Bending Stress Distribution in Bi-layer Cement Concrete Pavements

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Abstract: Determination of neutral plane position is the key in the analysis of bi-layer cement concrete slab on Winkler foundation; however, there are no solutions for it when the interface of bi-layer slab is semi-bonded. Based on the Kirchhoff hypotheses, a coefficient c was introduced to characterize the interface condition and the equation of neutral plane was formulated in this paper. The change of position of the neutral plane for the bi-layer semi-bonded slab was investigated. In addition, the ratio of the stress distribution along the depth is derived, which is of great importance in the design of bi-layer cement concrete pavements.

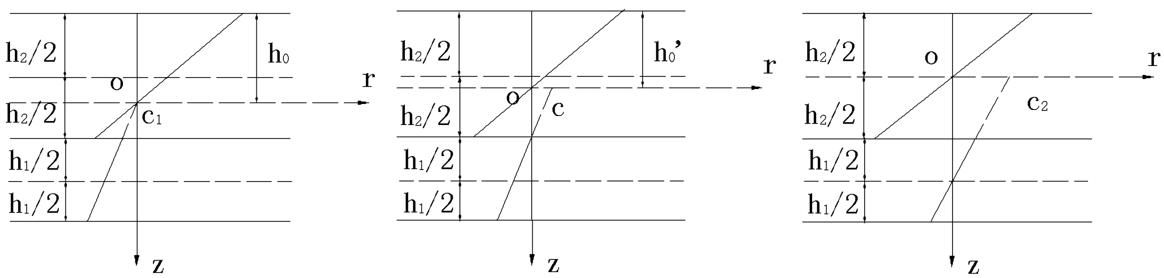
Keywords: Bi-layer concrete pavement, Bending stress distribution, Neutral planes, Interface condition

1. INTRODUCTION

It is well known that the concrete pavement is designed with the theory of slab on the Winkler foundation. The determination of the neutral plane of the slab is essential to the analysis of cement concrete pavements. For pavement maintenance, a new cement concrete layer is overlaid on the existing concrete pavement, thus bi-layer slab pavement model is proposed. The interface condition between the upper layer and lower layer depends on the art of construction, which presents a big change bending stress distribution of the bi-layer. The neutral plane can be easily positioned for the both unbounded bi-layer slab and bi-layer bonded slabs on Winkler foundation (ZHENG, 1992; YU, 2007). However, neutral plane can not be found easily for bi-layer semi-bonded slab, only empirical models were proposed (HU, 1999; TAN, 2001). The authors of this paper introduce a new coefficient c to characterize the interface condition ranging from bonded to unbounded condition and derive the governing equation for the determination of the neutral plane for the semi-bonded slab based on the Kirchhoff hypothesis in ZHENG *et al* (2003), and make further discussion in the bending stress distribution in the bi-layer.

2 THE EQAUTION OF NEUTRAL PLANES OF THE SLABS ON WINKLER FOUNATIONS

The bi-layer slab on Winkler's foundation is shown in Figure 1, the modulus of the lower and upper slabs are E_1 and E_2 respectively, μ_1, μ_2 and h_1, h_2 are the Poisson ratios, and thicknesses,



k is modulus of the Winkler foundation, cylindrical coordinates is selected in the following derivations

（a） （b） （c）

Figure 1. The neutral plane in bi-layer slabs for bounded (a), semi-bonded (b) and unbounded interface(c)

From Figure 1, it is clear that there are one neutral plane for bonded slab (a), two neural plane for unbounded slab (c). The number of the neutral plane for the semi-bonded slab depends on the bending stress of the upper layer on the interface, which is dependent on the interface condition. There is a jump for bending stress of the upper layer and bending stress of the lower layer at the interface. It is reasonable to introduce a coefficient to characterize the interface condition; mathematical c means the intersection of the lower bending stress distribution.

$$\sigma_r^{(1)} = -\frac{E_1 z - c}{1-\mu^2}\left(\frac{\partial^2 \varpi}{\partial r^2} + \frac{\mu}{r}\frac{\partial \varpi}{\partial r}\right) \tag{1}$$

$$\sigma_r^{(2)} = -\frac{E_2 z}{1-\mu^2}\left(\frac{\partial^2 \varpi}{\partial r^2} + \frac{\mu}{r}\frac{\partial \varpi}{\partial r}\right) \tag{2}$$

In which：

ϖ =deflection of the slab， is a function of r;

h_0 =position of neutral plane， is a function of r.

For bonded bi-layer slab , $c_1 = 0$;

For unbound bi-layer slab, $c_2 = \dfrac{E_1(h_1 + h_2)}{2}$

According to the Kirchhoff hypothesis, the conditions for determination of the neutral plane is that the horizontal stress along the thickness should be zero by ZHENG *et al* (2003).

$$\int_{-h_0}^{h_2-h_0} \sigma_r^{(2)} \cdot 2\pi r dz + \int_{h_2-h_0}^{h_1+h_2-h_0} \sigma_r^{(1)} \cdot 2\pi r dz = 0 \qquad (3)$$

Substitution of (1) and (2) into (3) yields

$$h_0 = \frac{E_1 h_1^2 + E_2 h_2^2 + 2E_1 h_1 h_2 - 2ch_1}{2(E_1 h_1 + E_2 h_2)} \qquad (4)$$

The number of the neutral plane for the bi-layer slab depends on tensile stress at interface.
On the interface z=h_2-h_0

$$\sigma_r^{(1)} = -\frac{E_1(h_2 - h_0) - c}{1-\mu^2}\left(\frac{\partial^2 \varpi}{\partial r^2} + \frac{\mu}{r}\frac{\partial \varpi}{\partial r}\right) = 0 \qquad (5)$$

Therefore

$$c_Z = \frac{E_1 E_2 h_2^2 - E_1^2 h_1^2}{2E_2 h_2} \qquad (6)$$

3 ANALYSIS OF THE NEUTRAL PALNE IN THE BI-LAYER SLAB

In the following example, a tired load is assumed to apply on the slab.

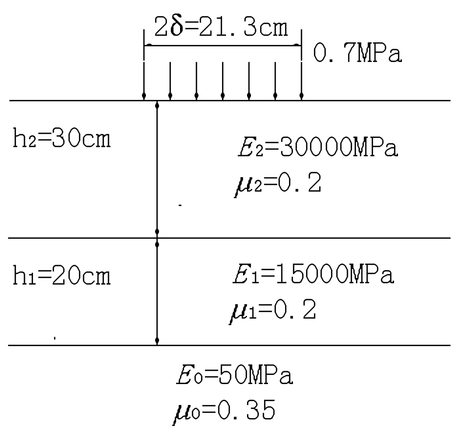


Figure 2. An example for the bi-layer slab

For the given example

$$c_1 = 0$$

$$c_2 = \frac{E_1(h_1 + h_2)}{2} = 3.75 \times 10^9 \ N/m$$

$$c_Z = \frac{E_1 E_2 h_2^2 - E_1^2 h_1^2}{2E_2 h_2} = 1.75 \times 10^9 \ N/m$$

When c increases from 0 to 1.75×10^{9}N/m， one the neutral plane becomes two. Thus it is not correct to assume that there are two neutral planes for the semi-bonded slab (HU 1999, TAN 2001). For the example given, about one third of the semi-bonded condition is for the one neutral plane.

From equation (6), it is easily proved that

Only when $$E_1 h_1^2 = E_2 h_2^2 \tag{7}$$

c_Z =0

That is to say, two neutral planes exist in the bi-layer slab if condition (7) is satisfied, indicting that the upper slab will be in compression fully.

4 BENDING STRESS DISTRIBUTION OF THE BI-LAYER SLAB

The ratio of the bending stress of lower layer of the bending stress of upper layer at the interface can be obtained from equation (1) and (2) for the bi-layer slab with semi-bonded interface.

For bonded bi-layer slab $$\frac{\sigma_r^{(1)}}{\sigma_r^{(2)}} = \frac{E_1}{E_2} \tag{8}$$

For semi-bonded bi-layer slab $$\frac{\sigma_r^{(1)}}{\sigma_r^{(2)}} = \frac{E_1(h_2 - h_0^{'}) - c}{E_2(h_2 - h_0^{'})} \tag{9}$$

In which：

$$h_0^{'} = \frac{E_1 h_1^2 + E_2 h_2^2 + 2E_1 h_1 h_2 - 2ch_1}{2(E_1 h_1 + E_2 h_2)}$$

For unbound bi-layer slab $$\frac{\sigma_r^{(1)}}{\sigma_r^{(2)}} = -\frac{E_1 h_1}{E_2 h_2} \tag{10}$$

Equation (8) (9) (10) show that the stress increase with the increase of the coefficient c_Z

in order to reduce the bending stress of the lower layer, c_Z should be carefully selected. c_Z can be measured with method proposed in reference(ZHENG,2007) . If $\sigma_r^{(3)}$ is assumed to be the bending stress of lower layer at the bottom surface, the ratio of the bending stress of each layer at the bottom surface $\frac{\sigma_r^{(3)}}{\sigma_r^{(2)}}$ also can be obtained as follows。

For bonded bi-layer slab
$$\frac{\sigma_r^{(3)}}{\sigma_r^{(2)}} = \frac{E_1(h_2 - h_0 + h_1)}{E_2(h_2 - h_0)} \tag{11}$$

For semi-bonded bi-layer slab
$$\frac{\sigma_r^{(3)}}{\sigma_r^{(2)}} = \frac{E_1(h_2 - h_0^{'} + h_1) - c}{E_2(h_2 - h_0^{'})} \tag{12}$$

For unbound bi-layer slab
$$\frac{\sigma_r^{(3)}}{\sigma_r^{(2)}} = \frac{E_1 h_1}{E_2 h_2} \tag{13}$$

When c increases from 0 to 3.75×10^9N/m，variation of $h_0^{'}$ along with c is shown in Figure 3. Variation of $\frac{\sigma_r^{(1)}}{\sigma_r^{(2)}}$ and $\frac{\sigma_r^{(3)}}{\sigma_r^{(2)}}$ with c is shown in Figure 4 and Figure 5. Variation of $\sigma_r^{(1)}$ and $\sigma_r^{(3)}$ with c is shown in Figure 6 and Figure 7. Figure 6 shows that the top surface of lower layer changes from tensile to compressive along with c increasing. When c is $1.75\times10^9\ N/m$, the bending stress of lower layer at the top surface is zero. Figure 7 shows that the bending stress at the bottom surface of lower layer always decreases along with c increasing. The bottom surface of lower layer is always tensile.

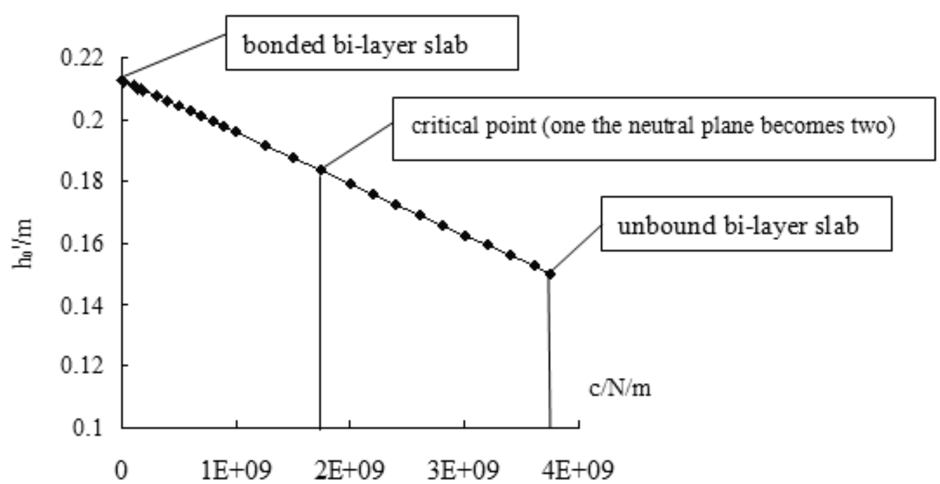


Figure 3. Variation of $h_0^{'}$ along with c

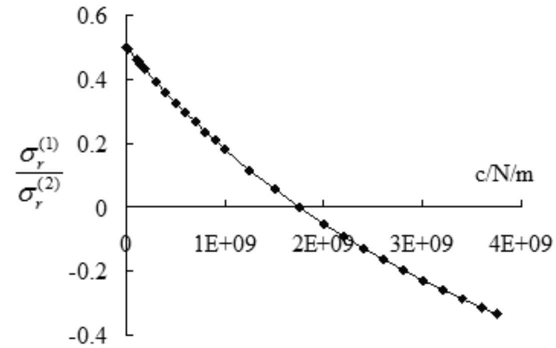


Figure 4. Variation of $\frac{\sigma_r^{(1)}}{\sigma_r^{(2)}}$ with c

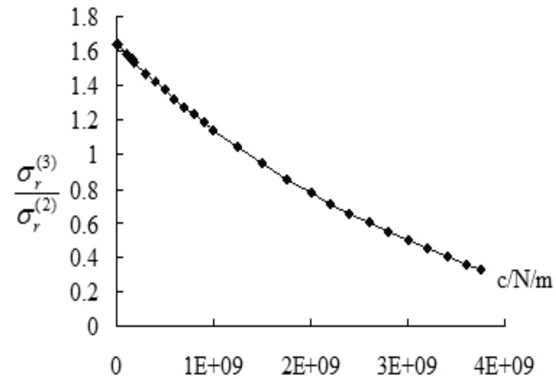


Figure 5. Variation of $\frac{\sigma_r^{(3)}}{\sigma_r^{(2)}}$ with c

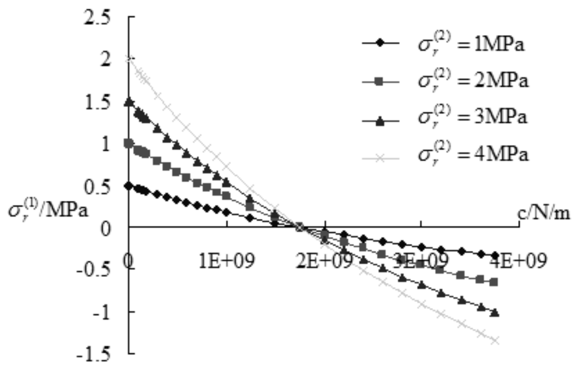


Figure 6. Variation of $\sigma_r^{(1)}$ with c

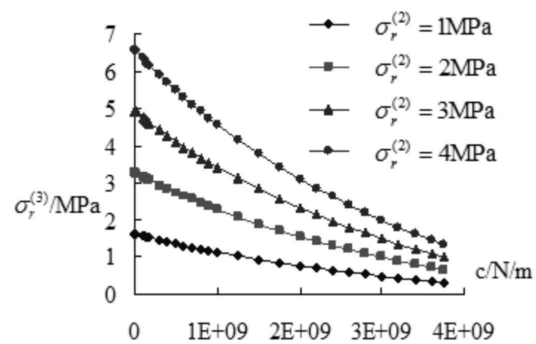


Figure 7. Variation of $\sigma_r^{(3)}$ with c

In the design of bi-layer rigid pavement, bending stresses at the bottom of upper layer and lower layer is easy to obtain from Figure 6 and Figure 7. In order to balance the loading capacity of the bi-layer rigid pavement, a proper interface condition should be carefully selected.

5 CONCLUSIONS

1) A new coefficient c is introduced to characterize the interface condition for the bi-layer cement concrete pavement. The variation of the neutral plane of the bi-layer cement concrete pavement can be determined theoretically. The condition is derived for bi-layer slab when two neutral planes exist.
2) The bending stress distribution along the depth of the bi-layer slab was derived, which is of great reference for the design of cement concrete pavement. The neutral plane of the mutli-layer semi-bonded slab can also be determined by the method proposed in this paper.
3）When c increases from 0 to 3.75×10^9N/m，the top surface of lower layer changes from tensile to compressive，the tensile stress at the bottom surface of lower layer decreases with increasing c, a proper interface condition should be carefully selected to balance the loading capacity of the bi-layer rigid pavement.

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