Synchronisation of Public Transport Services: Comparison of Methods

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Abstract: Synchronisation of timetables is a vital aspect of integration in public transport systems. Due to its importance, the schedule synchronisation has drawn the attention of researchers over the past years and various methods and models have been developed in this regard. Despite the diversity of the approaches and the models, few research attempts have been dedicated to comparing these methods and their fundamentals. This paper aims to present an insight into the transit schedule synchronisation and its different aspects. The existing approaches for the schedule synchronisation are compared and the current gaps and deficiencies in this domain are discussed in detail in this study.

Keywords: Public Transport, Transit, Schedule Synchronisation, Timetabling

1. INTRODUCTION

Integration is widely recognised as a key factor in public transport operation which can enable it to compete with private modes in urban transportation. Parallel to the increasing of public transport importance, there has been growing interest in developing integrated public transport systems over the recent years. Such systems are intended to facilitate transfers between different services and to provide seamless services for the passengers who need to take more than one service. Integration in public transportation has different sides and the temporal coordination of services, widely known as *schedule synchronisation*, is a vital aspect in this regard.

Schedule synchronisation concerns with minimising the delay caused by the phenomenon of transfer in public transport systems. Public transport journeys can often require the combination of several modes or services. This involves transferring between different services and results in delay. This delay, which is called transfer waiting time, is strongly dependent on synchronisation of intersecting transit services. It might be believed that transfers should be avoided in transit systems whenever possible. However, the economic reasons impose the necessity of transfer to public transport systems as it is impossible to cover all origin-destination demands by direct connections. Transfers improve transit operational flexibility and efficiency because each transit line can be planned according to its demand volume and characteristics. Moreover, a range of studies have emphasised that the transit systems which support more transfers and offer wider travel choices can generate higher levels of patronage (Dadson, 2011). Therefore, if passenger transfers are planned properly, the negative effects on passengers will be so small and compensated by the benefits of transfers (Vuchic, 2005).

As an important element in public transport planning, schedule synchronisation has drawn the attention of researchers especially over the past decades. This problem has been tackled via various approaches and several models and methods have been developed in this regard. While some of the approaches found in the literature have some fundamentals in common, others seem basically different. Despite this diversity of methods, very few research attempts have been dedicated in the literature to compare these methods and approaches. This study aims to present insights into the characteristics of transit schedule synchronisation problem and investigate the objectives and the methods presented in the literature in this regard.

The structure of this paper is as follows. First, we explain the methodology of this study. Afterwards, we describe the transfer waiting time and its importance in transit planning. Next, we discuss about the different approaches and the models arisen in the literature for the schedule synchronisation. In the end, a discussion will be presented about these methods and the existing deficiencies in the way of coordinating transit timetables.

2. RESEARCH METHOD

In order to evaluate the literature in relation to the transit schedule synchronisation, we explored the international scientific databases to find the reference papers in this area of research. In addition, we conducted free web search to cover other studies which are not contained in the international databases. This approach provided us a broader access to the literature. The search terms were selected so that most of the papers related to the area of transit timetabling were covered in this step. Since this study concentrates only on timetabling problem, the studies which consider other aspects of transit planning (e.g. frequency setting and network design) were excluded. Then, we conducted an ancestry approach to recognise and add the reference papers which were mostly cited in the literature. This step enriched our database significantly and prevented us neglecting most of the valuable studies in this research area.

We studied more than 60 research articles in relation to transit schedule synchronisation in this study. Considering the numerous studies on public transport, it should not be denied that our database is not exhaustive and some works may not be included in this study. However, we attempted to gather most of the associated studies together. After collecting the literature, we classified the papers according to their approaches, objectives and models in order to discover their similarities and differences. The results of this investigation are presented in the rest of the paper.

3. TRANSFER WAITING TIME

Travel time is an important factor in a potential user’s decision to use transit on a regular basis, as well as for the existing transit users. Travel time for transit users consists of different components, including walking time from the passenger’s origin to the first stop and from the last stop to the final destination, in-vehicle travel time, initial waiting time and any transfer time from one service to another, if required.

Passenger transfers between lines occur where two or more transit lines intersect or terminate at one point. Transfers may also happen between the lines which are relatively close to each other and can be accessed via short walking. The phenomenon of transferring from one transit service to another imposes transfer waiting times to the passengers. Each transfer adds a transfer time to one’s travel time because of the wait required for the next service. Transfers also may lead to a missed connection when the passenger misses the related service, which leads to longer waiting time.

The importance of travel time components is not equal for transit users and varies from one person to another. A wide range of studies have shown that out-of-vehicle times (initial waiting time and transfer time) are more annoying for passengers in comparison with in-vehicle time. More recent modelling efforts have also demonstrated that out-of-vehicle time components are between twice and four times as important as in-vehicle travel time (TRB, 2003; TRB, 2004). Studies have also shown that transfer time is even more important than initial waiting time in most cases (TRB, 2004). The fact is that if transit service is reasonably reliable, passengers can reduce the negative impact of the initial waiting time by adjusting their arrivals more closely to the schedule. However, transfer waiting time cannot be estimated and managed by passengers.

Transfer waiting time becomes more critical when the headways are not short enough (Daduna & Voß, 1995). When the headway of a related service is relatively short (generally, ≤6 minutes), the transfer time is usually short regardless to the headway of the first service. In contrast, transferring to a route which has long headway (say, > 10 minutes) may result in a delay which can be as long as the long headway of the related service (Vuchic, 2005). This case regularly happens in dispersed urban areas. In dense cities, transit frequencies tend to be high and missing a connection only increases passengers' transfer waiting time by a relatively short interval (Chakroborty, 2003). In contrast, in lower density areas where transit frequencies are lower, missing a connection results in longer delays and the absence of synchronisation may even discourage people from using public transport at all (Yan & Chen, 2002).

Transfer time itself consists of the walking time between two related services and the waiting time for arrival of the next service. While the former is influenced by the physical factors (e.g. distance between stops and ease of access), the latter is strongly dependent of the temporal coordination of the services, i.e. the arrival of the first service and the departure of the related service. In other words, the transfer waiting time is a direct consequence of schedule synchronisation (Teodorović & Lučić, 2005).

4. THE OBJECTIVES AND APPROACHES

Transit synchronisation problem concerns with setting timetables for transit lines of a pre-designed transit network. In practice, it may involve several practical tactics to improve transit services synchronisation, like adding or removing some services, reducing or increasing cycle times, short-turning, stop skipping and so on. However, shifting the departure times is a typical approach used at the stage of service planning and timetable setting. This approach can be performed statically and dynamically. In the dynamic approach, the real-time operational information is used and the departures and arrivals of services are adjusted continuously in order to minimise the transfer waiting time between related services. However, the static case is performed where transit services are intended to operate based on a fixed, pre-planned timetables. In such situation, the timetables are set at the planning phase and then published to the public. In other words, this is a sort of pre-planning approach which aims to synchronise the timetables on a fixed basis for different planning periods. This is what is widely called as *schedule synchronisation* or *transfer optimisation* in the literature. This study aims to focus on this approach and whatever presented in the rest of this paper is about this type of schedule synchronisation.

The entire process is to manipulate the departure times of transit lines so that the transfer waiting times for related services become minimised. This process can be either on changing the service frequencies or keeping the pre-determined frequencies for the lines

(Guihaire & Hao, 2008). Altering the line frequencies for the purpose of schedule synchronisation can result in changing the required fleet size (e.g. adding more buses to a bus route). However, synchronisation without altering line frequencies does not affect the fleet size and enables planners to reduce transfer waiting time by existing resources.

Transit timetable synchronisation has been tackled by different approaches so far and various methods have been introduced in the literature in this regard. These methods can be classified into two main categories in accordance with their objectives. The main approaches arisen in the literature so far are known as *timed transfers* and *transfer optimisation*. In the former approach, transit vehicles from different lines are scheduled to meet at certain transfer points whereas in the latter approach lines are scheduled to minimise the total passengers' transfer waiting time in the network (Guihaire & Hao, 2008; Castelli *et al.*, 2004). These approaches are described as follows.

4.1 Timed Transfer System

The main objective in the timed-transfer systems is to minimise the transfer waiting time via maximising the simultaneous arrivals of vehicles, known as *trip meets*, at some specific transfer points in the network. A timed transfer system (TTS) is a transit system consisting of transit lines and several transit centres at which transit vehicles from all intersecting lines arrive simultaneously and allow passengers to transfer easily in all directions (Vuchic, 2005). The term *transit centre* is generally referred to where multiple transit lines converge, allowing transfers among lines. This system is designed so that transit vehicles on all or most of the lines are scheduled to arrive at a transit centre simultaneously and depart after a short time, called *transfer window* (TRB, 2003). The transfer window is a short layover which may be provided at transit centres to ensure that connections can be achieved even if vehicles are running slightly behind schedule. Such systems vary from the basic form which consists of only one transit centre to the more sophisticated systems that include several transit centres.

In a timed transfer system the arrivals and departures of transit vehicles from different lines at a transit centre occur at time intervals referred to as the pulse headway h_p. Vuchik (2005) presents the basic relationship for specifying h_p for a timed transfer system which includes one transit centre as follow:

$$h_p = j_1 (T_1/N_1) = j_2 (T_2/N_2) = \ldots = j_i (T_i / N_i) \qquad (1)$$

Where, T_i is the cycle time of line i, N_i is the number of transit vehicles in line i, and j is an integer number. This relationship shows that the headways are set so that the headways for all intersecting lines in the network are integer multiples of the pulse headway. When headways are fairly similar, a common headway could be allocated to all lines. In contrast, when the network comprises lines with notably different frequencies, headway values could be fixed on integer multiples of the smallest headway (Ting & Schonfeld, 2005). Since h_p is dependent on the cycle time, its value is different for peak and off-peak periods. Timed transfers have become more attractive with the growth of hub-and-spoke network designs (TRB, 2009).

4.2 Transfer Optimisation

While the timed transfer system aims to maximise the simultaneous arrivals of transit vehicles at transit centres, transfer optimisation method is intended to minimise the total transfer waiting time all over the transit network. The total transfer waiting time is considered as the sum of all transfer waiting time spent by transferring passengers in a planning period. This approach considers all feasible transfers between transit lines in the network and attempts to set the lines timetables so that the total transfer waiting times becomes minimised in the entire network. In fact, TTS eliminates a large number of transfer points and focuses on maximal synchronisation (Ceder *et al.*, 2001). Nevertheless, transfer optimisation method considers all feasible transfers in the network in all direction.

Although minimising the total transfer waiting time is the main objective in transfer optimisation approach, other modified objectives have also been considered in the literature. *Minimising the maximal waiting time* is a sort of modified objective for this approach. This objective is based on the idea of preventing extremely long waiting times for transferring passengers, which surely discourage transit users (Daduna, 1995). This objective can be considered either by itself or in addition to the minimising the total transfer waiting time as a multi-objective approach. Assuming different time-values in the transfer optimisation approach is another modified objective considered in a few of the previous studies. This objective is based on classifying passengers into different groups and assigning different time-values to their waiting times.

Regardless to the objective, transit schedule synchronisation is an optimisation problem in which an objective function, either simultaneous arrivals or total transfer waiting time, is to be optimised subject to a set of constraints. These constraints are usually the operational limitations imposed by system characteristics, such as line frequencies, service hours and so on. In other words, trade-offs need to be made between passengers' perspective, which is shorter transfer waiting times, and operators' perspective, which is operation costs (Guihaire & Hao, 2008). The problem specifications are presented in the following section.

5. SCHEDULE SYNCHRONISATION AS AN OPTIMISATION PROBLEM

The synchronisation is the most difficult task for transit planners and schedulers (Ceder et al. 2001). This task is sometimes accomplished intuitively in practice by simplifying the problem in the favour of coordination in a few key points in the network. However, a network-wide synchronisation is a complex task by nature. As discussed in a wide range of previous studies, like (Guihaire & Hao, 2008 ; Shafahi & Khani, 2010 ; Ceder *et al.*, 2001 ; Cevallos & Zhao, 2006 ; Castelli *et al*, 2004 ; Shrivastava & Dhingra, 2002), transit timetable synchronisation in any form (timed transfers or transfer optimisation) is a complex optimisation problem.

While it is often desirable at a localised level to shift a particular scheduled arrival time of transit vehicles, it may be undesirable and impractical on a network wide basis because this shift affects other coordinated arrival times along the line. This also can change even headway of the lines to uneven headways, which leads to uneven service performance for waiting passengers and uneven loading of services for operators (Currie & Bromley, 2005). In fact, network-wide synchronisation imposes plenty of considerations and constraints to the problem. Moreover, trade-offs need to be made between passengers' requirement (shorter transfer waiting times) and operators' perspective (operational costs). Hence, schedule synchronisation is a complex optimisation problem and relies on the operations research (OR) methods.

Similar to any optimisation problem, the schedule synchronisation problem has three vital aspects: (1) selecting controllable parameter(s) as decision variable(s), (2) formulating the objective function and the set of constraints, and (3) specifying an efficient solution method. The decision variable which has been unanimously selected in the previous research is the departure time from the first stop in each line in the selected planning period. Since the departure times are set in minute in practice, this variable only takes integer values. However, the other influencing parameters may take either real or integer values. Therefore, the problem falls within the Integer Programming (IP) problems, which are by far harder to solve in comparison with Linear Programming (LP) problems. In terms of formulation, the schedule optimisation problem has been modelled variously so far. The diversity in the problem formulation in the literature is mainly due to the difference in underlying assumptions and simplifications, resource limitations (very case-specific) and the influencing factors selected as model parameters. However, two classical forms for this problem in the literature are Mixed-Integer Programming (MIP) and Mixed-Integer Nonlinear Programming (MINP). The MINP cases have been arisen in the literature when the linearity assumptions were violated in the objective functions, the sets of constraints or the both. The schedule synchronisation problem has been also modelled as an assignment problem (quadratic assignment or quadratic semi-assignment problem) in a few of studies, like in (Daduna, 1995). However, MIP and MINP are the main forms for this problem in the literature.

The intractability of the synchronisation problem is because of the need to search for the optimum solution in a very large search space made up by all possible solutions. In other words, permutation and combination of all possible departure times for all lines in a transit network creates a very large search space and make this optimisation problem extremely difficult to solve for real-world transit networks. Let us imagine a simple network consisting of three intersecting line-directions with different frequencies (Figure 1). Let us consider the departure time from the first stop in each line (*d*) as the decision variable by which we intend to minimise the transfer waiting time at the transfer points. This variable can take any integer value within the headways (*h*). Therefore, there are *n.m.p* possible permutations for setting the timetables. Considering equal headway (*h*) for such a network consisting of *r* intersecting line-directions, there are h^r possible settings for the first departure times. Therefore, the complexity of this problem is increased exponentially by increasing the network size (e.g. number of transit lines and transit centres) so that it becomes a large combinatorial problem even for small transit networks. Ceder *et al.* (2001) discuss that finding the optimum solution for even a small transit network using current computing resources requires even days of running time. Other research attempts, like (Shrivastava & Dhingra, 2002 ; Cevallos & Zhao, 2004) , also emphasise that this problem is an extremely difficult case even for small transit networks. In addition to the problem size, the nonlinearity adds to the problem complexity whenever it is formulated as a nonlinear problem.

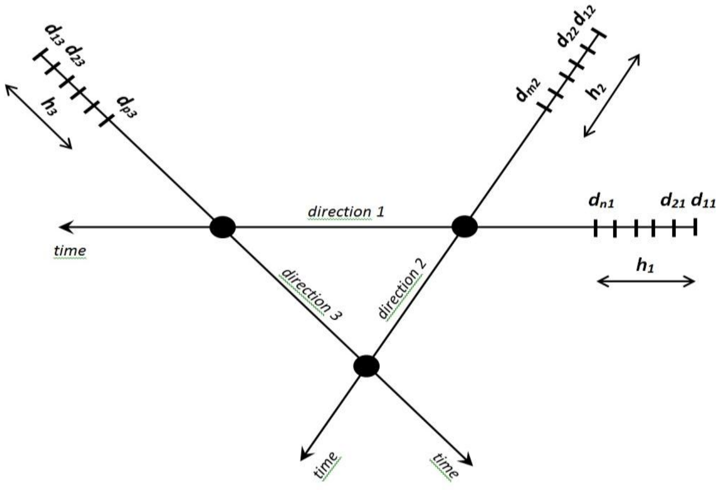

Figure 1. Possible departure times from the first stops in a simple transit network

There is a general agreement in the literature that an accurate mathematical definition of a realistic transit timetable problem with complete search space will result in a NP-hard problem, i.e. the hard problem in non-deterministic polynomial problem class (Cevallos & Zhao, 2006). Such an optimisation problem is unlikely to be solved with conventional computing resources. In other words, if an optimisation problem belongs to the NP-hard class, it is almost impossible to find an efficient algorithm which results in its exact solution. Rojas and Solis (2012) address that NP-hardness of this problem is not a negative result. Rather, it is an important result because researchers will seek approximate algorithms other ways to tackle the problem, instead of spending time on searching for an algorithm to find the exact solution.

Two classes of algorithms are available for the solution of combinatorial optimisation problems: exact and approximate algorithms. The application of exact algorithms to NP-hard problems in practice suffers from a sharp rise in computation time when the problem size increases and their use quickly becomes infeasible most often. Therefore, approximate algorithms are used to seek near-optimum solutions in a relatively short computational time. In other words, the only possibility for solving such problems is to trade optimality for efficiency (Dorigo & Stutzle, 2004). Two groups of approximate algorithms have been used in the literature so far in order to tackle the schedule synchronisation problem. The first group is the heuristic algorithms which have been developed based on the problem characteristics (i.e. problem-specific algorithms). The second group comprises the general-purpose heuristic algorithms (*metaheuristics*) which have been fitted to this problem in the previous studies. The following section presents a summary of such attempts in coping with this problem.

6. MODELLING ATTEMPTS IN THE LITERATURE

As mentioned earlier, transit schedule synchronisation has been tackled in the recent years via various approaches and methods. This section aims to present an insight into the previous studies in relation to the transit schedule synchronisation in order to investigate their approaches and method. Of course, evaluation of all of the previous attempts in detail is out of the limitations of this paper. However, we try to present the main aspects of the studies for the purpose of comparing their approaches.

Ceder *et al.* (2001) presented a Mixed Integer Programming (MIP) model for timetable synchronisation to maximise the number of simultaneous arrivals of buses from different lines at some transit centres. In this model, the headways are not fixed and they considered the variable headways which can vary within a defined range between minimum and maximum headways $[H_{min}, H_{max}]$ as discrete variables. The decision variable, X_{ik}, is also defined as the departure time of *i*th vehicle in line *k*. The difference between X_{ik} and $X_{(i+1)k}$ should not be smaller than $H_{min\ k}$ or greater than $H_{max\ k}$. The frequency of departures in the planning duration is determined for the headway for each line. The presented model is as follow:

$$\max \sum_{k-1}^{M-1} \sum_{q=k+1}^{M} Y_{kq} \tag{2}$$

s.t.

$$X_{ik} \leq Hmax_k\,, \quad 1 \leq k \leq M,$$
$$X_{F_k K} \leq T, \quad 1 \leq k \leq M,$$
$$Hmin_k \leq X_{(i+1)k} - X_{ik} \leq Hmax_k\,, \quad 1 \leq k \leq M,\ 1 \leq i \leq F_k - 1,$$

$$Y_{kq} = \sum_{n \in Akq} \sum_{i=1}^{Fk} \sum_{j=1}^{Fq} \max[\,1 - \left|(X_{ik} + T_{kn}) - (X_{jq} + T_{qn})\right|, 0] \quad (3)$$

Where,
A : the set of bus routes,
T_{kj} : the running time from the starting point of route *k* to node *j*,
M : the number of bus routes in the network,
F_k : the number of departures to be scheduled for route *k* during the interval [0,T], and
Y_{kq} : overall number of simultaneous arrivals of buses in route *k* with buses in route *q*.
In this model, running times are considered deterministic and referred to the mean of running times. They created a heuristic algorithm using Turbo-Pascal to solve this problem for large networks.

Rojas and Solis (2012) tried to improve the model above in order to reduce bus bunching and optimising passengers' transfer. The objective of this model is to maximise the number of simultaneous arrivals of transit vehicles at transit centres during a planning period. In this new model, the synchronisation is defined as the arrivals of two trips with a separation time within a time window to make a flexible formulation. The objective function and constraints in this model are determined mostly to suit a selected case study situation. The Branch and Bound algorithm, as well as multi-start iterated local search algorithm were employed in order to solve this optimisation problem.

While the models above belong to the timed transfer approach, other models have been developed for the transfer optimisation approach. Domschke (1989) modelled the problem as a quadratic assignment problem in order to minimise the total transfer waiting time in a mass transit network, as follow:

$$\min Z(x) = \sum_{hijk} w_{hijk}\ x_{hi}\, x_{jk} \quad (4)$$

s.t.

$$\sum_{i \in Th} x_{hi} = 1 \quad \mathit{for\ all\ routes}\ h = 1, \dots, m$$
$$x_{hi} \in \{0, 1\} \quad \mathit{for\ all}\ h\ \mathit{and}\ i$$

Where, w_{hijk} is sum of waiting times for all passengers who want to change from route *h* to route *j* or vice versa, if route *h (j)* departs at the *i-th (k-th)* possible time within its cycle. Binary variables x_{hi} (and x_{jk}) can take 1 if route *h* should depart at its *i-th* time (otherwise 0). In this model, *m* is the number of different routes and T_h is the number of possible different departure times of route *h*. This problem was solve using heuristic algorithms including regret methods and simulated annealing. Klemt and Stemme (1988) also showed that heuristic methods are more efficient for coping this problem. A modified version of this model was presented in (Daduna 1995) which considers the weight parameters in order to reflect the uncertainties frequently happening in real-world. The model was solved using the tabu search.

Voβ (1992) utilised the similar concept and developed a model for synchronisation of mass transit networks via determination of departure times. The model was aimed to minimise the waiting times of passengers at certain transfer stations in a network. An additional objective in this model considers the security distances between the vehicles where different lines partly use the same tracks. This model was considered as network optimisation problem and solved by the tabu search.

Wong *et al.* (2004) presented a timetabling method to maximise synchronisation between railway lines and enabling smooth transfers with minimum waiting time. They considered known transfer times at each interchange station for all passengers. They also made some simplifying assumptions such as unlimited vehicle capacity, as well as exact adherence to the schedule. A mixed integer programming optimisation model is proposed with running times, dwell times and dispatch time of each train as decision variables. This problem is solved using a MIP solver (CPLEX) to obtain values of integer variables. The resulting LP formulation is then solved to determine timetables for all trains.

Shrivastava and Dhingra (2002) developed a model for synchronising the timetables between a train line and feeder buses. Their objective function is the minimisation of transfer time between the services and bus operating costs. They developed a penalised objective function to find the optimum sets of frequencies considering minimisation of transfer time between bus and train and operator’s costs. In their model, penalties are applied if one or more constraints are violated in order to reflect the passengers’ satisfaction from the system. Their objective function and constraints make the problem nonlinear so that it is difficult to solve by classical approaches. Therefore, they used GA for this optimisation problem. The results of this study showed that GA can be used as an efficient approach to cope with transit timetabling problem.

Jansen et al. (2002) proposed a method to synchronise bus timetables in order to minimise passengers transfer time given a network with fixed headways. Stopping time and running time were assumed constant and deterministic and the model aims to set departure times for the first run of each line. First, a non-linear mixed integer model was developed and then, the Tabu search was applied to solve the problem. The model was tested on the city of Copenhagen.

Chakroborty et al. (1997) put the focus on the application of genetic algorithm to determine departure times in a transit network so as to minimise the passengers’ total waiting time considering predetermined fleet size, policy headway and bounds on the stopping time and maximum transfer time. A mixed non-linear program was formulated to model the problem and the genetic algorithm was chosen as the solution method. The genetic representation of a complete schedule is composed of a series of binary digits representing headways and stopping times.

Cevallos and Zhao (2006) developed a model to modify an existing timetable for bus network in order to minimise the total transfer waiting time for the entire network. Their approach is aimed to find the shift for the existing departure times of all bus routes so that the total transfer waiting time becomes minimal. This model considers deviation from the timetable (schedule adherence) and walking time between services. This model also filters unfeasible transfers between lines in the network. In order to prevent imposing more loads on operators, the headways are considered fixed in this model. The proposed model for a bus network with *N* bus routes, *D* transit centres along each route and *B* buses on each route is as follow:

$$\min \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{Di} R_k \sum_{l=1}^{Dj} C_{ijkl} \sum_{p=1}^{Bi} \sum_{q=1}^{Bj} [\,(\, d_{jlq} + \Delta_{jlq} + S_j) - (\, a_{ikp} + \Delta_{ikp} + S_i + w_{ijkl})] \tag{5}$$

s.t.

$$-b_i \le S_i \le b_i \qquad b_i \le \frac{1}{2}\, h(i)_{min}$$

$$-b_j \le S_j \le b_j \qquad b_j \le \frac{1}{2}\, h(j)_{min}$$

$$d_{jlq} \ge (\, a_{ikp} + w_{ijkl})$$

Where,
a_{ikp} : the arrival time of bus p for route i at transfer point k,
d_{jlq} : the departure time of bus q for route j at transfer point l,
Δ_{ikp} and Δ_{jlq} : deviation from schedule,
w_{ijkl} : walking time between routes i and j,
S : the time shift, b is time interval for time shifts, and
R_k : the ridership demand at transfer point k.

C_{ijkl} is 0 if there is no connectivity at transfer points from route i to route j and equals 1 there is connectivity between routes i and j at transfer points k and l. They used the genetic algorithm to find the optimum solution for the problem. This model was tested on a medium size bus network with 40 routes as a case study and revealed the improvement in the quality of transfers in the bus network.

Shafahi and Khani (2010) used genetic algorithm to tackle timetable synchronisation problem. They formulated new mixed integer programming (MIP) models for creating a timetable for any type of transit network. Their objective function aims to minimise the total transfer waiting time in the transit network. They assumed uniform headways and deterministic running times and presented the following model for specifying the departure time of vehicles from the first stop for a network with R transit lines, S transit centres:

$$\min z = \sum_{s}\sum_{ij}\left(\frac{D}{h_i}\, p_{ij}^{s}\, AWT_{ij}^{s}\right) \tag{6}$$

s.t.

$$WT_{ij}^{s} = \left(X_j + t_j^s + dt_j^s + h_j Y2_{ij}^{s}\right) - \left(X_i + t_i^s + tt_j^s + h_1 Y2_{ij}^{s}\right);\ \ \forall\, i \in R, j \in R, s \in S$$
$$AWT_{ij}^{s} = WT_{ij}^{s} + \left(\frac{h_j - g_{ij}}{2}\right);\ \ \forall\, i \in R,\ \ j \in R,\ \ s \in S$$
$$WT_{ij}^{s} \geq 0;\ \forall\, i \in R,\ \ j \in R,\ \ s \in S$$
$$WT_{ij}^{s} < 0;\ \forall\, i \in R,\ \ j \in R,\ \ s \in S$$
$$AWT_{ij}^{s} < h_j;\ \forall\, i \in R,\ \ j \in R,\ \ s \in S_j$$
$$X_k < h_k;\ \ \forall\, k \in R$$
$$X_k \geq 0;\ \ \forall\, k \in R$$
$$Y1_{ij}^{s}, Y2_{ij}^{s} = integer;\ \forall\, i \in R,\ \ j \in R,\ \ s \in S_j$$

Where,
D : the planning duration,
h : headway,
p_{ij} : the number of transferring passengers,
AWT : the average transfer waiting time,
X : the departure time of the first vehicle,
t : the running time of transit vehicles,
dt : the stopping time of vehicles,
Y : integer variables and g is the greatest common divisor of h_i and h_j.

They employed genetic algorithm to find the first departure times by which the total transfer waiting time for the whole network becomes minimal. They tested their model on a medium size network and a big size network. The results showed around 15% reduction in total transfer waiting time in the network.

Fleurent *et al*. (2004) describe the concepts that are implemented in the commercial software *Hastus* to generate synchronised transit timetables. Their study aimed to find an objective function which reflects more concerns of schedulers, particularly minimum, maximum and ideal waiting times, as well as the importance of transfers with respect to different times, places, routes and directions. The concept of trip meet was introduced in this study as the possible connection between an on trip (the first trip) and a related trip (the second one) at a transit centre. For each transfer, a weight factor as well as minimum, maximum and ideal waiting times, were provided in this study. An individual quality index (QI) is defined to assess each trip meet. A global synchronisation quality index (SQI) is also defined to measure the quality of synchronisation for the whole network. The Lagrangian relaxation, as well as several heuristic algorithms, were used to solve the problem. The discussion about these approaches and models will be presented in the following section.
Table 1 presents the comparison of some research attempts conducted so far on transit synchronisation problem in accordance with their objectives, optimisation methods and application scales.

Table 1. Comparison of studies on transit schedule synchronisation

Author(s)	Year	Objective	Optimisation method	Application
Chakroborty *et al.*	1995 1997	Minimising passengers' total waiting time	Heuristic (branch and bound)	Network wide
Chakraborty *et al.*	2001	Minimising passengers' waiting time and fleet size	Metaheuristic (genetic algorithm)	Network wide
Ceder *et al.*	2001	Maximising simultaneous arrivals	Heuristic	Bus network
Shirvastava *et al.*	2002	Minimising total transfer waiting time and operator's cost	Metaheuristic (genetic algorithm)	Train and feeder buses
Jansen *et al.*	2002	Minimising total transfer waiting time	Metaheuristic (Tabu search)	Bus network
Wong *et al.*	2004	Minimising waiting time	Mathematical (CPLEX)	Railway systems
Castelli *et al.*	2004	Minimising total transfer waiting time and operator's cost	Lagrangian heuristic	Different group of passengers
Fleurent *et al.*	2004	Timetable synchronisation quality	Software (Hastus)	Network wide
Currie *et al.*	2005	Timetable synchronisation quality	-	Network wide
Cevallos *et al.*	2006	Minimising total transfer waiting time	Metaheuristic (genetic algorithm)	Bus network
Shafahi *et al.*	2010	Minimising total transfer waiting time	Metaheuristic (genetic algorithm)	Network wide
Ibarra-Rojas *et al.*	2012	Maximising simultaneous arrivals and reducing bus bunching	Heuristic (Branch and bound, MILS)	Bus network

7. DISCUSSION

As presented in the previous sections, transit schedule synchronisation has been tackled via various approaches and methods so far. In spite of those valuable attempts, there are some gaps in this area of research, still requiring more research efforts. Evaluation of the existing methods reveals that they are basically different according to their objectives, formulations and solution methods. This section aims to present a discussion about the important features of these methods.

7.1 The best objective

Evaluation of the previous studies has shown that the most appropriate objective for the timetable synchronisation is still vague. In fact, it is not clear which objective (timed transfer or transfer optimisation) is more efficient for synchronising transit timetables. Some studies have focused on maximising trip meets (simultaneous arrivals) at certain transfer points as an effective timetabling technique. Those emphasise the importance of eliminating a large number of transfer points due to their adverse effect on the users. Furthermore, some research, like (Currie & Bromley, 2005), discuss that minimising transfer waiting time can be a wrong objective although it is attractive. They emphasise that if times are too short, some groups of passengers, particularly those have mobility difficulties, may not be able to make a given transfer. In contrast, transfer optimisation approach has been strongly supported by other researchers. They have tried to minimise the total transfer waiting time in the network by taking all feasible transfers at all transfer points into account. It is often believed that the timed transfer approach tends to target the major interchanges and ignore the issue of coordination at second and third level interchanges. Moreover, some studies addressed that the use of timed transfers is mainly due to the lack of practical methodologies for achieving synchronisation at all transfer points in a transit networks. Hence, it seems essential to apply these two approaches to the same cases for the purpose of comparing their strengths and weaknesses. During this study, however, no research has been found about comparing these different objectives.

7.2 Assumptions and formulations

Regardless to what approach is selected, the mathematical programming models developed in the previous studies still require improvements in order to lead to more reliable results. The existing deficiencies mostly rely on the underlying assumptions. The problematic assumptions in the schedule synchronisation models can be summarised as follows:

i. The running times between stations/stops are known and fixed
ii. All transfer times are known and fixed for all groups of passengers at all transfer points
iii. The number of transferring passengers are known and fixed
iv. The number of transfers is independent of the waiting time
v. The vehicle capacity is sufficient at any time to embark all entering passengers

Even though such assumptions reduce the complexity of the problem and make it tractable, they result in vulnerable models which cannot represent plenty of uncertainties in transit systems. For instance, the vehicle running time between successive stops/stations has been considered deterministic in almost all of the models. Even if a few models consider a parameter for schedule deviation, this parameter is assumed deterministic and needs to be determined by schedulers. However, it is obvious that running time is stochastic by nature and is more likely to deviate from the planned timetable. Therefore, it is essential to reflect service reliability in the models by considering stochastic variables. Overall, utilising deterministic models for transit timetabling seems useful where systems work on a reasonably reliable basis (e.g. rail modes). However, a lot of considerations for other transit modes, like bus system. It should be noted here that entering stochastic variables into the models will add more complexity to the synchronisation problem and make it harder to solve. However, neglecting uncertainties in transit systems may lead to a solution which may be even worse than an un-planned system.

Transfer importance could be another critical factor, which has not been considered in most of the previous models. In reality, some transfers can be logically more important than others according to the transfer locations and times. The transfer importance can be affected by travel pattern and network configuration in urban areas. Considering different time-values, a few of the previous studies have tried to reflect the importance factor corresponding to different groups of user. Assigning importance factor to transfers can result in more flexible models for synchronising timetables according to different planning purposes.

7.3 Model validation

Model validation is a critical step in operation research. This process comprises a range of sensitivity analysis in order to determine the sensitivity of the optimum solution to model specifications. In fact, such analysis demonstrates how robust the optimum solution is under inaccuracies in input data and structural assumptions (Murty, 1995). The necessity of validation is due to the likeliness of change in model parameters under difference conditions (e.g. change in resource limitations). Even though most of the models developed so far have been applied to real-world transit networks, they have not been validated under variation of model parameters. The operational characteristics of transit systems are very likely to change under different circumstances (e.g. peak and off-peak hours). Therefore, validation of the schedule synchronisation models is crucial to ensure their reliability. Amongst all the models investigated in this study, only the model developed by (Wong, 2008) has been validated by varying the model parameters, including dwell times, headways and running times.

7.4 Solution methods

As discusses in the previous sections, transit timetable synchronisation is a complex optimisation problem for which the exact solution methods cannot be used efficiently. Therefore, the quality of the optimum solutions (i.e. timetables) is significantly affected by the efficiency of the selected approximate method. As presented in Section 6, different mathematical and heuristic algorithms have been applied to this problem so far. However, more attempts are needed to find more efficient algorithms to solve this problem.

Similar to other complex optimisation problems, general-purpose heuristics (e.g. genetic algorithms, tabu search, etc.) have drawn the attention of researchers over the past years to cope with this problem. The efficiency of these algorithms is strongly dependent on their basic operators and parameters. In fact, fitting such algorithms to an optimisation problem requires a wide range of research, itself. Nevertheless, few studies have paid attention on specifying the most effective operators and parameters through parametric analysis. In other words, the main attention has paid on model construction rather than on improving the solution methods although it is a crucial aspect in coping with any optimisation problem.

8. CONCLUSION

Transit timetable synchronisation is an important element in transit planning, which has been approached via different methods over the past years. This paper presents an insight into the schedule synchronisation problem and discusses about the fundamentals of these methods. A wide range of research has been collected and studied deeply in order to investigate the similarities and the differences in the methods developed in the literature in this regard.

As discussed in this paper, extensive research efforts have been dedicated to reducing the negative effects of transfers in transit networks. However, this is still an active domain of research and requires more attempts to remove the existing deficiencies. The most appropriate objective for this problem is not unanimously known yet. More studies are still needed to disclose the strength and the weaknesses of different objectives for this problem. This study also shows that the existing deterministic models cannot purely represent the transit operation in real world. Therefore, considering uncertainties in the operational parameters (e.g. running time, dwell time, transit time, transfer numbers, etc.) can lead to more realistic and applicable models, as emphasised in the literature. Validation of the existing models, which has been neglected in most of the studies, is also a critical aspect in the schedule synchronisation, making the models more reliable under different operational conditions. Furthermore, the solution methods, also require more attention in order to attain high quality solutions.

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