alternately instead of an individual one, i.e., let N_1 trains in A to B direction go first and alternate with N_2 trains in B to A direction go (N_1:N_2 ratio) as shown in Fig. 5. Equation 3 shows the result of railway capacity based on this strategy.

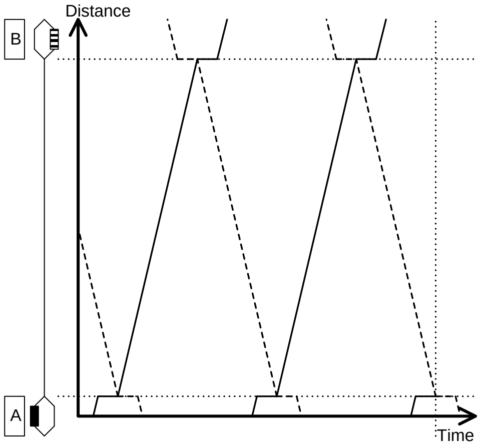

Figure 4. Time-space diagram for single-track system without a side track (1:1 ratio)

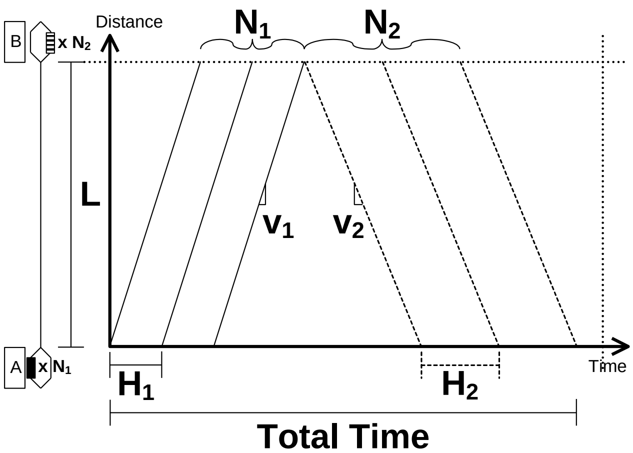

Figure 5. Time-space diagram for single-track system without a side track (N_1:N_2 ratio)

$$Railway\,Capacity = \frac{N_1 + N_2}{(N_1 - 1)\left(w + \frac{B + l_1}{v_1}\right) + (N_2 - 1)\left(w + \frac{B + l_2}{v_2}\right) + \frac{L}{v_1} + \frac{L}{v_2}} \qquad (3)$$

From Equation 3, it should be noted that if $N_1 >> N_2$, the capacity of single-track system will approach the double-track system since no opposing train can travel on this track. In addition, the storage for waiting trains at each station is limited. In practice, the "total time", the maximum allowable time to change the train direction, must be given.

Side track system is a way to reduce the headway between trains and increase railway capacity. Fig. 6 shows that a side track in the middle between two train stations could double railway capacity, and could increase the capacity up to the double-track capacity. However, this case is not always true if the positions of side tracks are not equally located in strategic manners.

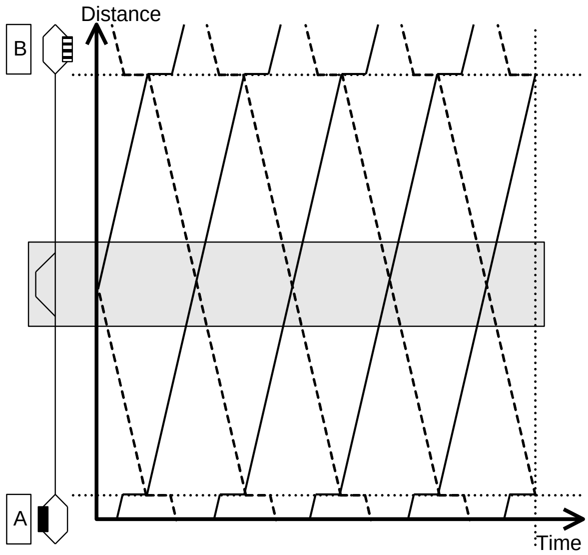

Figure 6. Effect of a side track to railway capacity

From Fig. 6, we could imply that if trains on both directions travel at the same speed, side tracks must be locate at equal distance along the single track. If the number of side tracks are not fitted or not evenly located, the effect of capacity increase will be reduced. Equation 4 shows the number of side tracks (*n*) that would fit for the operations as follows.

$$n = \frac{L}{H_2}\left(\frac{1}{v_1} + \frac{1}{v_2}\right) \tag{4}$$

where,

H_2 = Train headway for the less-priority direction (higher headway)

Since the distance between side tracks should be even for maximum effect, the ratio between the headway for the higher-priority direction and the one for the less-priority must be an integer, i.e., $H_2 = mH_1$, where m is a positive integer. However, since $H_1 = \frac{1}{w + \frac{B + l_1}{v_1}}$, therefore we can determine H_2 from the Equation 5 below.

$$H_2 = min\left\{ \frac{1}{w + \frac{B + l_2}{v_2}},\ m\left(\frac{1}{w + \frac{B + l_1}{v_1}}\right) \right\} \tag{5}$$

After that, the railway capacity for homogeneous single-track system with side tracks can be determined by substituted H_1 and H_2 in Equation 2.

4.2 Analysis of Non-Homogeneous System

The non-homogenous system is a boarder and more complex case when trains on the same direction travel in different speeds and need to overtake one another to increase higher capacity. To simplify this system, we assume that there are only two types of trains, i.e., freight or "slow" trains and passenger or "fast" trains. Each track scenario is shown below.

Double-track scenario

When a fast train is followed by a slow one, it can overtake a slow one through three strategies as follows.

(1) Use an opposing track for overtaking This strategy is to use the opposing track when no train is coming. It is similar to traffic operations on a two-lane highway. Fig. 7 shows this strategy on a time-space diagram. The benefit of this strategy is the increase in railway capacity without building more tracks. However, it requires high automated precision in track alteration; otherwise, it could lead to a major train accident. This is not feasible for an existing Thailand's train system since most operations are manually done and regularly experience substantial delays.

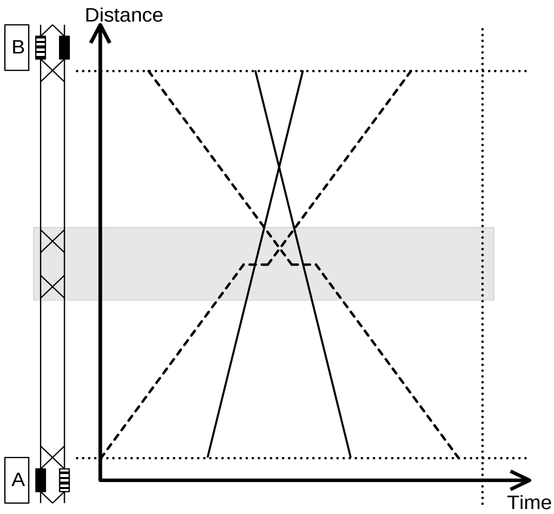

Figure 7. Time-space diagram for using an opposing track for overtaking

(2) Overtake at stations only In this strategy, fast trains are allowed to overtake slow ones at stations only. The calculation of one-way capacity with fast and slow trains is analogous to the one of two-way capacity in homogeneous single track without a side track scenario. The strategy to maximize capacity is to group the same train types, i.e., a group of fast trains and a group of slow trains, together and alternate them, according to the illustration in Fig. 8. The railway capacity in this case can be calculated by Equation 6 below.

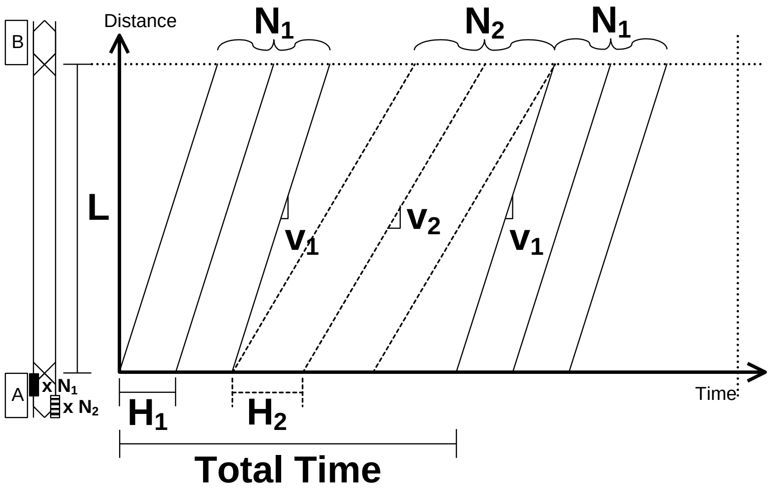

Figure 8. Time-space diagram for double-track system without a side track

$$Railway\,Capacity = \frac{N_1 + N_2}{(N_1)\left(w + \frac{B + l_1}{v_1}\right) + (N_2)\left(w + \frac{B + l_2}{v_2}\right) + \frac{L}{v_1} - \frac{L}{v_2}} \tag{6}$$

(3) Use side tracks This strategy applies side tracks for passing opportunities in the same direction as shown in Fig. 9. The concept is similar to the adding of a passing lane on a two-lane highway. It reduces headways between trains and increase capacity. Similarly, side tracks must be purposefully located evenly in appropriate numbers to maximize the capacity. The number of side tracks (*n*) that would fit for the operations are shown in Equation 7:

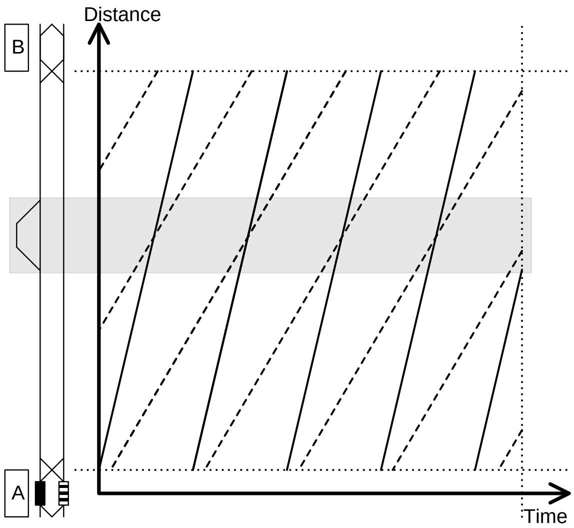

Figure 9. Time-space diagram for one-way double-track system with a side track

$$n = \frac{L}{H_2}\left(\frac{1}{v_2} - \frac{1}{v_1}\right) \quad (7)$$

where,

H_2 = Train headway for the less-priority direction (higher headway)

In this case, the ratio between the headway for the higher-priority direction and the one for the less-priority must be an integer, i.e., $H_2 = mH_1$, where m is a positive integer. Since $H_1 = \frac{1}{w + \frac{B + l_1}{v_1}}$, therefore we can determine H_2 from the Equation 8 below.

$$H_2 = min\left\{\frac{1}{w + \frac{B + l_2}{v_2}},\ m\left(\frac{1}{w + \frac{B + l_1}{v_1}}\right)\right\} \quad (8)$$

After that, the railway capacity for non-homogeneous double-track system with side tracks can be determined by substituted H_1 and H_2 in Equation 2.

Single-track scenario

This scenario is the general case on Thai single-track railway system. Without a side track, the operation is similar to the homogenous single-track without a side track in Fig. 9 since all fast trains must follow a slow one until they reach a station. However, if side tracks are built as shown in Fig. 10, the problem is much more complex since passenger trains can overtake a freight train at a side track or a station only and other trains could not use the track until it is available. This strategy is quite difficult to manage since one side track must be used for both train directions and required automated control to avoid train accidents, which would not occur in Thailand presently.

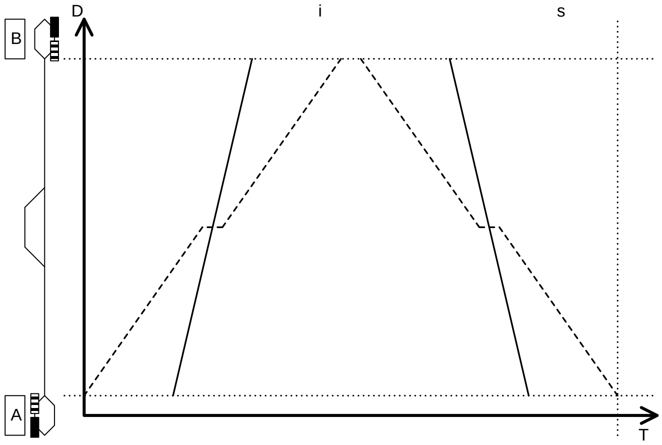

Figure 10. Time-space diagram for non-homogeneous single-track system with a side track

5. EMPIRICAL RESULTS

Table 2 shows empirical capacity results based on the substitution the values of variables from Table 1 or actual operating data in Equations 2-8. This table assumes 2 hours of the maximum allowable time to change the train direction, or “total time” in Figs. 5 and 8, and that the positions and numbers of side tracks are ideal and yield maximum possible rail capacity.

Table 2. Empirical results of railway capacity (train/day) from actual operating data

System	High Priority	Low Priority	Total	High Priority	Low Priority	Total	High Priority	Low Priority	Total
Homogeneous System	PT vs PT			FT vs FT			PT vs FT		
Double-track (Eq. 2)	111	111	222	105	105	210	111	105	216
Single-track without side tracks* (Eq. 3)	63	12	75	25	12	37	39	13	52
Single-track with side tracks** (Eqs. 2, 4, 5)	111	111	222	105	105	210	111	105	216
Non-homogeneous System	PT vs PT			FT vs FT			PT vs FT		
Double-track without side tracks*	Not applicable						90	90	180
Double-track with side tracks on both sides (Eqs. 2, 7,8)	Not applicable						216	216	432

Note:

PT = Passenger train;

FT = Freight train

*Assume that the maximum allowable time to change the train direction, or “total time”, is 2 hours

** Assume that side tracks are located strategically and evenly to yield maximum possible rail capacity.

The empirical results show that double-track system yield almost three times as much as the one for single-track without side tracks. With side tracks, it is possible that the capacity of single-track system could be increased as much as the double-track one. Nevertheless, these simple calculations do not take the delay due to deceleration or waiting time at the side tracks into consideration. Also, it might not be feasible to build side tracks at all strategic locations due to geographic, land-use, or financial constraints. Therefore, for real situations, the capacity of single-track system with side tracks could be 30-50% much less than the ideal condition.

6. CONCLUDING REMARKS

In summary, this research shows the time-space diagram analysis technique for creating models comparing single- and double-track railway system capacities, with the empirical data from State Railway of Thailand. It uses graphical illustration to determine the relationships among operating variables into a set of simple equations. These equations could be used to roughly determine railway capacities in different scenarios. We found that the capacity of single-track system with side tracks could be 30-50% much less than the ideal condition and the capacity of double-track system is approximately three times as much as one of single-track system. Although the results are somewhat rough, they are quite useful for rail operators and planners in Thailand.

This research has some limitation. First, some variables that affect operations such as slow-down when moving to side tracks were excluded. Second, the number of side tracks in Equations 4 or 8 might not be a whole number or side tracks are not located in equidistance manner, this would significantly reduce capacities. In addition, this research did not take the stochastic effects of train speeds, waiting times at stations, etc, into the calculations.

For future research direction, more related operating variables should be added for more realistic scenarios. Computer simulation might be used to investigate the stochastic effect of independent variables as well as do sensitivity analysis. In practice, side tracks could be expensive and could not be built at all strategic locations. The benefit-cost analysis of adding more side tracks and finding the most economically suitable locations could be useful for rail transportation planners and designers.

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