Robust Network Design with Supply and Demand Uncertainties in Humanitarian Logistics

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Abstract: Humanitarian logistics has gain attention as an important tool in disaster management. We propose a network design for relief distribution under several uncertain parameters based on robust optimization. The model has the solution robustness and model robustness properties. Furthermore, we present a methodology to reduce the number of variables when an equality constraint and objective function contain same variables. Our model attempts to minimize total cost of the system as well as the variance of total cost. We examine a case study on the earthquake scenarios in Bangladesh to show the applicability of the model. Our findings show that the model is robust in relief distribution planning. We analyze sensitivity of several parameters and compare several models to show the superiority of stochastic model.

Keywords: Bangladesh, Humanitarian Logistics, Relief, Robust, Uncertainty

1. INTRODUCTION

A disaster is an unforeseen and often sudden event that causes damage, destruction and human suffering (Interagency standing committee, 2006). Donor societies (e.g. non-government organization, non-profit organization, country government, and donation foundation) play key roles to reduce victim's suffering by providing relief (e.g. food, water, and shelter). The timely and effective distribution of relief is essential for aiding victims. It is natural that pre-disaster preparations strongly affect post-disaster activities. Hence, strategic planning is recommended. A decision maker chooses an option for disaster preparation in strategic stage from several available alternatives. The selected option is subject to a certain number of constraints, and the goal is to optimize certain given criterion. However, some parameters of the problem are unknown at the time of selecting the option. Generally, it is assumed that the decision maker is given a description of these unknown parameters in terms of a well-determined probability law. This sort of problems is called decision making under uncertainty or stochastic programming problem (Roger and Wets, 1974). The stochastic nature is relevant to humanitarian logistics and ensuring the robustness of the solution for all scenarios is required.

The objective of humanitarian logistics is to provide relief to victims after disaster, to minimize human suffering and death (Balcik and Beamon, 2008). Several authors define humanitarian logistics in different ways and the terms ‘disaster relief’, ‘emergency logistics’ and ‘humanitarian logistics’ are used interchangeably (Kovacs and Spens 2007). Sheu (2007) defines humanitarian logistics as a process of planning, managing, and controlling the efficient flows of relief, information, services from the points of origin to point of destination to meet the urgent needs of the affected people under emergency situation. Very recently after acknowledging the importance of logistics planning in disaster, few studies (Barbarosoğlu *et al*. 2002, Balcik and Beamon, 2008, Rawls and Turnquist, 2010) have proposed model incorporating uncertainty. The stated study addresses mostly demand uncertainty in humanitarian logistics. Demand uncertainty is apparent due to disaster per se parameters, for instance location, time, and intensity, cannot be predicted. Addition with it, disaster affects supplier's capacity. The reasons of the reduction of supplier's capacity are transportation link disruption and capacity limitation. The post-disaster supplier's capacity strongly affects the post-disaster procurement and need to be considered in strategic planning. In this regard, the proposed model incorporates demand uncertainty and supply uncertainty in the framework of robust optimization. The purposes of this study are to resolve a humanitarian logistics network for each of candidate location and inventory level through incorporating supply uncertainty and demand uncertainty based on robust optimization. This trade-off between the pre-disaster cost and the post-disaster cost is also presented here. To put light on our model, generally network model generates a large number of variables and is difficult to solve. We show a method to minimize the number of variables in the model to keep the model tractable. The main contribution of this paper are summarized as follows

- The proposed model incorporates demand and supply uncertainty and provides robust solution. Model robustness has also been ensured.
- A network model generally produces a large number of variables. A particular technique is adopted to decrease the total variables in model.

We have applied the model for a case study to show the applicability. The remainder of the paper is organized as follows. In Section 2, we introduce the problems and our assumptions to formulate the problem. Section 3 formulate mathematical model and explain the distinct features. A numerical example is explained in Section 4 and the results of the case study are discussed. Finally, Section 5 summarizes the main contributions and concludes the study.

2. PROBLEM DESCRIPTION AND ASSUMPTIONS

Our problem posits possible affected areas that may be hit by a disaster like earthquake and candidate relief distribution centers (RDCs) where resources already exist and /or can be pre-positioned. The proposed humanitarian logistics network becomes of three stages and two echelons as shown in Figure 1.

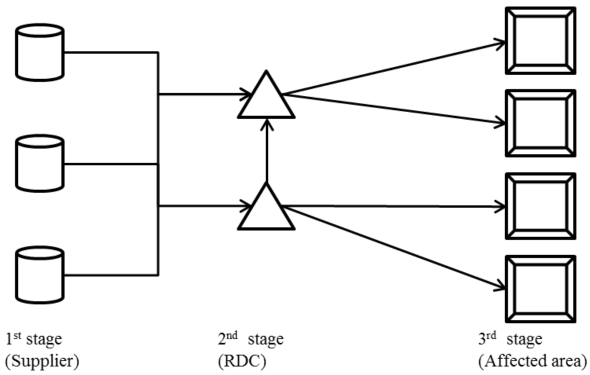

Figure 1. The relief chain structure

The first stage is the set of suppliers, the second stage contains RDCs and the last stage consists of affected areas. Concerning the selection of the RDC locations from a set of candidate RDCs, certain issues have to be addressed, namely, (1) the storage capacity of the RDCs (2) the distance to the affected people that keep the transportation costs at minimum and (3) post-disaster supply with respect to supplier's capacity.

Before the mathematical formulation is considered, we make the following assumptions on the problem:

(1) The capability of suppliers may be partially disrupted by a disaster through damage to the roads and/or miscellaneous reasons
(2) All affected area nodes are candidate for the pre-positioned of RDCs
(3) Transportation cost is not scenario dependent
(4) Each demand point may be served by multiple RDCs
(5) The locations specified are cities
(6) Two disasters will not occur simultaneously
(7) The relief demand is dependent on population density and earthquake intensity

With the above assumptions in place, we consider total cost minimization for the network model. Although cost minimization is not sole objective of humanitarian logistics, total cost is a good measure to compare different outcomes. Note that, total cost is not real monetary value in our model; it is a complex value where delay and human suffering are incorporated through parameter changing.

3. MATHEMATICAL FORMULATION

In this section, we introduce two-stage, stochastic mixed-integer model. This is a location model with the features of linearity and robustness. We explain our model in two steps. First we explain the framework of the model and system properties. All variables and parameters are also introduced here. In the second step, we introduce our mathematical model starting with basic stochastic model.

3.1 Model Framework

In the aftermath of a disaster, there will be demand for relief at specific location $k \in K$. The demand for commodity c at location k is uncertain at the planning stage. Uncertainty is modeled by the set S of discrete scenarios indexed by $s \in S$, each with a probability of occurrence p_s. The definition of a scenario includes the forecasted demand by commodity and location d_{kcs}.

Relief can be pre-positioned at a location j if RDC is made available there. For costing purpose, we define facilities to be in one of discrete set L of size categories indexed by $l \in L$. The overall capacity of a RDC in category l is N_l and choosing to open a RDC of size category l in location j incurs a fixed cost F_{jl}. Let z_{jl} be a binary decision variable equal to 1 if there is a RDC of capacity category l located at node j, and 0 otherwise. This is one of first stage decision in the two-stage model.

If a RDC is made available at location j, various commodities can be stocked there, subject to the capacity limits of the RDC. Let b_c be the unit volume for commodity c and q_{ijc} be the amount of commodity c pre-positioned at location j supplied from supplier i. The q_{ijc} is another first stage decision in our model. The RDC and stocking decisions are made before knowledge of any disaster scenario is available.

After a disaster occurs, the stocks of the various commodities are distributed across a transportation network to meet demands. Commodity c that is not used in scenario s, denoted o_{jcs}, incurs unit overflow cost θ_{kc}. On the other hand, if demand for particular commodity cannot be met in scenario s, denoted u_{jcs}, incurs unit shortage cost $\emptyset_{kc}$ for the commodity c. To reflect the connection between the RDC location and the transportation elements, we assume that the RDC locations are at nodes in the transportation network. Let TC'_c be the post-disaster transportation cost of commodity c from supplier to RDC and TCR'_c is the post-disaster transportation cost from RDC to affected area. Let x_{ijcs} be the amount of commodity c procured from supplier i and transferred to RDC j in the scenario s. Table 1–Table 3 show the explanation of sets, parameters and variables. Units are stated within square brackets (.) at the end of each of the definitions. Table 1 is the collection of all sets definition. Table 2 is the definition of all parameters. d_{kcs}, p_s and ρ_{ics} are scenario dependent parameters.

Table 1. Indices and index sets

Set	Definition
C	set of commodities indexed by $c \in C$
I	set of suppliers indexed by $i \in I$
J	set of candidate RDCs indexed by $j \in J$
K	set of affected areas indexed by $k \in K$
L	set of size of RDC indexed by $l \in L$
S	set of scenarios indexed by $s \in S$

Table 2. Deterministic and stochastic parameters

Type	Symbol	Definition
Pre-disaster parameter	F_{jl}	fixed cost of opening a RDC of size l at location j ($)
	N_l	capacity of RDC size l
	b_c	volume of a unit commodity c (m^3)
	SC_{ic}	delivery capacity of supplier i of commodity c
	PC_c	procuring cost of a unit commodity c before disaster ($ per unit)
	TC_c	transportation cost for a unit commodity c before disaster ($ per unit of c)
Post-disaster parameter	PC'_c	procuring cost of a unit commodity c after disaster ($ per unit of c)
	TC'_c	transportation cost for a unit commodity c after disaster from supplier to RDC($ per unit of c)
	TCR'_c	transportation cost for a unit commodity c after disaster from RDC to affected area ($ per unit of c)
	θ_{kc}	unit overflow cost for commodity c at affected area k ($ per unit of c at k)
	$\emptyset_{kc}$	unit shortage cost for commodity c at affected area k ($ per unit of c at k)
	λ	parameter for post-disaster deviation-cost
	γ	parameter for balance control ($)
	M	a very large positive number
Stochastic parameter	d_{kcs}	amount of demand for commodity c at affected area k in scenario s (unit)
	p_s	probability of scenario s
	ρ_{ics}	ratio of capacity of commodity c at the supplier i in scenario s

Table 3. Decision variables

Symbol	Definition
z_{jl}	1 if RDC with capacity category *l* is located at candidate RDC *j*; 0 otherwise
q_{ijc}	amount of commodity *c* procured from supplier *i* and stored at the RDC *j* ()
u_{kcs}	amount of shortage commodity *c* observed in scenario *s* at affected area *k*
o_{kcs}	amount of extra commodity c delivered in scenario *s* at affected area *k*
x_{ijcs}	amount of commodity *c* transferred from supplier *i* to RDC *j* in scenario *s*
y_{jkcs}	amount of commodity *c* transferred from RDC *j* to affected area *k* in scenario *s*. If *j*=*k*, it represents both RDC and affected area in same location
ω_s	cost variability for scenario *s*
φ_{jcs}	amount of deviation of commodity *c* at RDC *j* in scenario *s*

Table 3 is the list of decision variables. Here, z_{jl} and q_{ijc} are first stage decisions of our model and the variables in last block in Table 3 are second stage variables.

Table 4. Combination of analogous variables

Symbol	Definition
Bt	pre-disaster cost (i.e. Eq. (5))
A_s	summation of post-disaster procurement cost and transportation cost (i.e. Eq. (7))

Table 4 presents the terms that combine analogous variables. Those terms are introduced to make the equation simpler.

3.2 Formulation

First, we explain the basic structure of the model aiming to simplification of the presentation of the model.

$$min\, Bt + E_{\xi}[Q(t,\xi)] \tag{1}$$

$$\text{s.t.} \quad At \geq b \tag{2}$$

$$h(\omega) - T(\omega)t = Wy \tag{3}$$

$$t \geq 0 \tag{4}$$

The objective function in Eq. (1) expresses the cumulative cost of pre- and post-disaster circumstances. First term represents pre-disaster cost (Bt) and second term is post-disaster cost ($E_{\xi}[Q(t,\xi)]$). The pre-disaster cost consists of setup cost (FC_{jl}), procurement cost (PC_c), and transport cost (TC_c). Thus pre-disaster cost is defined as follows.

$$Bt = \sum_{j\in J, l\in L} z_{jl}\, FC_{jl} + \sum_{i\in I, j\in J, c\in C} PC_c q_{ijc} + \sum_{i\in I, j\in J, c\in C} TC_c q_{ijc} \tag{5}$$

Now, the post-disaster cost is scenario-dependent cost which includes procurement cost (PC_c'), transport cost (TC_c' and TRC_c') and deviation-cost. We have defined the summation of procurement cost and transportation cost in post-disaster as A_s. In the first place, we ignore the deviation-cost for simplification of explanation. Thus, the expected post-disaster cost is greater or equal to the summation of procurement cost and transport cost. So

$$E_{\xi}[Q(t,\xi)] \geq \sum_{s\in S} p_s A_s \tag{6}$$

where,

$$A_s = \sum_{i\in I, j\in J, c\in C}(PC_c' + TC_c')x_{ijcs} + \sum_{j\in J, k\in K, c\in C} TRC_c'\, y_{jkcs} \tag{7}$$

The deviation-cost generates from two sources. One source is the differences of post-disaster cost from the average post-disaster cost for all scenarios. The treatment of this sort of deviation-cost is adopted from Li (1996) and is added ω_s in Eq. (6) (underlying method of parameter setting of ω_s presents in appendices). Another source of deviation-cost generates from the balance constraint of commodity. Mulvey and Ruszczynski (1995) suggested adding φ_{jcs} to treat the deviation-cost. In this way, our model gains robustness characteristics. After addition of deviation-cost in the Eq. (6), we get the Eq. (8)

$$E_{\xi}[Q(t,\xi)] = \sum_{s\in S} p_s\, A_s + \lambda \sum_{s\in S} p_s \left[\left(A_s - \sum_{s\in S} p_s\, A_s\right) + 2\omega_s\right] + \sum_{s\in S, j\in J, c\in C} \gamma p_s\, \varphi_{jcs} \tag{8}$$

As shown above, the objective function of our stochastic model becomes as follows with addition of penalty cost

$$\begin{aligned} min\, Bt + \sum_{s\in S} p_s\, A_s + \lambda \sum_{s\in S} p_s \left[\left(A_s - \sum_{s\in S} p_s\, A_s\right) + 2\omega_s\right] + \sum_{s\in S, j\in J, c\in C} \gamma p_s\, \varphi_{jcs} \\ + \sum_{k\in K, c\in C, s\in S} (\emptyset_{kc}\, u_{kcs} + \theta_{kc} o_{kcs}) \end{aligned} \tag{9}$$

The constraints of this model are as follows

Balance control:

$$\sum_{i\in I} x_{ijcs} + \sum_{i\in I} q_{ijc} - \sum_{k\in K} y_{jkcs} = \varphi_{jcs} \quad \forall j\in J, c\in C, s\in S \tag{10}$$

RDC location:

$$\sum_{l\in L} z_{jl} \leq 1 \quad \forall \mathrm{j} \in \mathrm{J} \tag{11}$$

$$y_{jjcs} \leq M d_{jcs} \sum_{l\in L} z_{jl} \quad \forall j\in J, c\in C, s\in S \tag{12}$$

$$\sum_{k\in K} y_{jkcs} \leq M \sum_{l\in L} z_{jl} \quad \forall j\in J, c\in C, s\in S \tag{13}$$

$$\sum_{i\in I} x_{ijcs} \leq M \sum_{l\in L} z_{jl} \quad \forall j\in J, c\in C, s\in S \tag{14}$$

RDC capacity:

$$\sum_{i\in I\; c\in C} b_c q_{ijc} \leq \sum_{l\in L} N_l z_{jl} \quad \forall j\in J \tag{15}$$

Post-disaster demand:

$$y_{jkcs} \le M\left(\sum_{l\in L} z_{kl} + d_{kcs}\right) \ \forall\, j \in J, k \in K, c \in C, s \in S \tag{16}$$

Post-disaster supplier's capacity:

$$\sum_{j\in J} x_{ijcs} \le \rho_{ics} SC_{ic} \ \ \forall\, i \in I, c \in C, s \in S \tag{17}$$

Mean absolute value:

$$A_s - \sum_{s\in S} p_s\, A_s + \omega_s \ge 0 \ \ \forall s \in S \tag{18}$$

Non-negativity constraint:

$$z_{jl} \in \{0,1\} \ \ \forall\, j \in J, l \in L \tag{19}$$

$$q_{ijc}, x_{ijcs}, y_{jkcs}, \omega_s, \varphi_{jcs} \ge 0 \ \ \ \forall\, i \in I, j \in J, k \in K, c \in C, s \in S \tag{20}$$

Penalty function:

$$y_{kkcs} + \sum_{k\ne j\in J} y_{jkcs} - d_{kcs} + u_{kcs} - o_{kcs} = 0 \ \ \forall\, k \in K, c \in C, s \in S \tag{21}$$

The above mentioned two-stage model makes the trade-off between the pre-disaster costs and the post-disaster costs. The objective function of our model is Eq. (9) and the constraints include Eq. (10) – (21).

Eq. (10) is a balance control constraint of the in-coming flow and the out-going flow of relief. One RDC cannot delivery relief more than the summation of inventory and post-disaster procurement. The constraints Eq. (11) – (14) represent feasibility of RDC locations and deliver-ability from RDC. The constraint Eq. (12) explains that one RDC will not deliver more than the demand in same location. The Eq. (15) bound maximum storage limitation. It cannot be more than the RDC capacity. Eq. (17) bounds the post-disaster procurement and right hand sight of this constraint is scenario dependent. In other words, supplier’s capacity is scenario-dependent. The Demand management Eq. (16) restricts the flow more than the demand at affected area. The Eq. (18) shows post-disaster cost variability. This constraint aims to reduce the post-disaster cost variation in different scenarios. The Eq. (19) – (20) are non-negativity and variable type restriction. The penalty function Eq. (21) adds cost for either shortage or overflow.

Both objective Eq. (9) and constraint Eq. (21) contain shortage unit (u_{kcs}), and over-supply unit (o_{kcs}) and Eq. (21) is an equality constraint. These properties force us to add artificial variables and using 'two phase' or 'big M' (Scharge, 1991) method to solve the model. However, those methods will add many extra variables. To solve the model, we have changed the objective function and penalty function in line with Yu and Li (2000).

The objective function turns to

$$\begin{aligned} \min Bt &+ \sum_{s\in S, j\in J, c\in C} \gamma p_s\, \varphi_{jcs} + \sum_{s\in S} p_s\, A_s + \lambda \sum_{s\in S} p_s \left[\left(A_s - \sum_{s\in S} p_s\, A_s\right) + 2\omega_s\right] \\ &+ \sum_{c\in C, s\in S} p_s \left(\sum_{k\in K}\left(\theta_{kc}\left(y_{kkcs} + \sum_{k\ne j\in J} y_{jkcs} - d_{kcs} + \delta_{kcs}\right) + \emptyset_{kc}\delta_{kcs}\right)\right) \end{aligned} \tag{22}$$

Eq. (21) turns to

$$-y_{kkcs} - \sum_{k \neq j \in J} y_{jkcs} + d_{kcs} - \delta_{kcs} \leq 0 \;\; \forall \, k \in K, c \in C, s \in S \tag{23}$$

$$\delta_{kcs} \geq 0 \;\; \forall \;\, k \in K, c \in C, s \in S \tag{24}$$

The Eq. (21) transform to Eq. (23) with introducing single variable δ_{kcs}. After transformation of the Eq. (21), the variables u_{kcs} and o_{kcs} turns to single variable δ_{kcs} and thus the number of variables are reduced in the whole system. The Eq. (24) is added to ensure the positive value. In our final model, the objective function is Eq. (22) and the constraints are Eq. (10) – (20) and Eq. (23) – (24).

4. CASE STUDY

4.1 Study Area

This study selected Bangladesh for case study which is surrounded by several active tectonic faults. The Major faults are Himalyan arc, Shillong and Dauki fault system in the north, Burmese arc and Accretionary wedges in the east, and Naga- Disang-Haflong thrust zone in the north-east. The earthquake records suggest that since 1900 more than 100 moderate to large earthquake occurred in Bangladesh, out of which 65 earthquakes occurred after 1960. The recent earthquake activity in Bangladesh indicates the fresh tectonic activity of propagation of fractures from adjacent seismic zones (Khan *et al.* 2001). In a study by Villacis *et al*. (1999) on 20 cities of the world, Dhaka appeared to have one of the highest values of earthquake disaster risk index (EDRI) mainly due to its inherent vulnerability of building

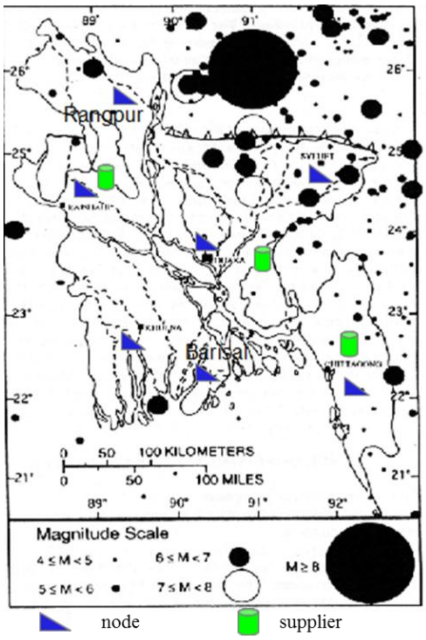

Figure 2. Location of earthquake epicenter in Bangladesh period 1750 to 2000 (source: United States geological survey; adapted from Khan *et al.* 2001), node and supplier added

infrastructure which lacks earthquake resistant features, high population density and poor emergency response and recovery capability.According to Figure 2, we consider three suppliers, named supp1, …, supp3 (Dhaka (Dhk), Chittagong (Ctg), and Rajshahi (Raj)) and 7 demand points, named dem1, …, dem7 (Dhaka (Dhk), Chittagong (Ctg), Rajshahi (Raj), Rangpur (Ran), Barisal (Bar), Khulna (Kul), and Sylhet (Syl)) spread geographically over the entire map. Alam *et al.* (2011) analyzed the earthquake scenarios in Bangladesh and we consider four scenarios, s1, …, s4 with occurrence probabilities of 0.4, 0.3, 0.2 and 0.1, respectively. Alam *et al.* (2011) reported five scenarios for representing earthquake scenarios. We remove one scenario from the list that has the lowest earthquake magnitude; because, there is no relief demand after the lowest magnitude earthquake. In this way, we keep the number of variables tractable without losing the generality.

4.2 Data

Two commodities, namely prod1 and prod2, that may be pre-positioned in RDC. In the example, we assume prod1 represents water and prod2 is shelter. One unit of prod1 consists of 1000 liter of water and one unit of prod2 consists of 1000 unit of shelter. We assume the RDC sizes are available with specific cost as shown in Table 5. RDC setup cost depends on the storage capacity.

Table 5. RDC setup cost and capacity

Size	$F_{jl}(10^3\$)$	N_l (10^3 m^3)
small	500	10
medium	800	16
large	1200	24

Procurement price and transportation cost per unit distance are calculated based on local currency (afterwards, converted to USD). Procurement price in post-disaster situation is more than that of pre-disaster situation. Transportation cost in post-disaster is also higher than the pre-disaster transportation cost. The higher cost in post-disaster situations can also be considered as proxy of delay cost and human suffering. Costs of different items are shown in Table 6.

Table 6. Unit procurement price, transportation cost, and volume of commodity

Commodity (10^3)	PC_c (10^3\$/unit)	TC_c (10^3\$/unit-km)	b_c (m^3/unit)
prod1	0.5	0.6	4.5
prod2	20	1.8	120

We assumed the demand for each scenario by using the population density and earthquake intensity, collected from Alam *et al*. (2011). Note that there is no well accepted methodology for relief demand estimation and literature (Akkihal, 2006, Balcik and Beamon, 2008) suggests using historical relief demand for earthquake disaster. The demand data are shown in Table 7.

Table 7. Demand data

	Dhk (prod1, prod2)	Ctg (prod1, prod2)	Raj (prod1, prod2)	Ran (prod1, prod2)	Bar (prod1, prod2)	Kul (prod1, prod2)	Syl (prod1, prod2)
s1	(319,106)	(222,74)	(238,79)	(225,75)	(0,0)	(0,0)	(579,193)
s2	(476,143)	(1339,446)	(0,0)	(0,0)	(75,25)	(30, 10)	(20,7)
s3	(76,10)	(187,62)	(0,0)	(0,0)	(100,33)	(100,33)	(0,0)
s4	(177,59)	(166,55)	(990,330)	(1654,551)	(21,7)	(20,7)	(94,31)

In the response phase, the available supplier's capacity is scenario dependent and shown in Table 8. It is assumed that supplier’s capacity changed for both commodities.

Table 8. Fraction of available supplier's capacity

	Dhk	Ctg	Raj
s1	0.94	0.95	0.9
s2	0.95	0.95	1
s3	1	0.99	1
s4	0.99	1	0.9

The post-disaster procurement prices are assumed to be 1.5 times of the pre-disaster procurement price and the increment of procurement price also represents delay of delivery of the commodity. The post-disaster unit transportation cost from supplier to RDC is assumed to be 1.8 times of that of the pre-disaster phase and from RDC to affected area is 2.0 times. These data are assumed to be fixed among scenarios. The cost of transportation between nodes is dependent of distance between two nodes. We have collected distances between different nodes using car-route option from the Google Map. It is natural that unit overflow cost (θ) is lower than the unit shortage cost ($\varnothing$). The unit overflow cost is assumed to be kept equal to pre-disaster procurement price of the corresponding commodity. The unit shortage cost is assumed to be the ten times the pre-disaster procurement price of the corresponding commodity (Raws and Turnquist, 2010.) The value of λ is equal to 2. It is a weight parameter for difference between the mean-value of A_s and the A_s for each scenario among different scenarios.

4.3 Results

In this section, we present computational results and analyze the behavior of proposed model. We solve the problem using the mixed-integer linear programming solver 'Gurobi' from neos-server (Czyzyk *et al.* 1998). Gurobi uses branch and cut algorithm for solving mixed-integer problem. We ran the model and the results are described in this section. Table 9 shows that three of five opened RDC are specialized for storing prod1 and prod2. The remaining two RDC do not maintain inventories and assist relief distribution in different scenarios. The total cost of designing the distribution network is 8.3 million dollar.

Table 9. Location and inventory

RDC	Size	prod1	prod2
Dhk	small	323	10
Ctg	small	184.5	62
Raj	small	-	-
Syl	small	20	7
Kul	small	-	-

Table 9 also explains the quantity of each commodity that will be stored in pre-disaster period. The supplier city (see Table 8) in which a RDC is located can take advantage of its relief commodities from supplier to RDC in lower cost. One exception is in Sylhet where supplier is not present but established the RDC and maintain inventory. Table 10 represents the relief distribution in scenario 4

Table 10. Relief commodities transferred from RDCs to demand points (for Scenario 4)

	Dhk (pd1,pd2)	Ctg (pd1,pd2)	Raj (pd1,pd2)	Ran (pd1,pd2)	Kul (pd1,pd2)	Bar (pd1,pd2)	Syl (pd1,pd2)
Dhk	(177,59)	-	-	(146,0)	-	-	-
Ctg	-	(166,55)	(0,7)	-	(21,0)	-	-
Raj	-	-	(550,100)	(254,200)	-	-	-
Syl	-	-	-	-	-	-	(94,31)
Kul	-	-	-	-	(0,7)	(21,7)	-

We analyze the sensitivity with the number of RDC in Figure 3. It can be seen that the objective value decreases when the possible number of RDCs increases until a certain number. After passing the threshold number, the objective value increases again. Thus it concludes that the best value of RDCs is five. In order to arrive at an appropriate solution such that the decision maker will be able to see trade-off between the pre-disaster cost and the post-disaster cost.

In Figure 4 and Figure 5, sensitivity analysis is performed for solution and model robustness against the multiplier of gamma. Figure 4 shows expected cost increases exponentially by increasing the value of gamma. On the other hand Figure 5 demonstrates the penalty cost $p_s \varphi_{kc} \gamma$ will eventually drop to zero with an increase in the value of gamma. Both figures indicate that decision maker can choose gamma value based on the preference. It is suggested to decision maker to select higher gamma value to avoid risk of shortage of relief. Then, we have performed the sensitivity of lambda (λ) value. The model is run for lambda values of '1', '2', '3', '5', and '10'. The objective value of model does not differ noticeably because we have only four scenarios.

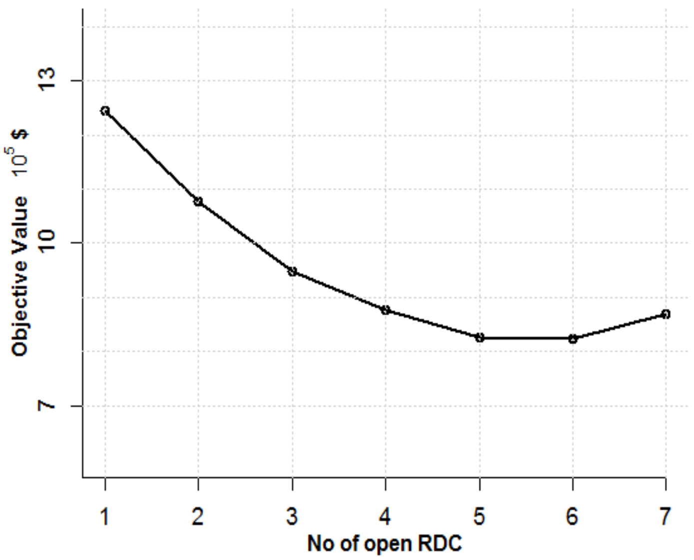

Figure 3. Sensitivity of total cost with the No of open RDC

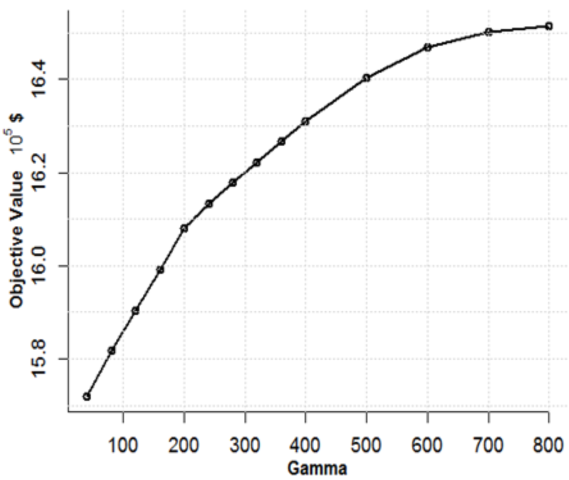

Figure 4. Sensitivity of solution robustness with respect to gamma

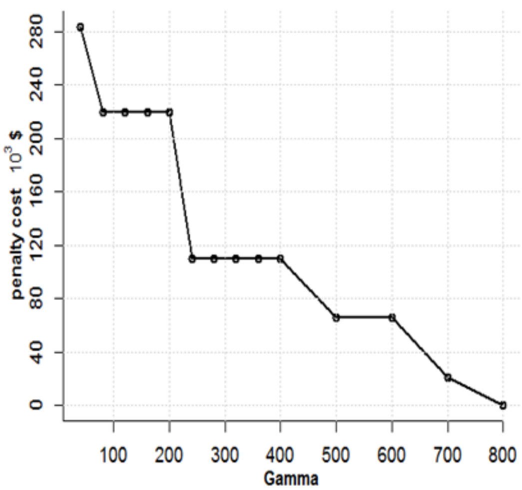

Figure 5. Sensitivity of model robustness with respect to gamma

To highlight the role of uncertainty in modeling, we compare here, three models result: deterministic demand and deterministic supply (DDS), deterministic demand and stochastic supply (DDSS), and stochastic demand and stochastic supply (SDSS). In DDS model, we assume that demand and supply parameters are known certainty. While DDSS model is designed with assumption that demand parameter are known certainly (demand parameters (d_{kcs}) are not scenario dependent), SDSS model represents complete stochasticity of demand (d_{kcs}) and supply (ρ_{kcs}) parameters. This comparison is made to show the benefit of considering stochastic parameters. To quantify the cost saving by considering the various sources of uncertainty, each typical model is solved for the case problem and results are shown in Figure 6. The cost of relief distribution is much higher than the SDSS. The DDS model have little cost benefit is scenario three. The remaining three scenarios cause much higher cost in DDS compare with all uncertain models. The similar phenomenon is also observed in DDSS model which gains lower cost compare with DDS model. It can be said that stochastic model gain cost benefits. By doing this analysis, we can also calculate the value of stochastic solution (*VSS*). The *VSS* provides relative advantage of stochastic model. In situations in which one cannot gather more information about the future, however, it may be more pertinent for decision makers to know how well the deterministic model solutions perform relative to solutions from more complicated stochastic programs (Birge, 1982).

$$VSS = C_{deterministic} - C_{stochastic} \tag{25}$$

where, first term in right hand side of Eq. (25) represents average solution of DDS model and second term is that of SDSS model. In our example, the VSS is 0.34 million dollar.

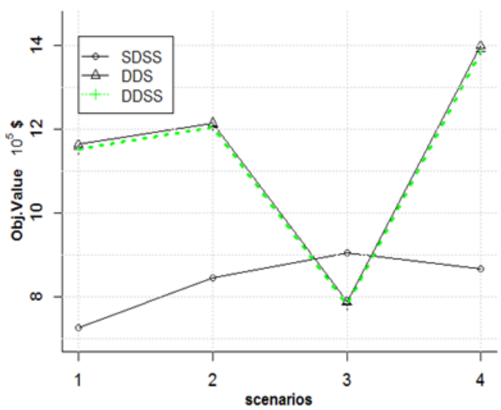

Figure 6. Comparison of different models in different scenarios

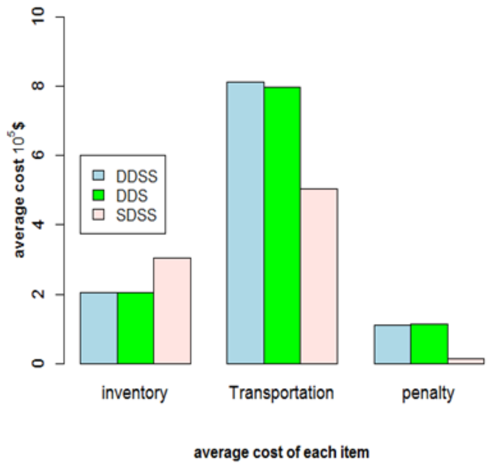

Figure 7. Comparison of cost items in different models

In the last, Figure 7 shows the components of the average cost in three different models explained above. The SDSS model incurs higher inventories cost compare with other two models. The SDSS model gain benefit in post-disaster situations and transportation cost is much lower in SDSS model. The penalty cost is also much lower in SDSS model which show the robustness of this model.

The stochastic nature of supply and demand parameters are formulated in this research and implemented in a narrow set of experiments. The results show that this consideration can gain cost benefits over deterministic models. Although stochastic models require a large number of data sets and to solve complex model, it is worth to apply stochastic model in strategic logistics planning for relief distribution.

5. CONCLUSIONS

We propose two-stage, stochastic, mixed-integer location model with the incorporation of demand uncertainty and supply uncertainty. This model and the solutions have robustness feature. Deterministic model is suitable for contexts, where all parameters are known certainly. Deterministic model is easy to solve and highly sensitive to parameter changes. On the other hand, stochastic model is superior over deterministic model in terms of rational decision. Stochastic model is difficult to solve and requires sufficient amount of data. In this research, stochastic parameters were presented under scenario approach. The first stage decisions were location of RDC and inventory level in each RDC, and the second stage decisions were distribution of relief in different locations and procurement of relief. Our model aimed to minimize the penalty cost, distribution cost with the operational constraints. This model showed the trade-off between the pre-disaster cost and the post-disaster cost. The model also selects two RDCs (Raj and Kul) that do not maintain inventory. It is worth to mention that this model is easy to solve via open-source solver and decision maker does not need to spend money for buying commercial software to solve the model.

The case study was performed to provide insights of the model. Sensitivity analyses were also performed to show the validity of the model. We also proposed the robustness of the model for several uncertainties. Our model showed that decision maker could save 0.34 million dollar by adopting stochastic model over deterministic model. Some parameter values, for instance penalty factor, robustness factor, and oversupply cost, are subject to decision maker's view to risk. Risk adverse decision maker can select higher value of parameter. Finally, this model is a generic model and possible to extend for business logistics. However, network model with supply uncertainty is highly appropriate for humanitarian logistics.

At the end, we make the following relevant suggestion for further research: (1) we have not explicitly considered the uncertainty of transport link. The incorporation of transport link uncertainty enables using different types of transport modes. The research in this avenue can provide some interesting results. (2) Although the open-source solver can solve a large network, it will be beneficial to propose heuristic algorithm for solving large network problem with lower time-duration. (3) The result is highly dependent on scenario selection. We used only four scenarios in our case study. A new research direction can be to produce scenarios of earthquake for a nation.

APPENDICES

Parameter of ω_s :

Li (1996) proposed a model for minimizing deviation cost. The overall purpose of this model is to minimize the deviations between the achievement of the goals (in this paper scenario) and their aspiration levels.

(P1) $min \sum_{s=1}^{s}(\omega_s^+ + \omega_s^-)$ (.a)

s.t. $f_s(x) - \omega_s^+ + \omega_s^- - g_s = 0 \;\; \forall s = 1, \dots, s$ (.b)

$x \in F, x \geq 0$ (.c)

$\omega_s^+, \omega_s^- \geq 0 \;\; \forall s = 1, \dots, s$ (.d)

where

$f_s(x)$ = linear function of the s th scenario

g_s = aspiration level of the s th scenario

after introducing the artificial variable in Problem (P1) and using big M method

(P2) $min \sum_{s=1}^{s}(\omega_s^+ + \omega_s^-) + M \sum_{s=1}^{s} S_s$ (.e)

s.t. $f_s(x) - \omega_s^+ + \omega_s^- + S_s = g_s \;\; \forall s = 1, \dots, s$ (.f)

$$x \in F, x \geq 0 \tag{.g}$$
$$\omega_s^+, \omega_s^-, S_s \geq 0 \ \ \forall s = 1, \dots, s \tag{.h}$$

observing the constraint (.b) in (P1)

$$\omega_s^- = -f_s(x) + \omega_s^+ + g_s \geq 0 \tag{.i}$$

substituting the constraint (.i) in objective function and constraints, denoting ω_s^+ as ω_s the equivalent formulation of (P2) is

$$\text{(P3)} \ \min \textstyle\sum_{s=1}^{s} (2\omega_s - f_s(x)) \tag{.j}$$
$$\text{s.t.} \ \ -f_s(x) + \omega_s + g_s \geq 0 \ \ \forall s = 1, \dots, s \tag{.k}$$
$$x \in F, x \geq 0 \tag{.l}$$
$$\omega_s \geq 0 \ \ \forall s = 1, \dots, s \tag{.m}$$

Thus, the parameter of ω_s is ‘2’.

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