OPTIMIZING MODEL DEVELOPMENT APPLIED TO THE BUS SYSTEM OF HANOI CITY

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Abstract

The urbanization with the continuing increase of population in big cities in developing countries always raise many problems for the authorities related to infrastructure. In case of transportation system, expansion of the road network is always in urgent need. However, due to limitation on the city's size, the expansion of the road network alone cannot cope with the increasing travel demand in the whole city if an appropriate public transportation system is not established. In this paper, the establishment of bus system is investigated. Multinomial logit model is used to present the behavior of traveler in selection of transport mode and a mathematical model is proposed to solved for finding adequate fare level and frequency of bus. Future requirement on bus share, which is planned by the government, is the only target of this paper in the context of a subsidy - minimization program.

Key words: *Public Transport, Road Network, Transport Model, Minimization, Bus Route*

1. INTRODUCTION

The important role of public transportation system in urban area is evident for sustainable development of major cities. Raising the standard of public transportation system regards to the improvement of urban environment, traffic safety, traffic congestion, social welfare, living standard, healthy life of urban citizens, etc. However, at the present time, the bus service company get loss on business and the bus system of Hanoi falls short of demand due to its disadvantages such as long travel time, long waiting time, inappropriate bus route, etc. A look at the current transportation situation of Hanoi City reveals the problem. A remarkable characteristic of Hanoi's traffic growth at present is the rapidly increasing proportion of two-wheel vehicles especially motorcycles with around 60 - 65% of the total transportation demand and a concurrent decreasing share for the public transportation with around 2 - 3% among the total. In this scenario, the subsidy of bus system from the government is required. The objective of the newly designed effectively operated system is to satisfy the demand and attract usage. The most effective way to do is a fare and frequency setting policy so that the bus system may attract users up to a certain proportion of total travel demand that set by the government in the next 15 years while the subsidy is minimized.

In the paper, the bus network with 33 bus routes for Hanoi City is offered by the government. The demand function based on the multinomial logit model is used to represent the demand of traveler among transport modes respecting to bus fare and frequency. A mathematical model is proposed to solve for finding adequate fare level and frequency of bus in such a way to attract switching of travel demand from private transport modes toward bus mode in order to minimize subsidy from the government. The government plans future bus demand for the three five-year periods. The result of the research can be used in conjunction with other non-qualified objectives to help the government in making final decisions for improving urban transportation situation.

2. BUS DEMAND DETERMINATION

The basic problem confronted by transport mode choice analysis of consumer's behavior is the modeling of choice function from a set of mutually exclusive and collectively exhaustive alternatives. The analysis of transport mode choice is conducted based on the principle of utility maximization. In brief, it is assumed that a person will select the alternative with the highest utility among those available at the time choice must be made. Particularly in the case of transportation network, the transport mode with the best combination of travel time, travel cost, and comfort so that the utility of that mode is the largest will be selected. Here the concept "value of time" was introduced. Basically, it is assumed that a minute on travel time has the same marginal effect with that on working time. That is, a traveler spending a minute for travel will loss an monetary amount equivalent to the amount that he/she can get from his/her working time.

2.1 Sets of notation

- P_e : probability that traveler choose transport mode e,
- $d(x, f)$: the market-share function of bus system
- U_e : utility function of the transport mode e
  $e = 1, 2, 3, 4$ for bus, car, motorcycle, and bicycle, respectively.
- G_1 : bus fare (unit:10000VND)
- G_2 : operating cost of car mode (unit:10000VND)
- G_3 : operating cost of motorcycle mode including fuel cost, depreciation cost, maintain and small repairmen cost and insurance cost (unit: 10000VND).
- G_4 : operating cost of bicycle mode including depreciation (unit:10000VND)
- W_T : average waiting time of bus service (hour); $W_T = \dfrac{1}{2f}$
- T_1 : time in vehicle of bus (hour)
- T_2 : time in vehicle of car (hour)
- T_3 : time in vehicle of motorcycle (hour)
- T_4 : time in vehicle of bicycle (hour)
- a, b, c : parameters
- VOT : value of time (monetary evaluation of time) (unit: 10000VND/hour)
  VOT is estimated by average monthly income divided for total working hours
- d_{ij} : total travel demand per day from node i to node j in the first year of the planning horizon
- d_{ij}^t : total travel demand per day from node i to node j in year t.

2.2 Multinomial logit model form

The multinomial logit model for determining market share can be applied to determine the travel demand of a transport mode among total travel demand. That is, the probability that the travelers choose a transport mode according to the multinomial logit model formulated as:

$$P_e = \frac{\exp(U_e)}{\sum_e \exp(U_e)} = d(x,f) \qquad (1)$$

In the above formulation, U_e is the utility function of transport mode e and it can be expressed as a linear function of independent variable as follows:

$$\begin{aligned} U_1 &= aG_1 + bW_T VOT + cT_1 VOT \\ U_2 &= aG_2 + cT_2 VOT \\ U_3 &= aG_3 + cT_3 VOT \\ U_4 &= aG_4 + cT_4 VOT \end{aligned} \qquad (2)$$

Values of parameters can be estimated based on Levenberg-Marquardt method for nonlinear regression (see Press et al., 1992). The Data Fit package, the commercial software developed for nonlinear regression basing on the above method, will be used in this research for the determination of parameters.

2.3 The parameters determination

Data to calculate the parameters were taken from distributing and collecting 1500 questionnaire these aim to investigate the bus demand at a certain level of waiting time and ticket price. Time in vehicle of transport modes are assumed known, we have

$T_1 = 28/60$ (h); $T_2 = 15/60$ (h); $T_3 = 19/60$ (h); $T_4 = 36/60$ (h)
$C_2 = 4.2$ (10000VND); $C_3 = 2.4$ (10000VND); $C_4 = 0.06$ (10000VND)

- The first period: $VOT = 0.4808$ (VND)
  Result: $a = -14.9278$; $b = -19.9524$; $c = -35.4153$
- The second period: The annual economic growth rate is 1.07 then $VOT = 0.6743$ (VND)
  Result: $a = -14.9278$; $b = -14.2289$; $c = -25.2187$
- The third period: $VOT = 0.9457$ (VND)
  Result: $a = -14.9278$; $b = -10.1450$; $c = -18.0027$

2.4 Bus demand function

The demand of bus from the origin i to the destination j calculated for year t is:

$$d_{bus} = d(x,f)d_{ij}^t \qquad (3)$$

d_{ij}^t is calculated as total demand in the first year of the horizon study multiplied by the average demand growth rate per year $d_{ij}^t = d_{ij}\alpha^{t-1}$ (4)

3. OPTIMIZATION MODEL DEVELOPMENT

The objective of the government is to provide a good public transportation system in the inner city in such a way that the total subsidy is minimized while encouraging people usage, which is very low at the present time. By providing a "cheap" and "high frequent" (which means convenient) bus system, it is possible to expect that motorcycle, bicycle and car users will shift day by day to the public transport mode.

It is noted that fare level and frequency are considered as key factors to help attracting bus usage and will be treated as variables in the model. The planning bus route network from the government will be taken into consideration and other alternatives of bus routes will not be considered in this research. In order to reduce the total required investment on buses at the beginning of the planning horizon, the planning horizon will be separated into 3 periods with the length of 5 years. The mathematical model will be developed based on these three periods.

3.1 Assumptions

The following assumptions are used in the development of the model

1. All of transport modes (car, motorcycle, bicycle) will take the same path with bus.

2. Fare level is considered as constant in each period regardless of the route and the length that a customer will travel along the route.

3. Annual growth rate of transportation demand in the whole network is constant for the planning horizon of 15 years.

4. If there is more than one route connecting the two nodes (i, j) in the network then the traveler will select the first bus coming to his/her waiting place regardless of bus type and bus route.
5. For a pair of nodes (i, j) that has no connection by any single bus route, if a traveler decides to travel by bus between these two nodes, he/she will select a path among available paths based on two criteria with the priority order as follows:

Criteria 1: The traveler will try to minimize the total fare that he/she has to pay, i.e., he/she will select the path with the smallest number of transfers.

Criteria 2: The traveler will select the path with the shortest distance, or equivalently, the shortest travel time

3.2 Set of notations

- i, j : nodes on the bus route network.
- (i, j) : a pair of nodes on the bus route network.
- ij : a direct link between two node i, j on the bus route network.
- x_1, x_2, x_3 : bus fare for period 1, period 2 and period 3.
- r : rate of return.
- α : demand growth rate.
- m : index of bus type (m = 1, 2, ...,M).
- l : index of route (l = 1, 2, ..., L).
- p_m : price of bus type m (in constant dollar).
- S_m : maximum capacity of bus type m.

- C_m : average hourly cost of operating bus type m which includes depreciation, direct and indirect salary, fuel cost, maintenance and assurance cost .
- T : daily working time of bus system (hours).
- T_l : cycle time of bus on route l.
- f_{ml}^k : frequency of bus type m on route l in period k.
- f_l^k : Frequency of bus on route l in period k.
- $f_{(i,j)}^k$: frequency of bus between two nodes i, j in period k.
- f_{ij}^k : frequency of bus on an direct link ij.
- d_{ij} : total demand from node i to node j in the first year of the planning horizon.
- D_{ij}^k : total number of passengers going through link ij during the last year of period k
- $n_{(i,j)}$: an index matrix which is defined as follows

  $n_{(i,j)} = 1$ if there is at least one route connecting node i and node j.

  $n_{(i,j)} = k$ if the passenger has to take k bus route in order to go from i to j.

Then we have the relationship between some notations as follows:

a. The frequency of bus on route l is the sum of frequencies of all type of buses on that route
b.

$$f_l^k = \sum_{m=1}^{M} f_{ml}^k$$

b. The frequency of bus between two nodes i, j can be expressed through frequencies of bus through routes as follows

- If there is at least one route connecting i and j then $f_{(i,j)}^k = \sum_{l|ij \in l} f_l^k$

Where the sum is taken on all routes l which go through node i and node j.

- If there is no route connecting i and j, and the customer has to transfer to some routes in order to go from i to j. Suppose the set of start nodes and end nodes on the path of the customer is $(i, a_1, a_2, \ldots, a_n, j)$ then $f_{(i,j)}^k$ can be defined as

$$f_{(i,j)}^k = Min(f_{(i,a_1)}^k, f_{(i,a_2)}^k, \ldots, f_{(i,a_j)}^k)$$

3.3 Detailed development

Objective function

The objective function of the model is

Minimize Subsidy = Minimize (Investment cost + Operation cost - Revenue)

a. <u>The first period</u>

- The investment cost can be developed as follows:

The number of bus type m needed on route l is: $T_l * f_{ml}^1$

Investment for bus type m on route l is: $p_m * T_l * f_{ml}^1$

Investment for all type of bus on route l is: $\sum_{m=1}^{M} p_m * T_l * f_{ml}^1$

Therefore, total investment cost can be expressed by: $\sum_{l=1}^{L}\sum_{m=1}^{M} p_m * T_l * f_{ml}^1$

- The operation cost can be developed as follows:

The expense of one bus type m in a day is: $C_m * T$

The expense of all bus type m on route l in a day is: $T_l * f_{ml}^1 * C_m * T$

Yearly expense of all bus type m on route l is: $365 * T_l * f_{ml}^1 * C_m * T$

Yearly expense of all buses is: $\sum_{l=1}^{L}\sum_{m=1}^{M} 365 * T_l * f_{ml}^1 * C_m * T$

Therefore, total operation cost in the first period can be expressed by:

$$\sum_{t=1}^{5} \frac{1}{(1+r)^t} \sum_{l=1}^{L} \sum_{m=1}^{M} 365 * T_l * f_{ml}^1 * C_m * T$$

- The revenue can be developed as follows:

Bus demand from i to j for the first year is: $d(n_{i,j)} x_1, f_{(i,j)}^1) * d_{(i,j)}$

Where $d(n_{i,j)} x_1, f_{(i,j)}^1) * d_{(i,j)}$ is the market share of bus from i to j (demand function).

Daily revenue received by transportation from i to j is:

$n_{(i,j)} * x_1 * d(n_{(i,j)} * x_1, f_{(i,j)}^1) * d_{(i,j)}$

Therefore, total revenue for the first period is:

$$\sum_{t=1}^{5} \sum_{(i,j)} \frac{365 * n_{(i,j)} * x_1 * d(n_{(i,j)} * x_1, f_{(i,j)}^1) * d_{(i,j)} (1+\alpha)^{t-1}}{(1+r)^t}$$

b. <u>The second period</u>

Similarly, we have:

Total investment cost: $\frac{1}{(1+r)^5} \sum_{l=1}^{L}\sum_{m=1}^{M} p_m * T_l * (f_{ml}^2 - f_{ml}^1)$

Total operation cost: $\sum_{t=6}^{10} \frac{1}{(1+r)^t} \sum_{l=1}^{L} \sum_{m=1}^{M} 365 * T_l * f_{ml}^2 * C_m * T$

Total revenue: $\sum_{t=6}^{10} \sum_{(i,j)} \frac{365 * n_{(i,j)} * x_2 * d(n_{(i,j)} * x_2, f_{(i,j)}^2) * d_{(i,j)} (1+\alpha)^{t-1}}{(1+r)^t}$

c. For the third period

Similarly, we have

Total investment cost: $\frac{1}{(1+r)^{10}} \sum_{l=1}^{L} \sum_{m=1}^{M} p_m * T_l * (f_{ml}^3 - f_{ml}^2)$

Total operation cost: $\sum_{t=11}^{15} \frac{1}{(1+r)^t} \sum_{l=1}^{L} \sum_{m=1}^{M} 365 * T_l * f_{ml}^3 * C_m * T$

Total revenue:

$$\sum_{t=11}^{15} \sum_{(i,j)} \frac{365 * n_{(i,j)} * x_3 * d(n_{(i,j)} * x_3, f_{(i,j)}^3) * d_{(i,j)} (1+\alpha)^{t-1}}{(1+r)^t}$$

Set of constraints

a. The first set of constraints

For each period k, the total capacity of bus on each direct link ij must equal or greater than the total number of passengers going through that link:

$$\sum_{l|ij \in l} \sum_{m=1}^{M} f_{ml}^k * T * S_m \geq D_{ij}^k$$

Where the first sum on the left-hand side is taken over all routes l that go through i and j.

b. The second set of constraint

Based on the planning of the government, the total number of bus customer must be at least 15%, 20% and 30% of the total transportation demand for each period. Therefore, another set of constraints has to be introduced into the formulation of the model.

For the first period, we must have $\sum_{(i,j)} d(n_{(i,j)} x_1, f_{(i,j)}^1) * d_{(i,j)} \geq 0.15 * \sum_{(i,j)} d_{(i,j)}$

For the second period, we have $\sum_{(i,j)} d(n_{(i,j)} x_2, f_{(i,j)}^2) * d_{(i,j)} \geq 0.20 * \sum_{(i,j)} d_{(i,j)}$

For the third period, we have $\sum_{(i,j)} d(n_{(i,j)} x_1, f_{(i,j)}^3) * d_{(i,j)} \geq 0.30 * \sum_{(i,j)} d_{(i,j)}$

c. Third set of constraint

It is assumed that buses, which are working on one period, will continue its operation for the neat period. Therefore, the following constraints are incurred $f^2_{ml} \geq f^1_{ml}\ and\ f^3_{ml} \geq f^2_{ml}$

The full formulation of the program can now be expressed as:

$$\text{Min} \quad \sum_{l=1}^{L}\sum_{m=1}^{M} p_m * T_l * f^1_{ml} \quad + \quad \sum_{t=1}^{5}\frac{1}{(1+r)^t}\sum_{l=1}^{L}\sum_{m=1}^{M} 365 * T_l * f^1_{ml} * C_m * T$$

$$- \sum_{t=1}^{5}\sum_{(i,j)}\frac{365 * n_{(i,j)} * x_1 * d(n_{(i,j)} * x_1, f^1_{(i,j)}) * d_{(i,j)}(1+\alpha)^{t-1}}{(1+r)^t}$$

$$+ \quad \frac{1}{(1+r)^5}\sum_{l=1}^{L}\sum_{m=1}^{M} p_m * T_l * (f^2_{ml} - f^1_{ml}) \quad + \quad \sum_{t=6}^{10}\frac{1}{(1+r)^t}\sum_{l=1}^{L}\sum_{m=1}^{M} 365 * T_l * f^2_{ml} * C_m * T$$

$$- \sum_{t=6}^{10}\sum_{(i,j)}\frac{365 * n_{(i,j)} * x_2 * d(n_{(i,j)} * x_2, f^2_{(i,j)}) * d_{(i,j)}(1+\alpha)^{t-1}}{(1+r)^t}$$

$$+ \quad \frac{1}{(1+r)^{10}}\sum_{l=1}^{L}\sum_{m=1}^{M} p_m * T_l * (f^3_{ml} - f^2_{ml}) \quad + \quad \sum_{t=11}^{15}\frac{1}{(1+r)^t}\sum_{l=1}^{L}\sum_{m=1}^{M} 365 * T_l * f^3_{ml} * C_m * T$$

$$- \sum_{t=11}^{15}\sum_{(i,j)}\frac{365 * n_{(i,j)} * x_3 * d(n_{(i,j)} * x_3, f^3_{(i,j)}) * d_{(i,j)}(1+\alpha)^{t-1}}{(1+r)^t}$$

Subject to

1. $\sum_{l|ij \in l}\sum_{m=1}^{M} f^k_{ml} * T * S_m \geq D^k_{ij}$ $\quad \forall direct\ link\ ij$, and $\forall k$
2. $\sum_{(i,j)} d(n_{(i,j)}x_k, f^k_{(i,j)}) * d_{(i,j)} \geq (percentage(k) * \sum_{(i,j)} d_{(i,j)} \quad \forall k$
3. $f^2_{ml} \geq f^1_{ml}\ and\ f^3_{ml} \geq f^2_{ml}$ $\quad (m = 1,2,\ldots,M;\ l = 1,2,\ldots,L)$
4. $f^1_{ml} \geq 0$ $\quad (m = 1,2,\ldots,M;\ l = 1,2,\ldots,L)$
5. $x_k \geq 0$ $\quad (k = 1,2,3)$

where $Pecentage(k) = 0.15,\ 0.20,\ 0.3$ for k =1, 2, 3 respectively.

Decision variables of the program is x_k and f^k_{ml}. With three types of buses, 35 bus routes with 106 direct links are considered, the above program has 318 variables; 210 linear constraints; 321 nonlinear constraints.

4. RESULT OF THE OPTIMAL MODEL

Running the non-linear programming we get the result of bus fare level and frequencies as shown in Table 1 and Table 2. Bus frequencies increases timely while fare level decrease to attract people usage from 15% of the total travel demand in the first period to 20% of total travel demand in the second period and then up to 30% of total travel demand in the third period. Results:

- The subsidy for the first period is: **-98,580,631,620** VND
- The subsidy for the first period is: **-99,574,480,840** VND
- The subsidy for the first period is: **21,828,969,170** VND

Those results mean that in the first and the second period we get benefit from operation of bus system. Subsidy will be paid for the third period only to attract bus users.

Table 1. Bus Fare level

The first period	The second period	The third period
819.57	664.91	598.79

Table 2. Frequencies of bus

	The first period			The second period			The third period		
Route	Bus type 1	Bus type 2	Bus type 3	Bus type 1	Bus type 2	Bus type 3	Bus type 1	Bus type 2	Bus type 3
1	2.94	1.00	1.08	5.65	1.07	1.08	8.47	4.67	1.61
2	8.79	2.00	3.00	10.47	4.00	5.00	9.41	6.24	7.65
3	4.49	4.21	1.06	4.49	7.62	2.06	6.10	8.50	4.38
4	3.00	2.17	1.02	5.00	6.31	2.02	7.78	8.26	2.82
5	2.33	3.10	1.00	4.50	6.19	3.00	7.32	7.22	4.18
6	3.18	3.07	2.00	5.18	5.31	4.00	8.66	10.84	4.25
7	3.00	5.00	3.00	5.00	7.80	4.50	7.40	9.44	10.44
8	4.00	6.20	4.56	8.15	8.25	9.47	12.56	15.78	13.20
9	4.21	5.16	3.28	7.56	8.05	5.60	9.23	8.06	7.55
10	2.12	3.15	1.00	4.15	6.07	5.23	7.06	6.09	8.56
11	2.13	4.52	1.05	2.13	8.04	2.05	2.98	12.62	3.47
12	4.81	1.00	3.20	4.81	5.73	5.27	7.70	10.51	8.05
13	2.03	2.02	3.06	3.06	4.52	4.67	7.60	5.98	6.24
14	3.06	4.37	2.65	6.45	6.23	5.32	8.56	9.55	8.32
15	3.65	3.24	1.00	3.98	4.27	2.00	4.56	4.05	3.09
16	3.00	2.07	1.00	3.00	3.56	2.65	6.67	7.62	3.50
17	3.90	1.10	2.00	6.73	1.10	2.56	8.67	3.52	2.87
18	2.05	6.65	1.00	3.09	8.67	2.68	5.65	9.65	3.65
19	2.06	2.64	1.00	3.65	3.98	2.06	5.62	5.67	4.56
20	3.65	3.68	2.65	4.47	5.62	3.98	5.62	6.53	5.45
21	1.70	2.67	4.32	3.55	4.82	5.69	5.96	7.56	7.68
22	2.20	3.67	2.56	2.29	4.13	3.00	4.08	6.86	5.87
23	2.09	1.09	2.56	3.90	1.19	3.67	5.65	4.05	3.65
24	4.24	7.09	1.10	4.24	9.06	3.02	8.49	10.06	5.07
25	3.81	1.09	3.02	3.81	3.54	5.06	5.98	5.67	7.28
26	3.00	1.03	1.00	4.28	3.65	3.6	5.71	5.62	6.30
27	3.04	2.85	1.02	3.04	3.06	3.16	4.78	5.62	5.17
28	1.97	2.65	3.05	3.91	3.65	5.62	5.28	4.35	6.62
29	3.06	2.65	1.00	4.56	4.65	3.05	4.56	5.56	4.67
30	4.67	2.55	3.65	5.56	3.21	4.20	6.65	5.86	4.56
31	3.02	2.00	1.00	4.27	3.06	1.00	5.96	5.67	2.68
32	3.09	2.95	1.04	5.06	3.38	2.87	7.62	4.65	4.32
33	2.57	1.05	2.01	2.61	3.61	4.32	4.67	4.23	5.62
34	3.02	2.65	1.00	4.89	4.49	2.06	5.89	5.47	3.61
35	3.56	2.68	1.07	5.82	4.65	1.07	8.89	5.53	2.13

Table 3. Costs and revenues estimating for three periods

	Investment cost	Operating cost	Revenue	Subsidy
The first period	75,458,602,220	47,326,630,100	221,365,863,900	-98,580,631,620
The second period	74,209,132,430	47,100,408,030	220,884,021,300	-99,574,480,840
The third period	178,759,458,700	43,035,931,350	199,966,420,900	21,828,969,170

5. CONCLUSIONS

This research deals with a complicated problem raising in establishment of the public transportation system in Hanoi City - Vietnam. Through a subsidy-based minimization program, although optimal solution cannot be guaranteed, a good policy of bus fare level and bus frequencies for the planning horizon of 15 years can be proposed. It has been found that the target of the government for encouraging bus usage can be archived for the future planning with an acceptable level of bus fare and investment on bus system. The result of this research can be used in conjunction with other non-quantified objectives to help the government in making final decision.

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