\ R_{b2}e^{R\tau_2}+C_3^*e^{R\tau_2} \qquad ,if\ x_2\geq b_2(t_2)\end{cases} \tag{18}$$

$$u_3(x_3,\tau_3)=\begin{cases}\int_{-\infty}^{b_4(\tau_4)}u_4(x_4,\tau_4)G^*(x,0;x_3,\tau_3)dx+\int_{\tau_4}^{\tau_3}u_{b3}(b_3(\tau),\tau)\left(-\dfrac{\sigma^2}{2}G_3^*(b_3(\tau),\tau;x_2,\tau_2)\right)d\tau\ ,if\ x_3<b_3(t_3)\\ R_{b3}e^{R\tau_3}+C_4^*e^{R\tau_3} \qquad ,if\ x_3\geq b_3(t_3)\end{cases} \tag{19}$$

$$u_4(x_4,\tau_4)=\begin{cases}\int_{-\infty}^{b_4(0)}u_0(x,0)G^*(x,0;x_4,\tau_4)dx+\int_{0}^{\tau_4}u_{b4}(b_4(\tau),\tau)\left(-\dfrac{\sigma^2}{2}G_4^*(b_4(\tau),\tau;x_3,\tau_3)\right)d\tau\ ,if\ x_4<b_4(t_4)\\ R_{b4}e^{R\tau_4}+C^*e^{R\tau_4} \qquad ,if\ x_4\geq b_4(t_4)\end{cases} \tag{20}$$

where $u_i(x_i,\tau_i), i=1-4$ means the TOT lane option value from exercised time to maturity. $\tau_i, i=1-4$ denotes the exercised date of the four quarters we exercise. $x_i<b_i(t_i), i=1-4$ indicates the traffic flow under the boundary level that the flow would not exceed the boundary during probation period. Otherwise, $x_i\geq b_i(t_i), i=1-4$ implies that the traffic flow exceeds the boundary during the probation period.

When the traffic flow exceeds the boundary, the value of this TOT lane obtains rebates R_{bi} from government who proposed the plus value of next TOT lanes option C_{i+1}^*. If holders do not excise at τ_1 period at the first quarter, they will retain the exercise condition and have a higher price at the next period. Therefore, the last quarter (the fourth quarter) is the maturity date and the payoff at this period is a general TOT lane option C^*.

Finally, we evaluate the value of the date with exercising right of purchasing date and the integral representation is as follows:

$$u_0(x_0,\tau_0)=\int_{-\infty}^{\infty}u_1(x_1,\tau_1)G^*(x_1,\tau_1;x_0,\tau_0)dx_1 \quad ,if\ x_0<b_0(t_0) \tag{21}$$

Therefore, we obtain the TOT lane option value at the purchased, which date is:

$$C(S_0,t_0)=u\left(\ln S_0+\left(R-\frac{\sigma^2}{2}\right)\tau_0, T-t_0\right)e^{-R\tau_0} \tag{22}$$

where $e^{-R\tau_0}$ denotes a discount rate. $C(S_0,t_0)$ is the reasonable value of the TOT lane options on purchasing date.

4. CONCLUSION

Governments allocate TOT lanes for controlling traffic congestion on highways, but they usually ignore the voice from freight industry. Setting up reasonable pricing policy for TOT lanes could increase government's revenue, enhance delivery speed, thus leading to tangible yield, and intangible value for sustainability. For these reasons, focusing on the application of transportation option application, in this study, we proposed the call option pricing model in order to communicate their freight transportation users for their willingness to assume an alternative way.

Through the establishment of an optional and mathematical pricing model for the traffic flow, the holders can evaluate the contract for finding optimal practicing decision and considering the effectiveness of the delivery strategy. Another advantage of optional pricing is that the holders can employ this analytical model not only to reduce the loss but also to avoid delivery uncertainty. . This model can also be regarded as an investment since it can help deal with the potential risks of unreliable traveling time on highways.

We also proposed an idea of using financial derivation to measure the road price on TOT lanes. In this study, we focus only on formulating theoretical modeling and setting rational variables. Following current study, we will collect and analyze empirically the actual data from highway and carrier industry. Furthermore, we will put optional concept, expand it to TOT lane markets, and eventually gain maximizing equity values between the holders and the government.

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