Addressing a New Equity Issue in Road Network Pricing

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Abstract: In congestion pricing, the social equity issue between the poor and the rich drivers by paying the same toll charge has been debated intermittently in the literature and has often been used as an argument to justify the political unacceptability of road pricing. This paper poses and addresses a new spatial equity issue arising from congestion pricing. It stems from the fact that the changes of the generalized travel costs of drivers travelling between different O-D pairs may be significantly different when tolls are charged at some links. We propose bilevel programming models for the network toll design problem by explicitly incorporating the equity constraint in terms of the maximum relative increase of the generalized equilibrium O-D travel costs between all O-D pairs. A penalty function approach by embodying a simulated annealing method is applied for solving the equity-constrained toll design problem and demonstrated with a simple network example.

Key Words: congestion pricing, equity, second best, O-D pairs, simulated annealing

1. INTRODUCTION

Road pricing is recently motivated by the need to improve the economic efficiency of the transportation system, and implemented in many metropolises around the word to reduce traffic congestion and pollution. In addition, the revenue from road pricing provides a basis for investment decision in transport infrastructure, such as expanding the road capacity, providing better maintenance, improving public transport. The advanced technology of electronic road pricing mechanisms offers lower extra cost and new possibilities for road pricing systems. So far, many countries or regions have built pricing systems successfully such as Norway, Singapore and Hong Kong.

Traditionally, the first-best congestion pricing theory, namely, the theory of marginal cost pricing is well established and widely advocated by economists. In line with this theory, a toll that is equal to the difference between marginal social cost and marginal private cost is charged on each link, so as to achieve a system optimum flow pattern in the network (Beckmann, 1965; Dafermos and Sparrow, 1971; Smith, 1979). Investigations are made into how this classical economic principle would work on a general congested road network with queuing (Yang and Huang, 1998) and on a congested network in a stochastic equilibrium (Yang, 1999). In spite of its perfect theoretical basis, the principle of marginal cost pricing can be difficult to apply in real situations. Apart from the public and political resistance, a primary reason is due to the high extra cost spent on the equipment for toll collection in the entire network. This has motivated a number of researchers to consider various forms of second-best pricing schemes where only a subset of links are subjected to toll charge. A typical simple example of the second-best pricing involves the two parallel route problem where an untolled alternative exists. This problem has been investigated for both static and dynamic situations by, for example, Braid (1996); Verhoef et al. (1996); Liu and McDonald (1999); and De Palma and Lindsey (2000). Optimal determination of tolls for a subset of links in a general network are studied by Yang and Lam (1996) for system optimum with fixed demand, for traffic restraint (Ferrari; 1995; Yang and Bell, 1997); for minimization of toll links subject to a user equilibrium (Hearn and Ramana, 1998), for revenue minimization (Dial, 1999c, 2000) and for private highway modeling (Yang and Meng, 2000; Yang and Woo, 2000). The second-best pricing for users with discrete or continuous time value distributions are investigated by Dial (1999a & 1999b); Leurent (1993, 1998); Yang et al. (2000) and Yang and Chow (2000).

Whereas congestion pricing is theoretically and technologically easy to implement, it has long been viewed as a political issue. A common criticism is that road use charge makes unequivocally distributional impacts on travelers with different incomes. Generally speaking, the equity implications of congestion pricing are complex because of all the different options facing travelers under a congestion pricing scheme (Richardson and Bae, 1998). People who continue to use the highway after the toll is imposed pay the toll, but also have a lower time cost: the toll decreases traffic volume, which decreases travel time. Some travelers with very high values of time would find that they are made better off (the reduced congestion can more than compensate the travelers for the extra cost of toll charges). Whereas those with low values of time and still using the roads are generally made much worse off than before. People who stop using the highway avoid the toll, but forgo the benefits associated with using the highway and experience the inconvenience of switching to another mode of transport. This type of social quity problem between the poor and the rich travelers have continued to receive attention (Foster, 1975; Small, 1983; Hau, 1992; Johansson and Mattsson, 1995; Button and Verhoef, 1998), and have often been used as an argument to justify the political unacceptability of road pricing (Giuliano, 1992).

Although a direct redistribution of the revenue generated by the congestion charge among travelers in equal or unequal shares could partially or completely resolve the aforementioned inequity problem, a practicable redistribution mechanism is not established and adopted anywhere.

The aforementioned equity issue among different social classes of travelers is often the primary focus of road pricing arguments, nevertheless, the spatial equity issue among travelers travelling between different locations is blatantly ignored in the literature. It is evident that after introducing congestion pricing in a road network, the changes in travel costs (inclusive of toll charges) between different origin-destination (O-D) pairs can be substantially different, depending the amounts and locations of toll charges. This paper poses and addresses this new equity issues arising from road pricing in a road network. Without loss of generality, our study focuses on the practically meaningful second-best pricing, namely, we consider the case where not all links in the network are subjected to toll charge. After demonstration of the inequity problem with a simple network, we formally propose mathematical programming models for the network toll design problem by explicitly incorporating the equity constraint in terms of the maximum relative increase of the generalized equilibrium O-D travel costs between all O-D pairs. A penalty function approach by embodying a simulated annealing method is applied for solving the equity-constrained toll design problem and illustrated with a simple network example. Finally, conclusions and suggestions for future studies are provided.

2. NEW INEQUITY CONSIDERATION IN ROAD PRICING: A SIMPLE EXAMPLE

When marginal-cost tolls are charged on all links or suitable tolls are charged only on a subset of links in a network, the inequity problem among drivers travelling between different O-D pairs may occur in both cases. Here we show this inequity problem using a simple network depicted in Figure 1. This network consists of 3 nodes and 3 links. Let q_{13} =100 and q_{23} =10 be the traffic demands from 1 to 3 and 2 to 3, respectively. The performance function of each link is given below:

$$t_1(v_1) = 10.2 + \frac{v_1}{500};\; t_2(v_2) = 3 + \frac{v_2}{100};\; t_3(v_3) = 4 + \frac{v_4}{50}.$$

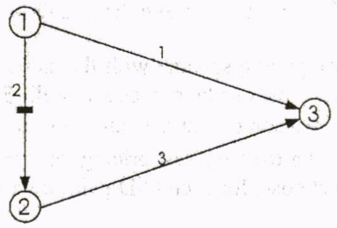

FIGURE 1. A Simple Network

Assuming that the route choice behavior of drivers follows the principle of deterministic user equilibrium, it is straightforward to obtain the traffic volume and travel time cost of each link as follows:

$$\tilde{v}_1 = 0, \tilde{v}_2 = 100,\ \tilde{v}_3 = 110$$

$$\tilde{t}_1 = 10.2,\ \tilde{t}_2 = 4,\ \tilde{t}_3 = 6.2$$

The corresponding equilibrium travel cost for each O-D pair is

$$\tilde{\mu}_{13} = 10.2,\ \tilde{\mu}_{23} = 6.2$$

Under the assumption of user equilibrium, the total network travel time cost is 1082.

After we charge a marginal cost toll on each link, we can obtain the system-optimal traffic flow distribution. The flow and travel time cost (not including toll charge) of each link are

$$\bar{v}_1 = 50, \bar{v}_2 = 50,\ \bar{v}_3 = 60;$$

$$\bar{t}_1 = 10.3,\ \bar{t}_2 = 3.5,\ \bar{t}_3 = 5.2$$

The tolls to be charged on each link are

$$\bar{y}_1 = 0.1,\ \bar{y}_2 = 0.5,\ \bar{y}_3 = 1.2$$

The generalized travel costs (inclusive of toll) for each O-D pair are

$$\bar{\mu}_{13} = 10.4,\ \bar{\mu}_{23} = 6.4$$

Under the first-best pricing scheme, the total network travel time cost decreases to 1002, and the total revenue of 102 is attained. To compare the variations in generalized travel costs, the corresponding ratios of the generalized O-D travel costs after and before introduction of the first-best pricing for each O-D pair are calculated below.

$$\frac{\bar{\mu}_{13}}{\tilde{\mu}_{13}} = 1.02,\ \frac{\bar{\mu}_{23}}{\tilde{\mu}_{23}} = 1.03$$

It is clear that the travel costs of both O-D pairs increase, and the magnitudes of increase are different, so an inequity problem occurs between the two O-D pairs.

We next look at a second-best pricing scheme with the same network. If a toll is charged on link 2, total network travel time cost can be reduced as well. Specifically, when the amount of toll charge $\hat{y}_2$ is set to be 1.6, we can obtain the same system-optimal traffic flow pattern (the lowest network travel time) under the first-best pricing, the revenue, however, decreases to 80. The resulting generalized travel costs for each O-D pair now become

$$\hat{\mu}_{13} = 10.3,\ \hat{\mu}_{23} = 5.2$$

In this case, the corresponding ratios of the generalized O-D travel costs after and before the toll charge are

$$\frac{\hat{\mu}_{13}}{\tilde{\mu}_{13}} = 1.01,\ \frac{\hat{\mu}_{23}}{\tilde{\mu}_{23}} = 0.84$$

It is obvious that the inequity problem becomes more significant than the first-best case. The travel costs between O-D pair 1→3 increases by 1%, but it decreases by about 16% between O-D pair 2→3. This toll charge will bring negative and positive effects on travelers travelling between the two O-D pairs, respectively. Thus, for equity and fairness considerations of congestion pricing, the amounts and locations of toll charges for a pricing scheme in a road network must be selected cautiously.

3. MODEL FORMULATION

3.1 Bilevel Network Toll Design Model without Equity Constraint

Congestion pricing problem can be represented as a leader-follower game where the leader is the system manager and the followers are the travelers (Yang and Lam, 1996). The manager cannot control, but can influence route choices of network users by setting alternative toll patterns. In response to any toll charge scenario, network users are assumed to have complete knowledge of the travel time and toll charge pattern on the network and choose the shortest path in term of the generalized travel cost (inclusive of toll charge). Suppose the system manager aims to minimize the total network travel time for given and fixed O-D demands by implementing a second-best pricing scheme (selection of tolls for a given subset of links in the network). Then the second-best network toll design problem subject to a standard user equilibrium constraint can be formulated as the following bi-level programming problem (a complete list of notations used here is given in Appendix II).

$$\min_{\mathbf{y}} \sum_{a \in A} t_a(v_a(\mathbf{y}))v_a(\mathbf{y}) \tag{1}$$

subject to

$$y_a^{\min} \leq y_a \leq y_a^{\max}, \; a \in A^* \tag{2}$$

Here $v_a(\mathbf{y}), a \in A$ is the solution of the following lower-level user-equilibrium program:

$$\min_{\mathbf{v}} \sum_{a \in A} \int_0^{v_a} c_a(x, y_a)dx \tag{3}$$

subject to

$$\sum_{r \in R_w} f_r^w = q_w, \; w \in W \tag{4}$$

$$\sum_{w \in W} \sum_{r \in R_w} f_r^w \delta_{ar} = v_a, \; a \in A \tag{5}$$

$$f_r^w \geq 0, r \in R_w, w \in W \tag{6}$$

The objective of the upper-lever program is to minimize the total travel time cost and the lower-lever problems is a standard network equilibrium problem (Sheffi, 1985) that describes the route choice behavior of network users. Note that $c_a(v_a, y_a) = t_a(v_a) + y_a$ if $a \in A^*$ and $c_a(v_a, y_a) = t_a(v_a)$ otherwise, A^* is a subset of tolled links in the network, $t_a(v_a)$ is a strictly increasing and continuous function of its link flow v_a, $a \in A$.

3.2 Specification of Equity Constraint

We now consider how to deal with the equity issue by incorporating an equity constraint in the upper-level problem. The equity can be measured as the relative change of the generalized O-D travel cost (inclusive of toll charge) and thus the equity constraint can be specified as follows:

$$\frac{\mu_w(\mathbf{y})}{\tilde{\mu}_w} \leq \phi_w, \; w \in W \tag{7}$$

The term $\tilde{\mu}_w$ is the original user equilibrium O-D travel time cost without pricing, and $\mu_w(\mathbf{y})$ is the generalized equilibrium O-D travel cost (inclusive of toll charge) after introducing the second-best pricing. ϕ_w is designated here as an equity parameter that dictates the degree of tolerance of the inequity associated with the pricing scheme. A meaningful selection of the value of this parameter is given by the following equation:

$$\phi_w = \begin{cases} 1+\varphi\left(\dfrac{\bar{\mu}_w}{\tilde{\mu}_w}-1\right) & \text{if } \dfrac{\bar{\mu}_w}{\tilde{\mu}_w}>1 \\ 1 & \text{otherwise} \end{cases} \tag{8}$$

Here the term $\bar{\mu}_w$ is the O-D travel cost after the best-first or marginal-cost pricing scheme is implemented. φ is a decision variable satisfying $0 \le \varphi \le 1$. Clearly, $1 \le \phi_w \le \bar{\mu}_w/\tilde{\mu}_w$, $w \in W$ for $0 \le \varphi \le 1$.

When φ is equal to 1 we have $\phi_w = \bar{\mu}_w/\tilde{\mu}_w$. This means that ϕ_w takes its largest value given as the ratio of the equilibrium O-D travel cost after introducing marginal-cost pricing to the original O-D travel time cost without pricing (if this ratio is greater than 1.0). In other words, the degree of inequity in a second-best pricing scheme is bounded by the first-best pricing case, or the generalized travel cost between each O-D pair under a second-best pricing scheme cannot exceed that under the first-best pricing scheme. This specification of the upper bound of the equity parameter is quite justifiable and meaningful. The theoretical foundation of congestion pricing stems from the fundamental economic principle of marginal cost pricing, which states that road users using congested roads should pay a toll equal to the difference between the marginal social cost and the marginal private cost so as to maximize economic benefit. Thus, by requiring to pay a marginal-cost toll, an individual driver will bear the full social cost generated by him or her in using a congested road. Toll charge higher than this amount seems to be irrational economically and unfair to drivers.

When φ is equal to 0, ϕ_w becomes 1. This means that the generalized O-D travel cost cannot exceed the equilibrium O-D travel time cost before pricing. Namely, all the O-D travel costs decrease or remain unchanged. Since in a real network, a congestion pricing scheme is very unlikely to lead to decline in all generalized O-D travel costs (inclusive of toll), the zero toll charges (do-noting alternative) may become the only feasible solution when $\varphi = 0$.

To summarize, the φ is an appropriate decision variable that can be used by the system manager to adjust the spatial level of equity for consideration in establishing a fair and reasonable pricing scheme. A higher value of φ permits a greater negative inequity impact on drivers, and a lower value of φ places a stricter inequity constraint.

3.3 Bilevel Network Toll Design Model with Equity Constraint

With the aforementioned spatial equity consideration, the network toll design problem with an equity constraint can now be formulated as the following bilevel programming problem.

Model M1:

$$\min_{\mathbf{y}} \ F_1(\mathbf{y}, \mathbf{v}(\mathbf{y})) = \sum_{a \in A} t_a(v_a(\mathbf{y}))v_a(\mathbf{y}) \tag{9}$$

subject to

$$\frac{\mu_w(\mathbf{y})}{\tilde{\mu}_w} \le \phi_w, \ w \in W \tag{10}$$

$$y_a^{\min} \le y_a \le y_a^{\max}, \ a \in A^* \tag{11}$$

Here $\phi_w = 1 + \varphi(\bar{\mu}_w/\tilde{\mu}_w - 1)$ if $\bar{\mu}_w/\tilde{\mu}_w > 1$ and otherwise $\phi_w = 1$, parameter φ is a given appropriate constant satisfying $0 \le \varphi \le 1$. $v_a(\mathbf{y}), a \in A$ and $\mu_w(\mathbf{y}), w \in W$ are the equilibrium link flow and the equilibrium O-D travel cost obtained from the model (3)-(6).

Note that the parameter φ introduced before reflects the allowable degree of inequity in terms of the increase in the equilibrium O-D travel cost after and before implementing a pricing scheme. This parameter is selected by the decision-maker and can be, in fact, treated as a decision variable in the programming model. By incorporating this equity decision parameter φ, we have the bi-level programming model with dual upper-level objective functions:

$$\min_{\mathbf{y},\varphi} F_2(\mathbf{y},\varphi,\mathbf{v}(\mathbf{y},\varphi)) = \begin{pmatrix} F_2^1 = \dfrac{\sum_{a \in A} t_a(v_a(\mathbf{y},\varphi))v_a(\mathbf{y},\varphi)}{\sum_{w \in W} q_w \tilde{\mu}_w} \\ F_2^2 = \varphi \end{pmatrix} \tag{12}$$

subject to (10) and (11) and $0 \le \varphi \le 1$.

A mathematically well-defined optimal solution does not exist for this multi-objective programming model. Therefore, a considerate trade off is necessary to balance the need of decreasing the total network travel time and the requirement of avoiding the great negative inequity impact. The problem thus becomes how to select a non-dominated efficient solution using an appropriate method. A widely used utility function approach is adopted here to transfer the two upper-level objectives into a single objective function. Thus objective vector (12) now becomes as follows:

$$F_3(\mathbf{y},\varphi,\mathbf{v}(\mathbf{y},\varphi)) = \omega F_2^1 + (1-\omega)F_2^2$$

$$= \omega \frac{\sum_{a \in A} t_a(v_a(\mathbf{y},\varphi))v_a(\mathbf{y},\varphi)}{\sum_{w \in W} q_w \tilde{\mu}_w} + (1-\omega)\varphi \tag{13}$$

Here, ω is a weighting parameter satisfying $0 < \omega < 1$. After the transformation, the previous bilevel model with an equity constraint becomes the following standard bi-level programming problem:

Model M2:

$$\min_{\mathbf{y},\varphi} F_3(\mathbf{y},\varphi,\mathbf{v}(\mathbf{y},\varphi)) = \omega \frac{\sum_{a \in A} t_a(v_a(\mathbf{y},\varphi))v_a(\mathbf{y},\varphi)}{\sum_{w \in W} q_w \tilde{\mu}_w} + (1-\omega)\varphi \tag{14}$$

subject to

$$\frac{\mu_w(\mathbf{y},\varphi)}{\tilde{\mu}_w} \leq \phi_w, \; w \in W \tag{15}$$

$$y_a^{\min} \leq y_a \leq y_a^{\max}, \; a \in A^* \tag{16}$$

$$0 \leq \varphi \leq 1 \tag{17}$$

where $v_a(\mathbf{y},\varphi)$, $a \in A$ and $\mu_w(\mathbf{y},\varphi)$, $w \in W$ are again the solutions of the lower-level user-equilibrium problem (3)-(6) for a given $\mathbf{y}$ and φ.

4. SOLUTION APPROACH FOR THE MODELS

We have proposed the bi-level programming model M1 to characterize the congestion-pricing problem with an equity constraint, and the bi-level programming model M2 to treat the equity reference parameter as an endogenous decision variable. The proposed bilevel programming models, just like any other form of bi-level mathematical programming problems, are intrinsically non-convex, and hence might be difficult to solve for a global optimum (Friesz *et al*, 1990; Meng and Yang, 2000). The difficulty comes from the fact that the equilibrium link flow, $v_a(\mathbf{y})$, $a \in A$ and the generalized O-D travel cost, $\mu_w(\mathbf{y})$, $w \in W$ generally are non-convex, continuous and non-differentiable functions with respect to $\mathbf{y}$.

In view of the difficulty in applying the standard algorithmic approaches for search of the global optimum, we adopt the Simulated Annealing (SA) method (Dekkers and Aarts, 1991; Romeijin and Smith, 1994), which is particularly suitable for the models proposed here. The SA method has been successfully applied to solve the continuous network design problems (Friesz et al., 1992; Meng and Yang, 2000). It has the ability to obtain the global optimal solution without the requirement for differentiability.

Now we consider how to solve the model M1 (solution for M2 is discussed later). The nonlinear, implicit constraint (10) is incorporated into the objective function by using an inner penalty function approach in applying the SA method. For the sake of clarity, let

$$\Omega = \left\{ \mathbf{y} \,\middle|\, y_a^{\min} \leq y_a \leq y_a^{\max}, a \in A^* \right\} \tag{18}$$

denote the feasible set of $\mathbf{y}$ dictated by its lower and upper bound.

4.1 Solution for Model M1

The procedure of the penalty function approach in conjunction with the SA method for solving **M1** is presented below.

The inner penalty function approach

Step 0. *Initialization.* Set a stop tolerance ε, an initial penalty multiplier λ_0, a scale parameter $\rho > 1$ and an initial point $\mathbf{y}^{(0)} \in \Omega$. Let $k = 0$.

Step 1. *Finding an optimal solution.* Starting with $\mathbf{y}^{(k)}$, solving the following problem through the SA method and let $\mathbf{y}^{(k+1)}$ be the optimal solution

$$\min_{\mathbf{y}\in\Omega} \hat{F}(\mathbf{y}) = F(\mathbf{y}) + \lambda_k \alpha(\mathbf{y}) \tag{19}$$

where

$$\alpha(\mathbf{y}) = \sum_{w\in W} \max\left\{ \frac{\mu_w(\mathbf{y})}{\tilde{\mu}_w} - \phi_w,\ 0 \right\} \tag{20}$$

Step 2. *Verifying the stop criterion.* If $\lambda_k \alpha(\mathbf{y}^{(k+1)}) < \varepsilon$, stop. Otherwise, set $\lambda_{k+1} = \rho\lambda_k$ and $k := k+1$, go to Step 1.

For any given vector $\mathbf{y}$ of the toll pattern, the equilibrium traffic assignment procedure is used to find the link flow $v_a(\mathbf{y})$ and the generalized O-D travel cost $\mu_w(\mathbf{y})$, and so it is straightforward to calculate the objective function value $\hat{F}(\mathbf{y})$. With this in mind, the procedure of simulated annealing method for solving problem M1 with any fixed penalty parameter λ_k is given below.

Simulated annealing method

Step 0. *Initialization.* Given an initial point $\mathbf{y}^{(0)} \in \Omega$ and the parameter $0 < \chi_0 < 1.0$, $0 < \delta < 1.0$, integer L_0, m_0 and T_s (stop tolerance of temperature). Set $k = k_1 = 0$.

Step 1. *Finding an initial temperature.* Uniformly generate at random m_0 points denoted by $\mathbf{z}^{(i)}$ $(i = 1,\dots,m_0)$ over the feasible set Ω. For each $\mathbf{z}^{(i)}$ use equilibrium traffic assignment to obtain the equilibrium link flow and O-D travel cost associated with $\mathbf{z}^{(i)}$ and then calculate the corresponding function value $\hat{F}(\mathbf{z}^{(i)})$. Let m_2 denote the number of points $\mathbf{z}^{(i)}$ with $\hat{F}(\mathbf{z}^{(i)}) - \hat{F}(\mathbf{y}^{(0)}) \ge 0$ and $\overline{\Delta\hat{F}^{+}}$ the average value of those $\hat{F}(\mathbf{z}^{(i)}) - \hat{F}(\mathbf{y}^{(0)})$, for which $\hat{F}(\mathbf{z}^{(i)}) - \hat{F}(\mathbf{y}^{(0)}) \ge 0$. Then the initial temperature T_0 is calculated as below

$$T^{(0)} = \overline{\Delta\hat{F}^{+}} \left(\ln \frac{m_2}{m_2\chi_0 + (1-\chi_0)(m_0 - m_2)} \right)^{-1} \tag{21}$$

Step 2. *Verifying the termination.* If $T^{(k)} < T_s$, then stop. Otherwise, go to Step 3.

Step 3. *Checking the termination of a Markov chain.* If $k_1 > L_0 N$ (N represents the number of decision variables), then go to Step 6. Otherwise, go to Step 4.

Step 4. *Generation of points.* Uniformly generate at random a number denoted by t_{random} from the internal [0,1). If $t_{\text{random}} > t$ then use the method of Hooke and Jeeves with the discrete step (pp.288, Bazaraa et al., 1993) from the point $\mathbf{y}^{(k_1)}$ as a local search procedure to find a local solution for the problem denoted by x. If $t_{\text{random}} \le t$, then uniformly generate at random a point denoted by x over Ω.

Step 5. *Metropolis' rule.* If $\hat{F}(\mathbf{x}) < \hat{F}(\mathbf{y}^{(k_1)})$, then $y^{(k_1+1)} = x$, set $k_1 := k_1 + 1$ and go to Step 3. If $\exp(-(\hat{F}(y^{(k_1)}) - \hat{F}(x))/T^{(k)}) > \text{random}[0,1]$ then $y^{(k_1+1)} = x$, set $k_1 := k_1 + 1$ and go to Step 3. Otherwise, $\mathbf{y}^{(k_1+1)} = \mathbf{y}^{(k_1)}$, $k_1 := k_1 + 1$ and go to Step 3.

Step 6. *Cooling schedules.* Calculate the standard derivation of the values of the objective function $\hat{F}(\mathbf{y}^{(k_1)})$ $(k_1 = 0, \ldots, L_0 N)$, denoted by $\sigma(T^{(k)})$. Set the temperature as follows

$$T^{(k+1)} = T^{(k)}\left(1 + \frac{T^{(k)}\ln(1+\delta)}{3\sigma(T^{(k)})}\right)^{-1} \tag{22}$$

$k := k+1$, $\mathbf{y}^{(0)} = \mathbf{y}^{(L_0 N)}$, $k_1 = 0$ and go to Step 2.

The convergence property of this simulated annealing algorithm is proved by Dekkers and Aarts (1991). Note that the original Hooke-Jeeves method is designed for the unconstrained optimization problem. In the problem examined here, there are simple bound constraints, $y_a^{\min} \le y_a \le y_a^{\max}$, $a \in A^*$ only, we can slightly modify the Hooke-Jeeves method to deal with this situation by projecting the trial point in the method onto the region Ω defined by the bound constraints. Furthermore, the objective function evaluation is necessarily required in the Hooke-Jeeves method. This means that we need to perform a user equilibrium traffic assignment procedure at each trial point in this local search method.

4.2 Solution for Model M2

We first analyze how the two components F_2^1 and F_2^2 of the objective function F_3 vary with φ. When φ is equal to 0, or when no travel cost increase is allowed between any O-D pair, the tolls have to be set equal to 0. The travel cost between each O-D pair remains unchanged so $F_2^1 = 1.0$. When φ increases to a value greater than 0, non-zero tolls are charged in certain links, and the total travel time in the network must decrease, otherwise, the solution is not optimal. In other words, the component F_2^1 will decrease to a number smaller than 1.0. Of course, the term $(1-\omega)F_2^2$ or the term $(1-\omega)\varphi$ linearly increases with φ. In summary, one component F_2^1 of the objective function F_3 decreases with φ, the other component F_2^2 of F_3 increases with φ.

Furthermore, the value of variable φ is restricted within the limited interval $[0,1]$. For any given value of φ in the interval $0 \le \varphi \le 1$, constraint (17) disappears, in view of the fact that q_w, $\bar{\mu}_w$ and ω together with φ are constant, minimization of F_3 is equivalent to minimization of F_1. In other words, model M2 reduces to M1 for a given value of φ. Based on this observation, we can use a one dimensional search method in conjunction with the algorithm for M1 to solve M2. At each new point of the one-dimensional search for φ, a sub-program M1 is solved to find the objective function value F_3, and the search is continued when both optimal φ and $\mathbf{y}$ are identified.

5. A NUMERICAL EXAMPLE

The road network shown in Figure 2 is made up of 7 nodes and 11 links, of which link 3 and link 4 are subject to toll changes. The link travel time function is

$$t_a(v_a) = t_a^0 \left\{ 1.0 + 0.5 \left(\frac{v_a}{C_a} \right)^2 \right\} \tag{23}$$

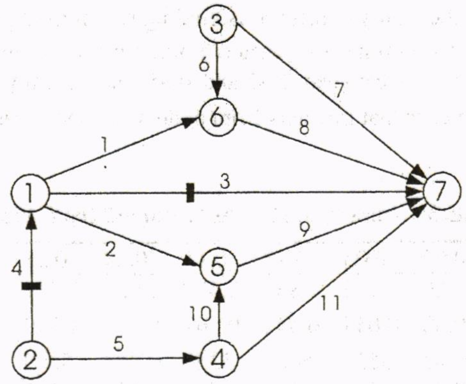

FIGURE 2. The Network Used in the Numerical Example

The values of t_a^0 and C_a for each link are given in Table 1. Let $q_{17} = 200$, $q_{27} = 50$, $q_{37} = 20$, $q_{47} = 5$ be the demands of the four O-D pairs (1→7, 2→7, 3→7, 4→7).

TABLE 1. Input Data for the Test Network in Figure 2

Link	1	2	3	4	5	6	7	8	9	10	11
t_a^0	19.0	20.0	14.0	7.0	28.8	0.3	1.4	1.0	1.0	0.2	1.5
C_a	30	800	220	50	800	20	15	150	100	50	30

Without implementation of a road pricing scheme, the O-D travel time costs from a user equilibrium assignment are presented in Table 2. The total travel time cost in the network is 5743. When a marginal cost toll is charged on each link, the resulting generalized O-D travel costs are also listed in the same Table and the total travel time cost decreases to a minimum value 5172, the resulting revenue generated from toll charge is 918. Table 2 also shows the ratios of the generalized O-D travel costs after and before marginal-cost pricing is introduced. It can be seen from the table that travel costs between all the four O-D pairs increases after the marginal costs tolls are charged. In particular, the travel cost between O-D pair (4→7) increases most.

TABLE 2. Change of the O-D Travel Costs Before and After Pricing

O-D pair	1→7	2→7	3→7	4→7
Travel cost without pricing	21.031	30.046	1.426	1.243
Travel cost with marginal-cost pricing	22.416	31.252	1.603	2.393
Ratio of the equilibrium O-D travel costs	1.066	1.040	1.124	1.925

Now we consider a second-best pricing regime with link 1 and link 2 subjected to toll charge. Table 3 shows the numerical results for 9 different values of the inequity threshold φ. The term $R_{ij}, i=1,\ldots,4, j=7$, denotes the ratios of the generalized O-D travel cost after and before the road pricing scheme is implemented. The last row of the table lists the O-D pairs, for which the constraint, $\mu_w(\mathbf{y})/\bar{\mu}_w \le \phi_w$, is binding. The following observations are made from this table. As the value of φ increases or as the equity constraint becomes less tight, toll charge on each link becomes higher, the resulting revenue increases, and the total travel time cost decreases significantly. When φ equals 0, no toll can be charged, and the total travel time cost is the highest, the equity constraint is binding for all O-D pairs. When φ is equal to or larger than 0.35, total travel time cost reaches a minimum value and all inequity constraints becomes inactive. When φ is between 0.00 and 0.35, the inequity constraint for O-D pair (4→7) is binding. This means that travelers from node 4 to node 7 are likely to suffer from the road-pricing scheme.

TABLE 3. Numerical Results Obtained from Model M1

φ	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
y_1	0.00	1.146	1.834	2.428	2.927	3.711	4.303	4.694	4.694
y_2	0.00	0.044	0.442	0.567	0.713	1.049	1.239	1.362	1.362
Total travel time	5743	5525	5408	5330	5277	5209	5182	5177	5177
Total revenue	0.00	234	367	460	529	616	662	681	681
$R_{1\to7}$	1.00	1.003	1.006	1.009	1.012	1.016	1.019	1.022	1.022
$R_{2\to7}$	1.00	1.002	0.119	1.006	1.008	1.010	1.012	1.014	1.014
$R_{3\to7}$	1.00	1.000	1.000	1.001	1.001	1.007	1.014	1.018	1.018
$R_{4\to7}$	1.00	1.046	1.093	1.138	1.185	1.232	1.278	1.315	1.315
Binding O-D pair	All	4→7	4→7	4→7	4→7	4→7	4→7	No	No

Table 4 provides the numerical results obtained from model M2 for 7 different values of weighting factor ω. As the value of ω increases, or as more emphasis is placed on reduction in total travel time than on the equity constraint, it is naturally that the value of φ increases, and the toll charge of each link becomes higher, the revenue increases and the total travel time decreases. Note that when ω is below a certain value, parameter φ will dominate the objective function, and hence φ is obtained to be 0.

TABLE 4. Numerical Results Obtained from Model M2

ω	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
φ	0.000	0.000	0.000	0.001	0.005	0.031	0.069	0.135	0.260
y_1	0.000	0.000	0.000	0.037	0.164	0.774	1.415	2.256	3.826
y_2	0.000	0.000	0.000	0.000	0.012	0.016	0.253	0.543	1.242
F_3	0.300	0.300	0.300	0.400	0.499	0.596	0.688	0.772	0.841
Total revenue	0	0	0	8	36	162	290	435	628
Total travel time	5743	5743	5743	5735	5709	5591	5474	5351	5200

6. CONCLUSIONS

We have addressed the spatial equity issue that can arise in congestion pricing in term of the different absolute and relative changes in the generalized O-D travel cost between different O-D pairs after and before the toll charges. Bilevel programming models with an equity constraint are proposed that can allow for selection of a fair and reasonable toll patterns. The selected toll pattern aims to reduce total network travel time, while attempting not to bring too much negative inequity impact on certain groups of users. Penalty function method in conjunction with a simulated annealing approach is applied to solve the proposed models. The rational of the proposed model is demonstrated with a network example.

Works are continued to select both amounts and locations of toll charges for a second-best pricing scheme and use a multi-class network equilibrium model with heterogeneous users in terms of their values of time. This will allow simultaneous consideration of the spatial equity issue posed in this paper and the conventional social equity issue between the rich and the poor travelers associated with road congestion pricing schemes. And the operating costs for different types of vehicles are expected to be incorporated into the generalized costs.

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APPENDIX NOTATIONS

The following symbols are used in this paper:

A:	the set of links in the network
A^*:	a subset of toll links in the network
W:	the set of O-D pairs
R_w:	the set of routes between O-D pair $w \in W$
f_r^w:	the path flow on route $r \in R_w$
v_a:	the link flow on link $a \in A$
$\mathbf{v}$:	the vector of all link flow, $\mathbf{v}=(\cdots, v_a, \cdots)^T$
y_a:	the charge of toll on link $a \in A^*$
$y_a^{\max}$:	the upper bound of toll charge on link a
$y_a^{\min}$:	the minimum of toll charge on link a
$\mathbf{y}$:	the vector of all toll charges
$t_a(v_a)$:	the travel cost on link $a \in A$, which is a function of link flow v_a
$c_a(v_a, y_a)$:	the generalized travel cost on link a, which is a function of link flow v_a and link toll charge y_a on link a
q_w:	the demand between O-D pair $w \in W$
$\mu_w(\mathbf{y})$:	the generalized equilibrium O-D travel cost with pricing
$\tilde{\mu}_w$:	the generalized equilibrium O-D travel cost without pricing
δ_{ar}^w:	1 if route r between O-D pair w uses link a, and 0 otherwise