

Analysis of Correlation between Car-Following Platoon Size and Macroscopic Traffic Stream Models

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Abstract: This paper presents vehicular movements of a dynamic car-following platoon consisting of ten vehicles that falls between macroscopic and microscopic models. The data included acceleration from stop condition, deceleration to stop condition, and car-following condition with some speed perturbations. In addition to the analysis of individual leader/follower car-following relationships, two platoon models were developed for analyzing inter-vehicle relationships among any vehicles in the platoon. The results of our study showed that the correlations of the platoon models increase as the number of vehicle increases in the analysis platoon. The trajectory data of the entire platoon were also applied to investigate the data fittings to the several major macroscopic models. Many of the major macroscopic traffic stream models fitted well on the platoon data sets with high correlation ratios as showing appropriate values of the maximum flow rates and jam densities.

Keywords: Macroscopic, Microscopic, Traffic Stream Models, Car-Following Platoon, Platoon Size

1. INTRODUCTION

Many macroscopic traffic stream models have been developed based on the variables observing tens of thousands of vehicles at real freeway segments. At the same time, microscopic models have been developed based on experiments observing variables of only two vehicles called a leader and a follower. Several macroscopic and microscopic models have been mathematically connected together and have common models representing both traffic flow aspects. However, there have not been many studies between these two different aspects due to the lack of the data sets.

Recently, due to the technology development, observation and data collection of car-following conditions became available with using GPS. There were almost no data sets of a car-following platoon consisting of more than several vehicles in past. However, the advanced GPS technologies enabled us to observe a long car-following platoon with good accuracy in these days. In this paper, RTK GPS was utilized and ten-vehicle dynamic car-following data was observed with tracking all vehicles' trajectories. The data sets were supposed to fall in the region between microscopic and macroscopic traffic flow aspects. Therefore, the car-following data sets were analyzed as a platoon instead of a leader/follower car-following relationship between two vehicles in this study. The relationships between the platoon size and correlations to the traffic stream models were investigated in order to measure the connection strengths to the macroscopic and microscopic traffic stream models.

2. BACKGROUND

Microscopic car-following models started with a simple linear relationship between driving speed and distance headway with the vehicle running in front of the subject vehicle introduced by Pipes (1953). Forbes and Zagorsk (1958) introduced the same car-following model, but using speed and time headway instead of space headway. After these speed-distance models, researchers at General Motors Company (GM) established a series of car-following models called stimulus-response system. In this car-following system, stimulus is represented by relative speed between a leader and a follower and response is represented by acceleration of the follower as shown in equation (1).

$$\ddot{x}_n(t+T) = \lambda [\dot{x}_{n-1}(t) - \dot{x}_n(t)] \quad (1)$$

where $x_n(t)$ is the position of the n^{th} vehicle at time t , T is a reaction time of the n^{th} vehicle, and λ is the sensitivity factor.

GM researchers, Gazis, et al. (1961), developed further to generalize car-following models. They considered that the sensitivity factor can be as simple as a constant value, but also can consider the influence of vehicle speed and distance headway upon the sensitivity as shown in equation (2). This generalized car-following equation is called the GM model.

$$\ddot{x}_n(t+T) = \alpha \frac{[\dot{x}_n(t+T)]^m}{[x_{n-1}(t) - x_n(t)]^l} [\dot{x}_{n-1}(t) - \dot{x}_n(t)] \quad (2)$$

where α , l , and m are model parameters. It is known that equation (2) can represent many of car-following models introduced in past. For example, the equation becomes Pipes' and Forbes' models when $l = 0$ and $m = 0$. In case $l = 1$ and $m = 0$, the equation becomes the GM's third model and it becomes the GM's fourth model with $l = 1$ and $m = 1$. This well-known GM's car-following model is also famous of making its connection to the macroscopic traffic stream models.

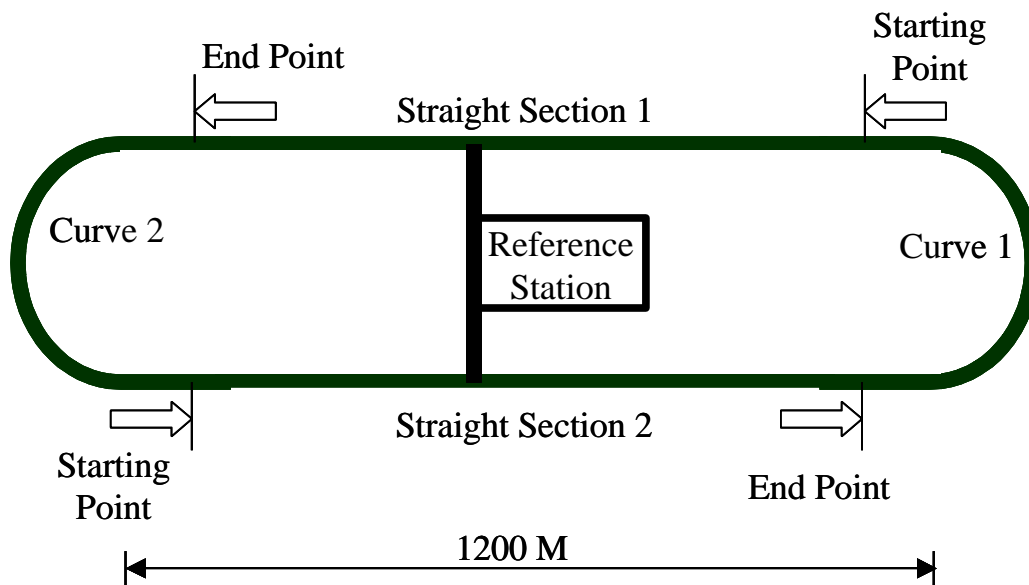
On the other hand, macroscopic traffic stream models also started with a simple linear relationship model between density and speed proposed by Greenshields (1935). Since then, macroscopic traffic stream models were further developed to better fit observed macroscopic data sets in various places in the US. These macroscopic traffic stream models were developed by Greenberg (1959), Underwood (1961), Drake, et al. (1967) showing non-linear relationships between density and speed. Finally, the macroscopic traffic stream model by GM researchers, Gazis, et al. (1959; 1961), became a well-known model representing most of the single regime models with different parameters. Equation (3) shows the well-known GM's traffic stream model.

$$V = V_f \left\{ 1 - \left(\frac{K}{K_j} \right)^{l-1} \right\}^{\frac{1}{1-m}} \quad \text{for } 0 \leq m < 1, l > 1 \quad (3)$$

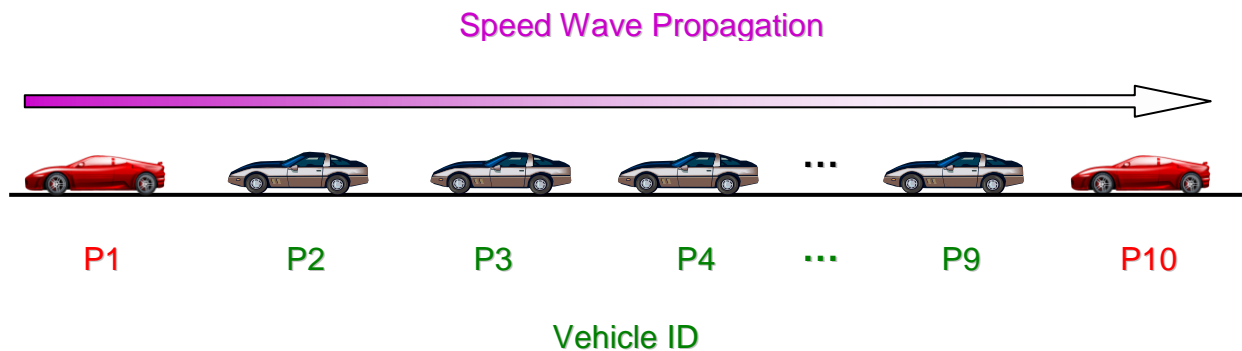
where l , and m are model parameters. This equation is originally derived from the car-following equation (2) and special cases are applied to transform logarithmic and exponential

functions when $m = 1$ and $l = 0$. Equation (3) can represent most of previously introduced traffic stream models such as Greenshields model when $l = 0$ and $m = 2$, Greenberg model when $l = 1$, $m = 0$, Underwood model when $l = 2$ and $m = 1$, and Northwestern model when $l = 3$, $m = 1$.

While many researchers were proposing single regime models, some researchers were developing models with multi-regimes such as Edie (1961) and Quandt (1958; 1960). After introduction of multi-regime models, other unique traffic stream models were developed with free flow and congested flow separation such as the three dimensional catastrophe model by Persaud and Hall (1989), the three-phase traffic flow model by Kerner (2004), and the neural network model by Nakatsuji *et al.* (1995). These models emphasize the difference of characteristics between free flow and congested flow and show data jumps between the two different traffic flow conditions in macroscopic traffic flow variables.

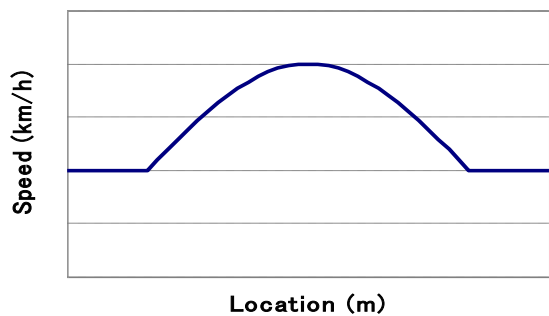


a) Schematic Layout of the Test Track

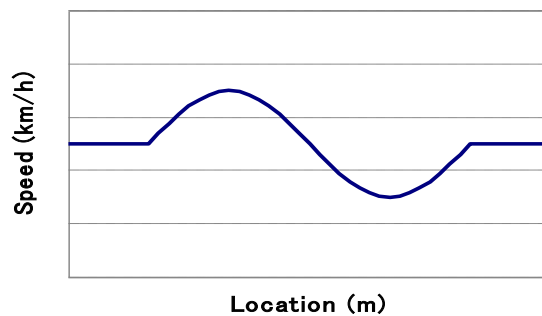


b) Vehicle Location ID in the Ten Vehicle Car-Following Platoon

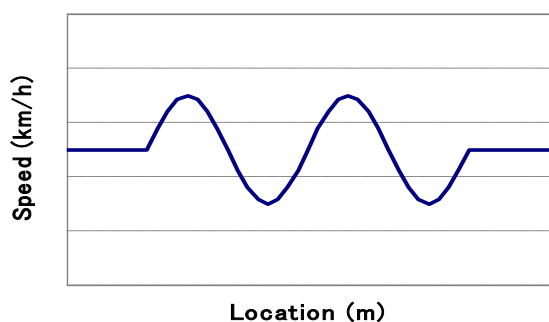
Figure 1. Test Track and Ten Vehicle Car-Following Platoon in the Experiment



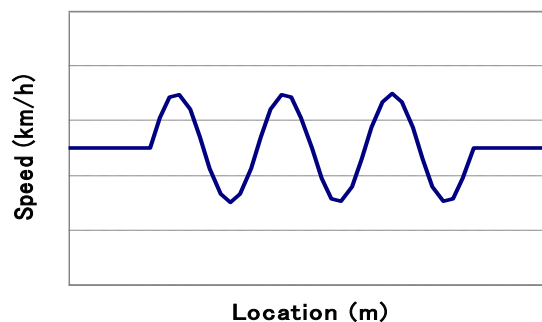
(a) Half Wave Pattern (Car-Following)



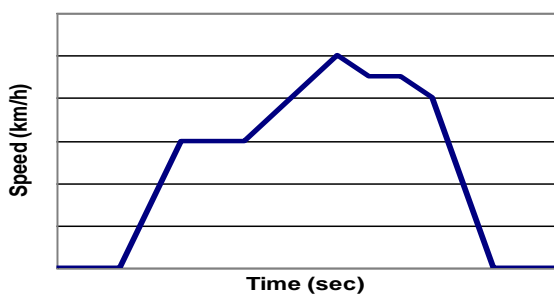
(b) One Wave Pattern (Car-Following)



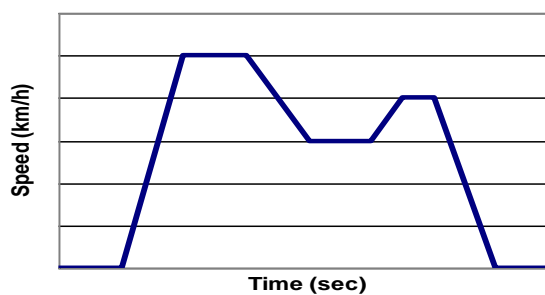
(c) Two Wave Pattern (Car-Following)



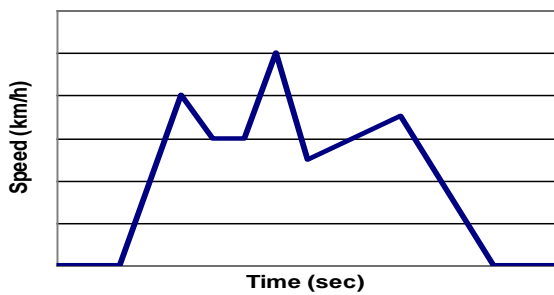
(d) Three Wave Pattern (Car-Following)



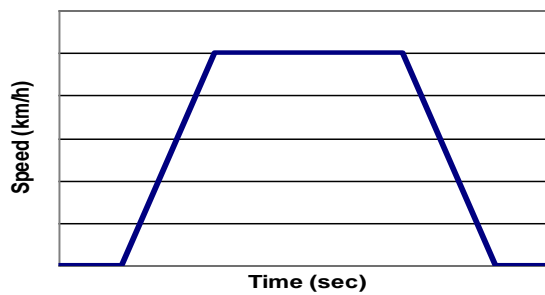
(e) One Peak Pattern (Start & Stop)



(f) Two Peaks Pattern (Start & Stop)



(g) Three Peaks Pattern (Start & Stop)



(h) Flat Top Pattern (Start & Stop)

Figure 2. The Speed Wave Patterns of the First Vehicle in the Platoon

3. CAR-FOLLOWING PLATOON DATA SETS

The car-following platoon data was recorded in a test track of Civil Engineering Research Institute of Hokkaido, Japan in October 2000. The test track consisted of two 1.2 km straight sections with two 0.3 km semicircular curves. Only the data on straight sections was used for eliminating geometry effects by curves. Figure 1(a) shows a schematic layout of the test track. Location and speed of each vehicle were recorded at every 0.1 second interval throughout the experiment. RTK GPS (Real-Time Kinematic GPS) was equipped on each of the ten test vehicles and the trajectories the vehicle were recorded. The RTK GPS receivers used in this experiment had 10mm+2ppm position accuracy and less than 0.2 km/h speed accuracy at every 0.1 second interval. Gurusinge et al. (2002) examined the accuracy of the experiment data and confirmed these data sets are sufficiently accurate.

Two different types of data have been recorded in the experiment; one for car-following conditions without stops and the other for the vehicle movements from start to stop in short distances. The car-following data portion has been used for studies of several researchers such as Gurusinge et al. (2002), Ranjitkar *et al.* (2003; 2004; 2005), and Tanaka et al. (2008)

There were a total of ten drivers in the car-following platoon. In this study, the ten drivers were identified as D1 through D10. There were two types of driver arrangements in the experiment. The Type-A arrangement was in the sequence of D1, D2, D3, D4, D5, D6, D7, D8, D9, and D10 from the first vehicle to the last vehicle in the platoon. Type-B arrangement was in the sequence of D1, D8, D7, D6, D5, D4, D3, D2, D9, and D10. In both driver arrangements, D1, D5, D9, and D10 had exactly the same positions. The positions were identified as P1 through P10 from the first vehicle to the last vehicle in the platoon as seen in Figure 1(b).

The driver of the first vehicle in the platoon, D1 on the vehicle location P1, initiated several speed patterns in the experiment. In the car-following conditions without stops, the speed patterns were called Half Wave, One Wave, Two Waves, Three Waves, and Constant Speeds due to the speed trajectory of the leading vehicle. Most of the car-following speed patterns were set to start of 40-50 km/h, have a speed wave pattern, and end of the same initial speed within the straight sections. In the stop and go conditions, the speed patterns were called One Speed Peak, Two Speed Peaks, Three Speed Peaks, and Flat Top. Most of the start & stop speed patterns were roughly designed to accelerate up to an expected optimum speed of 40-50 km/h, travel with speed fluctuations, and decelerate to complete stop. Figures 2(a) through 2(d) schematically illustrates the speed patterns for the car-following speed patterns and Figures 2(e) through 2(h) illustrates the start & stop speed patterns. There were a total of 92 data sets consisting of 46 car-following data sets and 46 start & stop data sets used in this analysis. Table 1 shows the number of data sets for each of the driver arrangements and the speed pattern combinations used in this analysis.

4. METHODOLOGY

First, the traffic flow models representing both macroscopic traffic stream conditions and microscopic car-following conditions were considered, and an appropriate model needed to be selected for the ten-vehicle platoon analysis. The well-known GM model was selected due to the capability of representing both microscopic and macroscopic traffic flow conditions and its characteristics of representing several various models with different parameters. As introduced previously, equations (2) and (3) represent the GM models in microscopic variables and in macroscopic variables, respectively.

Table 1. Speed Patterns and Number of Data Sets

Experiment Type	Speed Pattern	Driver Arrangement		Total
		Type A	Type B	
Car-Following	Half Wave	2	6	8
	One Wave	4	4	8
	Two Waves	2	5	7
	Three Waves	2	4	6
	Random	4	4	8
	Constant Speeds	7	2	9
Start & Stop	One Peak	13	12	25
	Two Peaks	9	4	13
	Three Peaks	2	0	2
	Flat Top	2	4	6
Total		47	45	92

The platoon analysis started with considering expanding the microscopic car-following model. The GM car-following model is designed for the inter-vehicle relationship between a leader and a follower. Assuming the platoon is more appropriately analyzed in steady-state speed-distance relationship rather than in second-by-second vehicular acceleration fluctuations responding to relative speed stimulus, reaction time T is eliminated in equation (2) and converted to speed-distance relationship equations. The model parameters of l and m change the final equation shapes of speed-distance relationship. Starting with the first car-following model, equation (4) expresses the linear speed-distance relationships converted from equation (2), called Pipes' and Forbes' model, with $l = 0$ and $m = 0$.

$$x_{n-1}(t) - x_n(t) = \alpha_0 \dot{x}_n(t) + C_0 \tag{4}$$

where α_0 and C_0 are constant model parameters.

For expanding this speed-distance model for the relationship among more than two vehicles in a platoon, individual vehicle variables need to be integrated. For representing the variables consisting of more than two vehicles in a car-following platoon, equation (5) is established for the vehicles from the r^{th} vehicle to the s^{th} vehicle in a car-following platoon. We define this model in equation (5) as the linear platoon model in this paper.

$$\sum_{i=r+1}^s \{x_{i-1}(t) - x_i(t)\} = \alpha_{ors} \frac{\sum_{i=r+1}^s \dot{x}_i(t)}{s-r} + C_{ors} \tag{5}$$

where $s \geq r+1$, α_{ors} and C_{ors} are constant model parameters. The left side of the equation represents the difference in vehicle positions between the r^{th} vehicle to the s^{th} vehicle and the first term of the right side includes space-mean speed of vehicles between the r^{th} vehicle and the s^{th} vehicle. When r equals to the first vehicle and s equals to the last vehicle of the platoon, equation (5) represents the relationship between mean-speed of the platoon and the platoon

length. This equation can be applied from two vehicles to infinite number of vehicles in a car-following platoon.

Assuming the platoon length is long enough to represent macroscopic perspective, platoon length can be converted to density and equation (5) can be transferred to the equation with macroscopic traffic flow variables using V , space-mean speed, and K , density.

$$\frac{1}{K} = \alpha_{ors} V + C_{ors} \quad (6)$$

Then, replace the coefficients with macroscopic traffic flow parameters.

$$V = V_o \left\{ \frac{1}{K} - \frac{1}{K_j} \right\} \quad (7)$$

where V_o is optimum flow speed and K_j is jam density. Finally, flow rate, Q , can be computed using the general relationship among the three macroscopic variables, $Q = KV$.

$$Q = V_o \left\{ 1 - \frac{K}{K_j} \right\} \quad (8)$$

Similarly, for the first non-linear GM car-following model, called the third GM model, equation (9) can be derived from equation (2) with $l = 1$ and $m = 0$.

$$x_{n-1}(t) - x_n(t) = e^{\frac{\dot{x}_n(t) - C_l}{\alpha_1}} \quad (9)$$

where $s \geq r+1$, α_1 and C_l are constant model parameters. Then, the equation (10) is derived for the speed-distance relationship of vehicles in a car-following platoon.

$$\sum_{i=r+1}^s \{x_{i-1}(t) - x_i(t)\} = e^{\frac{\sum_{i=r+1}^s \dot{x}_i(t) - C_{1rs}}{\alpha_{1rs}}} \quad (10)$$

where α_{1rs} and C_{1rs} are constant model parameters. We define this model in equation (10) as the exponential platoon model. This model represents the platoon length exponentially increases as the mean-speed increases in the platoon.

$$\frac{1}{K} = e \frac{V - C_{1rs}}{\alpha_{1rs}} \quad (11)$$

Then, replace the coefficients with macroscopic traffic flow parameters.

$$V = V_o \ln \left(\frac{K_j}{K} \right) \quad (12)$$

Table 2. R-Square Values of the Platoon Models Fit

(a) Linear Platoon Model

Linear Platoon Model		Following Vehicle in the Model								
		P2	P3	P4	P5	P6	P7	P8	P9	P10
Leading Vehicle in the Model	P1	0.643	0.748	0.782	0.832	0.865	0.876	0.879	0.889	0.890
	P2		0.673	0.743	0.814	0.854	0.868	0.872	0.886	0.887
	P3			0.665	0.779	0.841	0.863	0.869	0.886	0.886
	P4				0.664	0.794	0.833	0.849	0.872	0.876
	P5					0.686	0.816	0.840	0.866	0.874
	P6						0.687	0.771	0.823	0.840
	P7							0.706	0.807	0.837
	P8								0.723	0.809
	P9									0.708

(b) Exponential Platoon Model

Exponential Platoon Model		Following Vehicle in the Model								
		P2	P3	P4	P5	P6	P7	P8	P9	P10
Leading Vehicle in the Model	P1	0.753	0.831	0.846	0.878	0.899	0.917	0.921	0.927	0.926
	P2		0.771	0.815	0.868	0.893	0.916	0.920	0.927	0.926
	P3			0.736	0.838	0.878	0.913	0.920	0.928	0.928
	P4				0.787	0.858	0.908	0.918	0.927	0.928
	P5					0.763	0.891	0.910	0.923	0.926
	P6						0.828	0.879	0.904	0.911
	P7							0.807	0.881	0.899
	P8								0.826	0.878
	P9									0.787

As seen in equation (12), Geenberg traffic stream model can be derived from the exponential platoon model as we expected. In this study, several of macroscopic traffic stream models represented by different parameters in the generalized GM model will be used for the analysis. A list and formula of the models used in this study are summarized in Table 3.

5. ANALYSIS RESULTS

There were two kinds of driver arrangements and several speed patterns in the experiment as mentioned earlier. In this study, all different speed pattern types were added together to establish the relationships between mean-speed and platoon length. By adding all the separate experimental data together, the entire data for one vehicle became 80912 data points with 0.1

second intervals, which means 8091 seconds of trajectory data on each vehicle. The entire data set covered mean-speed from 0 km/h to 87 km/h. First, the differences between the two driver arrangements were reviewed and compared, however it was found that the difference by the arrangement is negligible in this case. All the data sets were added together by vehicle positions in the platoon regardless of driver arrangements.

Table 3. Regression Results in Macroscopic Traffic Stream Models

Model	Equation	R-Square	Max. Flow Rate (veh/hr)	Jam Density (veh/km)
Pipes/Forbes	$V = V_0 \left\{ \frac{1}{K} - \frac{1}{K_j} \right\}$	0.842	NA	157
Greenshields	$V = V_f \left\{ 1 - \left(\frac{K}{K_j} \right) \right\}$	0.819	2072	120
Greenberg/Edie	$V = V_o \ln \left(\frac{K_j}{K} \right)$	0.925	1850	125
Underwood	$V = V_f e^{\left(\frac{-K}{K_o} \right)}$	0.924	1830	NA
Northwestern	$V = V_f e^{\frac{1}{2} \left(\frac{K}{K_o} \right)^2}$	0.920	1895	NA
Pipes-Munjjal	$V = V_f \left\{ 1 - \left(\frac{K}{K_j} \right)^n \right\}$	0.922	1853	124
GM General	$V = V_f \left\{ 1 - \left(\frac{K}{K_j} \right)^{l-1} \right\}^{\frac{1}{1-m}}$	0.933	1838	162
		Minimum	1636	135
Observed Data Sets		Average	2017	143
		Maximum	2467	155

5.1 Relationship Establishment between Mean-Speed and Platoon Length using Platoon Models

The relationships between mean-speed and platoon length were examined among all vehicles in the platoon. Two vehicles were selected to obtain one set of relationship curves to determine the beginning and end of the platoon. All of the combinations of two vehicle positions were examined in the platoon. With 10 vehicles, there was a total of 45 two-vehicle combinations. The two proposed platoon models shown in equations (5) and (10) were used for the analysis. They are derived from Pipes’/Forbes’ linear model and GM’s third exponential model as explained. The least square method was utilized to find the parameters

for the best model fit. Tables 2(a) and 2(b) show the R-square values of the platoon models between the two vehicles in the platoon. It is seen that R-square values are low the relationship is only between adjacent vehicles, however the R-square value increases as the number of vehicles increases in both models.

Figure 3 summarizes the relationships between number of vehicles and R-square values in the analysis platoon in both models. It is clearly seen in this figure that R-square values increase as the number of vehicles increases in the platoon analysis, however it is also seen that the increase in R-square values reduce when the additional number of vehicles increases. Based on the figure, the five-vehicle analysis platoon almost reaches the highest R-square value in the case of the exponential platoon model. In this figure, generally the platoon model from the GM's third exponential model fits better to the data sets than the linear platoon model based on R-square values.

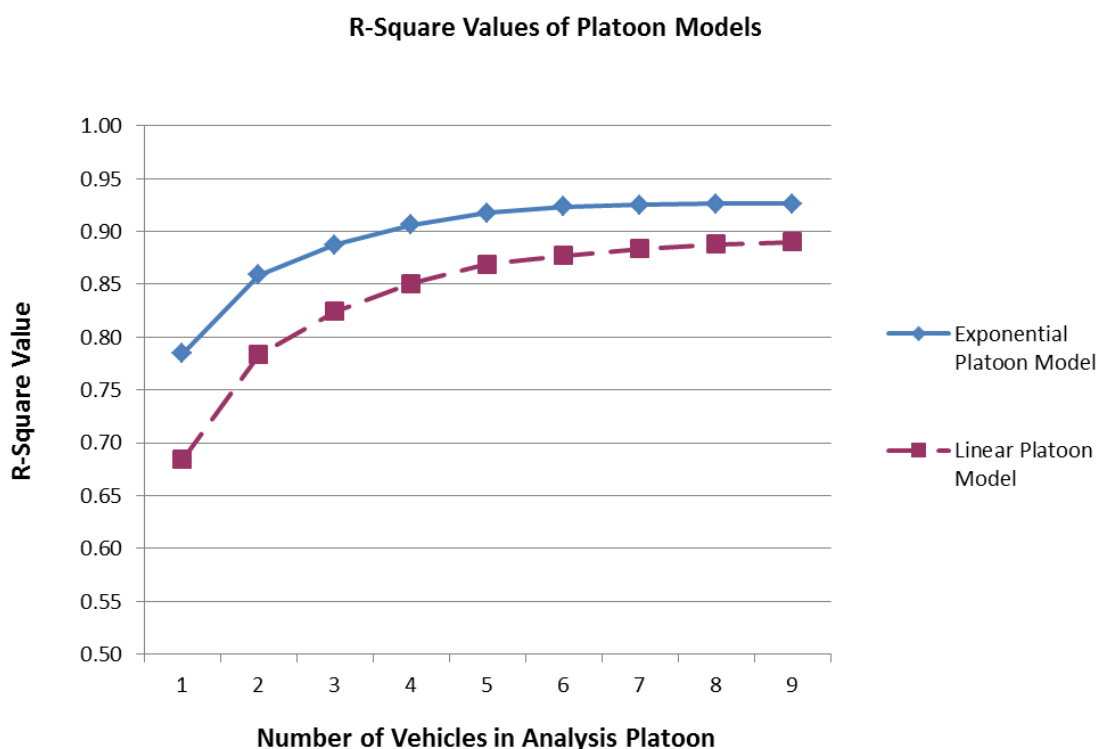
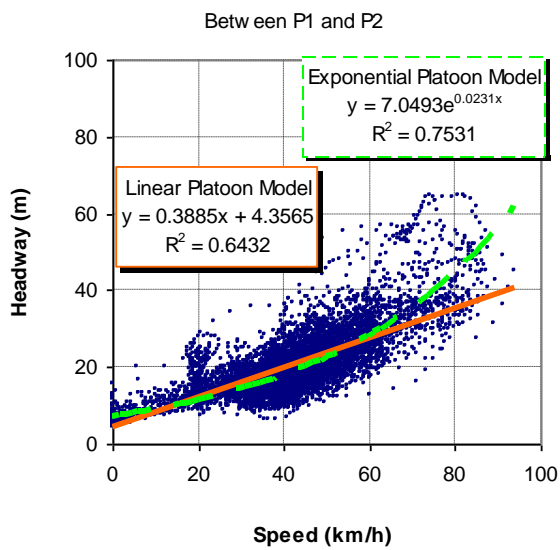


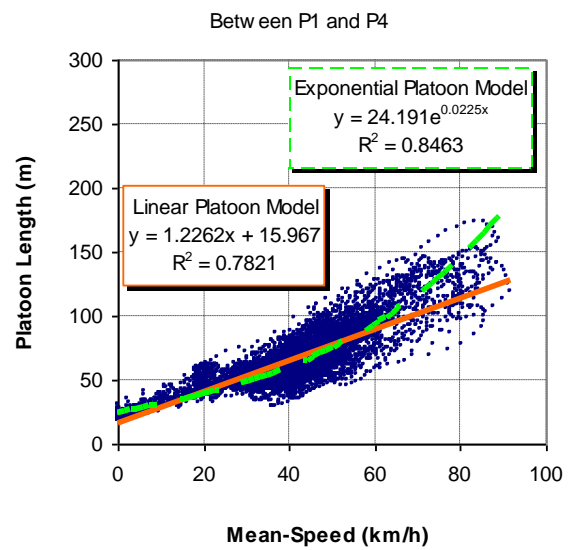
Figure 3. R-Square Values of Platoon Model Fit

Figures 4(a) through 4(d) graphically explains the relationships between mean-speed and platoon length to three different positions from the leading vehicle in the platoon, P1. It is seen that P2, the first follower behind the leading vehicle, has a large fluctuation of distance headway with the same speed and it tends to increase as the speed goes up as seen in Figure 4(a). The second figure, Figure 4(b), shows the mean-spread and platoon length relationship between P1 and P4, which includes three followers from the beginning to the middle position in the platoon. Vertical scale is different compared to the left chart for P2, however it is seen that the ratio of fluctuations to the platoon length is smaller with the same speed compared to P2. This implies that the fluctuation of average headway between P1 and P4 becomes smaller at the same mean-speed compared to the one between P1 and P2. The higher R-square value

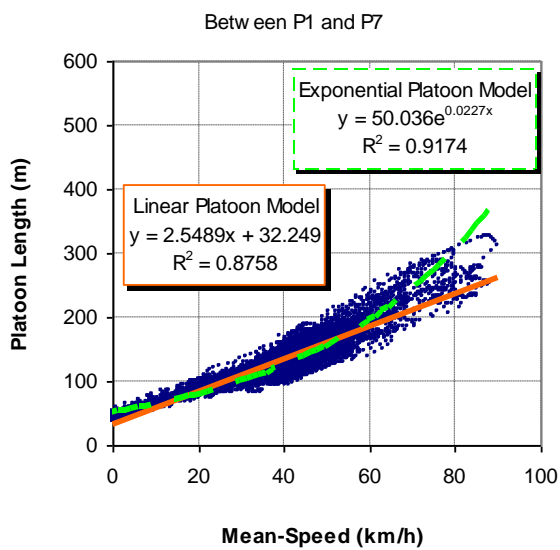
supports this phenomenon. The trend continues to the vehicle position P7, which is further located behind in the platoon, as seen in Figure 4(c). Then, finally, the trend continues to the last vehicle in the platoon, P10. Figure 4(d) shows the relationship between mean-speed and the entire platoon length in the experiment, which includes all vehicles from P1 to P10. Narrower bands of the data spread can be seen in the vertical data range in the chart and correlation values indicated by R-square also increase compared to the ones seen in the previous three charts for P1-P2, P1-P4, and P1-P7. Based on the results seen in Tables 2(a) and 2(b), which can be validated graphically with Figures 4(a) through 4(d), it is found that a platoon including the largest number of vehicles has the most stable relationship between mean-speed and the platoon length.



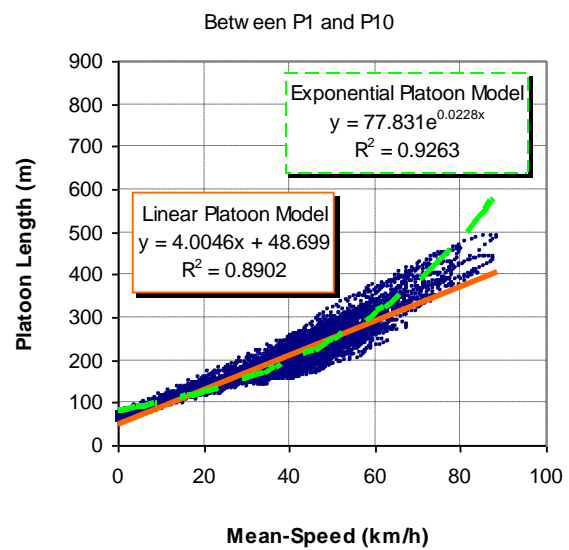
(a) Between P1 and P2



(b) Between P1 and P4

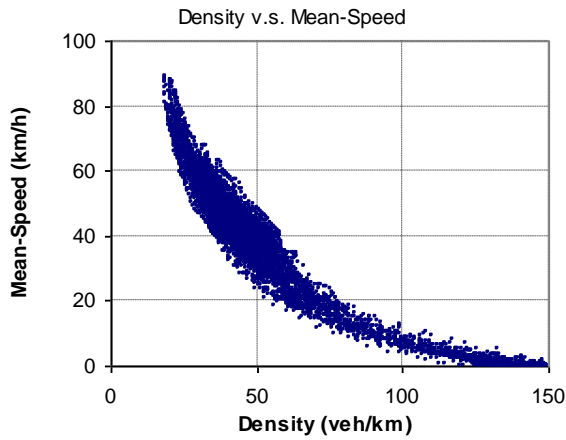


(c) Between P1 and P7

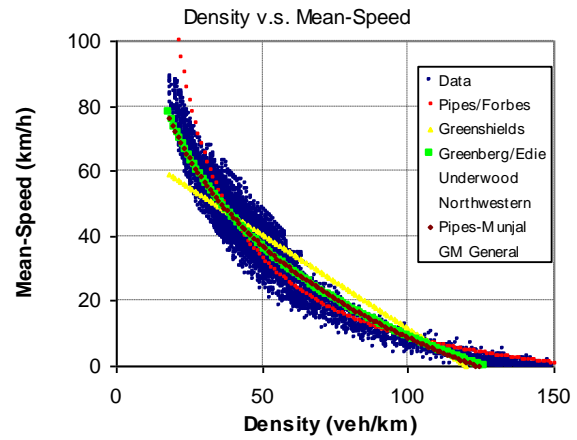


(d) Between P1 and P10

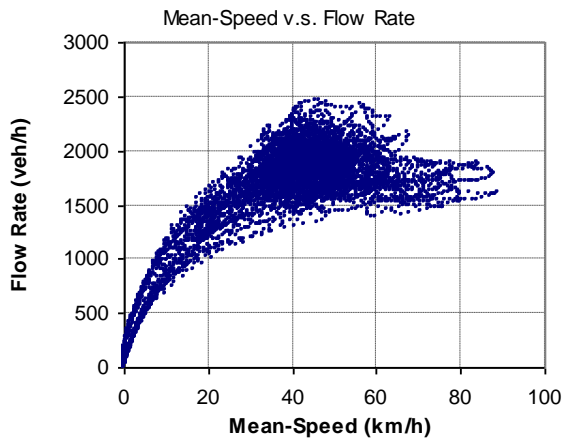
Figure 4. Platoon Models of Individual Vehicles with the Leading Vehicle P1



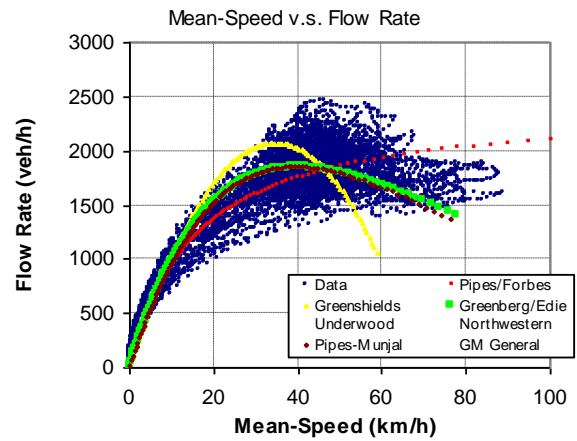
(a) Observed Data (Density vs Mean-Speed)



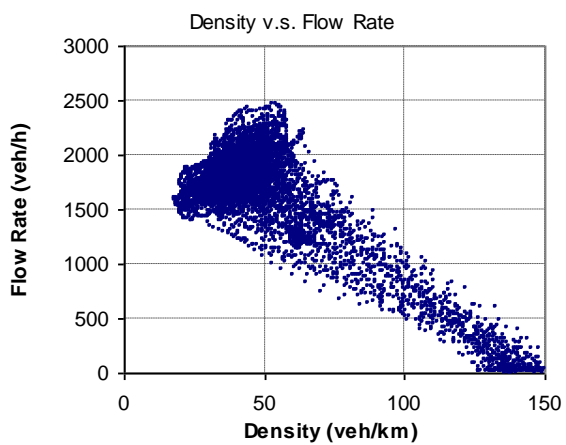
(b) Calibrated Models (Density vs Mean-Speed)



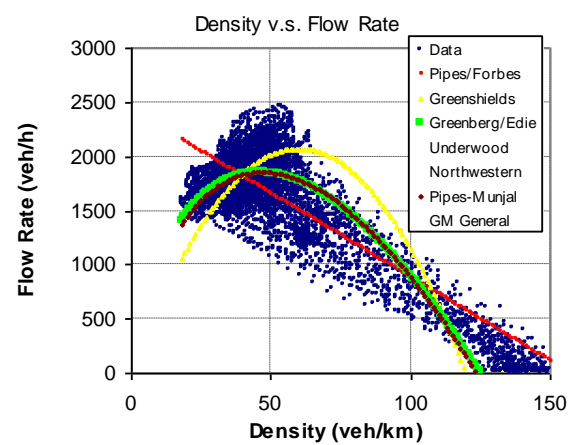
(c) Observed Data (Mean-Speed vs Flow Rate)



(d) Calibrated Models (Mean-Speed vs Flow Rate)



(e) Observed Data (Density v.s. Flow Rate)



(f) Calibrated Models (Density vs Flow Rate)

Figure 5. Observed Data Sets and Calibrated Macroscopic Traffic Stream Models

5.2 Regressions to Macroscopic Traffic Stream Models

The next step for analyzing platoon models is investigating the capability of applying them to macroscopic traffic stream models. In the previous section, the platoon models showed that the combination of P1 and P10 had the best correlation between mean-speed and platoon length among all the combinations of two vehicles in the car-following platoon. Due to this fact, mean-speed of all vehicles and the entire platoon length from P1 to P10 were applied towards examining macroscopic variables generated from the entire platoon. The platoon data plots created from the entire platoon lengths for macroscopic traffic stream models are shown in Figures 5(a), 5(c) and 5(e).

Several macroscopic traffic stream models were selected to apply the platoon data sets. Table 3 shows a list of the models used for the regression analysis in this study. Mainly the series of GM macroscopic models were examined. The models selected are Pipes/Forbes model, Greenberg model, Greenshields model, Underwood model, Northwestern model, Pipes-Munjal model, and finally the generalized GM model. The first two models, Pipes/Forbes model and Greenberg model, match the linear and the exponential platoon models examined in previous section. The least square method was again used to find the parameters for the best fitting to the data sets. Table 3 also presents the results of the regression to each model. As the values in the table indicates, most of the models show very high correlations to the platoon data sets. R-square values for most of the models show more than 0.92 except Pipes/Forbes model and Greenshields model. Figures 5(b), 5(d), 5(f) show all the calibrated models plotted on the observed platoon data. Table 3 also shows the maximum flow rates and jam densities. The values can be compared between observed and estimated by the models in the table. All models show similar maximum flow rate near 1850 veh/h. No models had the jam density within the observed data range between 135 and 155 km/h, but somewhat all models estimated jam density close to those values between 120 and 162 veh/km. Two models with exponential function, Underwood model and Northwestern model, had a disadvantage of no jam density. Considering the correlation values, estimated maximum flow rate, and jam density, we nominate Greenberg Model, Pipes-Munjal, and General GM models for good representing the platoon data sets in this study. However, among these three macroscopic models, we conclude that the best fitting model to the platoon data sets is the generalized GM's macroscopic model due to it's highest correlation value and model flexibility by adjusting parameters. It has been a long time since this model was developed, however our data fitting results still show the general GM model is the best for drawing one regime in the macroscopic traffic stream variables.

6. CONCLUSION

In this study, two platoon models were developed to explain the relationship between mean-speed and platoon length in the dynamic car-following platoon. The most important finding was that the correlations of the platoon models increased as the platoon size increased. However, the increase of the correlation by an additional vehicle gradually reduced as the platoon size increased. The strongest correlation among any two vehicle combination was seen in the longest platoon model between the first vehicle and the last vehicle in the platoon in the experiments.

The platoon data sets were also applied to several macroscopic traffic stream models. It was found that many of macroscopic traffic stream models well fitted on the platoon data sets

without any significant problems as showing reasonable estimated values of the maximum flow rates and jam densities.

Based on these findings, we conclude that the general GM's traffic stream models can well represent the variable relationships in a car-following platoon as well as the macroscopic traffic stream aspects. However, these relationships were only among the variables for steady flow conditions. Further study will be necessary to investigate how dynamic movements of the platoon can be well represented in microscopic traffic flow level.

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