Estimation of Dynamic Link Flows and Origin-Destination Matrices from Lower Polling Frequency Probe Vehicle Data

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Abstract: we present a methodology of estimating dynamic link flows and origin-destination matrices using lower polling frequency probe vehicle data (e.g. one point every 30-60s). Link travel time is first obtained from map-matched probe points using a method of proportional allocation. A derived speed-density function is then fitted for different types of roads. A Bayesian method that carefully incorporates prior information is used to estimate dynamic link flows from link travel speed. A bi-level generalized least-square (GLS) estimator is formulated so as to estimate dynamic OD matrices from estimated link flows. A traffic simulator in VISSIM is developed for a median size urban network using an open data set. The results validate the advantages of the proposed method for lower polling frequency probe vehicle data.

Keywords: Probe vehicle, Dynamic link flows, Dynamic origin-destination matrices

1. INTRODUCTION

Origin-destination (OD) matrix, which contains information on the number of travelers that commute or the amount of freight shipped between different zones of a region, is an essential element of network based traffic models. Specifically, dynamic OD matrices are one key input to dynamic traffic assignment (DTA) models for analysis and management of urban and suburban congestion problems (Ashok, 1993, Ashok, 2002). Usually, these matrices are estimated from time-dependent link traffic flows, since obtaining them directly is neither practical nor economically feasible. Link traffic flow is an important parameter used to quantify demand on a road, and it is commonly collected by using roadside (fixed) infrastructures such as inductive loops, radars, video cameras, etc. However, installation of these infrastructures needs an enormous expense, only a subset of links can be covered by them. Further, these infrastructures and gathered link counts are managed exclusively by road authorities; therefore, planners and researchers cannot use the data freely. Fortunately, this situation has been changing for years due to the availability of probe vehicle techniques and other mobile sensors.

With global positioning system (GPS) device installed, a moving probe vehicle can provide its information of link and path travel time, trajectory, origin and destination points. Obtaining these data is not limited in a subset of links by fixed infrastructures, but dynamic and widespread in the whole road network. There are two inevitable issues on using probe vehicle as traffic information collection tool. One is that the level of penetration is currently low (1%-3%) for probe vehicles (e.g. taxis, delivery vehicles). The widely used Smartphone in recent years brings a rapid increase of the level of penetration for cellular phone based probe vehicles. It is likely that a 10% of penetration will become a reality in coming years with the development of various technologies. The other is the polling frequency is usually low (e.g., one point every 1-2 minutes) for both dedicated and cellular phone based probe vehicles. This fact is less likely to change in near future, since high polling frequency would give rise to high expenses and fast energy consumption of battery in GPS device.

Probe vehicle data has been used to estimate link flows in a few studies. Yamamoto *et al.* (2009) proposed a Bayesian method to infer link flow from prior and current link speed distribution. Caceres *et al.* (2012) presented a set of models for inferring traffic volume by means of anonymous call data of phones. However, the inferred traffic volume actually is not link flow for a road segment but the number of vehicles from one cell to another.

More studies have been conducted to estimate OD matrices from probe vehicle data. Van Aerde *et al.* (1993) estimated the level of penetration by calculating the ratio of probe vehicles in the population in aggregated time intervals for links, and then estimated dynamic OD matrices using the origin and destination points of probe vehicles and the estimated penetration. Eisenman *et al.* (2004) proposed a method of estimated static OD matrices based on traffic assignment, in which link choice ratios are inferred from probe route data. Ásmundsdóttir *et al.* (2010) discussed the rules of determining origins and destinations, route choice and trip length distribution within probe data from taxies, and then proposed a method of estimating dynamic OD matrix from archived data and real time data. Nonetheless, the results didn't support the proposed method well. These methods attempt to develop a direct estimation process based on the assumption of random sampling. But in practice probe vehicles are from one or several types of vehicles (taxis, delivery vehicles, etc.) and consequently are not a random sample from the population. To avoid the assumption of random sampling, Yamamoto *et al.* (2009) suggested a two-step indirect framework. In the first step, they inferred link flows from probe vehicle speed using two Bayesian methods; then they updated the target OD matrix using an entropy maximization method. However, they didn't treat the variances of estimated link flows in a statistical manner and the results didn't validate the advantage of applying Bayesian methods in estimating the link flow. To address these deficiencies, Cao *et al.* (2013) improved Yamamoto's two-step method by formulating a bi-level generalized least-square estimator in the second step. The results validated the importance of carefully treating the variances of estimated link flows.

Almost all existing methods of estimating link flows and OD matrices using probe vehicle data either utilized the accurate link travel times from high polling frequency probe vehicle points, or ignored the effect of polling frequency on the derived link travel times (Van Aerde, 1993; Eisenman, 2004; Ásmundsdóttir, 2010; Yamamoto, 2009; Cao, 2013). However, It is less likely a probe vehicle directly records link travel times, since the two consecutive polled positions do not necessarily correspond to the end points of individual links. Methods have been proposed to decompose travel times measured by probe vehicles into individual road segments (Hellinga, 2008; Zheng and Van Zuylen, 2012). These researches indicate the link travel time become less reliable when it is derived from lower polling frequency probe vehicle points. This paper analyzes the effects of polling frequency and method of decomposing travel time on the derived travel time, and then explores method of estimating dyamic link flows and OD matrices from lower polling frequency probe vehicle data (e.g. one point every 30-60s).

This paper first discusses the issue of decomposing probe vehicle travel times into individual links, and then proposes a method of estimating dynamic link flows and OD Matrix sequentially. The proposed method is expected to give reliable estimates for lower polling frequency probe vehicle data.

2. METHODOLOGY

The raw probe vehicle data are series of track points including location, time. Usually, it requires procedures of map-matching and travel time allocation before link travel time being obtained. This research starts from the map-matched probe data, discusses travel time allocation that is directly related with polling frequency. To build a bridge between link travel time and link flow, we derive link performance function based on the density-versus-space mean speed (*k*-*v*) curve. Then dynamic link flow is inferred using Bayesian inference. Finally, a bi-level generalized least squares (GLS) estimator is used to obtain the dynamic OD matrices. Therefore, the methodology includes four steps: travel time allocation, link performance function fitting, dynamic link flows estimation and dynamic OD matrices estimation.

2.1 Step1-Travel Time Allocation

In principle, it's no necessary to allocate travel time from high polling frequency (e.g. one point every 1s) probe vehicle data. Whereas travel time allocation should be taken carefully for lower frequency data, since the two consecutive polled positions do not necessarily correspond to the end points of individual links. Hellinga *et al.* (2008) proposed an analytical method of travel time allocation by recognizing that vehicles are more likely to incur stopping delay at the downstream rather than upstream end of a link, especially when the link is influenced by a traffic control device. Soon afterwards, Zheng and Van Zuylen (2012) presented a three-layer Artificial Neural Network (ANN) model and got higher accuracy estimates than Hellinga's model. However, both models are not capable for network application in that it's difficult to determine the parameters for them. In Hellinga's model, there are two unknown model parameters that are used to reflect the stopping likelihood pattern of a link. In Zheng's model, all parameters are learned from very high polling frequency data (one point every 0.3s), while this kind of data are not available in practice.

To our knowledge, most existing probe vehicle systems still use a simple but practical method to obtain link travel time, in which uniform motion is assumed (Miwa *et al*. 2004). From the perspective of practical applications, we also use this method described as follows.

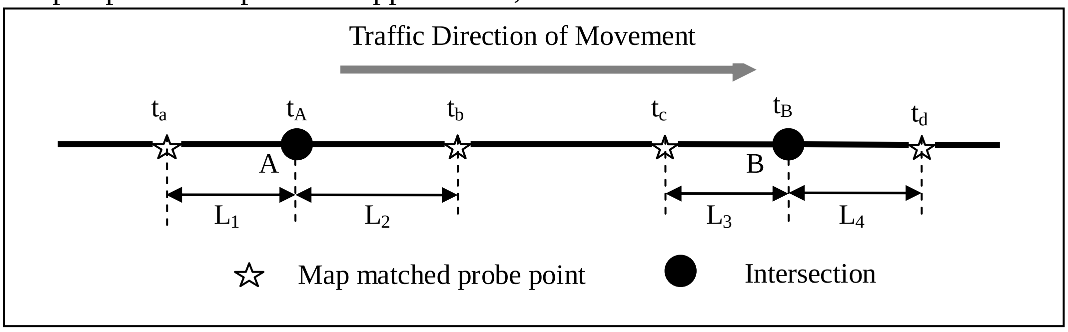


Figure 1 Calculation of link travel time using probe vehicle data

In Figure1, we assume raw probe data have been map-matched onto road, A and B are intersections which are controlled by traffic signals. t_a, t_b, t_c, and t_d are the time recorded by the probe vehicle. t_A and t_B are the time a probe vehicle departs intersection A and B respectively, which need to be estimated.

Assume a probe vehicle travels in uniform motion between t_a and t_b, and between t_c and t_d, then t_A and t_B can be estimated by:

$$\widehat{t_A} = \frac{t_a * L_1 + t_b * L_2}{L_1 + L_2} \tag{1}$$

$$\widehat{t_B} = \frac{t_c * L_3 + t_d * L_4}{L_3 + L_4} \tag{2}$$

Then the travel time of link AB is calculated by:

$$\widehat{T_{AB}} = \widehat{t_B} - \widehat{t_A} \tag{3}$$

This method actually allocates travel time according to the distance between probe point and intersection point, thus we call it proportional allocation in this paper. Proportional allocation can be easily applied to various kinds of probe vehicle systems in spite of probably resulting inaccurate link travel times. Moreover, it has a special function of reducing the variance of the derived link travel time. It can be illustrated with an example using Figure 1 in the following.

Note that the following calculation is just an illustration example. Without loss of generality, we assume the time of a vehicle entering in link AB t_A is known, the time of this vehicle leaving the link should be calculated using equation (2). Intersection B is controlled by a traffic signal with 30s red light of 60s cycle. A probe vehicle enters link AB at $t_a = 100s$, and runs at a constant speed of 10m/s or stops at the intersection B waiting for the red light. And the polling frequency is one point every 60s. The length of link AB is 300m, values of L_3 and L_4 depend on the time when probe vehicle sends data. Since the red light is the main factor causing a vehicle to stop, we consider two extreme situations: no stop, stop when red light begins.

Situation 1: No stop

In this situation, a vehicle always maintains a speed of 10m/s, and experiences the minimum delay. A probe vehicle sends one point on $t_c = 115s$ and $t_d = 175s$, then we can calculate $L_3 = 150\text{m}$, $L_4 = 450\text{m}$, $t_B = 130\text{s}$, thus $T_{AB} = t_B - t_A = 30\text{s}$. If only t_a and t_d are assumed known, t_B is estimated using equation (2), then $\widehat{t_B} = 130s$, $\widehat{T_{AB}} = 30s$. We can see the estimated link travel time $\widehat{T_{AB}}$ equals the true link travel time T_{AB} in this case

Situation 2: stop when red light begins

A probe vehicle travels at a speed of 10m/s, and then stops at intersection B waiting for 30s of red light, finally leaves link AB at the speed of 10m/s. In this situation, the vehicle experiences the maximum delay. The probe vehicle reports $t_c = 115\text{s}$ and $t_d = 175\text{s}$. We can calculate $L_3 = 150\text{m}$, $L_4 = 150\text{m}$, $t_B = 160\text{s}$, thus $T_{AB} = t_B - t_A = 60\,\text{s}$. Using equation (2-3), we can estimate t_B as $\widehat{t_B} = 145s$, T_{AB} as $\widehat{T_{AB}} = 45s$ Thus, in this case the estimated link travel time $\widehat{T_{AB}}$ is smaller than the true link travel time T_{AB}.

From the above calculation, we know the true link travel time T_{AB} is distributed in range of $[30s, 60s]$, whereas the estimated link travel time using proportional allocation $\widehat{T_{AB}}$ is distributed in a smaller range of $[30s, 45s]$. Therefore, the variance of estimated link travel time using proportional allocation is smaller than that of true link travel time.

2.2 Step2-Link Performance Function Fitting

Link performance function builds a bridge between link travel time and link flow. The average and variance of link flow can be estimated from link travel time if the performance function is given. We have known the basic relation:

$$q = k\bar{v} \tag{4}$$

where q is link flow, k is vehicle density, $\bar{v}$ is space mean speed.

The density k cannot be directly obtained from probe vehicle, but can be estimated from space mean speed using a *k-v* function. From Gazis's nonlinear follow-the-leader model (Gazis et al. 1961), we derive a *k-v* function:

$$\bar{v} = v_f exp(-\alpha \left(\frac{k}{k_j}\right)^{\beta}) \tag{5}$$

where k_j denotes the jam density, v_f denotes the free flow speed and α and β are parameters.

This formula is essentially a generalized version of the Underwood model (Underwood, 1961). As indicated by Wu *et al.* (2012), the fundamental diagram is significantly affected by signal timings in the case of an urban road, and thus the speed-density relationship is different from that for a highway. It should be noted that the derived function is a little different from the one given in Yamamoto *et al.* (2009), where link capacity C replaces jam density k_j, and α and β have different meanings. The above formula is applied in this study, because k_j can be easily set for a particular link as the length of the link divided by the average vehicle spacing.

In a probe system, link travel times and thus link speeds can be easily obtained in real-time in aggregated time intervals. However, the observed link travel speed of probe vehicle is not necessarily identical to the mean speed, since the speed of a probe vehicle depends on the arrival flow rate and distribution, traffic signal timings and arrival time (Hellinga and Fu, 1999), which are random variables. So the link speed of probe vehicle i in time interval t is regarded as a random variable distributed around the mean speed $\bar{v}_t$:

$$v_t^i = \bar{v}_t + \varepsilon_t^i \tag{6}$$

where ε_t^i is an error term.

Assume that ε_t^i follows a normal distribution $N(0, s_t)$, then the probability density function of v_t^i can be given as

$$f\left(v_t^i \middle| \bar{v}_t, {s_t}^2\right) = \frac{1}{\sqrt{2\pi} s_t} exp\left\{-\frac{\left(v_t^i - \bar{v}_t\right)^2}{2{s_t}^2}\right\} \tag{7}$$

A Lagrangian likelihood function can be constructed for equation (7) into which equation (5) is substituted, then parameters $v_f, \alpha,$ β and s_t can be estimated using the maximum likelihood method. The reader is recommended to refer to Yamamoto *et al.* (2009) for more details of this procedure.

2.3 Step3-Dynamic Link Flows Estimation

To utilize prior information that can be obtained from archived probe data, a method based on Bayes' inference theory, namely the Bayesian method, is used in this study. For each link at time interval t, the posterior mean and variance of the mean speed v_1, ${\sigma_1}^2$ are given as

$$v_1 = \frac{{\sigma_0}^{-2} \cdot v_0 + n \cdot s^{-2} \cdot \hat{v}}{{\sigma_0}^{-2} + n \cdot s^{-2}} \tag{8a}$$

$${\sigma_1}^{-2} = {\sigma_0}^{-2} + n \cdot s^{-2} \tag{8b}$$

where v_0, ${\sigma_0}^2$ are the prior mean and variance of the mean speed, respectively, and $\hat{v}$ is the average link speed of probe vehicles.

Then using the posterior distribution of link speed, the link performance function and the relationship among link flow, traffic density and link speed as in equation (4), the posterior mean and variance of link flow, $\bar{q}$ and ${\sigma_q}^2$, are given as

$$\bar{q} = \int_{v=0}^{\infty} K(v) \cdot v \cdot f(v|v_1, \sigma_1) dv \tag{9a}$$

$${\sigma_q}^2 = \int_{v=0}^{\infty} (K(v) \cdot v - \bar{q}) \cdot f(v|v_1, \sigma_1) dv \tag{9b}$$

where $K(v)$ is the function of density k with respect to speed v solved from equation (5).

Note that we obtain both the link flow and the variance using Bayesian method. The variances virtually reflect the difference in the reliability of link flow estimates among links. In practice, a huge number of data can be obtained after a probe vehicle system has been running for several months. This make it possible get enough data for particular time of particular day on particular road, thus provide enough prior information for link speed distribution.

2.4 Step4-Dynamic OD Matrices Estimation

Existing methods of estimating dynamic OD matrix can be categorized into two classes: DTA (dynamic traffic assignment) -based vs. non-DTA-based, depending on whether a DTA component is incorporated into the estimation model (Chang and Tao, 1999; Peeta and Ziliaskopoulos, 2001; Zhou and Mahmassani, 2007). We focus on the DTA-based in this research in that it is recommended when dealing with complex network (Frederix, 2010). In this section, we describe a DTA-based bi-level GLS model. This is an extension of the iterative bi-level estimation framework proposed by Tavana *et al.* (2001a), and we adopt the same notation for variables in the model formulation that follows.

We consider a traffic network where L is the number of sensed links, and I and J are the numbers of origins and destinations, respectively. We are interested in finding a feasible vector OD demand D for Γ aggregated time intervals, given a target demand vector $\widehat{D}$, and observed link flow vector $\hat{V}$ for T observation time intervals. The assignment of the OD matrix onto the links in the network is made according to the link-flow proportion matrix $P = \{p_{(l,t)(\tau,i,j)}\}, l = 1,2,\cdots,L; t = 1,2,\cdots,T; \tau = 1,2,\cdots,\Gamma; i = 1,2,\cdots,I; j = 1,2,\cdots,$ where each element $p_{(l,t)(\tau,i,j)}$ in the matrix represents the proportion of aggregated demand flow $d_{(\tau,i,j)}$ in aggregated time interval τ that flows on link l during observation time interval t. Further, $\hat{v}_{(l,t)}$ is the element of $\hat{V}$ representing the observed link flow for link l during observation time interval t, and $\hat{d}_{(\tau,i,j)}$ is the element of $\widehat{D}$ representing the target OD demand for trips originating in zone i in aggregated time interval τ with destination j. It is noteworthy that the duration of the aggregated time interval can be one or several departure time intervals, and the departure time interval is equal to the observation time interval.

Similarly to the static case (Cascetta, 1984), the dynamic bi-level GLS estimator can be formulated as

$$min\, F(D) = \omega\big(\widehat{D} - D\big)^T Z^{-1}\big(\widehat{D} - D\big) + (1-\omega)(\hat{V} - PD)^T W^{-1}(\hat{V} - PD) \qquad (10)$$

$$\text{subject to } P\text{=assignment } D \text{ from DTA} \qquad (11)$$

$$D \geq 0$$

where ω is a weight factor, Z and W are variance-covariance matrices.

Weight factors $(1-\omega)$ and ω are used to reflect the decision maker's preference or perceived importance for observed link flows and the target OD matrix. Generally speaking, if no target OD matrix or an unreliable OD matrix is provided, the value of ω should be small and vice versa. Usually, if no further perceptual information about observed link flows and the target OD matrix is known, ω is given a value of 0.5 for both terms to indicate no preference.

In the objective function, both the distance between the estimated and target OD matrices and the distance between the calculated and observed link flows are considered, if both Z and W are set to be the identity matrix I, our formulation will drop back to the ordinary least squares (OLS) presented by Zhou *et al.* (2003). Actually, W was set to I in Tavana's experiments, so the benefit of W has not been validated. In this paper, the merits of both Z and W will be carefully considered and implemented.

As already noted, this proposed model is a DTA-based bi-level GLS estimator. This model is solved using an iterative solution procedure presented by Zhou *et al.* (2003). In each iteration, we solve the upper level using a constrained algorithm which is first proposed for static OD estimation by Bell (1991) and extended to the dynamic case by Tavana (2001b), and for the lower level, we use the DTA module in VISSIM as the simulator.

3. NUMERICAL EXPERIMENT AND RESULTS

3.1 Study Network

The network studied in this research is a western part of central Tokyo, namely Kichijoji-Mitaka area, which extends about 2 km from east to west and 1 km from north to south (Figure 2). This network consists of 138 links and 57 nodes, including four major north-south streets and two major east-west streets. Horiguchi *et al.*(1998) carried out a precise traffic survey on this network in morning peak period 7:00 am-10:00 am on 30 Oct. 1996, and made an open data set. Link volume on 70 links were observed and totally 16,043 vehicle trajectories are identified and, after data cleaning, link flows and OD demands for each 10-minute period for the effective time interval 7:50 am to 10:00am are derived. There are 26 origins and 26 destinations identified in this network. In addition to these data, geometry of most intersections and all signal timings can be also found in the Kichijoji-Mitaka open data set. We use this network not only because of the rather complete data set, but also because there are multiple routes for many O-D pairs.

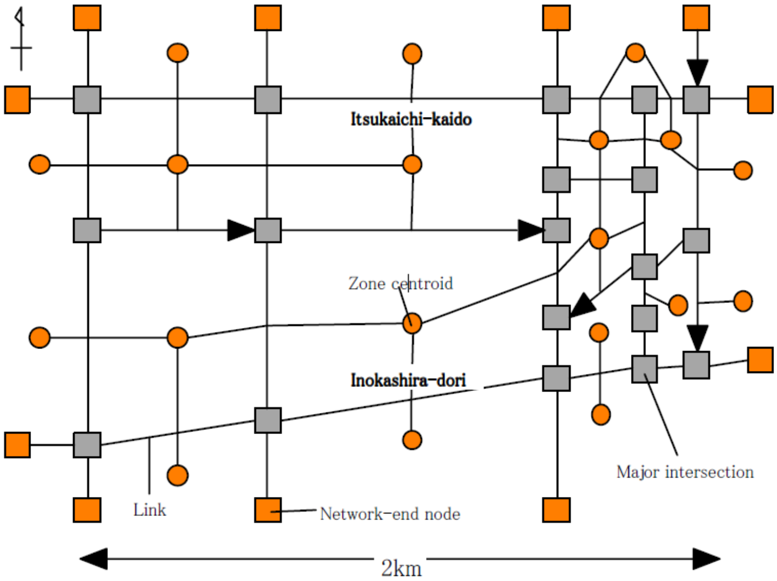


Figure 2 The Kichijoji-Mitaka network (Horiguchi, 1998)

3.2. Traffic Simulator

In order to reproduce the real traffic conditions, we develop a microscopic traffic simulator based on VISSIM using Kichijoji-Mitaka network and its data set. An equal scale road network is drawn on VISSIM. Network geometry including link lengths, number of lanes and link connectivity etc. is set the same as in reality. Parameters for the simulator, including signal timings and traffic controls are also set using the real data. The observed dynamic OD matrix is mapped onto the developed network to validate the simulator.

The comparison between the developed simulator and the real traffic network is shown in Figure 3. Figure 3a is the scatter plot of observed link flows given by the Kichijoji Benchmark Data Set and simulated link flows obtained from the simulator for each 10-minute period. The linear correlation coefficient between them is 0.8901, the slope of the linear fitted line is 0.9793, and the root mean square error (RMSE) is 13.56 vehs/10mins. We also select four links and observe dynamic link flows for various time intervals (Figure 3b). In Figure 3b, the red plots represent observed link flows while the blue plots represent simulated link flows. As we can see, the simulated link flows fits the observed values well. These results suggest that the simulator is able to reproduce real traffic conditions with high accuracy, although a better simulator would be obtained if more parameters of the true network, such as lane widths, stop line locations and road gradients were available.

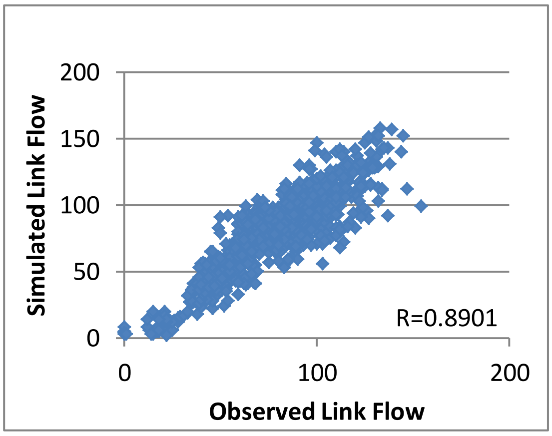


Figure 3a Scatter plot of observed and simulated link volume for each 10-minute period

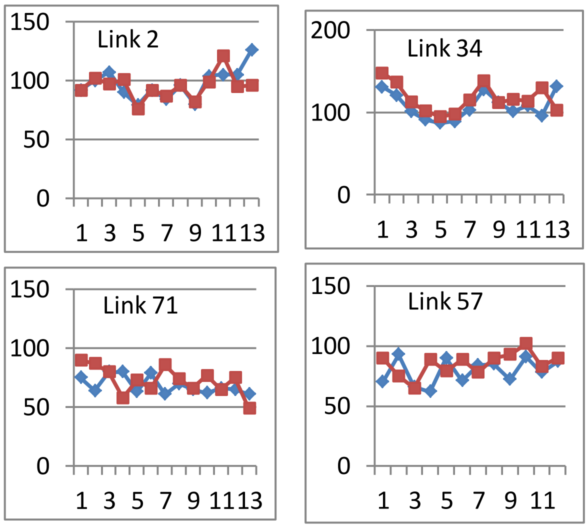


Figure 3b Link flows comparison for all time intervals. x-axis is the aggregated time interval, and y-axis is link flows per 10minuts

Figure 3 Validation of the developed simulator

3.3 Link Flow Estimation

Utilizing the developed VISSIM simulator, we can simulate the traffic within certain ratio of probe vehicles. Before implementing this simulator, we make assumptions of system as follows:

- Single class vehicles. All vehicles are assumed passenger vehicles with standard vehicle properties like length and width. Multi-class vehicles in the original network have been converted using vehicle conversion factors (Horiguchi, 1998).
- Driving behavior. We use the psycho-physical model of Wiedemann 74 (1974), which classifies four driving states: free driving, approaching, following and braking. Wiedemann 74 is applicable for inner urban road traffic. We assume free lane changing for vehicles which would change lane for larger traveling space or higher traveling speed. This means overtaking is permitted at any lane as long as conditions are satisfied.
- Dynamic traffic assignment. All vehicles are not equipped route guidance devices, thus make route choice decision only based on cost in previous iteration. We assume that driver chooses not only the optimal route but also a series of feasible routes.

The probe ratio is set 0.1 in the experiment. The simulation time is the morning peak period from 7:50am to 10:00 am, the length of aggregate time interval is assumed 10 minutes, and thus there are 13 time interval during the simulation time.

In principle every link has its own performance function, which is affected by factors like number of lanes, width and length of link and signal timings at intersections. However, in practice there is usually not enough data for determining performance function for each link. Considering the practical application, we choose number of lanes as the only criterion that distinguishes one link from another. Thus, two types of link are identified for the study network according to the number of lanes: roads with one lane and roads with two lanes in one direction. Then the link performance functions and *k*-*v* functions for them are obtained using method described section 2.2. The calibrated link performance functions and *k*-*v* functions are then used to estimate dynamic link flows using Bayesian method in section 2.3. In the Bayesian method, prior distributions of vehicle speed are aggregated over each 10 minutes.

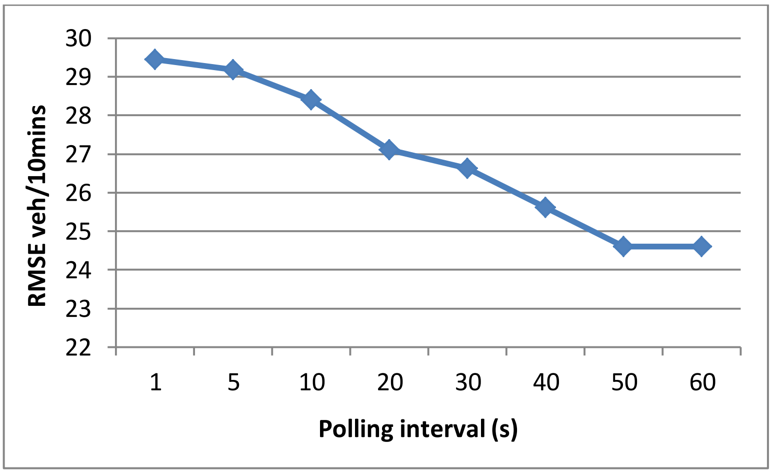
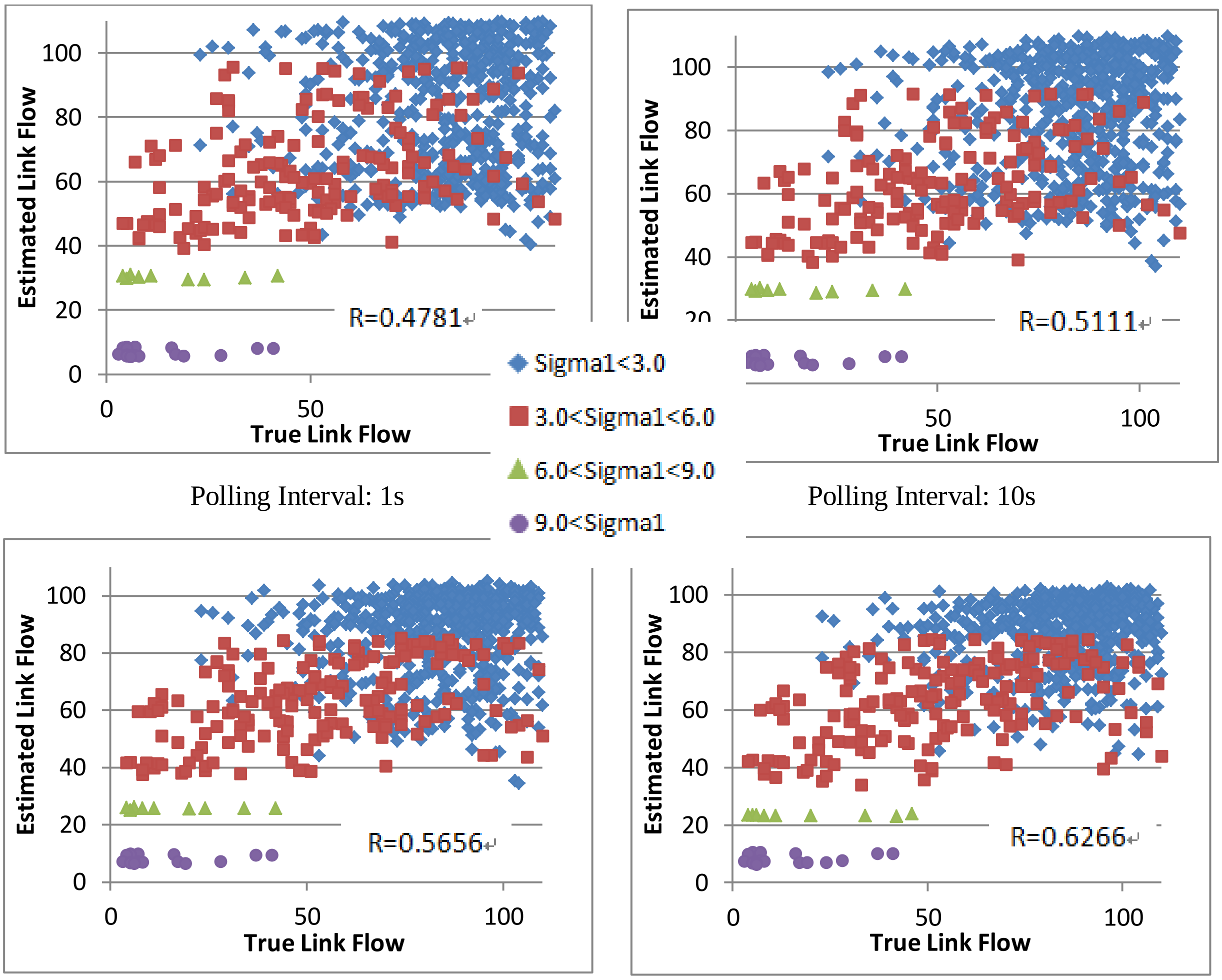


Figure 4 RMSE of estimated link flows

Figure 4 shows the RMSE of estimated link flows using various polling frequencies probe data. As we can see, the polling frequency does affect the accuracy of link flow estimation. Along with the polling interval becoming longer (i.e. the polling frequency becoming lower), the RMSE of estimated link flows decreases before polling interval is 50s, later begins to slow down. Further, we obtain acceptable estimates of link flows with RMSE 24.6 veh/10 minutes when polling interval is longer than 50s. In other words, using the proposed Bayesian method, the longer polling interval would produce higher accuracy estimated link flows.

In order to get insight of estimation results, we plot scatter figures of true link flow and estimated link flow for four different polling intervals: 1s, 10s, 30s and 60s (figure5). In figure 5, we also demonstrate the distribution of estimate with different σ_1 value ranges (the variance of posterior link speed distribution calculated by equation 8b) denoted by different tags. We can observe from these figures that the scatter plots become more concentrated on the diagonal line for each σ_1 value range with the increase of polling interval, and the correlation coefficient R becomes larger. Therefore, we can obtain the conclusion that the estimate link flows are more accuracy for longer polling intervals, which consists with that from figure 4. Additionally, from all plots in figure5 we can see the trend that the σ_1 increase with the decrease of link flow and this trend is clearer for longer polling interval. The reason can be found in equation (8b). There are more probe vehicles observed in certain time interval for links with higher volume of flow. The increase of number of probe vehicles *n* would bring the decrease of σ_1 in equation (8b).



Polling Interval: 30s Polling Interval: 60s

Figure 5 scatter plots of true and estimated link flows for various polling intervals

These results show that the link flow estimates are more accurate for longer polling intervals than shorter polling intervals. The reason lies in the proportional allocation in dealing with signalized intersection. Short polling interval probe data might incorrectly reflect the traffic condition in some specific cases. For example, even in the free flow situation, a probe vehicle, which arrives at the intersection when the red signal begins, will report a long link travel time (free travel time + red signal time, thus low link speed). The reported link travel time from short polling interval reflects actual travel time of the vehicle but it will lead misunderstanding of the traffic situation in some cases. Suppose that penetration of probe vehicle is 0.1 and there are 100 vehicles passing an intersection in one cycle. Even when 50 vehicles experience stop and 50 vehicles don't, it's possible that 8 probe vehicles experience stop and 2 probe vehicles don't stop. In this situation, the travel times from shorter interval data will lead biased traffic condition. In the long polling interval case, on the other hand, although the observed travel times will be relatively inaccurate, they are evened link by link and vehicle by vehicle and they can reflect actual traffic condition better than that in short polling interval case. As illustrated in section 2.1, the variance of derived link travel times from longer polling interval data using proportional allocation is smaller than that from shorter polling interval data regardless of free flow or congested flow. From these analyses, we can find that our conclusion is more suitable for the lower penetration of probe vehicle, since the randomness whether a vehicle experiences a stop is larger for small number of probe vehicles. Therefore, the proposed method is applicable for practice application, because penetration of probe vehicle is usually low in reality. Although our method performs better for longer polling interval (lower polling frequency), it doesn't mean that very low polling frequency data (e.g. one point every 2-5 minutes) can still give better results. In reality, polled points become sparser and map-matching becomes much more difficult and for lower frequency probe vehicle data, which results less reliable information. Based on this consideration, it’s better for the proposed Bayesian method to be applied in the situation of lower polling frequency probe vehicle data (e.g. one point every 30-60s).

3.4 OD Matrix Estimation

In the OD matrix estimation experiments, the matching link flow estimation results are used as the observed link flows defined in the bi-level GLS estimator. And the variances calculated from equation (9b) are used as the variances of the observed link flows in matrix W of equation (10). A noise level of 50% is added to the true OD matrix and the resulting OD matrix is used as the target matrix. Therefore, the 50% dispersion in true demand is regarded as the variance of the target OD demand in matrix Z of equation (10). Co-variances of both the observed link flow and the target OD demand are assumed to be 0. The value of ω is assumed to be 0.5 as in most studies. The aggregated time interval for OD demand is taken to be the same as the departure time interval and observation time interval for link flows, which is 10 minutes.

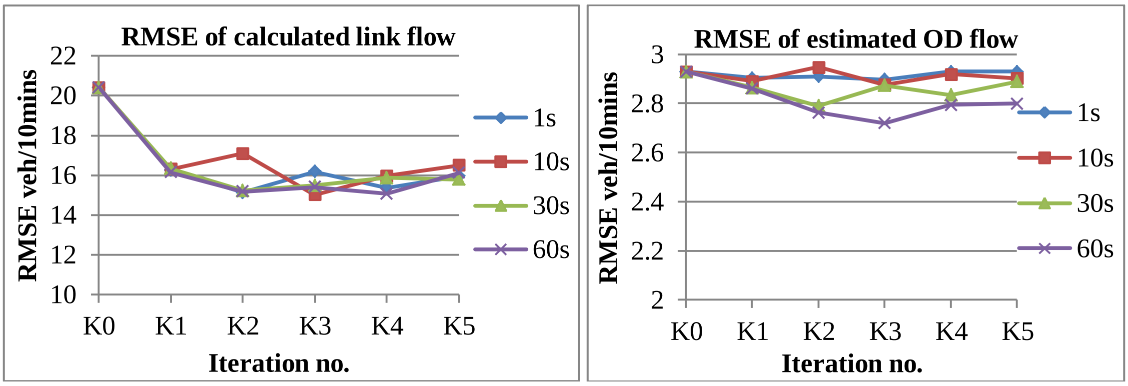


Figure 6 Results of dynamic OD matrices estimation

Various polling intervals probe data are tested in experiments. Figure 6 shows the iterative process of OD estimation. The RMSE of calculated link flow and the RMSE of estimated OD flow are calculated in each iteration. According to the left graph of Figure 6, the calculated link flow is improved using all kinds of polling intervals probe data. The calculated link flow using polling interval 60s is slightly better than 30s, 10s and 1s. This trend can be observed more significantly for the estimated OD matrix in the right graph of Figure 6. It is clear that the reason is that the estimated link flows from longer polling interval are more accuracy than that from shorter polling interval demonstrated in section 3.3. All in all, the network traffic assigned from dynamic OD matrix is improved using the proposed four-step methodology, and implement using lower polling frequency probe data performs better than that using higher polling frequency probe data.

4. CONCLUSION

In this paper, method of estimating dynamic link flows and OD matrices using lower polling frequency probe vehicle data is explored. The probe data is assumed the map-matched points on the road network. We proposed a four-step methodology including: travel time allocation, link performance function fitting, dynamic link flow estimation and dynamic OD matrices estimation.

In the first step, method of proportional allocation is used to decompose probe travel time onto individual links. This method can be easily carried out for network application, and it has a function of reducing the variance of the derived link travel time. In the second step, link performance function is obtained from a derived speed-density function. The speed –density function is derived from Gazis's nonlinear follow-the-leader model. The key feature of this function is that it only has four parameters, which are estimated using maximum likelihood method. We only classify links into two types by their number of lanes in the experiment from a view of practical application. If link performance function is known for each link, the estimate would be better. In order to estimate link flow, a Bayesian method that incorporates prior distribution of link speed is applied in third step. It has been shown that the Bayesian method can effectively use the prior distribution of vehicle speed accumulated from archived probe vehicle data, and produce an acceptable link flow estimate even if there is no probe vehicle observed in that link. In addition to the average value of link flow, the Bayesian method estimates the variance of estimated link flow at the same time (see equation 9b). Using the estimated link flow and its variance, we can implement the proposed bi-level GLS estimator to estimate dynamic OD matrices in the forth step. It is an extension of Tavana's model, both the distance between the estimated and target OD matrices and the distance between the calculated and observed link flows are considered in the objective function.

To make the proposed method more capable in application, we chose a commercial system VISSIM as the DTA simulator for a DTA-based dynamic OD matrix estimation model. This DTA simulator is developed according to a rather complete Kichijoji-Mitaka benchmark data set. Results show, the Bayesian method can give acceptable estimates of link flows, and bi-level GLS estimator can improve the original target OD matrices assumed in this research. Especially, the estimates become more accuracy for lower frequency probe data, which validates the proposed methodology is applicable to the cases of lower polling frequency probe data. As a whole, the proposed method can be applied in practice for estimating dyamic traffic state (link flow) and dynamic OD matrices using probe data, such as in probe vehicle-based dynamic route guidance system, or in situation that link counts are not available.

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