Analysis of Traffic Speed-Density Regression Models -A Case Study of Two Roadway Traffic Flows in China

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Abstract: Speed, flow and volume are the most important elements of the traffic flow. In this study, the speed-density regression models are compared between congested city road and the tolled highway based on traffic flow data on Beijing Third Ring Road (BTRR) and Jing Jin Tang Highway (JJTH) in China, which are collected by road detectors on each road. The speed-density regression curves are analyzed based on four the classical models: Greenshields, Greenberg, Underwood, and Bell-shape models. From data analysis, it can be seen that Greenberg and Underwood models for BTRR and JJTH are presented with higher confidence and less variation. Through the comparison of the two roads' regression models, it can be found that the free flow speed and the speed maximum flow level on highway is much higher than city road, but the density at maximum flow level and the jam density on highway is smaller than city road.

Keywords: Traffic flow; Speed-density relations; Highway; City road; Regression curve

1. INTRODUCTION

Since the Highway Capacity Manual published the first edition in 1965, the equilibrium speed-flow relationship under stationary states has become a significant component for the highway capacity planning. The HCM suggests speed-flow curves for traffic road under the free-flow conditions. Under the stationary traffic relation that the traffic flow equals space-mean speed times density, which implies the flow-density and speed-density curves under free-flow conditions. Many traffic researchers have investigated the functionally specifying and estimated these three variable relations. Greenshields (1935) proposed a parabolic function for the flow-density relation by suggesting a linear speed-density curves under the free-flow condition. Based on notions like fluid dynamics and car-following decisions, varieties of functions forms for the flow-density relationships have been studied. For example, Greenberg (1959) used a logarithmic form for speed-density relations, and Underwood (1961) proposed an exponential form to show the relations. Drake et al. (1967) extended the specification to the bell-shaped curve model, which corresponds to a speed-flow specification associated with the normal distribution curve.

Until now, the classical speed-flow curves such as Greenberg model, Greenshied-type model and Underwood type model have been used widely to characterize some traffic flows. In most models, the relationship of speed-flow or speed-density is dependent only on road type and a free flow speed. Researchers in many countries have investigated the relationship of the traffic flow, speed and density since 1934. Hall et al. (1986) have studied the flow-density relationship of Canada using an extensive data set collected on the Queen Elizabeth Way in Ontario. Ohta and

Harata (1989) developed the regression model for the three variable relationships for Japanese cities. Olszewski et al. (1995) developed an area-wide traffic speed-flow model for the Singapore CBD to get an analytical framework for traffic management measures evaluation. Lum et al. (1998) analyzed the speed-flow relationship of the arterial road using the traffic volume and travel time data at a number of arterial roads in Singapore. Nielsen and Jorgensen (2008) explored the speed-flow and flow-density relations based on a large data sample at the motorway network of the greater Copenhagen region in Denmark. Xie et al. (2011) investigated the traffic speed-flow relationship of different lanes under a given traffic condition and general levels of service based on Chengdu expressway data of China. However, most researchers focus on the character of the network roads, lacking of the comparison of different roads of different type.

The aim of the present paper is to compare the speed-density classical curves to estimate functional relationships based on the recorded data from Beijing Third Ring Road and Jing Jin Tang Highway. BTRR represent the congested city road and the JJTH represent the tolled highway, so they represent two different kinds of road. A large number of recorded speed, flow, and density measurements are examined to estimate the consistency of the relations between speed and flow. In addition, some comparisons are given on the analysis of the speed-flow relations under highly congested road or uncongested toll road.

In this paper, based on the traffic flow data of BTRR and the JJTH, several regression curves are compared to find which one suite the BTRR better and which one suits the JJTH better, and compare their differences. Based on these classical models, the basic variables such as the jam density, free flow speed, and density or speed at the maximum level can also be derived. Display of the appropriate speed-flow models can provide the traffic manages the traffic flow characters of different roads, which can help them to do more research on the traveler's travel behaviors, traffic ramp control, congestion analysis, and so on.

2. CLASSICAL SPEED-DENSITY MODELS

Traffic flow theory assumes that there is a fundamental relationship among the three principle variables of traffic flow q, speed V, and density K, as follows:

$$\mathrm{q} = \mathrm{V} \times \mathrm{K} \tag{1}$$

Eq. (1) can be derived to get the relationship between any two variables in the three principle variables (e.g., speed-density), and the other two relationships (density-flow, speed-flow) will be got automatically. Therefore, in this paper, we choose the speed density K as a function of speed V, then the speed-density function can be shown as $\mathrm{K} = \mathrm{K(V)}$.

The first traffic flow model war proposed by Greenshields in 1935. He suggested a linear relationship between the density and speed.

$$K = K_j \,(1 - \frac{V}{V_f}) \tag{2}$$

where K_j = jam density (corresponding to zero speed, i.e.$\mathrm{V} = 0$); V_f = free flow speed.

Many researchers have focus on the characteristic of traffic during the period of 1950-1970, especially in the United States (Dick, 1966; Drake et al., 1967; Gerlough and Huber, 1975; Greenberg, 1959; Underwood, 1961; Wardrop, 1952; Wardrop and Charlesworth, 1954). A number of mathematical models between the traffic speed and density were proposed and calibrated by fitting curves to empirical traffic data. May (1991) summarized some well-known classical speed flow relationships which are consistent with the fundamental relationship.

Greenberg: $K = K_j \ exp\left(-\frac{V}{V_0}\right)$ (3)

Underwood: $K = K_0 In\frac{V_f}{V}$ (4)

Bell-shape: $K = K_0(2In\frac{V_f}{V})^{1/2}$ (5)

where K_0 is the density at maximum flow level; V_0 is the speed at maximum flow level.

The common approach proposed by the traffic flow researchers was using the density and speed data to calibrate the mathematical models and then convert them into speed-flow model using Eq.(1). From these equation, we only need to know one of K_j or K_0, one of V_f or V_0, then we can fully get a speed-density relationship curve. Since there is no guarantee for the existence of K_j or free flow speed V_f, we also can use the traffic flow data and these fundamental models to deduce these variables.

3. DATA BACKGROUD

The data in this study come from Beijing Third Ring Road (BTRR) and Jing Jin Tang Highway (JJTH) in China. By the road direct recorded data processing, the three principle variables of traffic flow q, speed V, and density K can be obtained.

3.1 BTRR and JJTH Road

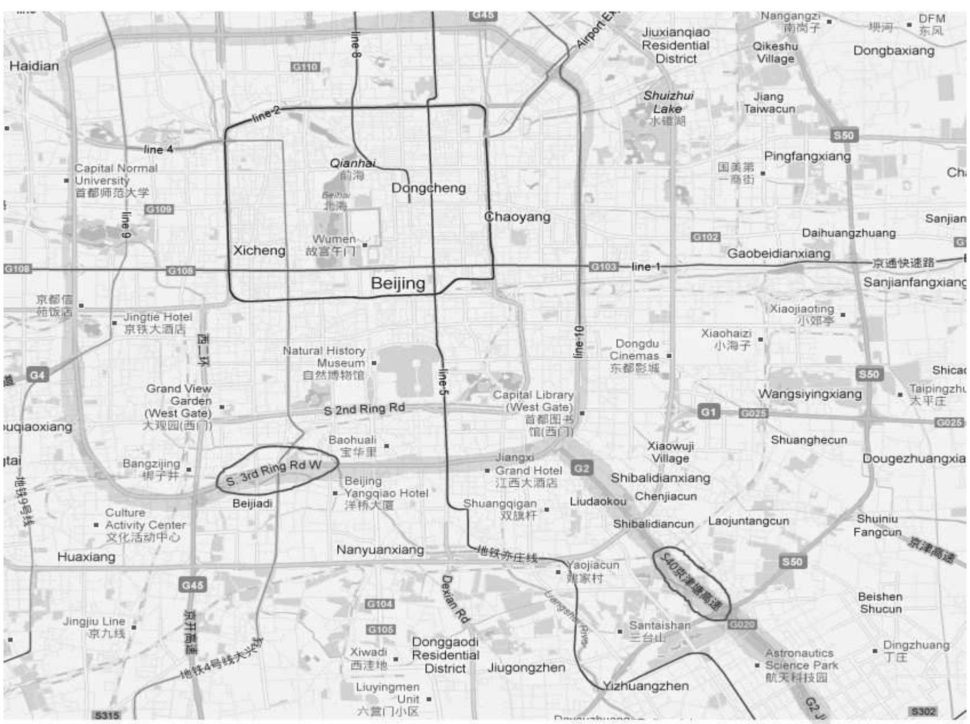

Figure 1. Road network map for BTRR and JJTR

The location of the two roads are shown in Figure 1 with highlighter. As the map shown, the Beijing Third Ring Road (BTRR) is a circle road that encircles the center of Beijing city. BTRR is 2.5 kilometers from the Second Ring Road and 5 kilometers from the city center. This city road have 6 lanes (3 up and 3 down), with the main width of 20m and side road width of 5m. “The BTRR goes through mostly residential and some commercial areas (except for the CBD) that were created during Beijing's first wave of rapid expansion.” The main vehicle type on BTRR is the private cars and public buses. As we all know, Beijing is the capital of China, which have a heavy traffic burden every day. The BTRR is notorious for its traffic jams. “The eastern segment, which runs through Beijing's central business district, is regularly gridlocked during rush hour. ” (Wikipedia, 2008)

Jing Jin Tang Highway, is a straight line road, which links Beijing via central Tianjin to the Tanggu District in eastern Tianjin. JJTH have 4 lanes(2 up and 2 down) with emergency belt. The heavy traffic period of JJTH is during weekends and mornings. The 4 lane width with emergency belt of JJTH is 26m, and the emergency belt takes 2.5m, which is a little lower than the normal standard 28m with 3.5 m emergency belt for 4-lane highway road. The main vehicle on JJTH is the private cars, trucks, and coach buses. JJTH is also a tolled highway. “Tolls apply as of Dayangfang near the Eastern 5th Ring Road in Beijing until the Tanggu/TEDA exit. The expressway uses a networked toll system across all jurisdictions and is managed by Huabei (North China) Expressways.” (Wikipedia, 2009)

Table 1. Introduction of the two road condition

Road Name	Category	Lane	Width	Length	Toll policy	Max speed limit	Main vehicle type
BTRR	City road	6	25m	48km	Free	80 km/h	Car, bus
JJTH	Highway	4	26m	143km	Toll	110 km/h	Car, truck, coach bus

Table 1 lists the differences of the two road condition. For example, BTRR is a congested city road and JJTR is a charged highway; the max speed limit for BTRR is 80 km/h and for JJTH is 110km. Therefore, the condition difference is obvious, then how about the variation in the traffic flow related parameters across the two road? What are the differences of the relationship for the traffic flow, density, and speed between the two different roads? We will make the regression analysis based on the collected data by the road detectors on the two roads.

3.2 Recoded Data Processing

The traffic volume and flow on the two research road are recorded every 2 minutes of each lane. The road traffic flow data are obtained by Hi-pro MTC-10 which is an instrument for collecting data of traffic flow. Table 2 gives an example of the recorded data form the detector. It illustrates a two minutes sample of the BTRR date in one detector. In this following table, the POSID means the detector number, and TIMEAG means the recorded time, the LANNO means the lane number. VOLUME means how many cars have passed this detector, which also called traffic flow number. For example, the last line date means there are 14 vehicles at speed 56 km/h on lane 2 in the time interval between 2005-12-31 00:00:49 and 00:02:48 detected by NO. 2035 Detector.

Table 2. A sample recorded data of 2 minutes of one detector

POSID	TIMETAG	LANENO	VOLUME	SPEED
2035	2005-12-31 00:00:02	13	4	85
2035	2005-12-31 00:00:02	12	10	54
2035	2005-12-31 00:00:02	11	6	49
2035	2005-12-31 00:00:49	3	1	44
2035	2005-12-31 00:00:49	1	19	51
2035	2005-12-31 00:00:49	2	15	60
2035	2005-12-31 00:02:01	13	1	77
2035	2005-12-31 00:02:01	12	11	46
2035	2005-12-31 00:02:01	11	10	58
2035	2005-12-31 00:02:48	3	1	75
2035	2005-12-31 00:02:48	2	14	56
…	…	…	…	…

Obviously, these data are needed to be processed to get the flow-density regression model data. With VOLUME and SPEED, the density can be calculated. In this paper, we choose Lane1 as the analysis lane, because there are always more cars on lane 1 than the other lane according some experience. Then every 10 minutes volume data of 1 lane will be summed up and multiplied 6 as the average flow date per hours, and analysis all day data of different detectors on lane 1. And the processed data are given in the following Table 3.

Table 3. The processed data on BTRR road for the regression

POSID	LANENO	Speed	Density	Flow
…	…	…	…	…
2038	1	47	27.493	1292.2
2038	1	48	26.543	1274.06
2038	1	49	25.277	1248.4
2038	1	52	23.409	1217.3
2038	1	55	20.776	1142.7
…	…	…	…	…
2038	1	74	13.745	997.1429
2038	1	75	13.888	1041.667
2038	1	76	12.878	978.75
2038	1	77	11.688	900
…	…	…	…	…

Figure 2 and Figure 3 show separately the matrix of two-dimensional scatter plots of the BTRR and the JJTH flow data. These displays are helpful in visualizing the relationships among multivariable data set. The two plots indicates the relationships among flow, speed, density are more closely to certain curves than certain line. Generally, when the density increases, the speed will slow down, but the traffic flow will increase to some extent and then decrease. From Figure 2, it can be seen that the change scope of flow density is high. BTRR have the biggest flow density when the traffic speed equals to 26km/hour and the flow equals 2490 vehicles. When the density is too high, the vehicles on BTRR cannot move fast, so traffic flow is lower; when the density is too lower, it means the vehicle number on BTRR is small, so the traffic flow is also lower though the traffic speed is higher. The density and flow change in JJTH is smaller than

BTRR according to Figure 3. As JJTH is a tolled highway, so the highway manager will control the vehicle flow and vehicle speed by road price and highway policy. So the scatter plots for the JJTH flow data are different with BTRR flow data.

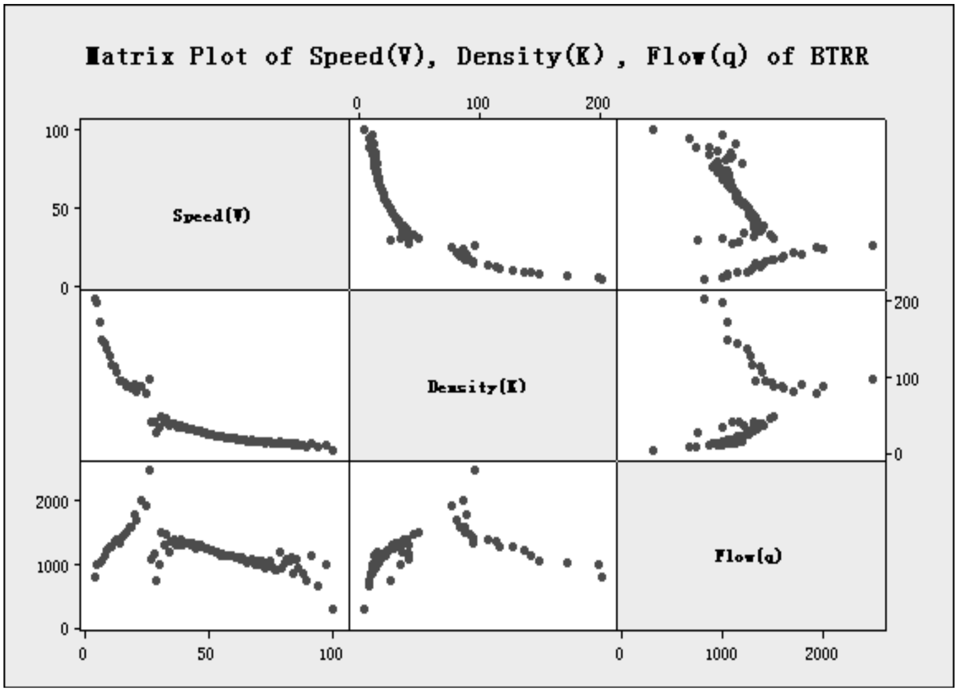

Figure 2. Matrix of scatter plots for the BTRR flow data.

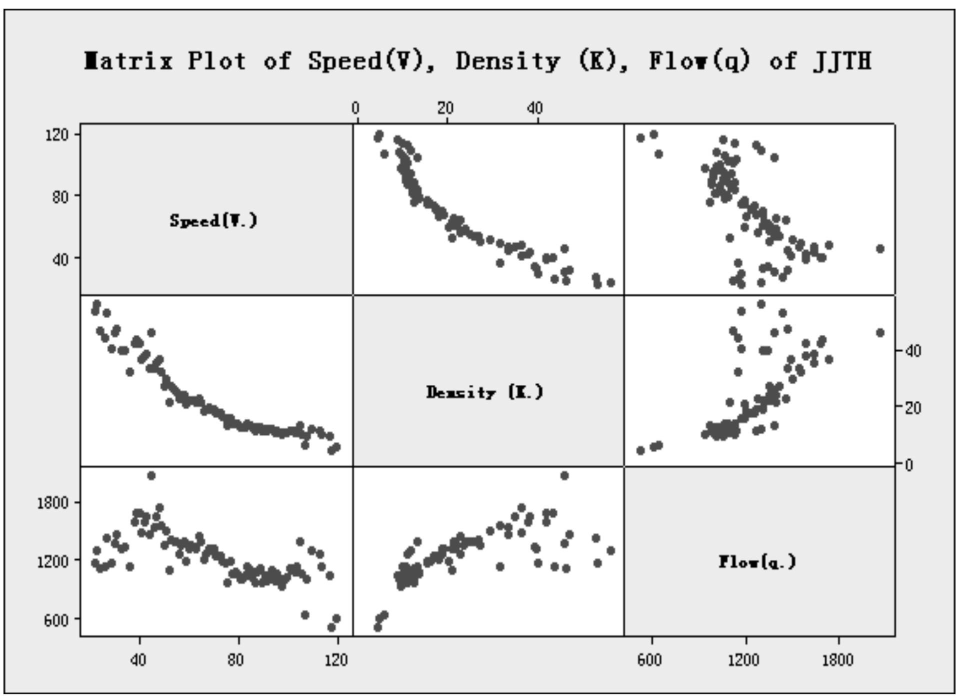

Figure 3. Matrix of scatter plots for the JJTH flow data.

4. REGRESSION MODEL ANALYSIS

In order to get the speed-density curves for BTRR and JJTH, regression analyses are performed by using the regression software. In this paper, the Minitab 16.0 is used to do the analysis. Some transformations will be very useful in the speed-density relationship analysis where the true relationship between the response density and the predictor speed may be not well approximated by a straight line. To the regression curve models by Minitab, the classical models can be equally transformed to the following format as shown in Table 4. Then the four classical speed-density models and their transformations will be used on BTRR and JJTH regression models in the following part.

Table 4. Classical Speed-Density Function and Transformation

Model	Speed-density Function	Transformation
a) Greenshields	$K = K_j\,(1 - \frac{V}{V_f})$	$K = K_j - \frac{K_j}{V_f}V$
b) Greenberg	$K = K_j\ \exp\left(-\frac{V}{V_0}\right)$	$In(K) = \mathrm{In}(\mathrm{K_j}) + (-\frac{1}{\mathrm{V_0}})\mathrm{V}$
c) Underwood	$K = K_0 In\frac{V_f}{V}$	$K = K_0 InV_f - K_0 InV$
d) Bell-shape	$K = K_0(2In\frac{V_f}{V})^{1/2}$	$K^2 = 2K_0^2 InV_f - 2K_0^2 InV$

4.1 Regression Models for BTRR

The Greenshield regression equation for BTRR traffic flow by the Minitab 16.0 is

$$K = K_j - \frac{K_j}{V_f}V = 114.15 - 1.42V \tag{6}$$

From the Eq.(6), the jam density K_j and free flow speed V_f can be calculated as $K_j = 114.15, V_f = 80.28$. So the Greenshield model for BTRR can be expressed as

$$K = 114.15 \times (1 - \frac{V}{80.28}) \tag{7}$$

The test for significant and the residual plots of Greenshields Regression are shown in Table 5 and Figure 4. The adjust R-square is used to check the “goodness of fit” adequacy of the regression model. Table 5 shows the adjusted R-square is 68.7% for this regression. P-value shows the related variable significance of regression, and P=0 means the variable is significant to the regression model and should be included.

Table 5. Test for Significant of the Greenshields Regression

Predictor	Coef	SE Coef	T	P
Constant	114.15	5.729	19.93	0
Speed(V)	-1.4178	0.1024	-13.85	0
S=25.3694	R-Sq=69%		R-Sq(adj)=68.7%	

Estimation of the model parameters requires the assumption that the errors are uncorrelated random variables with mean zero and constant variance. Therefore, test of hypotheses and interval estimation requires that the errors be normally distributed. Figure 4 shows the residual plots of the Greenshields curve for BTRR to do the resident normal distribution check.

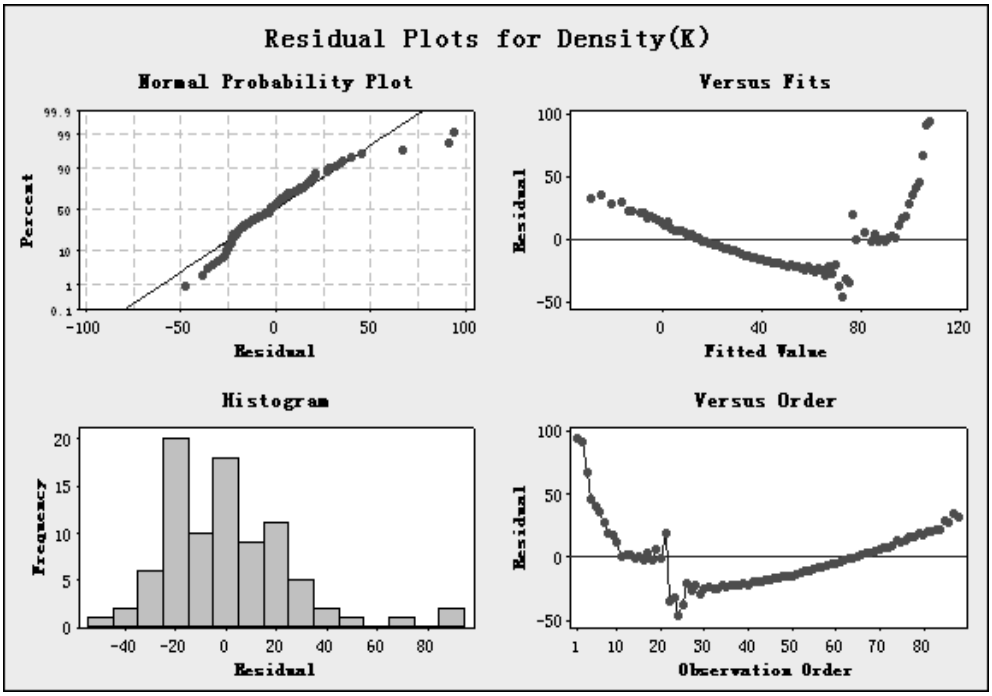

Figure 4. Residual plots of the Greenshields curve for BTRR

By using the same method, the other regression curve models for BTRR can also be obtained. The following Table 6 shows the four regression models for BTRR flow data.

Table 6. The regression curve models for BTRR

Model	Minitab Regression Curve	Speed-density Curve
a) Greenshields	$K = K_j\,(1 - \frac{V}{V_f}) = 114.15 - 1.42V$	$K = 114.15 \times (1 - \frac{V}{80.28})$
b) Greenberg	$In(K) = \mathrm{In}(\mathrm{K_j}) + \left(-\frac{1}{\mathrm{V_0}}\right) = 4.97 - 0.034\mathrm{V}$	$\mathrm{K} = 143.88 \times \exp(-\frac{\mathrm{V}}{30.8})$
c) Underwood	$K = K_0 InV_f - K_0 InV = 255.85 - 57.5InV$	$K = 57.5In(\frac{85.6}{V})$
d) Bell-shape	$K^2 = 2K_0^2 InV_f - 2K_0^2 InV = 37564 - 9123InV$	$K = 67.54 \times (2In\frac{61.55}{V})^{\frac{1}{2}}$

Figure 5 shows the four different regression velocity-density curves of BTRR. From the pictures (a)-(d), we can directly see that the regression curve of picture-(b) and picture-(c) are better than picture-(a) and picture-(d), which means the Greenberg and Underwood curves are more suitable on BTRR than the Greenshields and Bell-shape curves.

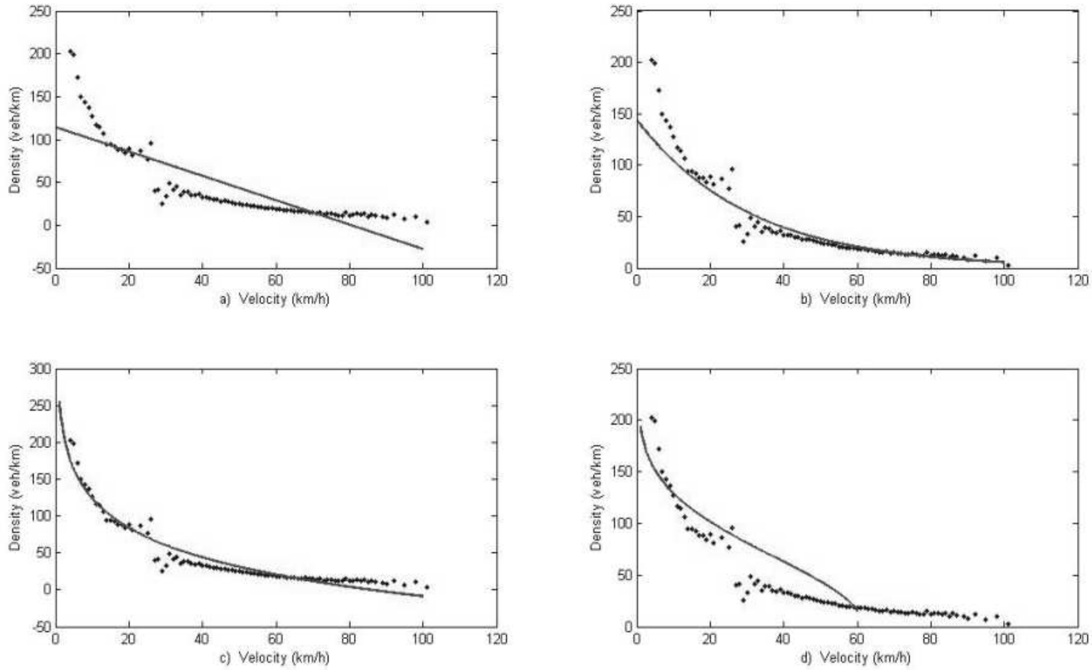

Figure 5. Four different regression curves of BTRR

Table 7 presents the relation between the traffic density and speed on the models of Greenshields, Greenberg, Underwood, and Bell-shape with correlation coefficient of related to itself. F value shows the significance of the regression model, which means the relations between speed and density here are obvious for all the four models in Table 7. The highest value of correlation coefficient among presented models is related to Greenberg model with $R^2 = 0.942$ and after that underwood model with $R^2 = 0.934$ has the most value of correlation coefficient. (Montgomery and Runger, 2011)

Table 7. The model summary and parameter estimates from BTRR data

	Model Summary		**Parameter Estimates**			
Model	R^2	**F**	V_f	V_0	K_0	K_j
a)Greenshields	0.690	191.77	80.28	\	\	114.15
b) Greenberg	0.932	1191.15	\	30.8	\	143.88
c) Underwood	0.934	1225.5	85.6	\	57.5	\
d) Bell-shape	0.769	286.15	61.5	\	67.54	\

The parameters of the four models are shown in Table 7. Comparing the four models' parameters, the free flow speed V_f on BTRR is close in model Greenshields and Underwood, at about 80.28~85.6. But in Bell-shape model, V_f only shows to be 61.6, which may be because its regression model is not very accurate, as the R^2 is only 0.769 in this model. V_0 can be assume to be 30.8 on BTRR, because there is only one estimation for this parameter and the R^2 is high to 0.934. K_0 is also assumed to be 57.5 as the underwood model is better than the Bell-shape model.

K_j may be more close to 143.88 because R^2 of Greenberg curve is higher than Bell-shape curve.

4.2 Regression Models for JJTH

The regression models for JJTH traffic flow data can be gained by regression software using the same method with BTRR. Table 8 presents the different velocity-density curves for JJTH under the Greenshields model, Greenberg model, Underwood model and Bell-shape model.

Table 8. The regression curve models for JJTH

Model	Minitab Regression Curve	Speed-density Curve
a) Greenshields	$K = K_j - \frac{K_j}{V_f}V = 54.1 - 0.456V$	$K = 54.1 \times (1 - \frac{V}{118.6})$
b) Greenberg	$In(K) = \ln(K_j) + \left(-\frac{1}{V_0}\right)V = 4.42 - 0.02V$	$K = 83.09 \times \exp(-\frac{V}{46.95})$
c) Underwood	$K = K_0 InV_f - K_0 InV = 143 - 29InV$	$K = 29 \times In(\frac{134.3}{V})$
d) Bell-shape	$K^2 = 2K_0^2 InV_f - 2K_0^2 InV = 7247 - 1582InV$	$K = 28.12 \times (2In\frac{97.6}{V})^{\frac{1}{2}}$

The four different regression velocity-density curves of JJTH are shown in Figure 6. From the pictures, we also can see directly that the Greenberg and Underwood curves are more suitable on JJTH than Greenshields and Bell-shape curves, because the regression curves of they can show the change trend of the traffic flow data on JJTH better.

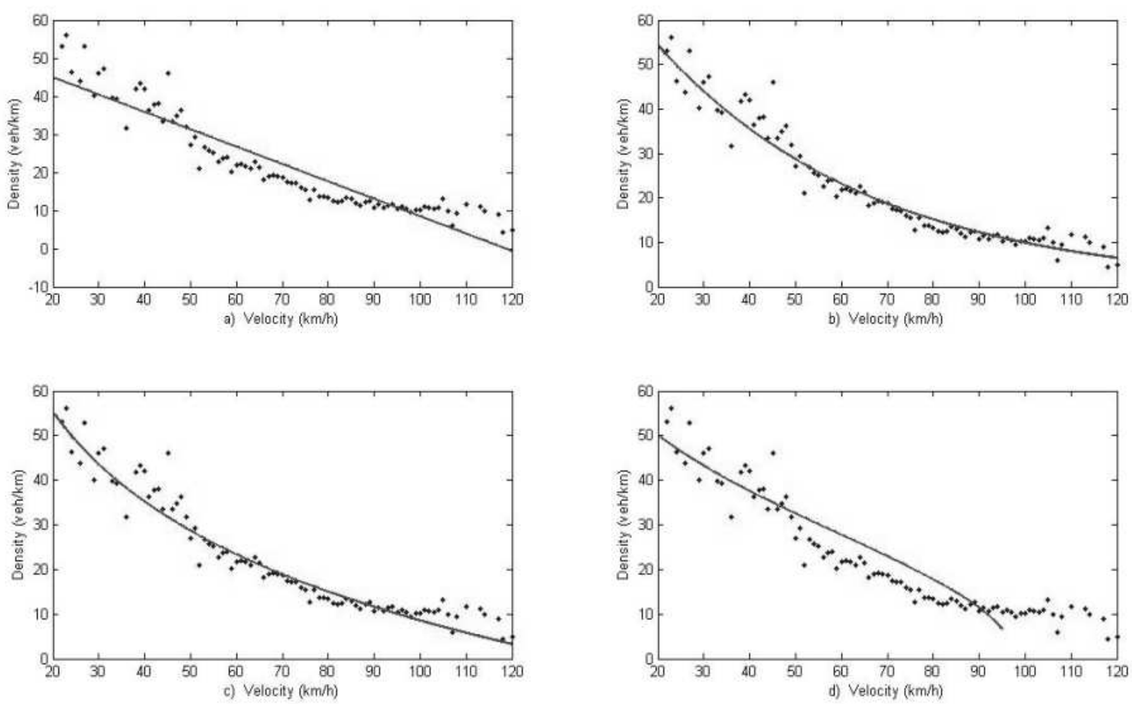

Figure 6. The four different regression curves of JJTH

Table 9 presents the relations between the traffic density and speed on the four models of JJTH. The R^2 in the Greenberg and Underwood model of JJTH are still better than the two others, but R^2 of Greenshields and Bell-shape for JJTH have increased to 0.864 and 0.859 bigger than on BTRR. So the Greenberg and Underwood curve is still preferred on the analysis velocity-density regression of JJTH. The same analysis method as the BTRR, we can get the free flow speed V_f on about 118.6~134.3, V_0 is 46.95, K_0 is 29 and K_j is assumed as 83.09.

Table 9. The model summary and parameter estimates from JJTH data

	Model Summary		**Parameter Estimates**			
Model	**R-square**	**F**	V_f	V_0	K_0	K_j
a)Greenshields	0.864	548.57	118.6	\	\	54.1
b) Greenberg	0.934	1191.02	\	46.95	\	83.09
c) Underwood	0.937	1268.94	134.3	\	29	\
d) Bell-shape	0.859	526.07	97.6	\	28.12	\

4.3 Comparisons between BTRR and JJTH

From the above analysis, we have known that both BTRR and JJTH are more suitable to use Greenberg and Underwood curve to regression their speed-density relation. From the regression process, we find out that their regression models are really different by comparing the traffic flow data recorded from BTRR and JJTH.

Table 10. Parameters of BTRR and JJTH

	V_f	V_0	K_0	K_j
BTRR	85.6	30.8	57.5	143.88
JJTH	134.3	46.95	29	83.09

Table 10 compares the differences of parameters on BTRR and JJTH. Because JJTH is a highway and BTRR is a city road, so the free flow speed V_f and the speed maximum flow level V_0 on highway is much higher than city road, but the density at maximum flow level K_0 and the jam density K_j on highway is smaller than city road.

The differences of the two regression curves are shown by comparing Figure 7 and Figure 8, where the curve for BTRR seems to be more curved than JJTH, and the curve for JJTH are more linear than BTRR. This may be because BTRR is a congested city road, so the congestion and un-congestion are both exist on the road. The speed-density ratio has a big difference between the traffic rush hour and the uncongested hours, so the BTRR curve is more curved. For the JJTH highway, the highway manage can control the traffic flow by the charging fee and government policy, and JJTH is the highway which connects two cities, so their rush hour may be not obvious. Therefore, speed-density ratios are more similar, the traffic flow data for JJTH is more homogenous.

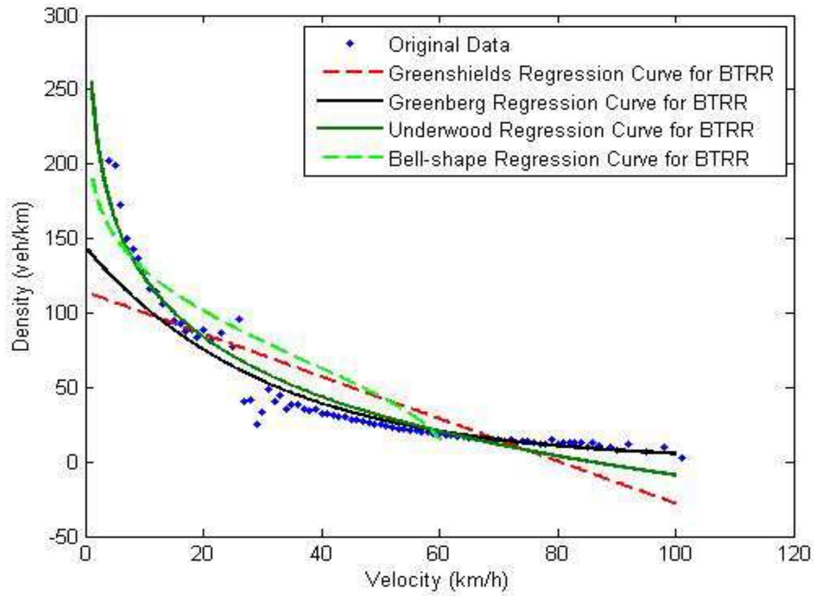

Figure 7. The regression curves on BTRR

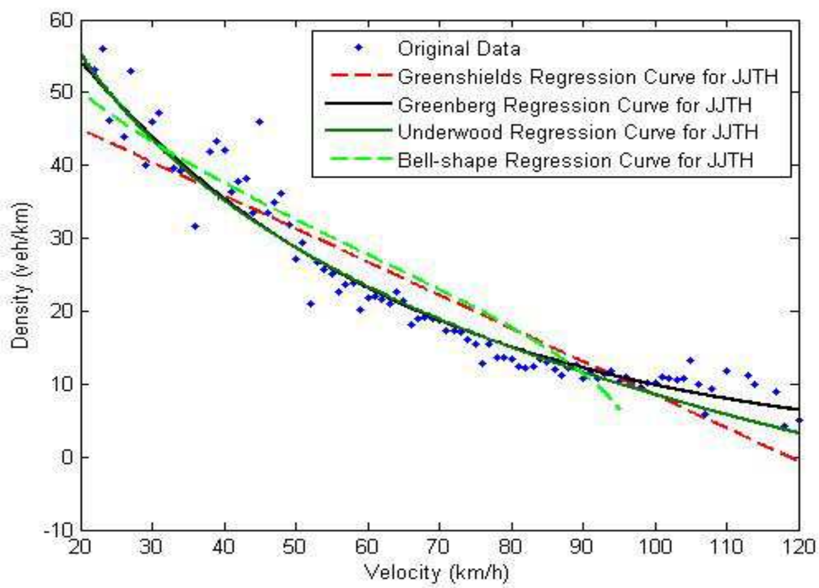

Figure 8. The regression curves on JJTH

5. CONCLUSION

In this paper, we investigated the speed-density regression models for BTRR and JJTH by using Minitab. Through these analyses, there are mainly three important conclusions generated from the two roads. Firstly, the recorded data verified the four classical models. Through the regression model, we can get the jam density, density at maximum flow level, free-flow speed and speed at maximum flow level. Trough comparing the four classical models, it can be concluded that

Greenberg and Underwood for BTRR and JJTH are presented with higher confidence and less variation than the other models. Secondly, the road type and free flow speed determined the traffic flow curve. In our study, through comparing the two different kinds of road, we proved that the free flow speed and the speed maximum flow level on highway is much higher than city road, but the density at maximum flow level and the jam density on highway is smaller than city road. Thirdly, the ratios between speed and density are also influenced by the road type, which makes the curve for BTRR seems are more curved than JJTH, and the curve for JJTH are more linear than BTRR. In conclusion, the characters of the city road and tolled highway are different, and the relations of its three traffic flow factors are also different. Therefore, it is highly recommended that each road should develop its own models according to its own flow characters.

As a regression analysis, there are also several limitations in this paper, which can be improved in the future work. First, we only used the four classic traffic flow-density models to do the analysis. The adjust R-square is acceptable in our analysis. Therefore, these regression models may be not the best regression models for the particular BTRR and JJTH. Secondly, this study developed the traffic speed-density relationship using 10-minute aggregate data. It may be deserve to do the further study to check the variability in the results by using different time intervals, such as 4-min, 6-min, 8-min, 12-min, and so on. It may be interesting to compare the magnitude of the traffic flow elements based on different time intervals.

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