# Time-Dependent Vehicle Routing Problems with Time Windows and Split Delivery in City Logistics

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**Abstract:**

In order to improve service quality and satisfy specific delivery requests from different kinds of customers, recently wholesalers are tending to provide more efficient and convenient distribution services rather than follow traditional approaches. Customers may have different preferred hours, and wholesalers must deliver goods in different time windows. In addition, parking restrictions and overwhelmed demands during the peak hours also increase the difficulties in urban logistics.

The study starts from a typical vehicle routing problem with time windows and then considers time-dependent constraints while the traffic flow changes and split delivery during the distribution processes. We solve a small case problem to check the feasibility with the optimization software CPLEX, and then further apply Genetic Algorithm to solve a large-scale network problem. A revised Solomon instance with added time-dependent parameters and real wholesalers' data is tested. The results are also linked with real maps and displayed with TransCAD.

*Keywords*: Vehicle Routing Problem with Time Windows (VRPTW), Time-Dependent, Split Delivery, Genetic Algorithm (GA), TransCAD.

## 1. INTRODUCTION

To meet different types of retailers daily's needs, the wholesalers must visit all scheduled retailers during different opening hours. Some retailers may further request visits within certain time windows. It is not always easy to meet the time window delivery requirement because the delivery processes are usually affected by traffic flow conditions. The traffic congestion during the rush hours might cause severe delays and could make the wholesalers' carriers fail the retailers' delivery requirement. In order to improve service quality in city logistics, this study seeks to solve several challenging missions simultaneously among wholesalers and retailers, including: 1. to satisfy retailers' specific time windows; 2. to approximate travel time affected by urban traffic; and 3. to schedule a split delivery if necessary.

This study extends a basic vehicle routing problem with time windows (VRPTW). Every vehicle starts from the depot of the wholesaler, visits all retailers within the requested time windows, and then returns to the depot. Most retailers' time windows correspond to their business hours, which means vehicles cannot arrive early or late based on the time window. In order to reflect effect of traffic flow on vehicle travel time, a time-dependent constraint is added. According to observations of real-world operations, a split delivery (i.e. in which every retailer can be served by more than one vehicle) is also considered in our model.

The VRPTW has been extensively examined and classified as a NP-hard problem (e.g. Fu, 2002; Meng et al., 2005). Solomon (1983) first presented a mix integer programming (MIP) for the VRPTW and introduced a set of well know benchmark problems now known as "Solomon Instances." He subsequently designed and analyzed algorithms for the VRPTW (Solomon, 1987). To consider traffic congestion, the time-dependent traveling time is added into the VRPTW as the time-dependent vehicle routing problem with time windows (TDVRPTW). Malandraki & Daskin (1992) discussed diversified traffic conditions at different times of the day; the time horizon is divided into M slices and then a constant travel time is assigned to each arc in every interval. The idea is sound; however, the discontinuous travel time settings may violate the first in, first out (FIFO) property. Hill & Benton (1992) also considered TDVRP without time windows but based on time-dependent travel speed. Ichoua et al. (2003) assigned a speed distribution to each arc during the time horizon and then obtained the travel time distribution by integration. Hence, the resulting travel time distribution was a continuous linear function and satisfied the FIFO property.

The other interesting variant of the VRPTW is called a spilt delivery vehicle routing problem with time windows (SDVRPTW). The SDVRPTW is considered to be a relaxation of the classical VRPTW (Dror & Trudeau, 1990; Archetti et al., 2005), where a fleet of homogeneous vehicle has to serve a set of customers. Each customer can be visited more than once. Archetti et al. (2006) proved that the cost of a solution can be potentially reduced by as much as 50%. In this study, we try to integrate above models as a time-dependent VRP with time windows and split delivery (TDVRPTWSD). Some formulations are referenced from Balseiro et al. (2011).

Many previous studies apply genetic algorithms (GAs), one well-known population-based heuristic approach, to solve the VRPTW problem. Thangiah et al. (1991) first applied GAs to solve VRPTW with a cluster-first, route-second strategy. Berger et al. (2003) and Homberger & Gehring (2005) developed hybrid GAs to solve the sample problem. Haghani & Jung (2005) proposed GAs to solve the TSVRP. Boudia et al. (2007) solved SDVRP by GAs, combined with a local search procedure for intensification and a distance measure to control for population diversity. Since GA is well suited for such problems, we also adapt it to solve the proposed TDVRPTWSD models in this study.

The remainder of this paper is organized as follows: the relevant optimization problems are described in detail. Through a series of cases solved with CPLEX (for a small case revised from the Solomon instances) and GAs (for a large scale network case), the model demonstrates its ability to evaluate the best routing decisions. Another case study with real input data is further tested. Results are linked with real maps and displayed with TransCAD. The paper concludes with a discussion of possible future research.

## 2. Model Assumptions and Formulations

A directed graph is given with a set of nodes and a set of directed arcs. Those directed arcs are represented by an ordered pair of nodes *(i, j)* in which *i* is called the origin and *j* is called the destination of the arc. We assume that there are *V* service vehicles departed from the depot, and each vehicle has to return to the depot after serving the assigned demand nodes. The candidate demand nodes are given, along with the required amount of cargos and delivery time window. The demand and the time window for the node *i* can be denoted as $d_i$ and [$e_i$, $l_i$], respectively. The cargo unloading time is also treated as the required service time, $s_i$. A capacity limitation for each vehicle is denoted as *Q*. Each demand node could be served by more than one vehicle if necessary. The depot also has its own business hours.

In addition, the link travel time may vary over time, especially during the rush hours. To simplify, we divide the whole time period into several intervals, Tw. The time-dependent traveling times, $c_{ij}^{w}$, for one OD pair (i, j) in different periods are assumed to form a step function. The following MILP model is revised from the VRP model introduced by Balseiro et al. (2011). One major revision is to relax the restriction which holds that every demand node is served by one vehicle only. The model is expressed as follows:

$$\text{Minimize} \quad \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{v=1}^{V}\sum_{w=1}^{W} c_v^w x_{ijv}^w + \sum_{i=1}^{N}\sum_{v=1}^{V} t_{iv}$$

(1) Subject to

$$\sum_{i=1}^{n}\sum_{v=1}^{v}\sum_{w=1}^{w} x_{ijv}^{w} \geq 1, \ \forall j = 2,..,n \tag{2}$$

$$\sum_{i=1}^{n}\sum_{w=1}^{w} x_{ipv}^{w} - \sum_{j=1}^{n}\sum_{w=1}^{w} x_{pjv}^{w} = 0, \ \forall v = 1,..,v, \ p = 1,..,n \tag{3}$$

$$\sum_{w=1}^{W}\sum_{v=1}^{V} y_{iv}^{w} = d_i, \ \forall i = 2,..,N \tag{4}$$

$$\sum_{w=1}^{W}\sum_{i=1}^{N} y_{iv}^{w} \leq Q, \ \forall v = 1,..,V \tag{5}$$

$$y_{iv}^{w} \leq d_i \sum_{j=1}^{N} x_{ijv}^{w}, \ \forall i = 2,..,N, i \neq j, v = 1,..,V, w = 1,..,W \tag{6}$$

$$t_{iv} + \alpha_{ij}^{w} + \beta_{ij}^{w} t_i + s_j - t_{jv} \leq M\left(1 - x_{ijv}^{w}\right), \ \forall i, j = 1,..,n, w = 1,..,w, v = 1,..,v \tag{7}$$

$$t_{iv} - T_w \leq M\left(1 - x_{ijv}^{w}\right), \ \forall i, j = 1,..,n, w = 1,..,w, v = 1,..,v \tag{8}$$

$$t_{iv} \geq T_{w-1} x_{ijv}^{w}, \ \forall i, j = 1,..,n, w = 1,..,w, v = 1,..,v \tag{9}$$

$$e_i + s_i \leq t_{iv} \leq l_i + s_i, \ \forall i = 1,..,n, v = 1,..,v \tag{10}$$

$$x_{ijv}^{w} = 0 \text{ or } 1,\ \forall i,\ j = 1,..,n,\ v = 1,..,v,\ w = 1,...,w \tag{11}$$

$$y_{iv}^{w} \in N,\ \forall i = 2,..,N, v = 1,..,V, w = 1,..,W \tag{12}$$

$$t_{iv} \geq 0,\ \forall i = 2,..,n,\ v = 1,..,v \tag{13}$$

The minimized objective function (Equation 1) is formulated as the sum of total vehicle dispatching costs and the sum of total travel time, service time and waiting time along the all service routes. In Equation 2, every demand node is served at least once. Equation 3 expresses the balance between in-and-out flows. Equation 4 specifies that every node's demand should be satisfied. Equation 5 ensures that total demand loaded on each vehicle should not exceed the vehicle capacity Q.

Equation 6 indicates the amount of demand split to more than one vehicle still equal to the total demand for each node. Equation 7 computes the departure time at node j. Equations 8 and 9 link the departure time tiv with the time slice Tw, hence the proper slice of traveling time function is employed. Equation 10 imposes the time windows that are defined in terms of the departure time at the node i. Equation 11 represents that if arc (i, j) is traversed by vehicle v during time slice w is 1, otherwise it will be 0. Equation 12 states that the fraction of a node's demand delivered by vehicle v at time slice w is between 0, 1. Equation 13 assumes that departure times at each node are non-negative.

All notations are listed as follows: N = the set of all nodes, $N = \{0,1,...,n\}$, 0 is the depot; V = the set of all vehicles, $V = \{1,...,v\}$; W = the set of all slices, $W = \{1,...,w\}$; $d_i$ = the demand of the node *i*; Q = the capacity of the vehicle; $\alpha_{ij}^{w}$ = *arc(i,j)* coefficient at slice *w*; $\beta_{ij}^{w}$ = *arc(i,j)* coefficient at slice *w*, $\beta_{ij}^{w} \geq -1$; $s_j$ = the serviced time of node *j*; M = a large enough number; $T_w$ = upper bound for slice *w*; $e_i$ = the earliest time to start to service node *i*; $l_i$ = the latest time to start to service node *i*; $c_v^w$ = the dispatching costs of vehicle v during w slice;

$$x_{ijv}^{w} = \begin{cases} 1, \text{ if link}(i, j) \text{ is traversed by vehicle } v \text{ during } w \text{ slice.} \\ 0, \text{ o.w.} \end{cases}$$

$y_{iv}^{w}$ = fraction of node's demand $i$ delivered by vehicle $v$ during $w$ slice.

$t_{iv}$ = the departure time at node $i$ by vehicle $v$.

## 3. Model Applications and Analytical Results

Through this work we seek to optimize the routing decisions for ready outbound vehicles at a dispatching center (i.e. the depot). A small case problem is solved by the optimization software CPLEX, and another large-scale network problem is solved by Genetic Algorithm (GA). These case studies also provide flexibility in delivery with split demands. The network configurations of the case studies are illustrated in Figures 1 and 2.

Case 1: Small Size Network Configurations

In Case 1, four trucks depart from the depot (node 0) and are assigned to the four retailers (nodes 1~4). The capacity of each truck is 165 boxes (i.e. single-wall corrugated with contents up to 95 lbs) and the entire time horizon is divided into three slices. The optimized results are illustrated in Figure 1.

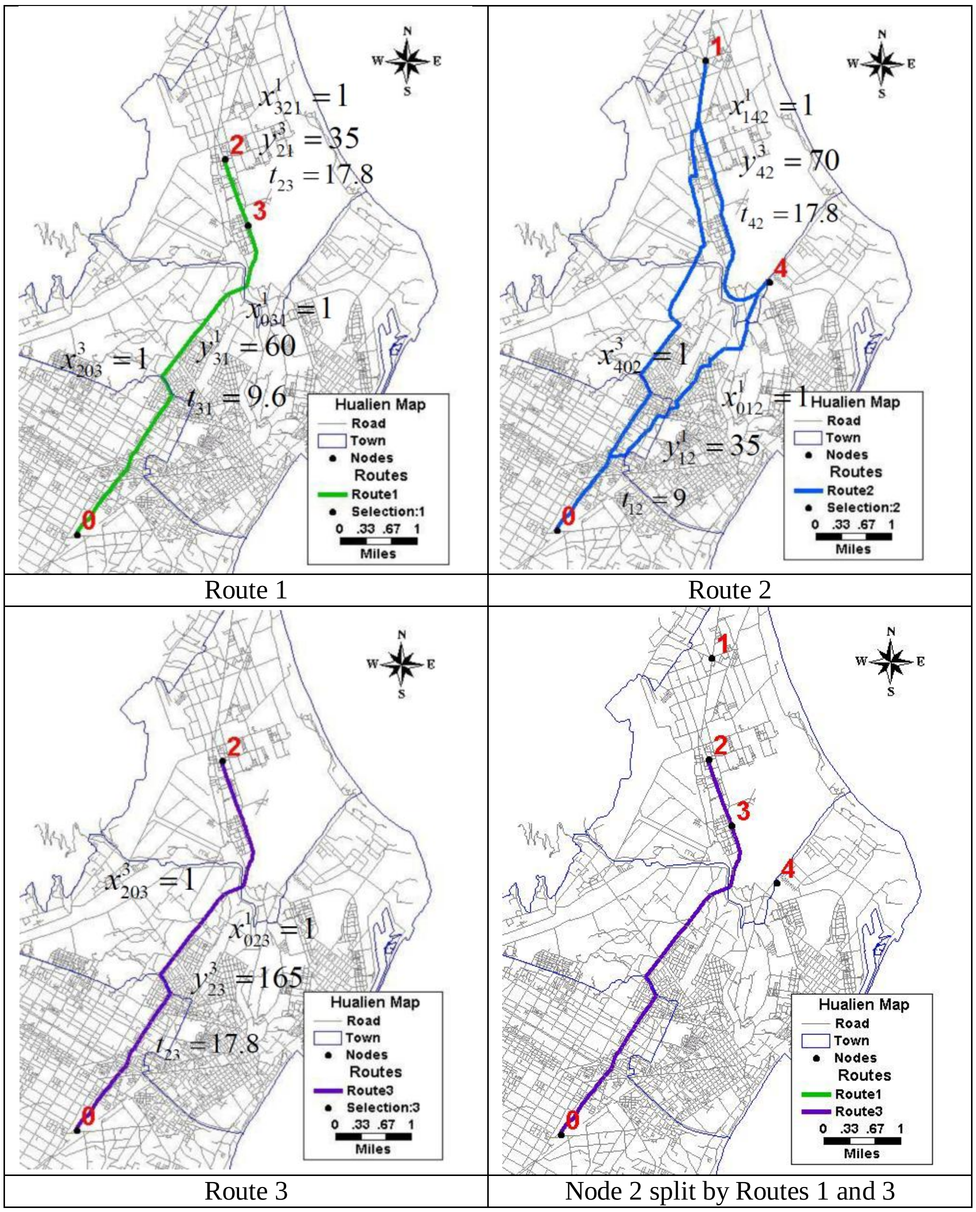



Figure 1 Optimized Results for Case 1

Three delivery routes are optimized in Case 1. Especially for the node 2, some demands are separately served by Vehicles 1 and 3. The optimized results show that eight vehicles / routes spend 248.8 minutes to complete all delivery requests.

Case 2: Large-Scale Network Configurations

We initially assume that the optimal solution is an infinite number, and a set of genetic sequences is generated. After calculating total travel time T of each sequence, the minimized $T^*$ in this set of sequences is defined as the current dominant solution. Another set of genetic sequences is then generated. If $T$ is smaller than the dominant solution T*, then we replace it until improvements become negligible or we reach our pre-specified stopping criteria (e.g. the maximum number of generations, $k$.)

In Case 2, there are 50 trucks serving 100 retailers, revised from the Solomon Instance C109, where customers' locations are clustered with much similarity to the environments of city logistics. The capacity of each truck is 200 boxes and the depot's opening time horizon is divided into five slices of equal durations between the depot opening and closing times [e0, l0]. The travel speed settings are revised from Figliozzi's (2012) study. Travel speed is up to 60 km/hr and drops to 24 km/hr with heavy traffic congestions.

Different travel speeds imply different vehicle travel times. Two types of extreme travel speed distributions are tested, where travel speeds of Types A and B in odd time slices are slower and higher than even ones, respectively. On average, speeds of Type A (including TD1a, TD2a, and TD3a) are relatively lower than those of Type B (including TD1b, TD2b, and TD3b), as shown in Table 1.

Table 1 Travel Time Settings in Case 2

| Time Slices | | Extreme Travel Speeds of Time Slices | | | | |
|---|---|---|---|---|---|---|
| Speed Types (km/hr) | | Slice 1 | Slice 2 | Slice 3 | Slice 4 | Slice 5 |
| Type A | $TD_{1a}$ | 24.0 | 38.4 | 25.2 | 38.4 | 24.0 |
| | $TD_{2a}$ | 24.0 | 48.0 | 36.0 | 48.0 | 24.0 |
| | $TD_{3a}$ | 24.0 | 60.0 | 42.0 | 60.0 | 24.0 |
| Type B | $TD_{1b}$ | 38.4 | 24.0 | 25.2 | 24.0 | 38.4 |
| | $TD_{2b}$ | 48.0 | 24.0 | 36.0 | 24.0 | 48.0 |
| | $TD_{3b}$ | 60.0 | 24.0 | 42.0 | 24.0 | 60.0 |

Under operations split between two vehicles, the total travel time can be reduced by up to 66%, which is even better than the results (i.e. 50%) claimed by Archetti et al. (2006). Based on the overall results listed in Table 2, implementing split delivery does not significantly increase the program computation time. It takes only a few more seconds to solve the TDVRPTWSD problem.

Table 2a Optimized Results of TDVRPTW with Type A Settings

| | With Split Delivery | | | | Without Split Delivery | | | Travel time reduction | Increase CPU time |
|---|---|---|---|---|---|---|---|---|---|
| | Number of vehicles | Number of split nodes | Total travel time | CPU time (s) | Number of vehicles | Total travel time | CPU time (s) | | |
| TD1 | 19 | 3 | 1099.1 | 48.4071 | 15 | 1961.7 | 41.4807 | 43.97% | 16.70% |
| TD2 | 17 | 1 | 741.7781 | 45.9423 | 13 | 1583.4 | 39.1875 | 53.15% | 17.24% |
| TD3 | 17 | 2 | 481.3311 | 46.1139 | 16 | 1429.9 | 42.9003 | 66.34% | 7.49% |

Table 2b Optimized Results of TDVRPTW with Type B Settings

| | With Split Delivery | | | | Without Split Delivery | | | Travel time reduction | Increase CPU time |
|---|---|---|---|---|---|---|---|---|---|
| | Number of vehicles | Number of split nodes | Total travel time | CPU time (s) | Number of vehicles | Total travel time | CPU time (s) | | |
| TD1 | 16 | 2 | 1726.9 | 44.0391 | 14 | 2534.6 | 39.4215 | 31.87% | 11.71% |
| TD2 | 18 | 1 | 809.8683 | 44.9907 | 16 | 2385 | 46.7691 | 66.04% | -3.80% |
| TD3 | 19 | 1 | 712.5908 | 46.6911 | 15 | 1863.5 | 42.8223 | 61.76% | 9.03% |

Case 3: Real-World Applications

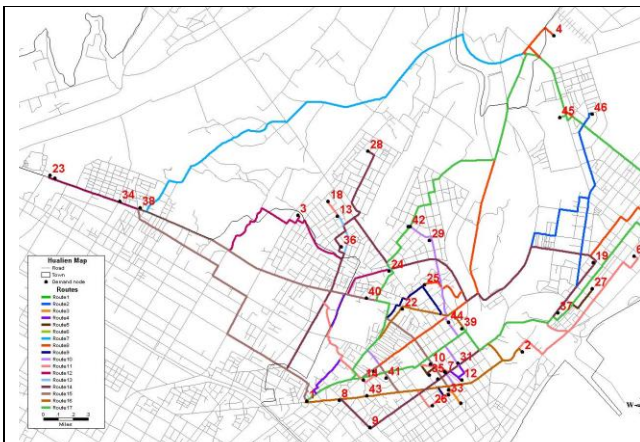
In order to enhance the capability of the proposed model, the real data of one food wholesaler r located in Hualien, Taiwan, is examined in Case 3. The demand unit is denoted by the specified box, which is more convenient for arranging goods and deliveries in practice. The wholesaler's depot opens at 7:00 am and closes at 5:00 pm. The service time for each node is 5 minutes. Here we still assume the entire time horizon equally divided into five parts and the corresponding average traveling speeds are 20km/hr, 50km/hr, 30km/hr, 50km/hr, and 20km/hr, respectively. The first and the last time slices represent the rush hours (i.e. the morning and afternoon peaks), so the speeds are slow. Detailed results are listed in Table 3.

Table 3 Overall Results of All Routes with Split Delivery

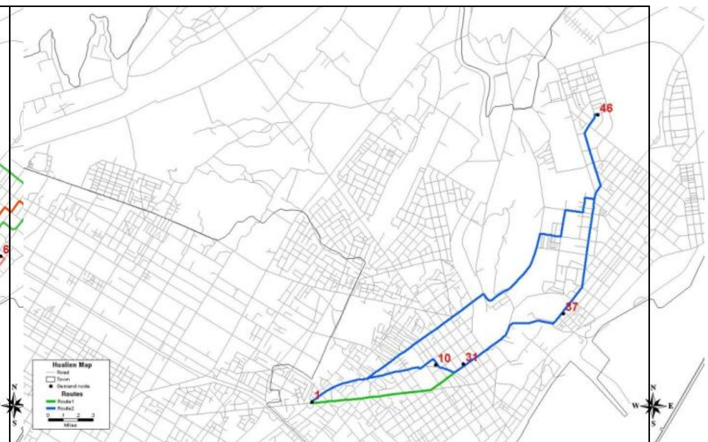
| | **Service Sequence** | **Delivery Nodes** | **Travel Time (minutes)** | **Start of Travel Time** | **End of Travel Time** |
|---|---|---|---|---|---|
| **Route1** | 1→37→46→1 | 29, 71 | 30.7866 | 12:01 | 12:31 |
| **Route2** | 1→46→31→10→1 | 53, 24, 17 | 35.9835 | 11:01 | 11:36 |
| **Route3** | 1→20→8→1 | 21, 11 | 16.5471 | 8:51 | 9:07 |
| **Route4** | 1→29→12→15→40→1 | 20, 17, 14, 19 | 51.6947 | 11:00 | 11:51 |
| **Route5** | 1→35→27→5→7→1 | 23, 29, 32, 12 | 32.0963 | 12:47 | 13:19 |
| **Route6** | 1→32→43→1 | 88, 12 | 13.0384 | 8:05 | 8:18 |
| **Route7** | 1→34→4→1 | 43, 57 | 24.221 | 7:17 | 7:41 |
| **Route8** | 1→4→21→25→1 | 45, 11, 44 | 28.0394 | 7:08 | 7:36 |
| **Route9** | 1→25→44→33→1 | 67, 11, 19 | 20.8074 | 7:08 | 7:28 |

| | | | | | |
|---|---|---|---|---|---|
| **Route10** | 1→41→42→26→1 | 22, 13, 65 | 22.0985 | 8:42 | 9:04 |
| **Route11** | 1→26→6→14→18→1 | 3, 56, 23, 14 | 33.5472 | 7:54 | 8:27 |
| **Route12** | 1→3→23→24→1 | 12, 28, 60 | 29.6259 | 13:19 | 13:48 |
| **Route13** | 1→24→13→1 | 45, 31 | 21.5243 | 12:33 | 12:54 |
| **Route14** | 1→36→28→19→9→1 | 24, 10, 28, 20 | 34.5373 | 8:52 | 9:26 |
| **Route15** | 1→11→30→38→1 | 30, 35, 21 | 29.4779 | 9:27 | 9:56 |
| **Route16** | 1→22→2→1 | 20, 77 | 17.5008 | 14:05 | 14:22 |
| **Route17** | 1→39→16→45→17→1 | 15, 10, 55, 12 | 35.0807 | 10:26 | 11:01 |

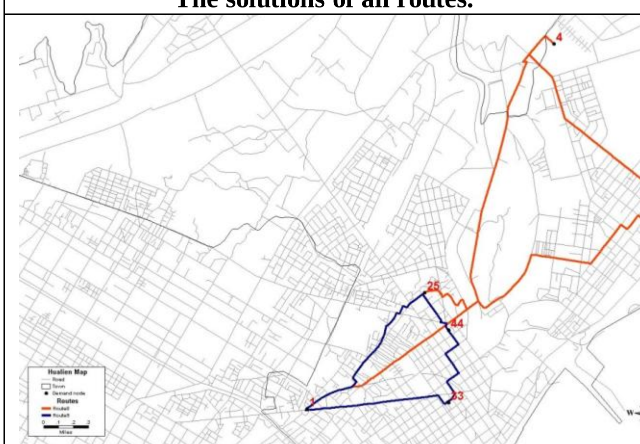
Split operations are applied at three nodes. Demands at Node 46 (124 boxes) are split between Routes 1 (71 boxes) and 2 (53 boxes); demands at Node 25 (111 boxes) are split between Routes 8 (44 boxes) and 9 (67 boxes); and demands at Node 26 (68 boxes) have been split between Routes 9 (3 boxes) and 10 (65 boxes). Detailed routing results are displayed through the software TransCAD, as shown in Figure 2.



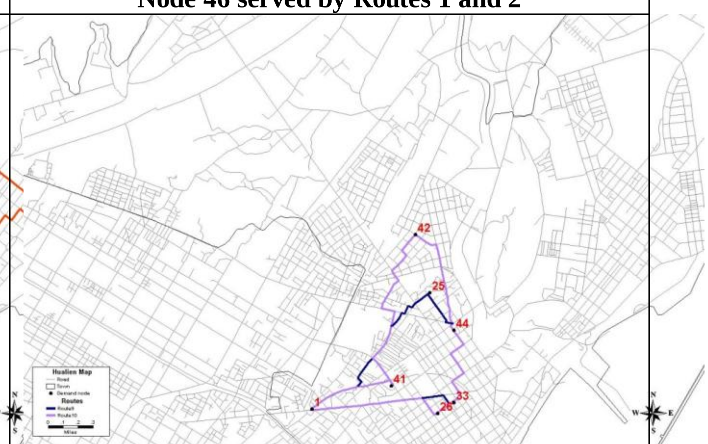

**The solutions of all routes.**




**Node 46 served by Routes 1 and 2**




**Node 25 served by Routes 8 and 9**




**Node 26 served by Routes 9 and 10**

Figure 2 Optimized Results for Case 3

In Case 3, total travel time is about 476 minutes and CPU time is only 14.5705 seconds. It is found that our proposed models yield better results than the traditional VRPTW models, and the extremely short computation time can improve the efficiency and increase the feasibility and flexibility during the real-time dispatching operations.

## 4. Conclusions

In this study, a mathematical model is developed for solving the TDVRPTWSD problem. Since the studied TDVRPTWSD problem belongs to the set of NP-hard problems, a heuristic algorithm, namely a Genetic Algorithm (GA), is chosen to solve it. In our case studies, the algorithm can solve the 46 demand nodes within only 14.57 seconds and 100 nodes within 40 seconds, which can provide a complex distribution and routing plan in a timely manner. The optimal vehicle routing results can be shown in a city map by TransCAD, which can be further linked to GIS/GPS technology.

The results show that the split delivery constitutes a significant percentage of the total travel time reduction, which is even better than the performance tested by Archetti et al. (2006). Although the GAs program running times are satisfactory in this study, some other hybrid metaheuristic techniques (e.g. Chen and Schonfeld, 2012) will be tested in the future. In addition, it might be worth to consider multi-trips while solving this problem, and to optimize the fleet size of service vehicles based on overall trade-off considerations.

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