

OPTIMAL TRANSPORTATION INFRASTRUCTURE INVESTMENT TIMING WITH UNCERTAINTY

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Abstract: This paper is motivated by a concern that the current economic evaluation for the transportation infrastructure investment does not adequately consider the timing of investments. The conventional economic evaluation procedure considers only built-now alternative by comparing evaluation index of each characteristic alternative at the same designed year. Another deficiency of the conventional transportation investment evaluation method is that it does not properly capture and take into account the uncertainty of the evaluation indices such as Net Present Value (NPV), Economic Internal Rate of Return (EIRR), and Benefit-Cost Ratio (B/C Ratio). The objective of this study is to propose a transportation infrastructure evaluation method which takes into account not only the optimal investment timing but also the uncertainty of benefits and costs. Monte Carlo simulation technique is utilized to capture the probability density function of evaluation index. The proposed approach is demonstrated with a transportation infrastructure project in Bangkok, Thailand.

Key Words: Economic Evaluation, Optimal Investment Timing, Risk Analysis, Uncertainty, Monte Carlo Simulation

1. INTRODUCTION

Every year, immense funds are invested in transportation infrastructure to support economic growth and enhance quality of life. To spend the huge amount of money more reasonably, the reliable tools that are able to predict whether a project should be implemented or not, and to ensure that the resource would be used most efficiently, are required. Project evaluation is a tool to evaluate appropriateness of a project. It is used to provide necessary information to help decision makers in planning process. Project evaluation is widely used for both public and private sectors.

This paper is interested to a concern that the current planning process for the transportation infrastructure investments does not adequately consider the timing of investments. Major investment studies typically look for performance in the design year, such as 30 years from now, which gives the result for only a build-now alternative. Another deficiency of the current transportation infrastructure investment decision procedure is that it uses a point estimate, which is only a single value based on the most likely value. It is not enough for decision makers to trust on the outcome. At least, they should be able to know how seriously the uncertainty affects the decisions.

The objective of this study is to propose a transportation infrastructure evaluation method that takes into account not only the optimal timing but also the uncertainty in predicting the future. The optimal timing is proposed to solve the problem that in some cases, if the project were postponed, it would make the investment more efficient whereas in conventional method realizing only on now or never basis (Chu *et al.*, 1998). However, future is still the future, which is subject to great uncertainty; that is, no one exactly knows. Therefore, the uncertainty is also explicitly considered in the methodology of this study. Since the uncertainty is coped by the risk analysis method (Lam *et al.*, 1998), the results from the analysis would be a probability density function, which is more precious than predicting only a single most likely value that cannot tell the whole story (Hertz, 1979).

This paper first defines and formulates the optimal infrastructure investment timing problem with uncertainty. The proposed methodology is then outlined followed by the example problem and study design. Lastly the results will be discussed.

2. METHODOLOGY

2.1 Problem Definition and Mathematical Formulation

The method to evaluate the infrastructure investment timing is similar to that of the general infrastructure project evaluation. The difference is that in identifying optimal investment timing, the project must be evaluated over a project's variable time horizon period rather than over a fixed year when the year of construction and operation is already fixed before evaluating. In the optimal timing evaluation method, the construction and operation periods are flexible and they can be shifted into any year in analysis period. Therefore, the analysis period of this method must be longer than the time horizon of the conventional method. The benefit and cost for each year must be calculated for each plan to start the project as called "Time Alternative". The service life for each starting time alternative should be the same (e.g. service life being equal to 25 years) to make a fair evaluation, no matter when the project would start.

The evaluation index such as NPV, EIRR, and B/C Ratio, which composes of many kinds of uncertainties, is calculated from benefit and cost of each year. The probability density function or stochastic value of it is used to represent the uncertainty of each key variable. Accordingly, the evaluation index would have the distribution and variance which provides more accurate and helpful information to planner. In this study, each key variable (i.e., benefit and cost) of each year for each alternative is assumed to have its own uncertainty as presented by the probability distribution and to be a time-dependent variable. Hence, by including the uncertainty of each stochastic variable, the expected value of NPV in the base year "O" when the project starts to operate at the year "S" would be calculated by equation (1):

$$E(NPV)_S = \int_S^{S+T} E(\text{Benefit}_{s,t}) \cdot e^{-rt} dt - \int_{S-U}^S E(\text{Cost}_{s,t}) \cdot e^{-rt} dt \quad (1)$$

Where,

- T = Operating period of the project (i.e., period from the project starting to operate to the end analysis period)
- U = Construction period of the project (i.e., period from the construction beginning to the starting year of the project)

- $e^{-E(r)t}$ = Discounting term to convert the value into base year "O". It is same as [P|F, r, t] which is considered in the discrete function.
- r = Discount rate
- $E(NPV)_s$ = The expected value of NPV at the starting year of operation "S"

To choose the optimal timing of the project investment, the considered time period is defined as in Figure 1 from the cash flow perspective.

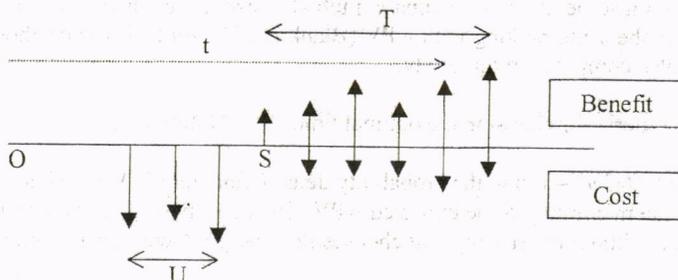


Figure 1. The cash flow diagram

Note that the starting time of construction of the project is S-U. Similarly, the S+T means the end of analysis period. In other words, U+T is the fixed horizontal time of cash flow in the conventional evaluation. From Equation (1), it can be noticed that NPV varies with the starting year, S, which emphasizes the significance of the study. That is, the NPV of the same project can be different depending on the starting year, S, and this should be the same with the EIRR and B/C Ratio.

2.2 Conceptual Framework

The conceptual framework to identify NPV in the base year "O" when the project starts to operate at the year "S", NPV_s , is depicted as shown Figure 2.

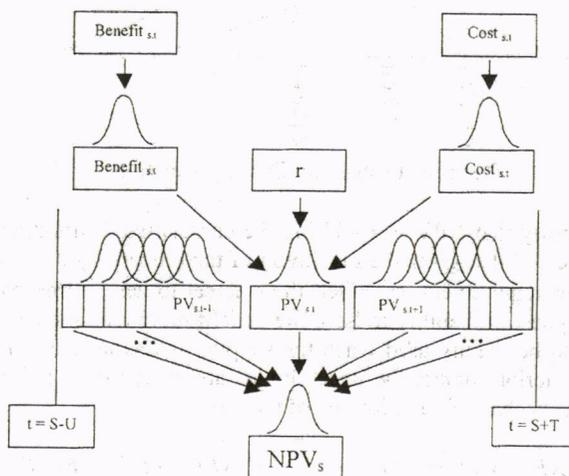


Figure 2. The conceptual framework to identify NPV

2.3 Criteria for Choosing

The NPV is chosen as the best evaluation index of this methodology because of its high potential for evaluation, which can give the best, actual result compared to other indices (Layard *et al.*, 1996). The NPV can also cope with the differences in the size, investment cost and returned benefit of projects for choosing the alternatives whereas the other familiar indices such as EIRR and B/C ratio cannot be used to compare the alternatives easily. EIRR and B/C ratio must be used to compare mutually exclusive alternatives by incremental analysis to give the same ranking with NPV (Blank *et al.*, 1998). However, those three kinds of results are also computed in this study.

There are two criteria for choosing the optimal time "S" as following;

- (1) Maximum $E[NPV]$ – Using the probability density function (PDF) to choose the time "S" that gives the maximum of the expected NPV. The concept of this criterion is the same as the problem without uncertainty that chooses the year producing the maximum of NPV.

$$\arg \max [E(NPV_1), E(NPV_2), \dots, E(NPV_S), \dots, E(NPV_n)]$$

Where,

$$E(NPV_S) = \text{The expected value of NPV in the base year "O" when the project starts to operate at the year S}$$

For example, in Figure 3, the alternative C (starting at the year C) is chosen since it has the maximum expected NPV.

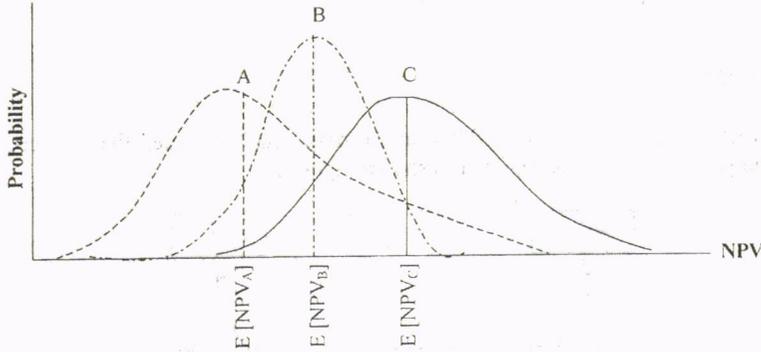


Figure 3. Probability Density Function

- (2) Maximum probability that $NPV > \alpha$ – Using the cumulative distribution function (CDF) to choose the time "S" that gives the maximum of the probability that the NPV is greater than the minimum requirement, α . When the α is set to zero, it means that the chosen project has the highest probability to have the benefit more than the cost. The concept of this criterion could be mainly used when the supplier is risk averse. The decision-maker cannot use this criterion, unless the uncertainties are analyzed by risk analysis method, which can give the probability distribution of the result.

$$\arg \max [P(NPV_1 > \alpha), P(NPV_2 > \alpha), \dots, P(NPV_S > \alpha), \dots, P(NPV_n > \alpha)]$$

Where,

$P(NPV_s > \alpha)$ = The probability that the NPV of the project when starts at the year S is more than α

For example, if the decision-maker is risk averse, the alternative E (starting at the year E) is chosen as shown in Figure 4 since it has the highest probability that the NPV is greater than the minimum requirement, $\alpha = 0$, which equals to $1 - 0.1 = 0.9$.

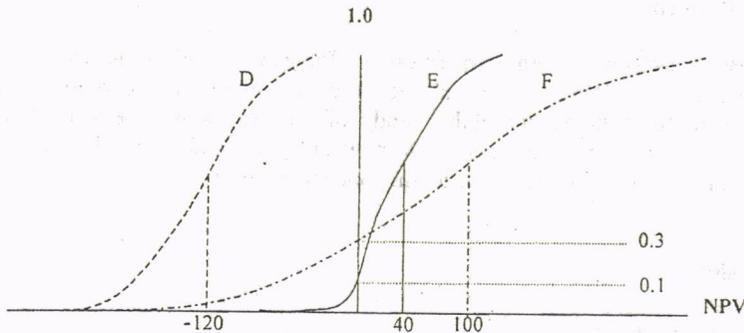


Figure 4. Cumulative Distribution Function

This criterion gives the extra information to decision-maker. According to Figure 4, the ordinary decision-maker must trade-off between these two criteria, one with the higher expected NPV (100, Alternative F) and the other with the higher probability that the expected NPV is greater than the minimum requirement (0.9, Alternative E).

2.4 Extra Alternatives

The proposed methodology will be able to consider multiple number of alternatives for evaluating projects with the concept of optimal investment timing as outlined below:

A_{11}	A_{12}	...	A_{1j}	...	A_{1n}
A_{21}			\vdots		\vdots
\vdots			\vdots		\vdots
A_{i1}	...		A_{ij}		\vdots
\vdots			\vdots		\vdots
A_{m1}	...		\vdots		A_{mn}

Where,

A_{ij} = The i^{th} alternative from m alternatives for choosing characteristics of the project such as route selection or size of project, and the j^{th} alternative from n alternatives for optimal investment timing

That is, the conventional evaluation procedure can solve the problem only in the first column, m alternatives, since it assumes a given starting year of the project and decides only which alternative should be implemented – including do-nothing alternative. In contrast, the proposed approach will consider $m \times n$ alternatives for choosing the plan to implement, if it has

n alternatives for choosing the best year to start operating. Not only which project should be implemented, but also when it should be answered. Moreover, by including the uncertainties in the evaluation procedure, it will give more information to the decision-maker.

3. EXAMPLE PROBLEM AND STUDY DESIGN

3.1 Example Problem

Srinakkarin-Bangna-Samutprakarn Expressway in Bangkok, Thailand is selected in this study as an example problem. It is planned to be a six-lane elevated expressway of 13.8 kilometers length. In 1999, the feasibility study of the project was performed with the conventional method by considering only build-now alternative and the result shows a single value of the evaluation index without considering uncertainty of it.

3.2 Study Design

3.2.1 General Assumptions

This study assumes that the benefit and cost can be estimated if the project is postponed. It is important to note that these values can be more realistic if the traffic model were run again for other time alternatives. Instead of that, the probability density functions (PDFs) are utilized to define the uncertainty of them. The Monte Carlo simulation technique is then used to estimate the probability of the outputs (NPV, EIRR, and B/C ratio).

This paper considers ten alternatives of different investment timing and there is no characteristic alternative such as route selection or size of project (i.e., $m=1$ and $n=10$). Each alternative is different in the starting year of operation (S). The first alternative (A1) means the starting time of construction of the project (S-U) is the year 1999, construction period assumed constantly four years, and the end of analysis period (S+T) is in the year 2028. The second alternative (A2) means the starting time of construction of the project (S-U) is the year 2000, and the end of analysis period (S+T) is in the year 2029, and so on. Thus, the last time alternative (A10) is considered from the year 2008 to 2037 as shown in Table 1.

All prices are based on the constant baht (Thai currency) in the year of 1998, which is also the base year of converting value calculation, year 0. Therefore, the NPV of the year of 1998 is assumed. Discount rate is assumed to be given by the decision-maker, normally 12% in Thailand. The estimated salvage value of each alternative is defined as equation (2) (ETA, 1999).

$$\text{Salvage Value} = \frac{\text{The remaining live of the project} * \text{The investment cost}}{\text{The project total live}} \quad (2)$$

Where the project's total life is assumed to be 50 years, the remaining life of the project is 24 years, and the investment cost is the sum of costs on the first four years. Hence the estimated value of the last year of each alternative for the last year is calculated from the salvage value plus the maintenance cost of that year, which is 39.91 million baht.

Table 1. Benefit and Cost of each year in analysis period

t	Year	Alternatives									
		A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
1	1999	C									
2	2000	C	C								
3	2001	C	C	C							
4	2002	C	C	C	C						
5	2003	B & C	C	C	C	C					
6	2004	B & C	B & C	C	C	C	C				
7	2005	B & C	B & C	B & C	C	C	C	C			
8	2006	B & C	B & C	B & C	B & C	C	C	C	C		
9	2007	B & C	B & C	B & C	B & C	B & C	C	C	C	C	
10	2008	B & C	B & C	B & C	B & C	B & C	B & C	C	C	C	C
11	2009	B & C	B & C	B & C	B & C	B & C	B & C	B & C	C	C	C
12	2010	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	C	C
13	2011	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	C
14	2012	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
15	2013	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
16	2014	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
17	2015	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
18	2016	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
19	2017	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
20	2018	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
21	2019	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
22	2020	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
23	2021	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
24	2022	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
25	2023	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
26	2024	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
27	2025	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
28	2026	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
29	2027	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
30	2028	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
31	2029		B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
32	2030			B & C	B & C	B & C	B & C	B & C	B & C	B & C	B & C
33	2031				B & C	B & C	B & C	B & C	B & C	B & C	B & C
34	2032					B & C	B & C	B & C	B & C	B & C	B & C
35	2033						B & C	B & C	B & C	B & C	B & C
36	2034							B & C	B & C	B & C	B & C
37	2035								B & C	B & C	B & C
38	2036									B & C	B & C
39	2037										B & C

Remark: B is benefit with probability density function.
 C is cost with probability density function.

3.2.2 Key Variable

Key variable is defined as the factor that has the effect on the output. To find the probability distribution of the output (i.e. NPV), the key variables are defined as benefit and cost. If one wants to consider benefit variations in every small benefit component that goes into a detailed estimate, the approach will be impractical. In this sense, only main items that are totally independent are modeled probabilistically. With this approach the possibility that some correlations of uncertainties (uncertainties at the intersection among those sub-items) are omitted or double-counted will decrease. However, note that each of these items comprises of those sub-items' uncertainties. Therefore, an integrated approach would assess the key

variables (i.e., benefit and cost) consistently and completely with the assumption that there is no correlation among these key variables. In other words, benefit does not depend on cost since the actual cost that is lower or higher than estimated cost has no effect on benefit and vice versa.

Since this study used the data from the feasibility study using the conventional project evaluation method, the estimated value of benefit and cost exist only for the A1 alternative. Accordingly, in this study the estimated value must be assumed for other alternatives starting later. There are existing data of benefit in the feasibility study (ETA, 1999) for 30 years (i.e., year 2028). The trend line is used to expand the benefit data until the year 39 (year 2037), which is the last year of evaluation as shown in Figure 5.

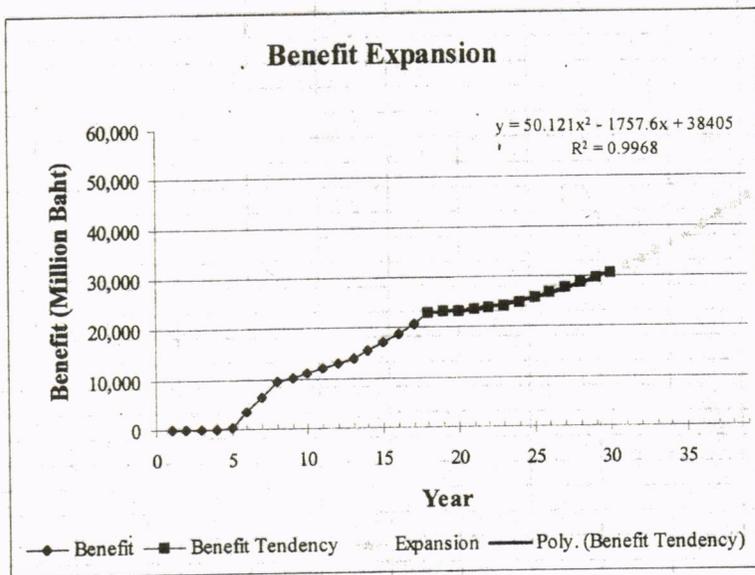


Figure 5. Benefit Expansion

The estimated benefit of each year is assumed to be the same across all alternatives. For example the benefit of the year 2007 is 20,808.46 Million Baht for all alternatives. It is assumed that the traffic demands of Bangkok would be the same no matter when the project started. In other words, the additional travel demand caused by the construction of the project would be marginal compared to the general increment of the travel demand. On the other hand, the estimated cost of the first year for all of the alternatives is assumed to be the same. It is because of the assumption that all cost were estimated in constant 1998 price, so when the project is postponed to A2, the first year of construction cost would still be the same in 1998 price. However, when it is converted to the base year 1998, the present value of the first year cost of A2 would be lower than that of A1 due to the discount rate. It is important to note that these estimated values of benefit and cost could be more realistically captured by repeating the conventional feasibility study for the different starting years.

3.2.3 Statistical Distribution of Key Variables

The statistical distribution of benefit and cost of each year is estimated by combining experts' opinion. The PERT distribution is used to model an expert's opinion because it requires only three parameters (i.e., minimum, most likely, maximum) and its shape is smooth, therefore, it is appropriate to represent the expert's opinion (Molak, 1997). Each expert was interviewed for the minimum, most likely, and maximum value. Each value is the actual value divided by the estimated value. For example 1.10 means that the actual value would be 10% higher than the estimated value. The PDFs of benefit and cost are assessed for every five years, and they are interpolated to create the PDFs of each intermediate year. There were five experts interviewed for the distribution of those three parameters of cost and benefit as shown in Figure 6.

From the previous projects, the actual costs seem to be higher than the estimated costs so the expert can predict the most likely value by using that tendency. In addition, due to the lack of reliability towards forecasting, it has a tendency that the range would be wider for the later year (Touran *et al.*, 1994). It is because the accuracy of estimation is directly related to the clarity of the project's scope and the amount of information available at the time of the estimates. For the benefit side, the model which forecasts the future traffic volume of the project must consider other planned projects that would be constructed in the future. However, when the model was run, the data of those new projects were put only to the year 2006. The years after 2006 do not include any new project, which may be misleading with the tendency of benefit over-estimation. In this case, the decision-makers can incorporate some possible projects that the model cannot cover by including their opinions into PDF. If the decision-makers trust estimation of the model, they can use the normal distribution to identify the PDF and use similar way to solve the problem.

After five experts were interviewed, their opinions were combined as shown in Figure 7. A discrete ($\{x_i\}, \{p_i\}$) distribution is used where the $\{x_i\}$ are expert opinions and the $\{p_i\}$ are weights given to each opinion according to the emphasis one wishes to place on them (Vose, 2000). In this example of analysis, there are five experts (i.e., A, B, C, D, and E); the expert A and B are double-weighted compared to the others considering their greater experience.

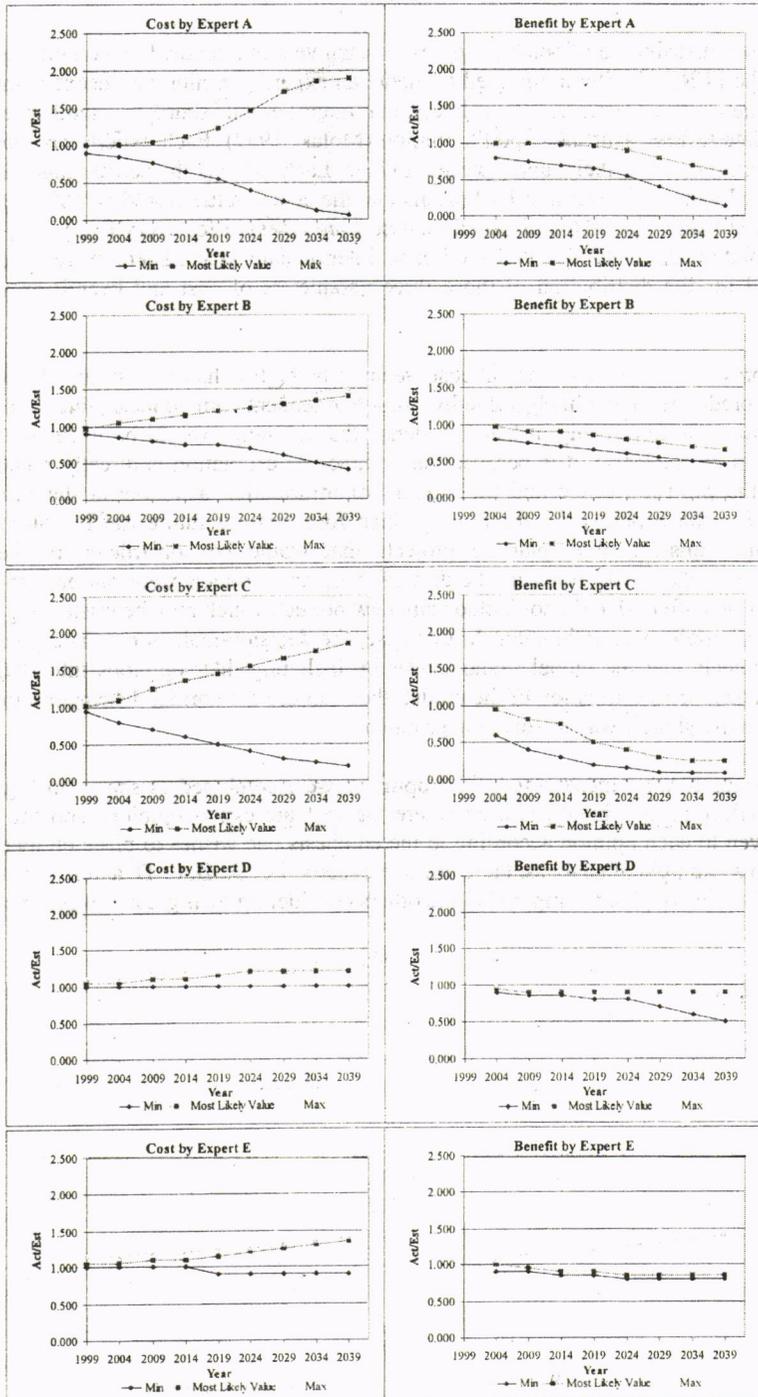


Figure 6. Interview Data from Five Experts

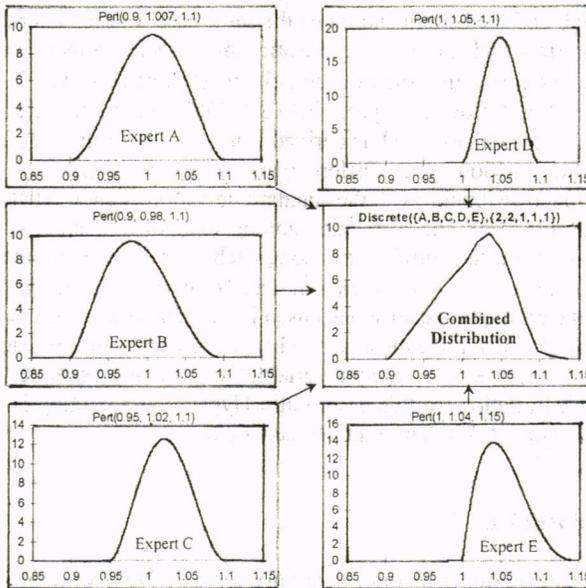


Figure7. Combining five expert opinions

The statistical distributions of cost and benefit represent the total uncertainty of them. They were multiplied by estimated values to obtain the total value of cost and benefit. The distributions depend on the time. Thus all alternatives have the same distribution of cost and benefit in the same year. For example, if the expert A gives the distribution PERT (0.400,1.451,1.700) for the cost in the year 2024 for A1, the distribution of the cost in the year 2024 of other alternatives from expert A should be the same.

3.2.4 Simulation

This study utilizes Monte Carlo simulation technique. Each benefit and cost of each year has its own distribution, and then each of them is selected at random based on their probability distributions. For each iteration, the error term of cost and benefit from each expert in each year are randomly picked up by its distribution. These values from five experts are combined by Discrete Function to estimate the total error. It is then multiplied by the estimated value to obtain the trial value of cost and benefit. Consequently, the trial value of benefit is subtracted by the trial value of cost in the same year to estimate net benefit of that year. Hence, the results are calculated for the iteration as tabulated in Table 2 showing an iteration of A5.

The procedure is repeated one thousand times and every time the values of the NPV, EIRR, B/C are computed. At the end, the results are apparent in term of stochastic value, the probability density function (PDF). The results can then be converted into a cumulative distribution function (CDF) so that the planner is able to use it as a criterion for choosing the best alternatives.

One of the most important things in simulation is the correlation among key variables. In this study no correlation between the cost and benefit is assumed as explained earlier. However, the costs of the different years have correlation. If the cost of this year is high, the cost of next

year tends to be high and the benefit should be the same case. In each alternative, there are six cost correlation matrices and six benefit correlation matrices since there are five experts. For example, there are six correlation matrices in A5 and each of them has 29×29 cells because the horizontal time of cash flow is 30 years between 2003 and 2032. For the last year, almost all values are depreciated values which is derived from the sum of the first four years so that it is not affected by the previous year. Different from other years, the cost of the last year is assumed to be normal distribution with the standard deviation equal to 10% of mean. The first five matrices are derived from the correlation among years of five experts' opinions. And the last matrix is derived from the correlation among each year of the combined distribution. On the benefit side, there are six 26×26 correlation matrices because in the first four year, there is no benefit. All of the correlation coefficients used in this analysis are equal to 1. It means that there is a strong correlation among the cost and benefit of each year. For each of one thousand iterations of each alternative, the 336 input variables are randomly sampled by its distribution together with their correlation. In this case, Latin Hypercube Sampling is used because it is more efficient than Monte Carlo Sampling (Vose, 2000).

4. ANALYSIS OF RESULTS

The results of the conventional method give only a single NPV value of 52,695.39 million baht, EIRR of 35.82%, and B/C ratio of 6.77 only for A1 (ETA, 1999). After the simulation is completed, the results (i.e., NPV, EIRR, and B/C) are obtained as shown in Figure 8 for A1. The mean values of the proposed method are different from the single point estimation of the conventional method since this study assumes that the expected mean from each expert may not be equal to the estimated value from the feasibility study. However, if the decision-makers indeed believe in the estimated value, they can identify the PDF by normal distribution which will give the mean of result equivalent to the value by conventional method but gives more clear picture from the PDF of the result, which can also be presented as the cumulative probability.

After the simulation for other alternatives was completed, the results are shown in Table 3, which briefly presents the 5th percentile, mean, 95th percentile and standard deviation (S.D.). Figure 9 shows the optimal timing among the ten time alternatives. This paper considered the 5th percentile and 95th percentile instead of minimum and maximum because they may vary for each simulation. It is because they are obtained from the tail of probability curve.

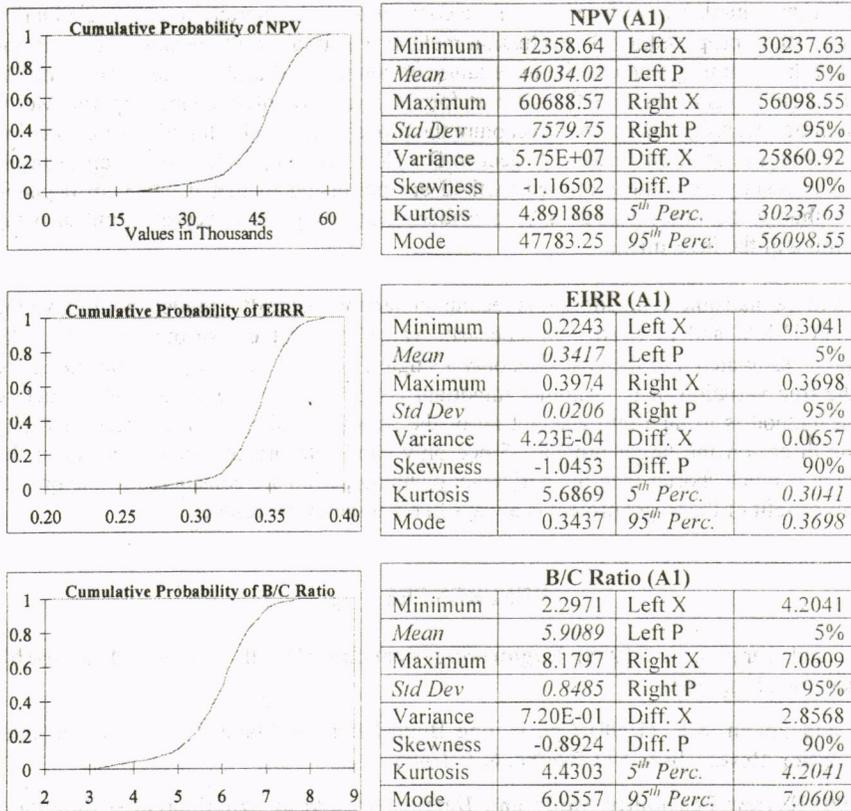


Figure 8. Results from Alternative 1

From Table 3 and Figure 9, it is clear that the optimal time of this project is A2, starting to construct in the year 2000, if the NPV is considered. However, if the EIRR and B/C ratio are considered, this project should be delayed. In this case, EIRR and B/C ratio do not provide the same ranking of alternatives as does NPV. This study would trust on the NPV because of the reasons discussed in the section of criteria for choosing. Therefore, A2 is the best alternative which provides the means of NPV= 47,430.41 million bahts, EIRR= 39.86%, and B/C ratio = 6.65.

Therefore, this project should be started to construct in the year 2000 and be operated in the year 2004. Whatever the possible result is, the project will give $E[NPV] > 0$, $E[EIRR] > 12\%$, and $E[B/C \text{ ratio}] > 1$. In addition this alternative makes the highest NPV among the ten time alternatives.

5. CONCLUDING REMARKS

Motivated from a number of deficiencies of the conventional transportation project evaluation method, this study proposed a new evaluation method which takes into account not only the optimal investment timing but also the uncertainty of benefits and costs. After estimating the statistical distributions of the benefit and cost for each year by interviewing experts, Monte Carlo simulation technique is utilized to capture the probability density function of evaluation index. A transportation infrastructure project in Bangkok, Thailand was used as an example project. The results from the proposed method are promising in that it can help decision makers to choose the best alternative by considering not only the uncertainty but also the optimal timing of the investment.

To make a more plausible evaluation, it is recommended that the traffic model should give the results of PCU-KM and PCU-HRS in stochastic value so that the distribution of the total benefit can be estimated. For the cost side, only a single estimated value may not be enough to capture the true variation of it. Another important issue of the transportation infrastructure evaluation method is to take into account all of the possible projects in the future that may have some effect on the target project. Since only candidate projects of few years in the future which is much shorter than the horizontal planning period are considered in the project evaluation, benefit of the target project is always likely to be overestimated.

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