# Vertical Alignment Optimization using Customized Polynomial Regression Model

Soknath MIL[a], Mongkut PIANTANAKULCHAI[b]

[a,b] *School of Civil Engineering and Technology, Sirindhorn International Institute of Technology, Thammasat University, P.O.Box 22, Pathum Thani 12121, Thailand.*
[a] *E-mail: soknath.siit@gmail.com*
[b] *E-mail: mongkutp@gmail.com*

**Abstract:** The earthwork optimization is essential for making optimal vertical alignment design. It should be minimized as low as possible from the cost and benefit viewpoint. However, the design should be subjected to design constraints (maximum allowable gradient and minimum vertical curve length). In this paper we present polynomial regression model to search for the profile of vertical alignment. Since polynomial regression model bases on least-square method, it provides the sense of minimize earthwork volume and also balancing cut and fill. In addition, we also create two more iterative algorithms to handle the design constraints. Finally, the model is tested with the real elevation data to investigate its efficiency. The result of earthwork volume shows the reasonable ratios of cut and fill volume and the value of R-square (average R-Square = 0.83). This proves the efficiency of designed optimization model.

*Keywords:* Vertical Alignment Optimization, Polynomial Regression Model, Earthwork Calculation, Iterative Algorithms, Least Square.

## 1. INTRODUCTION

The tasks of highway alignment design consist of horizontal and vertical alignment. Because of the difficulty of making design simultaneously, the problem always breaks into two stages of design, horizontal and vertical respectively. That means after determined the horizontal alignment, the vertical alignment design can be conducted (Schoon, 2000). Determination of vertical alignment not only depends on construction cost, but also traffic operation, driver visualization, and safety. Furthermore the environment may also interrupt such as change in natural landform and soil conservation. During the past decades, the analysis technique of selection the desirable vertical alignment becomes actively topic in research. The reviews of existing model regards the vertical alignments have been found as the following.

The common studies approach for vertical alignment are: *enumeration* (Easa, 1988*)*, *dynamic programming* (Puy Huarte, 1973; Murchland, 1973; and Fwa, Chan, and Sim 2002), *linear programming (*Revelle et al. 1996), *numerical search (*Hayman 1970; Goh et al., 1988; Robinson, 1973; and OCED 1973)*, and genetic algorithms* (Jong, 1998; Jha et al., 2006; Fwa et al., 2002; Jong and Schonfeld, 2003).

In this paper, the main design constraints inclusion of maximum allowable gradient, and the minimum required curve length (take into account the stopping sight distance) are considered. The classical optimization techniques, polynomial regression model and iterative algorithms, are adopted in this study. The following sections discusses in detail the polynomial regression model and iterative algorithms. And finally the numerical case study is presented at the end to show the efficiency of the optimization model.

## 2. POLYNOMIAL REGRESSION MODEL

Polynomial regression is a form of linear regression in which the relationship between the independent variable x and the dependent variable y is modeled as an $n^{th}$ order polynomial. It is usually fit base on the least-square technique. Least-square means minimization of the sum of the error. Thus the general form of least-square can be found as in eq. 1. In this study, the least-square is useful to handle the problem because it has a sense of minimizing earthwork volume and also balancing the amount of cutting and filling volume.

$$\min s = \sum_{i=1}^{m} (f_i - y_i)^2 \tag{1}$$

Where,

$f_i$ is the polynomial function $n^{th}$ degree of polynomial at a given data point $i$ (Eq. 2)

$y_i$ is the value of existing ground elevation a given data point $i$

$m$ is the number of data point i

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0 x^0 \tag{2}$$

Where,

$a_n$, $a_{n-1}$,…$a_1$, $a_0$ are the parameters

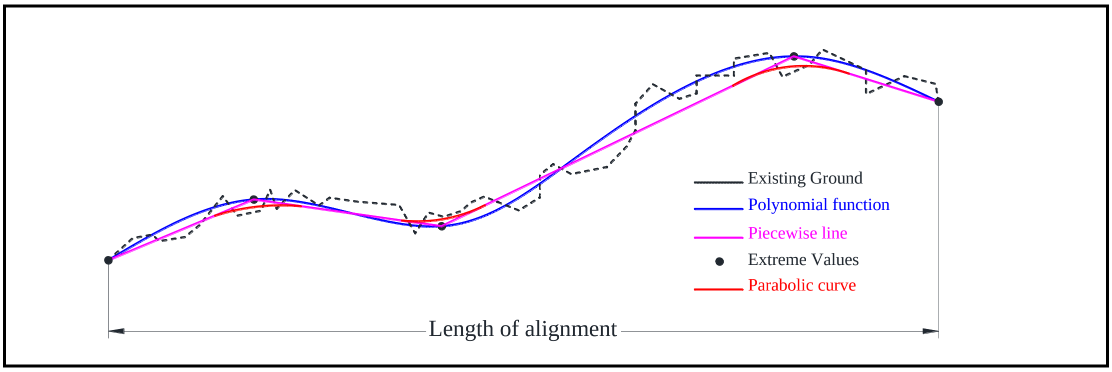


Figure 1 Sample of vertical curve with polynomial fit

Unlike the model proposed by Revelle et al., (1996), in this study we introduce the polynomial regression model as a tool to find a set of PVIs and later it was adjusted to satisfy the design constraints. Various degrees of polynomial from 1 to 10 use to fit the existing ground elevation. Finally the one which requires less amount of cut and fill is selected.

As describe in figure 1, we try to fit the existing ground elevation with the fourth degree polynomial function. So the maxima and minima points of this function are denoting as a set of PVIs which we have to take for further consideration in design constraints. Next section, we will discuss above the design constraints which have to be considered before any earthwork calculation can be made.

## 3. HANDLING DESIGN CONSTRAINTS

Vertical alignment should be as smooth as economically feasible for comfort and safety reasons. Thus, it is inevitable to have a cut or fill along the profile to satisfy with the design constraints. The following sub-sections present two main concerns in vertical alignment design: maximum allowable grade and minimum required vertical curve length.

### 3.1 Handling Maximum Allowable Constraint

Once a group of piecewise segments received, the design constraints have to be investigated before any earthwork volume calculation can be conducted. First, the maximum allowable grade is handled. Maximum grade basically depends on the type of terrain (level, rolling, mountainous and steep) with the design speed required. Base on the Asian Highway Handbook: Annex II Classification and Design Standards, the maximum allowable grade for all types of road with respect to different types of terrain are shown in Table 1.

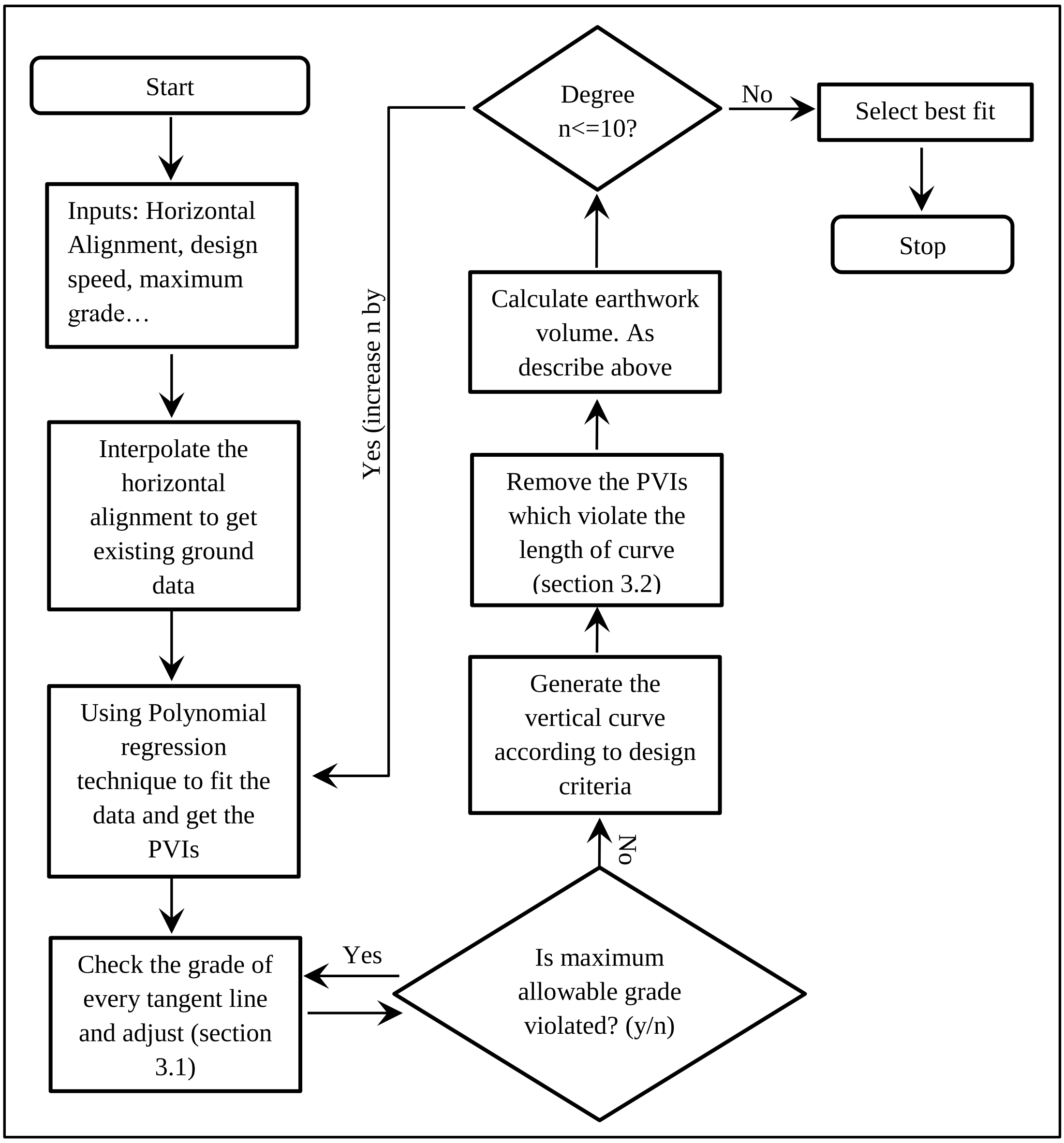

Figure 2 Model Workflow

Table 1 Maximum Vertical Grade

| Terrain classification | Maximum vertical grade |
|---|---|
| Level (L) | 4 per cent |
| Rolling (R) | 5 per cent |
| Mountainous (M) | 6 per cent |
| Steep (S) | 7 per cent |

Source: Asian Highway Handbook

To check and adjust the grade, the adjustment algorithm which is similar to the manual design is created. The detail procedure can be explained as the following steps:

Step 1: Check all grades of segments received from the peak and valley points of polynomial regression model.

Step 2: Start looping through all the segments except the last segment

The procedure for adjustment can be described as follows:

- If the grade exceeds the maximum allowable grade then reduce it to the maximum allowable grade by fixing the first point of the segment, and move the second point along the second segment direction in order to keep the grade of the second one unchanged.
- In case the algorithm cannot find the point that can stay on the subsequent segment, the next points of vertical intersect (PVIs) will automatically be removed from a set of PVIs.

Step 3: If the last segment violates the maximum allowable grade then the algorithm reversely check from the last segment back to the first.

Step 4: Repeat checking until no violation is found.

The result from the above algorithms is a group of PVIs which is not violated to the maximum allowable gradient. Next, the driver visualization has to be investigated for safety reason. The following subsection describes about the minimum required length of vertical curve.

### 3.2 Handling Minimum Curve Length Constraint

The second design constraint to handle is the vertical curve length. It is the functions of absolute value of different grade (A) and the stopping sight distance (S). According to ASSHTO (2004), the formulation of curve length can be written differently between crest and sag curve. With the assumption of driver and objective height of 1080 mm and 600 mm respectively, the formulation can be simply written:

For Crest Curve,

$$\begin{aligned} L &= \frac{AS^2}{658}, && \text{if } S < L \\ L &= 2S - \frac{658}{A}, && \text{if } S > L \end{aligned} \tag{3}$$

Where,

L is the minimum required length of vertical curve

S is the stopping sight distance required

A is the absolute value of different grade of the two consecutive segment

For Sag Curve,

$$L = \frac{AS^2}{(120 + 3.5S)}, \qquad if\ S < L$$
$$L = 2S - \frac{(120 + 3.5S)}{A}, \qquad if\ S > L \tag{4}$$

Once the minimum required length is determined, the overlap tangent length is checking to ensure there is no discoutinuity exist (fig. 3). The simple equation uses to check the tangent length as proposed by Jong (1998):

$$D = 0.5 * (L_v(i+1) + L_v(i)) - |PVI(i+1), PVI(i)| \tag{5}$$

Where,

$D$ is the overlap tangent length
$L_v$ is the length PVI to the start or ending point of curve (Fig. 3)

The iterative algorithms is similarly to the above algorithm, but this time once the overlap is found the uneccesary or too close PVIs to each others have to be removed. The step by step explaination can be found:

Step 1: Calculate the minimum required length of vertical curves using Eq. 3 and 4.
Step 2: Investigate if the consecutive vertical curves overlap or not by calculating the tangent length (D) as in Eq. 5.
In case the value D is less than 0, the vertical curves overlap.
Step 3: Remove unneeded PVIs if the vertical curve overlap (negative value of D).
Step 4: Re-calculate the tangent length (D) according to the new set of PVIs
Step 5: Repeat step 3 and 4 until a non-negative value of D is found.

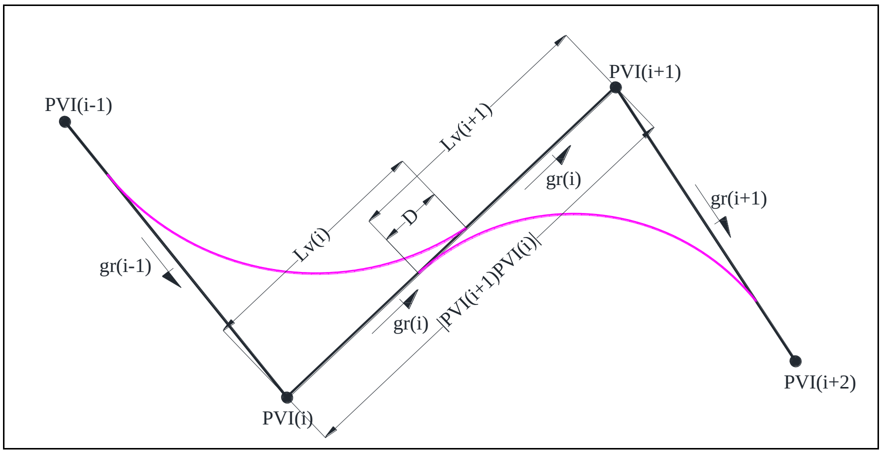


Figure 3 Overlap length of minimum required vertical curve

## 4. EARTHWORK CALCULATION

After receiving the vertical alignment from the above two constraints, the earthwork calculation can be conducted. There are several techniques which can handle volume calculation for earthwork. Those are parallelepipeds, prisms, and volume by average end area.

To simplify the calculation method for only optimization purpose, our model uses the average end area. In 1998, Jong also has used this method for his optimization technique (Jong, 1998).

$$2V = L(A_1 + A_2) \tag{6}$$

Where,

V = volume of the soil to be cut or fill between section $A_1$, $A_2$

L = the distance between two consecutive stations.

There are three types of cross-sections to calculate the earthwork volume:

1. Embankment only
2. Cutting only, and
3. Transitional (both cut and fill)

According to the above equation, the total sum between each station will provide the total amount of cut and fill for the proposed alignment. In this model, we assume the road as a flat area in which the shoulders of the road equally elevated to the centerline for whole cross-section of the alignment.

## 5. MODEL SUMMARY

In order to apply the polynomial regression model into vertical alignment, it needs to interpolate the existing ground elevation in advance. Using polynomial regression model with respected to the least-square value the preliminary alignment can be found. Next the design constraints have to be handled as the above sections. The repetition of using different degrees of polynomial is tried in order to get the best fit profile (less amount of cutting and filling volume). The whole framework of the model can be seen in figure 2.

## 6. NUMERICAL EXAMPLE

In order to investigate the efficiency of the model, the simple numerical study is conducted with real elevation data downloaded from http://www.mass.gov. The horizontal is provided as in figure 4 below, while the design parameters are provided in table 2. The study area was clipped from the whole elevation data at the lower boundary [83515 m, 928027 m] and upper boundary [84595 m, 928807m]. The starting point and ending point are [83750 m, 928250 m] and [8.4450 m, 928350 m] respectively.

Table 2: Design Parameter Inputs for This Vertical Alignment Model

| Design Variables | Values | Units |
|---|---|---|
| Design Speed | 50 | [Km/h] |
| Deceleration rate | 3.4 | [m/s$^2$] |
| Perception time | 2.5 | [s] |
| Maximum Allowable Grade | 0.065 | - |
| Station distance | 25 | [m] |

## 7. RESULTS AND DISCUSSION

The model codes in MATLAB (processor 2.3 GHz CPU and 4GB 1333 MHz DDR-3). The polynomial searching among the first 10 degree to provide the best fit after adjustment.

The result shows that the best fit is found at the polynomial of 9$^{th}$ degree with the least square value of 0.95 after adjustment. The final alignment can be found as in Fig. 6. The amount of cut and fill are 4341.4 m$^3$ and 3412.5 m$^3$ respectively. The ratio of cut and fill is 1.27. The result of polynomial fit and final vertical alignment plotted with existing ground elevation can be seen in figure 5 and 6. And finally the alignment in 3D view is displayed in figure 7. There are some parts of the alignment disappear in figure 7 because they are the excavation part. Its elevations are lower than the existing ground elevation. The other parts appear which mean the elevations are higher.

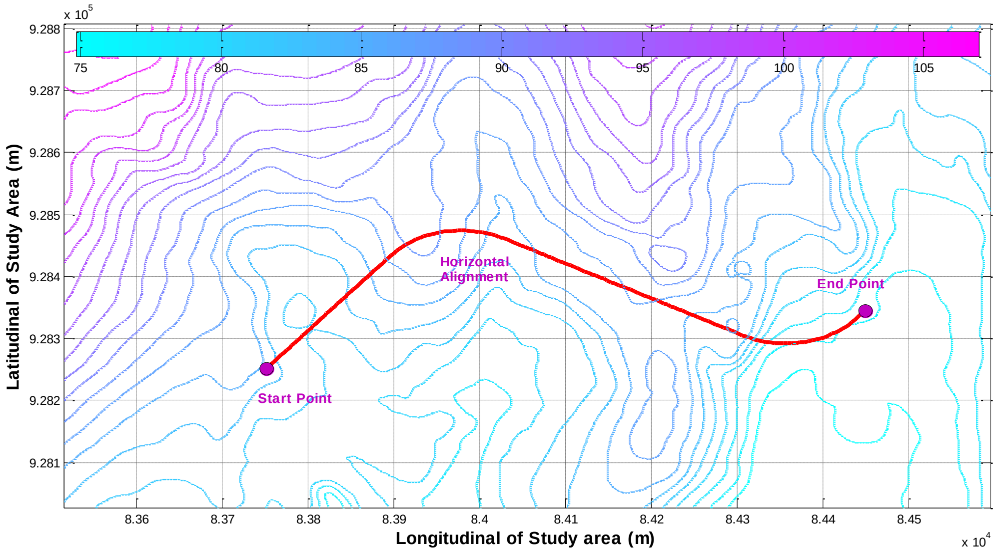


Figure 4 The contour map with the predetermined horizontal alignment (alignment no.1)

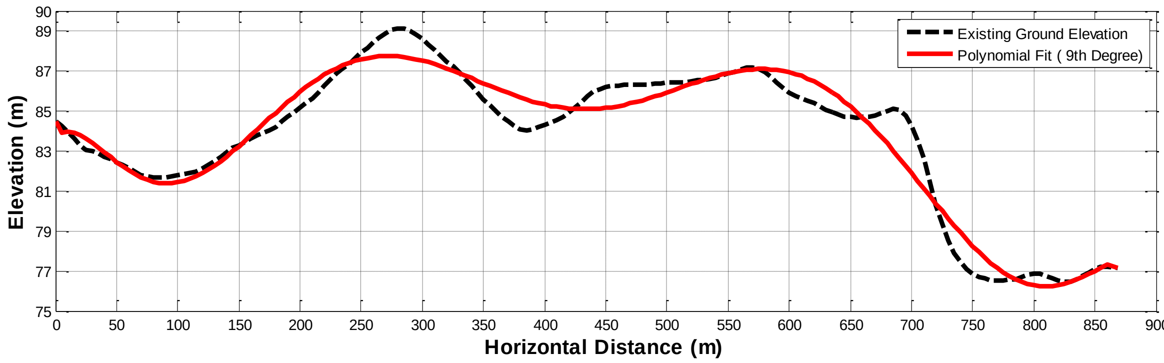


Figure 5 The existing Ground Elevation and the polynomial fit (alignment no.1)

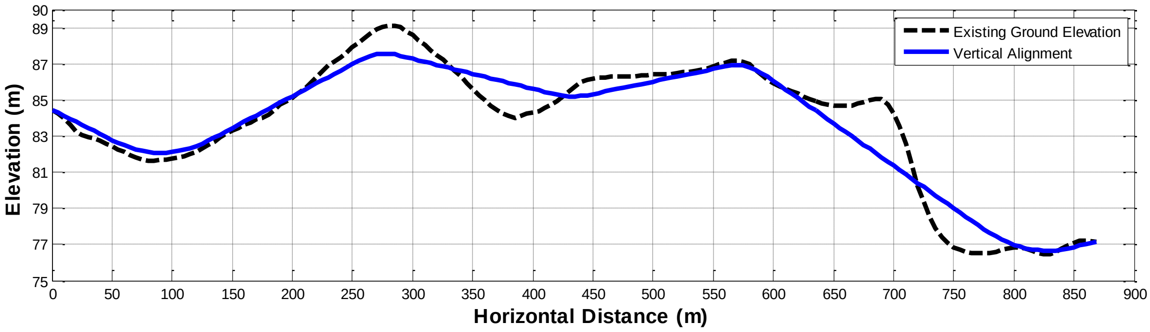

Figure 6 The existing ground elevation and final profile (alignment no.1)

In order to illustrate the efficiency of iterative algorithms, different horizontal alignments were provided as in Table 3. 20 of different horizontal alignments were assumed to test the model and to investigate the R-square values base on minimum earthwork volume after applying the iterative algorithms. The min and max value are 0.37 and 0.87 with the average value 0.83. Small value of standard deviation (0.14) presents the concentrate value of R-square around the means value.

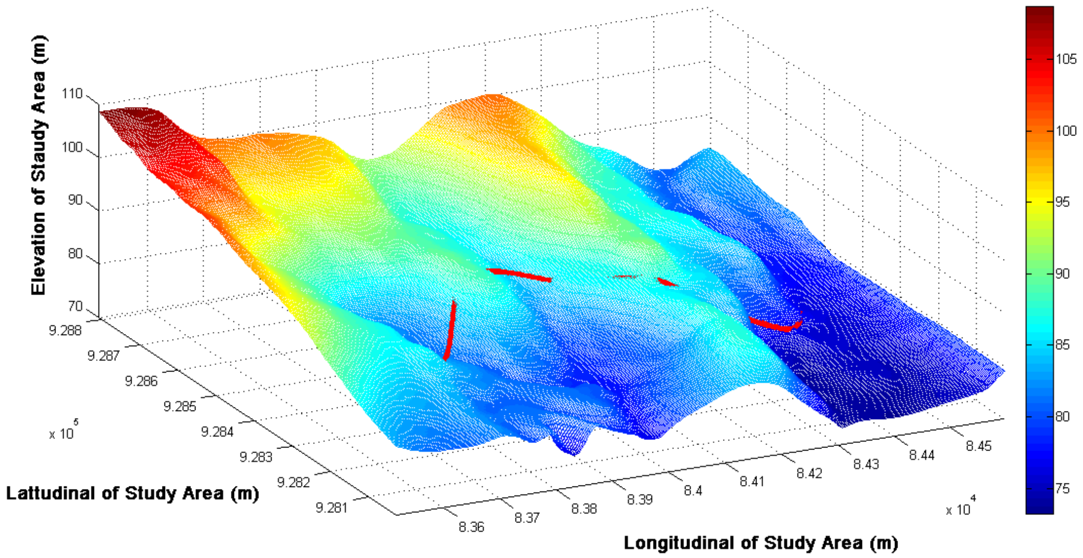


Figure 7 Final Alignment in 3D view

Table 3 The R-Square values of different horizontal alignments

| Alignment No. | Start Point_x (m) | Start Point_y (m) | End Point_x (m) | End Point_y (m) | n* | Total Earthwork Volume (1000 $m^3$) | R_square (before Adjustment) | R_square (After Adjustment) |
|---|---|---|---|---|---|---|---|---|
| 1 | 83,750 | 928,250 | 84,450 | 928,350 | 9 | 112.38 | 0.95 | 0.91 |
| 2 | 83,800 | 928,100 | 84,400 | 928,500 | 9 | 105.31 | 0.89 | 0.82 |
| 3 | 83,600 | 928,200 | 84,400 | 928,100 | 10 | 95.85 | 0.98 | 0.93 |
| 4 | 83,700 | 928,400 | 84,500 | 928,500 | 9 | 258.46 | 0.93 | 0.81 |
| 5 | 83,600 | 928,400 | 84,500 | 928,100 | 10 | 106.98 | 0.99 | 0.97 |
| 6 | 83,600 | 928,400 | 84,500 | 928,700 | 9 | 97.44 | 0.90 | 0.94 |
| 7 | 83,600 | 928,100 | 84,500 | 928,300 | 10 | 185.47 | 0.95 | 0.91 |
| 8 | 83,700 | 928,100 | 84,500 | 928,200 | 7 | 242.18 | 0.89 | 0.83 |
| 9 | 83,600 | 928,300 | 84,500 | 928,600 | 9 | 101.79 | 0.92 | 0.88 |
| 10 | 83,700 | 928,700 | 84,500 | 928,400 | 9 | 73.62 | 0.96 | 0.76 |
| 11 | 83,600 | 928,500 | 84,400 | 928,600 | 8 | 51.38 | 0.91 | 0.85 |
| 12 | 83,600 | 928,700 | 84,400 | 928,700 | 8 | 86.47 | 0.95 | 0.51 |
| 13 | 83,700 | 928,600 | 84,400 | 928,400 | 10 | 30.17 | 0.96 | 0.89 |
| 14 | 83,700 | 928,500 | 84,400 | 928,300 | 8 | 78.66 | 0.97 | 0.90 |
| 15 | 83,700 | 928,300 | 84,400 | 928,200 | 8 | 75.89 | 0.98 | 0.92 |
| 16 | 83,700 | 928,200 | 84,300 | 928,700 | 10 | 47.85 | 0.96 | 0.93 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ***17*** | ***83,800*** | ***928,700*** | ***84,300*** | ***928,100*** | ***5*** | ***209.86*** | ***0.89*** | ***0.37*** |
| 18 | 83,800 | 928,100 | 84,300 | 928,600 | 6 | 152.15 | 0.87 | 0.79 |
| 19 | 83,800 | 928,600 | 84,300 | 928,200 | 4 | 88.37 | 0.85 | 0.81 |
| 20 | 83,800 | 928,300 | 84,300 | 928,500 | 9 | 70.58 | 0.99 | 0.92 |
| | | | | | | Min | ***0.85*** | ***0.37*** |
| | | | | | | Max | ***0.99*** | ***0.87*** |
| | | | | | | Mean | ***0.96*** | ***0.83*** |
| | | | | | | Sta. Dev. | ***0.03*** | ***0.14*** |

***** n is the degree of polynomial fit

At alignment number 17, there is a big different of R-square value before and after adjustment. This is because the terrain is so steep which requires a lot of adjustment in order to satisfy the constraints (Fig. 8 and 9).

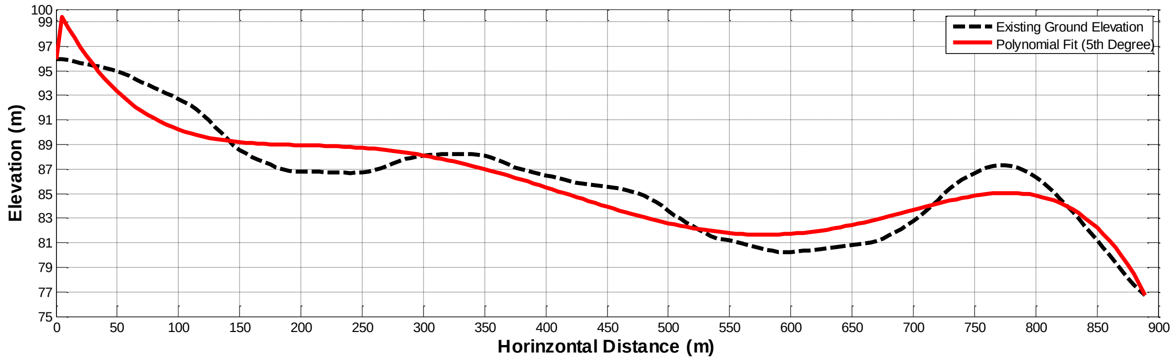

Figure 8 The existing ground elevation and polynomial fit (alignment no. 17)

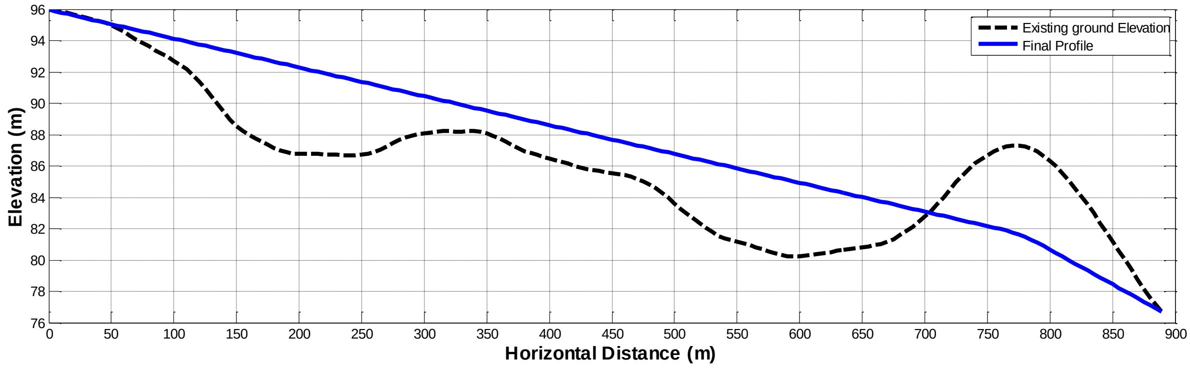

Figure 9 The existing ground elevation and final profile (alignment no. 17)

In this study, we allow the polynomial to increase up to 10 degrees since there is no matter of computational time. According to previous study by Revelle et al., (1996), the 5$^{th}$ degree of polynomial has been selected. However, there are several stopping criteria can be applied for this problem such as the maximum allowable degree of polynomial, the acceptance value of R-square and no improvement of earthwork volume.

## 7. CONCLUSION

The purpose of this study is to introduce the polynomial regression model with additional iterative algorithms to minimize the earthwork volume. The cost function in this study considers only earthwork cost, however other source of costs, such as pavement construction, land acquisition, and vehicle operating costs, can be involved in the objective function for further works.

Since the earthwork volume technique uses average end area. The more precise value of earthwork volume can be increased by inceasing the number of stations alone the profile.

The study can be extended to incorporate with horizontal alignment and simultanously optimize 3D alignment with modern algorithms such as genetic algorithms, particle swarm optimization and so on.

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