# **Quantification of Level-of-Service Index for Bus Routes in Developing Countries: A Case Study in India**

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**Abstract**: Quality plays an important role in the market-oriented economy and the success of any transit system depends upon its quality of service. Since, the demand for transit is quite high in comparison with the supply in most of the Indian cities; quite often the level of service is not given a priority by the service providers. However, to improve the quality of service, it is necessary to identify the parameters to be considered important by the commuters for knowing their satisfaction levels for the present service. This paper is focused on the results based on the observations made through reveled preference (RP) survey to evaluate the quality of service. The model uses various Multi criteria decision making tools such as Numerical rating approach; Fuzzy set approach, Analytic Hierarchy Process (AHP) and AHP Fuzzy. The results from different approaches are compared and justified for their appropriate use.

*Keywords*: Public Transit Attributes, Level-of-service Index, Numerical rating, Fuzzy sets, Analytic Hierarchy Process (AHP) and AHP- Fuzzy

## 1. INTRODUCTION

The urban transport systems have an enormous impact on the way people travel. However, increased urbanization and population growth, urban expansion, dispersal of amenities and activities have increased the demand and dependence on motorized transportation. Consequently, urban transportation problems like congestion, accidents, environmental degradation and urban sprawl have increased. Sustainable transportation development plans are thus replacing the routine approach of building more roads to alleviate congestion along with an integrated-mass-transport system, which is affordable, space and resource-efficient, and minimizes environmental impacts and transport nuisances. As a consequence, encouraging and improving public transport system in developing countries like India has got wider attention and has become an important strategy for sustainable transportation development.

The aim of any good transit system should be to provide an adequate Level-of-Service (LOS). However, the quality requirement of service varies with the expectation of the users. In developed countries, where the demand for transit is relatively low, and the service authorities try to attract passenger's attention by providing maximum possible LOS. However, the situation is completely different in developing countries, like, India. Here, the demand for

public transportation is very high in comparison to the supply. Operators do not care to provide reasonable LOS to the transit passengers. In the absence of an adequate and efficient bus transit system, the potential bus users shift to personal vehicles. The need to provide adequate LOS and its measurement has been felt as a very important step for the improvement of transit services. However, the quality requirement of service varies with the expectation of the users. Since, most of the people in developing countries like India are relatively poor and hence public transport is one of the most common transportation modes that can be accessed by the urban population. In Jaipur, the capital city of Rajasthan state in India, approximately 45 percent of the trips are made by walking and public transport. Hence, it is very important to quantify the LOS values on the basis of not only socioeconomic and trip related characteristics but also considering the relevant qualitative aspects regarding quality of bus service. This study is an attempt to demonstrate the approaches to quantify the LOS considering all the above aspects.

## 2. RESEARCH OBJECTIVES

The objectives of this paper are as follows:

- 1) To identify the service characteristics considered to be important by the city dwellers to determine the level-of-service provided by buses of Jaipur City Transport Service Limited (JCTSL), Rajasthan (India).
- 2) To determine the weights for the identified service characteristics through the interaction with the regular users of city buses.
- 3) To arrive at an overall index, named Level-of-Service Index (LOSI) through Numerical rating approach; Fuzzy set approach, Analytic Hierarchy Process (AHP) and Fuzzy AHP.
- 4) To compare the results of all four approaches for assessing their suitability and then to compare the LOSI provided by city buses running on different routes considered for this study.

## 3. SCOPE OF RESEARCH

Based on the research objectives, the scope of this study is outlined as given below:

- 1) A fuzzy based approach for analyzing service quality provided by public transit is developed based on the user perception, using fuzzy-set theory and AHP.
- 2) The developed fuzzy approach is used to quantify the service quality of buses in Jaipur.
- 3) The research work was limited for the five selected bus routes of Jaipur city.
- 4) The research work was limited to only the low-floor buses operated and maintained by JCTSL, Rajasthan (India).

## 4. LITERATURE REVIEW

Every State Transport Undertakings (STU) has its own method for evaluating the adequacy of its services. However, the evaluations by the owners of the services do not necessarily reflect the user's opinions. Also, no such evaluations can be considered adequate. There should be a mechanism by which the transit users can evaluate the service quality offered by the operators. Therefore, it is always needed to have a sound methodology using which the service quality can be measured appropriately to identify the deficiencies in the present

system as perceived by users of the service, which would help operators to take appropriate measures for improving the system. Public transport system has been evaluated from the point of view of level-of-service provided to the users (Botzow, 1974). Policy formulation, subsidy, public information and involvement, federal responsibility and transit management information are considered to be the main purposes of evaluation of services (Allen and Dicesare, 1976). A model was proposed for the integration of qualitative and quantitative evaluations, in which four primary elements, namely, cost, amount, impact and quality have been considered for evaluation of public transit services (Alter, 1976). Studies on the strategies of public transport operation, differentiating between all day service and peak hour operations have also been conducted (Baker, 1976). For this purpose, a general modal split formula, in which disutility of car is equated with the disutility of transit, is used to evaluate the relevant factors in the individual choice of transit mode. Studies have also been conducted to establish preferences for different aspects of the service, where attributes were selected keeping in view user's requirements. Value score curves were drawn for each attribute against marginal changes in the perceived value of the attribute (Dhingra and Bains, 1986). The quality of service provided by different categories of buses on two routes of Calcutta was also evaluated by Debasish and Sarkar (1994). In this, eleven attributes have been chosen for determining the level-of-services and they identified the weakness in different aspects of a quality, for a particular category of bus (Debasish and Sarkar, 1994). The existing routes are evaluated to compare the usage of these routes based on the various performance parameters. Analysis has been done for the known data, as how to allocate the buses to improve the present systems (Dhingra and Sharma, 1988). Proposals were also made to define level-ofservice with equivalent single value replacing multiple attributes (Gupta and Virat, 1981). A comparative evaluation of the quality of service provided transit systems under different ownerships in Delhi (Umrigar et al., 1988) has been documented. Psychometric analysis has been carried out based on the responses given through questionnaire to assign a weightage to each of the attributes and aspects of the quality. The methodology used for the study lends itself to suggest appropriate improvement of quality through weights of the attributes. The fuzzy weighted average is based on extended algebraic interval operations and the concept of a α-cut representation of fuzzy number (Dong and Wong, 1987). From the literature review, the need of providing adequate level-of-service and its measurement has been felt to be very important for the improvement in transit services. It is also required by the planners and operators to identify the service attributes, which are considered as important by the commuters so as to suggest improvement measures to provide better level-of-service as compared to the existing situation.

### 5. APPROACH AND METHODOLOGY

#### 5.1 Data Collection

In the present study, a fieldwork was carried out in the months of February and March 2012 from 9.00 A.M. to 8.00 P.M. in Jaipur, the capital city of Rajasthan. Primary survey was conducted on the selected routes of the city. Secondary sources were also referred for the purpose of knowing operational details transit systems in Jaipur. The routes were judiciously chosen so that it covers entire city areas with different land uses and important business and other activities. The data collection was planned in such a way that the commuters of different socio-economic and age groups are interviewed in peak and off peak hours for all the routes to have uniform data points.

In order to determine the level-of-service of public transit in Jaipur, after getting feedback from the regular users of bus, operators and experts through initial questionnaire survey, nineteen out of the preliminary chosen thirty attributes have been chosen judiciously. Then 'on-board' bus survey was conducted under a close supervision (authors of the paper), by the qualified enumerators through interview based questionnaire survey with passengers in low-floor city buses on five different routes. Out of the five routes chosen for the study, three routes are radial (one of the routes is having BRTS partially) and two are circular, the details of which are shown in Fig. 1 and Table 1. The opinions of the users were noted down for assessing relative weights and rating the existing service qualities, for all the attributes. The details about the sample size (three hundred and fifty) considered for this study are shown in Table 1. Respondents are chosen at random and care was taken to have responses from the users having different socio-economic backgrounds and age groups. Special permission to conduct on-board survey was granted by Jaipur Development Authority (JDA), Rajasthan (India).



Figure 1.Map showing Routes considered for the present study (Source: JDA, Rajasthan)

Routes Type of		Length	Male	Female	Total
	Route	(Km)	Respondents	Respondents	Respondents
Route 1	Radial	17	61	13	74
Route 3	Radial	20	90	9	99
Route 6	Radial	26	65	12	77
Route 8	Circular	40	44	13	57
Route 10	Circular	21	37	6	43
		Total	297	53	350

Table 1.Details of the Bus Routes of	considered for this study
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A group of passengers, who are regular users of the low-floor Jaipur city buses were requested to give the importance of all the nineteen attributes in terms of five descriptors: (i) Extremely important as (A), (ii) very important as (B), (iii) important as (C), (iv) important to some extent as (D), and not at all important as (E). Then other passengers were requested to rate the existing service qualities in regard to all the nineteen attributes in another questionnaire based on their individual level-of-satisfaction. The rating was done in terms of the five descriptors as very good, good, fair, satisfactory and poor.

#### **5.2 Data Analysis**

### 5.2.1 Numerical rating approach

Numerical rating approach is a simple conventional weighted average method used to calculate Level-of-Service (LOS).Level-of-Service-Index (LOSI) provided by a category of bus, which can be defined as the composite index calculated using the various service characteristics (attributes).Mathematically it is given as equation (1),

$$LOSI = \frac{\sum_{i=4}^{n} W_i \times R_i}{\sum_{i=4}^{n} W_i}$$
(1)

Where,

n : No. of attributes that define the overall LOSI.

- W<sub>i</sub> : Weight associated with the i<sup>th</sup> service attribute
- R<sub>i</sub> : Value score for the i<sup>th</sup> service attribute for the category of bus service for the existing situation.

For a particular service characteristic, the LOSI provided by an urban bus service is expressed as shown in equation (2) below:

$$LOSI = W_i \times R_i$$
<sup>(2)</sup>

If the scale for weight associated for the service characteristics is such that  $\sum_{i=1}^{n} W_i = 1$ , and the value score is expressed with respect to unity, then the maximum possible LOSI as per equation (1) is one. LOSI values closer to 1 indicate very high LOSI, whereas, closer to 0 indicates very poor service level. However, practically no bus system is expected to provide a LOSI as high as 1. Thus, it is important to know the accepted LOSI, by the users, in a developing country like India. From review of the literature, it has been observed that 0.6 may be used as the value for accepted service level (Dhingra and Bains, 1986; Gupta and Virat, 1981; Umrigaret al., 1988). The same value has been considered in this study also.

As per the objectives framed for this study, data analysis was done using conventional weighted average method for all the routes considered for this study, the detailed calculations and results of route-1 are shown in Tables 2, 3, and 4, as example. Passenger's opinion in linguistic expressions (columns 2, 3, 4, 5, and 6 of Table 2) are then averaged (column 7 of Table 2). The results shown in column 8 of Table 2 represent a weighted average opinion on importance of service quality of the nineteen attributes for city buses on a conventional rating scale of 1-5, with 1 being 'not at all important' and 5 being 'extremely important'. It is evident from the results showing relative weights in Table 2 that relatively more importance was felt on the safety and security aspect inside the bus by the respondents. In the same manner, the results shown in column 8 of Table 3 represent a weighted average opinion on service quality of the nineteen attributes for city buses plying on Route-1 on a conventional rating scale of 1-5, with 1 being 'poor' and 5 being 'Very Good'. In this category of service (Table 3), the service quality with respect to Space available for standing inside the bus (Service is less when the bus is overcrowded) attains the maximum score (0.88), whereas condition and cleanliness of buses receives less score (0.51). The service level of the attributes and its deficiency from the acceptance level (0.6) as indicated by the negative sign as per numerical rating technique (are presented for low-floor city buses on route-1(radial route as depicted in Fig 1) in Table 4. For example, in Table 4 for a radial route Route-1, the level of service of the attributes namely, seat availability for women and condition and cleanliness of buses are having deficiency from acceptance level as they are shown with negative sign. The service levels of the remaining seventeen attributes considered are at acceptable level as they are indicated with positive sign, with respect to sufficiency/deficiency, which is shown in Table 4. Thus, from this analysis, it can be thought that the overall LOSI of buses on a particular route can be improved by improving the service quality of the attributes, which are below the acceptance level.

As per typical characteristics of bus operation, attribute numbers 2, 6,7,8,9,10,11,13,14,17,18 and 19 pertain to riding operation. Attribute number 3 is the access and egress to and from bus stop. Attribute number 12, 15 and 16 defines characters of the bus stop. Attribute number 1 is classified as user cost.

S.No	Table 2. Relative Weightag           Attributes	-		er of pas	sengers pu hts on		Average Weightage	Relative Weight
			4**	3***	2****	1****	(X)	$W_i = \frac{X}{\sum X}$
1	Travel Cost (TC)	3	5	11	3	2	3.17	0.05
2	Total journey time (Min) (TT)	7	11	5	1	0	4.00	0.06
3	Walking distance both at the origin(home to bus stop and destination stop to destination)(Km) (WD)	6	9	6	2	1	3.70	0.05
4	Waiting time at bus stop (min) (WT)	5	9	6	4	0	3.60	0.05
5	Punctuality and Reliability (P)	9	8	3	3	1	3.88	0.06
6	Availability of seat (AoS)	8	4	4	6	2	3.41	0.05
7	Space available for standing inside the bus (Service is less when the bus is overcrowded) (Sp.S)	8	9	6	1	0	4.00	0.06
8	Comfort level of the seats (you can sit comfortably with enough leg room and side space) (ClS)	9	10	3	2	0	4.08	0.06
9	Ease of boarding & alighting from bus (E&A)	10	10	2	0	2	4.08	0.06
10	Sufficient number of seats for women (W)	8	3	6	4	3	3.37	0.05
11	Ventilation inside bus (i.e. air circulation) (V)	9	13	0	2	0	4.20	0.06
12	Quality of Bus-stop: (QoB)	4	10	5	4	1	3.50	0.05
13	Safety and security inside bus							
	(S&S)	11	9	2	1			
14	Bus driver, conductor & co- passenger behaviour (DC)	7	9	4	3	1	4.17	0.06
15	Route information (Route No. etc.) Written on outside body of the bus (RI)	10	7	5	1	1	3.75	0.05
16	Convenience in ticket purchasing system (CTP)	9	6	6	3	1	4.00	0.06
17	Condition & Cleanliness of Bus (C&C)	4	10	2	4	0	3.88	0.06
18	Noise level (both from vehicle and passengers inside the bus) (N)	3	10	4	4	4	3.25	0.05
19	Jerking inside the running bus (J)	6	8	3	3	4	3.38	0.05

Table 2. Relative Weightage of the attributes for city Buses (Route-1)

\*Extremely important, \*\*very important, \*\*\* important, \*\*\*\*important to some extent, \*\*\*\* not at all important

Attributes		Number of pa	assengers putti	ng Weights o		Average	Service Quality
	5*	4**	3***	2****	1*****	Weightage	w.r.to
						( X)	unity (X divided 5)
TC	4	28	34	5	3	3.34	0.67
TT	6	31	29	6	2	3.45	0.69
WD	9	22	29	11	3	3.31	0.66
WT	12	14	19	20	9	3.00	0.60
Р	14	24	23	8	5	3.46	0.69
AoS	15	17	17	22	3	3.26	0.65
Sp.S	40	26	7	1	0	4.42	0.88
CIS	33	22	13	6	0	4.11	0.82
B&A	38	24	6	5	1	4.26	0.85
W	2	10	23	21	18	2.42	0.48
V	37	29	6	2	0	4.36	0.87
QoB	21	28	17	5	3	3.80	0.76
S & S	27	33	10	3	1	4.11	0.82
DC	11	30	17	12	4	3.43	0.69
RI	15	39	11	8	1	3.80	0.76
СТР	32	23	14	4	1	4.09	0.82
C&C	13	9	9	17	26	2.54	0.51
Ν	10	31	27	4	2	3.58	0.72
J	9	35	17	12	1	3.53	0.71

Table 3. Service Quality with respect to Unity of the Attributes (Route-1)

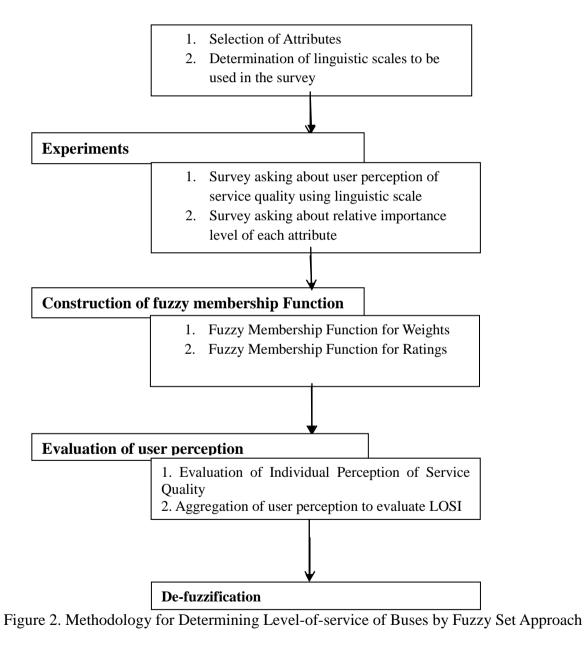
\*Very good, \*\*Good, \*\*\*Fair, \*\*\*\*Satisfactory, \*\*\*\*\*poor.

Attributes	Relative	Service Quality	LOSI	Acceptance Level	Deficiency from
	Weight (Scale	(w.r.t. unity)	(3) = (1) * (2)	(60% of Scale	acceptance Level
	Value)	(2)		Value)	(5) = (3) - (4)
	(1)			(4)	
TC	0.045	0.668	0.030	0.027	0.003
TT	0.057	0.689	0.039	0.034	0.005
WD	0.052	0.662	0.035	0.031	0.003
WT	0.051	0.600	0.031	0.031	0.000
Р	0.055	0.692	0.038	0.033	0.005
AoS	0.048	0.651	0.031	0.029	0.002
Sp.S	0.057	0.884	0.050	0.034	0.016
CIS	0.058	0.822	0.047	0.035	0.013
B&A	0.058	0.851	0.049	0.035	0.015
W	0.048	0.484	0.023	0.029	-0.006
V	0.060	0.873	0.052	0.036	0.016
QoB	0.049	0.759	0.038	0.030	0.008
S & S	0.059	0.822	0.048	0.035	0.013
DC	0.053	0.686	0.036	0.032	0.005
RI	0.057	0.759	0.043	0.034	0.009
СТР	0.055	0.819	0.045	0.033	0.012
C&C	0.046	0.508	0.023	0.028	-0.004
Ν	0.046	0.716	0.033	0.028	0.005
J	0.048	0.705	0.034	0.029	0.005

Table 4. Service Levels and their Deficiencies of Buses using Numerical Rating Approach (Radial Route-1)

## 5.2.2. Use of Fuzzy Sets to Evaluate Level of Service

Mathematical formulation of linguistic and heuristic procedure has become possible with the development of the concept of fuzzy-sets by LotfiZadeh in 1964 in connection with the control theory and system science (Zadeh, 1965; Juangand Amirkhanian, 1992). Since then, many successful applications of this conceptual framework have been made to real-world problems. The LOSI using Fuzzy set approach is then computed by a series of steps described as follows. To satisfy the objectives of the present study, the following structure of the methodology and the methodological steps have been adopted for quantifying LOSI using fuzzy-set approach, which is described below and also shown in the flowchart as Fig. 2.



Weights and Ratings: The weight and value score of each attribute was noted in terms of letter grades from the samples collected using two structured questionnaires, one for

determining the weights on the attributes and other for finding the satisfaction levels with the present service based on those attributes from the regular users of bus.

**Construction of Membership Function** The role of the membership function is to represent an individual and subjective human perception as a member of a fuzzy set. The fuzzy sets that represent the letter grades adopted in this study are characterized by their membership functions as shown in Fig. 3 and Table 5. The indication of intensity of belongingness is expressed in membership function, assigning to each element a number from the unit interval (some range) and no single value, as it is done in the case of numerical rating approach. Two membership functions were constructed with five scales of linguistic statement representing the importance and ratings of the chosen attributes. The triangular membership function (TFN) is used after an initial review of the data (Singh and Vidyarthi, 2008; Singh, Singh and Dubey) and through the opinions of the experts in the field of fuzzy logic and public transit evaluation.

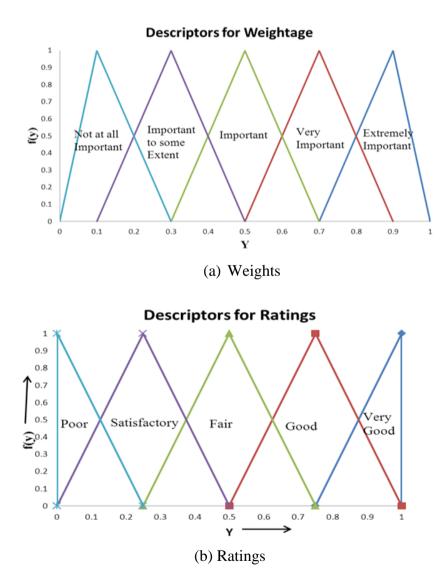


Figure 3. Membership Functions of Fuzzy Sets that represent letter grades

Triangular Fuzzy Number for Weights								
Descriptors for weightage	А	В	С					
Extremely Important	0.70	0.90	1.00					
Very Important	0.50	0.70	0.90					
Important	0.30	0.50	0.70					
Important to Some Extent	0.10	0.30	0.50					
Not at all Important	0.00	0.10	0.30					
Triangular Fu	zzy Number for Rating	gs						
Descriptors for Ratings	А	В	С					
Very Good	0.75	1.00	1.00					
Good	0.50	0.75	1.00					
Fair	0.25	0.50	0.75					
Satisfactory	0.00	0.25	0.50					
Poor	0.00	0.00	0.25					

#### Table 5. Membership Functions of Fuzzy Sets that represents letter Grades

**Evaluation of Individual Perception of Service Quality:** In the proposed methodology, the rating of a service quality according to a particular type of attribute, was assessed and recorded in terms of a linguistic grade. The advantages of using linguistic grades for ratings (and weights) in a predominately qualitative engineering evaluation are well documented (Eltonand Juang, 1988). However, it demands an effective method for processing and combining the qualitative information obtained. One such method is to process the information using the following equation (3) (Schmucker, 1984):

$$R = \frac{\sum_{i=1}^{n} W_i \times R_i}{\sum_{i=1}^{n} W_i} \tag{3}$$

Where,

- R : The overall rating of service level of a bus service,
- $R_i$  : The rating of the i<sup>th</sup> service quality of the category of bus service for the existing condition,
- W<sub>i</sub> : The weight of that service attribute i, and
- n :Number of attributes that define the overall service level.

Each term in the right-hand side of equation (3) is a linguistic grade or, simply, a letter grade-A, B, C, D and E. A rational approach to evaluate equation (3) is to represent these letter grades with fuzzy sets, rather than using a single number to represent a letter grade, as is done in the conventional numerical rating approach. A fuzzy set is a set of paired numbers that describes the degree of support to each service quality. In other words, a defined range of measured or calculated values is called fuzzy-sets. In describing the service quality, the attributes for which higher values represent higher level of satisfaction (e.g., comfort level of seats), have been represented Very Good (highly satisfactory) =A, Good =B, Fair (Standard value) =C, Satisfactory = D, Poor (worst condition) =E

Aggregation of the Individual Perceptions: The 350 individual perceptions of bus service evaluated above should be aggregated to represent the group's overall opinion. For aggregating the fuzzy number of the perceptions, an "arithmetic mean" of the fuzzy numbers, which represent all individual perceptions, was calculated using a fuzzy average operation based on the " $\alpha$ -cut" concept of fuzzy sets and an interval analysis. The outputs from this step are still fuzzy numbers, and they should be transformed into crisp numbers to be more easily understood.

**Defuzzification:** To transform the final fuzzy set that represents the group's overall opinion into crisp numbers, a defuzzification procedure was conducted. Out of several defuzzification methods, the defuzzified method using  $\alpha$ -cut (Juangand Amirkhanian, 1992) was used due to its simplicity and ease of computation. It is a mapping model for measuring fuzzy numbers using estimated utility as per the following equation (4):

$$LOSI = \frac{A_L - A_R + 1}{2} \tag{4}$$

Where,

The LOSI is defined as the composite index of various service characteristics provided by a JCTSL buses in Jaipur

- $A_L$  : the area enclosed to the left of the characteristic function that characterizes the fuzzy number,
- $A_R$  : the area enclosed to the right of the characteristic function that characterizes the fuzzy number.

The non-fuzzy outputs, which are the overall LOSI of the buses on a particular route after defuzzification, are shown in Table 6 using fuzzy-set theory approach. From the Table based on the present values of overall LOSI, it is evident that Route 8 has highest overall LOSI as compared to other routes as per both the approaches, which may be due to the fact that route is mostly passing/covering the places, which has less business and recreational activities, i.e. mostly it is passing through the less congested areas.

Route	$\alpha$ - cut intervals	Non-fuzzy outputs for	Overall LOSI
		Routes	
1	0	0.425, 0.842	0.635
	0.5	0.542, 0.750	
	1	0.658, 0.658	
3	0	0.300, 0.750	0.517
	0.5	0.405, 0.630	
	1	0.509, 0.509	
6	0	0.339, 0.788	0.562
	0.5	0.449, 0.674	
	1	0.560, 0.560	
8	0	0.432, 0.869	0.662
	0.5	0.552, 0.771	
	1	0.673, 0.673	
10	0	0.348, 0.784	0.569
	0.5	0.460, 0.680	
	1	0.572, 0.572	

Table 6. Non-Fuzzy Outputs for Overall Rating and LOSI using Fuzzy Set Approach

### 5.2.3. Analytic Hierarchy Process (AHP)

The analytic hierarchy process (AHP), which provides a proven, effective means to deal with complex decision making, was first introduced by Thomas Saaty in 1970's. It is used around the world in a wide variety of decision situations, in fields such as government, business, industry, healthcare and education. Fundamentally, the AHP works by developing priorities for alternatives and the criteria used to judge the alternatives. First, priorities are derived for the criteria in terms of their importance to achieve the goal, and then priorities are derived for the performance of the alternatives on each criterion. These priorities are derived based on pair-wise assessments using judgments, or ratios of measurements from a scale if one exists. Finally, a weighting and adding process is used to obtain overall priorities for the alternatives as to how they contribute to the goal (Saaty, 1977). The weights of Criteria and Alternatives are given using Saaty's Rating Table 7.

Intensity of	Definition	Explanation
importance		
1	Equal Importance	Two activities contribute equally to the objective
2	Weak or slight	
3	Moderate importance	Experience and judgment slightly favor one activity over another
4	Moderate plus	
5	Strong importance	Experience and judgment strongly favor one activity over another
6	Strong plus	
7	Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice
8	Very, very strong	
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation
Reciprocals of above	If activity I has one of the above non-zero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i	
1.1-1.9	If the activities are very close	May be difficult to assign the best value but when compared with other contrasting activities the size of the small numbers would not be too noticeable, yet they can still indicate the relative importance of the activities.

The n elements to be compared,  $C_1 \dots C_n$  and denote the relative 'weight' (or priority or significance) of  $C_i$  with respect to  $C_j$  by  $a_{ij}$  and form a square matrix  $A=(a_{ij})$  of order n with the constraints that  $a_{ij} = 1/a_{ji}$ , for  $i \neq j$ , and  $a_{ii} = 1$ , all i. Such a matrix is said to be a reciprocal matrix. The weights are consistent if they are transitive, that is  $a_{ik} = a_{ij}a_{jk}$  for all i, j, and k. Such a matrix might exist if the  $a_{ij}$  are calculated from exactly measured data. Then find a vector  $\omega$  of order n such that  $A_{\omega} = \lambda_{\omega}$ . For such a matrix,  $\omega$  is said to be an eigenvector (of order n) and  $\lambda$  is an eigenvalue. For a consistent matrix,  $\lambda = n$ .

For matrices involving human judgement, the condition  $a_{ik} = a_{ij}a_{jk}$  does not hold as human judgements are inconsistent to a greater or lesser degree. In such a case the  $\omega$  vector satisfies the equation  $A_{\omega} = \lambda_{max\omega}$  and  $\lambda_{max} \ge n$ . The difference, if any, between  $\lambda_{max}$  and n is an indication of the inconsistency of the judgements. If  $\lambda_{max} = n$  then the judgements have turned out to be consistent. Finally, a Consistency Index can be calculated from  $(\lambda_{max}-n)/(n_1)$ . That needs to be assessed against judgments made completely at random and Saaty has calculated large samples of random matrices of increasing order and the Consistency Indices of those matrices. A true Consistency Ratio is calculated by dividing the Consistency Index for the set of judgments by the Index for the corresponding random matrix. Saaty suggests that if that ratio exceeds 0.1 the set of judgments may be too inconsistent to be reliable. In practice, CRs of more than 0.1 sometimes have to be accepted. A CR of 0 means that the judgements are perfectly consistent.

Professional commercial software Expert Choice developed by Expert Choice Inc. is used to Simplify the implementation of the AHP's steps as it automates many of its computations. The relative ratings of criteria's are shown in the figure 4. The reciprocal ratings are shown in red colour. The pair-wise comparison of alternatives with respect to all criteria's is done. For example, the relative importance with respect to Travel cost is as shown in figure 5. The final level of service Index of different routes is as shown in figure 6.

	тс	Π	WD	WT	Р	AoS	Sp.S	CIS	E&B	w	۷	QoB	<b>S&amp;S</b>	DC	RI	СТР	C&C	N	J
тс		7.0	4.0	3.0	6.0	2.0	7.0	8.0	8.0	2.0	8.0	2.0	8.0	5.0	7.0	6.0	1.0	1.0	2.0
Π			2.0	3.0	1.0	5.0	1.0	1.0	1.0	5.0	2.0	4.0	2.0	2.0	1.0	1.0	6.0	6.0	5.0
WD				1.0	2.0	2.0	2.0	3.0	3.0	2.0	4.0	2.0	3.0	1.0	2.0	2.0	3.0	3.0	2.0
WT					2.0	1.0	3.0	3.0	3.0	2.0	5.0	1.0	4.0	1.0	3.0	2.0	3.0	3.0	2.0
Р						3.0	1.0	2.0	2.0	4.0	2.0	3.0	2.0	1.0	1.0	1.0	5.0	5.0	4.0
AoS							5.0	5.0	5.0	1.0	7.0	1.0	6.0	2.0	5.0	3.0	2.0	2.0	1.0
Sp.S								1.0	1.0	5.0	2.0	4.0	2.0	2.0	1.0	1.0	6.0	6.0	5.0
CIS									1.0	6.0	1.0	5.0	1.0	2.0	1.0	2.0	7.0	7.0	6.0
E&B										6.0	1.0	5.0	1.0	2.0	1.0	2.0	7.0	7.0	6.0
W											7.0	1.0	7.0	3.0	5.0	4.0	1.0	1.0	1.0
V												6.0	1.0	3.0	2.0	2.0	8.0	8.0	7.0
QoB													5.0	2.0	4.0	3.0	2.0	2.0	1.0
S&S														3.0	2.0	2.0	8.0	8.0	7.0
DC															2.0	1.0	4.0	4.0	4.0
RI																1.0	6.0	6.0	5.0
СТР																	5.0	5.0	4.0
C&C																		1.0	1.0
N																			1.0
J																			

Figure 4. Relative rating matrix of criteria's

	Route1	Route3	Route6	Route8	Route10
Route1		1.0	7.0	2.0	3.0
Route3			7.0	2.0	3.0
Route6				8.0	3.0
Route8					5.0
Route10	Incon: 0.01				

Figure 5.Pairwise comparison matrix of alternatives with respect to Travel cost

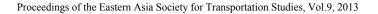




Figure 6. Final results shown as idealised priorities

### 5.2.4. Determination of Level of Bus Transport Service through Fuzzy-AHP

AHP uses pair-wise comparisons of criteria and alternatives to form a reciprocal decision matrix, thus transforming qualitative data to crisp or fuzzy ratios. The difference with respect to the crisp method is that Saaty's scale is fuzzified, all operations are fuzzy with triangular fuzzy numbers, and the final ranking of alternatives is performed after the defuzzification. The LOSI using Fuzzy AHP is then computed by a series of steps described as follows by Bojan and Yvonilde (2008).

**Fuzzifying Judgment Scale**: Fuzzy numbers are intuitively easy to use when expressing the decision maker's qualitative assessments. To facilitate the making of pair-wise comparisons in fuzzy AHP application, Saaty's original 9-point scale may be fuzzified. Membership functions for  $\tilde{1} \leq \tilde{x} \leq \tilde{9}$  are assumed to be symmetrically triangular, different for an internal pair and odd integers and adjusted for edge values along the scale. The crisp values of saaty's scale are fuzzified using Table 8. The ratings of the alternatives for all the criteria are fuzzified. For example, the rating of the alternatives with respect to Travel cost, are shown in Table 9. Fuzzified AHP rating for criteria are shown in Table 10.

Saaty's crisp values (x)	Judgement definition	Fuzzified Saaty's value
1	Equal importance	(1, 1, 1+δ*)
3	Week dominance	(3-δ, 3, 3+δ)
5	Strong dominance	(5-δ, 5, 5+δ)
7	Demonstrated dominance	(7-δ, 7, 7+δ)
9	Absolute dominance	(9-8, 9, 9)
2,4,6,8	Intermediate values	(x-1, x, x+1), x=2, 4, 6, 8

Table 8. Original and fuzzified saaty's scale for pair-wise comparisons

\* $\delta$  is fuzzy distance (0.5 $\leq \delta \leq 2$ )

Table 9. Fuzzified AHP r	ting of alternatives for Trave	el cost

Alternatives	а	b	с
Route1	0.050203	0.090226	0.191944
Route3	0.050203	0.090226	0.191944
Route6	0.327249	0.524080	0.776087
Route8	0.028917	0.046865	0.109979
Route10	0.137594	0.248602	0.413078

**Evaluating Criteria:** The procedure of ranking starts with the determination of the importance of criteria with respect to the formulated goal. By using a fuzzified scale, a fuzzy reciprocal judgment matrix for criteria is determined as:

$$\mathbf{A} = \begin{bmatrix} \tilde{a}_{11} & \cdots & \tilde{a}_{1M} \\ \vdots & \tilde{a}_{22} & \vdots \\ \tilde{a}_{M1} & \cdots & \tilde{a}_{MM} \end{bmatrix}$$

Where,

$$\tilde{a}_{ij}=1$$
 for all  $i = j$  (i,j =1, 2,..., M), and  $\tilde{a}_{ij}=1/\tilde{a}_{ij}$ 

By applying the fuzzy synthetic extent, corresponding weights of criteria can be determined as equation (5).

$$W_{i} = \sum_{j=1}^{M} \tilde{a}_{ij\times} \left[ \sum_{k=1}^{M} \sum_{l=1}^{M} \tilde{a}_{kl} \right]^{-1}, i = 1, \dots, M$$
(5)

All w<sub>i</sub>, i=1,..., M, are normalised fuzzy numbers with medium values equalling 1. It should be noted that fuzzy extent could be defined as the result of fuzzy arithmetic, or by using the extension principle. The second is slightly more difficult, but would lead to reduced uncertainty.

Table 10.Fuzzified AHP rating for criteria (weights)

Criteria	Weights	а	b	С
TC	W1	0.00504	0.010509	0.024687
TT	W2	0.035522	0.079050	0.180281
WD	W3	0.013833	0.036913	0.077950
WT	W4	0.012724	0.031493	0.068591
Р	W5	0.026918	0.062643	0.140078
AoS	W6	0.007994	0.020100	0.043490
Sp.S	W7	0.035522	0.079050	0.180281
ClS	W8	0.042461	0.093965	0.214831
B&A	W9	0.042461	0.093965	0.214831
W	W10	0.007787	0.016486	0.038455
V	W11	0.051619	0.114846	0.261943
QoB	W12	0.008667	0.021154	0.046345
S & S	W13	0.048289	0.108880	0.246868
DC	W14	0.020119	0.047976	0.106155
RI	W15	0.035522	0.079050	0.180281
CTP	W16	0.026918	0.062643	0.140078
C&C	W17	0.006328	0.012459	0.030065
Ν	W18	0.006328	0.012459	0.030065
J	W19	0.007745	0.016362	0.038204

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Evaluating Alternatives: The provided N alternatives are compared pair-wise, with respect to each of the K criteria. After obtaining K fuzzy judgment, the fuzzy extent produces the decision matrix.

$$W_{k} = \begin{bmatrix} \tilde{a}_{11} & \cdots & \tilde{a}_{1N} \\ \vdots & \tilde{a}_{22} & \vdots \\ \tilde{a}_{M1} & \cdots & \tilde{a}_{NN} \end{bmatrix}, k = 1, \dots, K$$

$$X_{ij} = \sum_{k=1}^{K} \tilde{a}_{ik} \times [\sum_{l=1}^{N} \sum_{m=1}^{k} \tilde{a}_{lm}]^{-1}, i = 1, \dots, N; j = 1, \dots, K$$

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1K} \\ \vdots & x_{22} & \vdots \\ x_{N1} & \cdots & x_{NK} \end{bmatrix}$$
(6)

In the decision matrix X,  $x_{ij}$  represents the resultant fuzzy performance assessment of the alternative A<sub>i</sub> (i=1, 2,..., N) with respect to the jth sub-criterion ( j=1, 2,..., K).

**Performance Matrix**: As proposed by Deng (1999), an overall performance of each alternative across all criteria may be represented by the fuzzy performance matrix. For example, the performance matrix of Route 1 is shown in Table 11.

$$\mathbf{Z} = \begin{bmatrix} x_{11} \times w_1 & \cdots & x_{1k} & \times & w_k \\ \vdots & \ddots & & \vdots \\ x_{N1} & \times & w_1 & \cdots & x_{NK} & \times & w_k \end{bmatrix}$$

It is obtained by multiplying the entries of the weighting vector by the related column values of the decision matrix and by applying fuzzy interval arithmetic.

 Table11: Performance matrix of Route 1

Alternatives	Route 1		
	а	b	с
TC	0.000253	0.000948	0.004738
TT	0.001815	0.008805	0.040361
WD	0.00138	0.005998	0.023255
WT	0.000489	0.002369	0.010937
Р	0.002623	0.009901	0.039128
AoS	0.001028	0.004167	0.015857
Sp.S	0.007665	0.026606	0.092424
ClS	0.010114	0.034052	0.112330
B&A	0.012377	0.040192	0.127192
W	0.000407	0.001598	0.008245
V	0.013322	0.045305	0.149243
QoB	0.001473	0.007032	0.024692
S & S	0.009632	0.037925	0.134848
DC	0.002304	0.009908	0.039072
RI	0.010303	0.038123	0.132884
СТР	0.006279	0.025234	0.090084
C&C	0.000243	0.000940	0.004538
Ν	0.001138	0.003870	0.015147
J	0.001296	0.004497	0.017764
Sum	0.084138	0.307470	1.082739

**Final Assessments and Synthesis**: Several methods have been proposed to aggregate the decision maker's assessments. The most commonly used are the mean, median, max, min and mixed operators (Buckley 1985). Additive synthesis has been assumed here and the final alternative performance weights with respect to overall goal are calculated by the summation of elements in the rows of the performance matrix to obtain final result. Final alternative performance weights are shown in Table 12.

$$F_{i} = \sum_{j=1}^{K} x_{ij} \times w_{j} i = 1, 2, \dots, N$$
(7)

Alternatives	а	b	с
Route1	0.084138	0.307470	1.082739
Route3	0.033482	0.129099	0.505330
Route6	0.027855	0.107276	0.460548
Route8	0.093872	0.340337	1.186790
Route10	0.030848	0.115818	0.476059

Table 12: Final alternative performance weight

In the case of fuzzy AHP, defuzzification is necessary at the end to obtain crisp weights and finally rank the alternatives. There are several methods that are able to do this; such as the centre of gravity method, the dominance measure method, the  $\alpha$ -cut with interval synthesis method, and the total integral value method. The last, the total integral value method (Liou and Wang 1992), is considered to be a good choice for performing the task efficiently and, therefore, has been proposed within this methodology. For the given triangular fuzzy number A= (a1, a2, a3), the total integral value is defined as equation 8.

$$I_{\tau}^{\lambda}(A) = (1/2)[\lambda a_3 + a_2 + (1-\lambda)a_1], \lambda \in [0, 1]$$
(8)

In above equation,  $\lambda$  represents an optimism index which expresses the decision maker's attitude toward risk. A larger value of  $\lambda$  indicates a higher degree of optimism. In practical applications, values 0, 0.5 and 1 are used respectively to represent the pessimistic, moderate and optimistic views of the decision maker. The defuzzified values using equation 8 are shown in Table 13.

Alternatives	Index of optimism			Final Rank
	$\lambda = 0$ (pessimistic)	$\lambda = 0.5$ (moderate)	$\lambda = 1$ (optimistic)	
Route1	0.19580	0.44545	0.39160	2
Route3	0.08129	0.19925	0.16258	3
Route6	0.06756	0.17573	0.13513	5
Route8	0.21710	0.49033	0.43420	1
Route10	0.07333	0.18463	0.14666	4

### 6. RESULTS AND DISCUSSIONS

The important findings of the study are given as follows:

 The results of the estimated LOSI using four different methods: (i) Fuzzy approach, (ii) Numerical Rating Approach, (iii) AHP approach, and (iv) Fuzzy-AHP approach are shown in the Table 14. The differences between the results (values of overall LOSI) by numerical rating and fuzzy set approaches were found to be between 10.1% and 14.8%. The differences between the results by AHP and Fuzzy-AHP were found to be between 8 % and 13.7%. This difference is significant (at 5% level of significance with a t value of -29.851 and 4 degrees of freedom) for the routes considered for this study, because each letter grade is given an appropriate range or a fuzzy-set (for fuzzy-set and Fuzzy-AHP approaches) and not just a single mean value like in the case of Numerical rating and AHP approaches. Hence, fuzzy-set theory and Fuzzy-AHP approaches may be considered as more accurate and reliable over the other two methods as they eliminate fuzziness (uncertainty in deciding the importance of the attributes). The results for Fuzzy and Numerical Rating approaches are comparable (closer), since the overall LOSI is calculated based on the weightage and rating of all the attributes together. Whereas, the results for AHP and Fuzzy-AHP approaches are comparable (closer), since the overall LOSI is calculated based on the pair-wise comparison of criteria's and alternatives.

Route	LOSI			
	Fuzzy approach	Numerical Rating Approach	AHP	Fuzzy-AHP
1	0.635	0.726	0.321	0.445
3	0.517	0.607	0.122	0.199
6	0.562	0.648	0.098	0.175
8	0.662	0.737	0.353	0.490
10	0.569	0.657	0.106	0.184

Table 14.Comparison of the LOSI results using different approaches

- 2) The values of overall LOSI for AHP and Fuzzy-AHP are lower as compared to numerical rating approach and Fuzzy set approach, because the original ratings have been normalized for easy and appropriate pair-wise comparison of criteria and alternatives. The acceptance level of 0.6 is not applicable for values obtained from AHP and Fuzzy-AHP.
- 3) If the LOSI is to be found approximately and quickly for the given data samples, the quick and easy methods that can be used are: Numerical rating approach and AHP. For more precise and detailed analysis of the given data samples, where lot of fuzziness is involved, Fuzzy set approach and Fuzzy- AHP can be applied. For prioritizing the routes, AHP and Fuzzy AHP are the best methods as the pair-wise comparisons are used.
- 4) Attributes such as condition, cleanliness and sufficient seats for women were found to be deficient for their service quality for all the five routes selected for this study, which may be because of improper maintenance, lack of focus on cleanliness by the operators and because of the policy of very limited number of seats reserved for women in all JCTSL buses. Also, during the survey time it was observed that majority of the time men had occupied seats reserved for women. In developing countries like India, based on the socio-economic background, the policy of 'reserving seats only for women' can be proved as very important and effective for attracting them to use transit services as they may feel more secured and safer to use public transit rather than any other mode.
- 5) Attributes such as, quality of bus stop and availability of seats were also found to be deficient against the accepted service quality for all the routes except for Route-1, which is partly a BRTS stretch (8 km) having newly built bus stops.
- 6) Of all the routes studied, using fuzzy and numerical rating approaches, Route-3 had a minimum overall LOSI, indicating a high commuter expectation level. Possibly as a result of the fact that the route passed through some of the busy and congested commercial places of Jaipur (Sawai Mansingh Stadium, JDA, Sanganeer Depot, etc.). Whereas, using AHP and Fuzzy-AHP approaches minimum overall LOSI was found for Route-6. This variability can be attributed to the usage of same individual ratings by experts both in AHP and Fuzzy AHP. Hence, it is strongly proposed that the individual rating of the attributes to be used in Fuzzy and Numerical weighted approaches and pairwise rating of different attributes to be used in AHP and Fuzzy-AHP.

- 7) From the table 14, it can be seen that, using all the four approaches, the Route-8 was found to have a maximum overall LOSI among all the routes selected for the study, which may be due to the fact that route is passing/covering the places, which are relatively less developed and thus are less congested as compared to the other routes.
- 8) By improving the service quality of low LOSI attributes, there is scope for improvement of overall LOSI. Significant efforts are needed to maintain high overall LOSI scores.

## 7. RESEARCH CONTRIBUTIONS

The major research contributions of the study are summarized below:

- 1. Methodology have been proposed to address inbound deficiency of conventional weighted average method in rating the transit service quality, using more robust fuzzy and fuzzy-AHP methods, which can be transferable to other State Transport Undertakings (STU's) in Indian cities, which are plying low-floor buses
- 2. The numerical rating and AHP approaches have the potential to show the amount of deficiencies/sufficiency in quality-of-service provided with respect to the various attributes, thereby it helps to identify the attributes, which need improvement and in turn helps in quality management.
- 3. The fuzzy-set and fuzzy-AHP approaches employed in the paper to evaluate LOSI can be used as a decision making tool by all stakeholders and the researchers, in an effort to identify the deficiency very accurately and to take necessary steps to improve the state of the service quality of public transit in India and other developing countries, which may help in improving the image of public transportation by attracting people towards public transit with improved service quality as compared to other modes of transportation.
- 4. Extensive efforts were taken to design RP survey questionnaire and also efforts were taken to conduct 'on-board' survey for five selected routes in regard with assessing LOS provided by a bus, which is the first of its kind in India. On the basis of interaction and opinions of regular public transit users, nineteen attributes were chosen judiciously.

## 8. CONCLUSION

In this paper, quality of transit service was evaluated using the numerical rating, fuzzy, AHP and Fuzzy AHP approaches. Generally, level-of-service is a very important indicator in evaluating the performance of public transit. However, it is difficult to measure, as the service quality that a human mind perceives is affected by various quantitative and qualitative factors and it can be represented in a better way by combining both the factors and not just using quantitative factors only. The proposed methods in this study make numerical evaluation of service quality of buses more feasible and realistic, as it attempts to combine quantitative and qualitative aspects (attributes) to estimate the overall LOSI.

In this study, the differences between the values of overall LOSI by numerical rating, fuzzy set, AHP and fuzzy-AHP approaches were found to be significant based on the different ways, the weightage and rating of different attributes considered for determining LOSI of buses. The fuzzy set approach, which takes into account the uncertainty associated with quantification of the linguistic grade hence, has lot of potential to assess and compare the level-of-service provided on different routes of the transit network of the city over a given time period. From practical point of view, the Numerical rating and AHP approach can be

readily employed by planners, decision makers and the funding agencies, to improve the quality of service of public transit in India, by identifying the deficiencies very quickly with less efforts and skills, but for more accuracy and reliability one may rely more on fuzzy approach or fuzzy-AHP. For prioritizing the routes, AHP and Fuzzy AHP are the best methods as the pair-wise comparisons are used.

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