DYNAMIC TRAFFIC ASSIGNMENT MODEL CONSIDERING THE EFFECTS OF TRAFFIC MANAGEMENT

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Abstract: This paper firstly analyzes the effects of traffic management on dynamic traffic properties. Then focused on signal control and management, the dynamic user optimal (DUO) model considering intersection signal control (ISC) is discussed and proposed in this paper. And aiming at this new model, we also issued the disadvantages of the model such as its unreasonable simplification to intersection signal control mode. So we proposed the improved scheme, which is to transfer the ISC mode into a series of virtual links and to integrate these links into the whole links. This method can solve the question of simulation of ISC well and easily. At last the application foreground of this model such as to discuss the effects of signal control management on the whole traffic network, to analysis traffic capacity and queue length of link at the intersection and assess them with this model is discussed in this paper.

Key Words: Dynamic Traffic Assignment, Traffic Management, Dynamic User Optimal (DUO), Intersection Signal Control (ISC), Intersection Transfer Model,

1. INTRODUCTION

Dynamic traffic assignment is an assignment method which views the continuously changed traffic demand as the research object. Until now most developed models can be divided into static traffic assignment class, whose method is to research the daily traffic volume and to gain the daily distribution of traffic volume for planning and design of transport network. However, static traffic assignment model can not solve the question of simulating a relatively short period traffic flow properties in the network including its spatial-temporal distribution model. Therefore, the dynamic analysis is undoubtedly required in order to simulate this various and short period traffic flow.

Dynamic traffic assignment theory has gone through the development for 20 years. Until present, there are third kinds of models from the point of research method, which respectively are computer-simulating method, numerical-simulating method and optimal-control method. It can also be divided into three aspects from the viewpoint, which are trip ratio research, assigning the trip ratio into road network and calculating both of them at the same time. From the tripper route-selecting behavior, there are two different models: dynamic user optimal (DUO) model and dynamic system optimal model. There are many researchers including Carey, Friesz, Ho, Smith, Boyce and Ran Bin who made a lot of progress in establishing DUO models. Because the modeling method of DUO is based on the reasonable assumption of tripper route-selecting behavior, we select the DUO model to discuss the effects of traffic management on it.

However, the history of dynamic traffic assignment theories and models is so short that there are many practical questions facing to us. And we think the followings are the future missions waiting for us to pursue.

- Forecasting the congested time and length.
- Forecasting travel time under the congestion network
- Impact assessment on speed obligation, work-time obligation.
- Assessment on route guiding between collateral routes.
- Assessment on entrance flow control of urban freeway.
- Assessment on signal control effects of network
- Impact assessment on elastic work and car sharing.

From the above, we can conclude that the static traffic assignment is one demand forecasting method for traffic network planning; on the other hand, dynamic traffic assignment model is one for traffic control and management viewing practical network traffic flow as object.

2. ANALYSIS OF VARIOUS TRAFFIC MANAGEMENT MEASURES ON ESTABLISHING DUO MODELS

At present, there are many traffic management measures in cities in order to make traffic system safer, speeder, more comfortable and more efficiently. However, how to assess the effects of these measures and how to establish the optimal synthetic strategies are the focus for many researchers. Dynamic traffic assignment proposes one possible method to assess them and forecast them.

As described above, dynamic traffic assignment model is usually used for the core part of decision support system in traffic control and management. And the traffic management will influence properties of dynamic traffic assignment models at a large extent:

- Effect of intersection signal control
- Effect of single lane and link capacity
- Effect of social cost integrating environment pollution and energy consumption
- Effect of traffic congestion tax or congestion cost

In a word, we can divide the effects of traffic management on DUO model into two classes. One is to influence the parameters of network of DUO model such as single lane set and link capacity limitation. The other one is to influence the traveler route-selecting behavior of DUO model such as traffic congestion tax and HOV measure.

In this paper we mainly discuss the DUO model considering the intersection signal control (ISC) expecting to simulate and assess the effects of ISC such as average queue length and network load ratio.

3. DYNAMIC USER OPTIMAL MODEL CONSIDERING INTERSECTION SIGNAL CONTROL

3.1 Link State Equation And Its Constraints

3.1.1 Link state equation

In this research, a traffic network with multiple origins and ends is expressed as follows:

$$G = (A, N) \tag{1}$$

where

A means the set of all links,

N means the set of all nodes,

a node is either a origin, an end or an intersection in the network.

For one trip, we define the origin as k, the end as m, the traffic assignment span as [0,T]. Other definitions are listed as follows:

$x_a(t)$: the traffic volume on Link a at time t;

$u_a(t)$: the traffic volume flowing into Link a at time t;

$v_a(t)$: the traffic volume flowing out of Link a at time;

$w^{km}(t)$: the traffic volume arriving End m from Origin k at time t;

$W^{km}(t)$: the traffic volume arriving End m from Origin k before time t;

$x_{ar}^{km}(t)$: part of the traffic volume on Link a at time t which flows from Origin k to

End m through Route r;

$u_{ar}^{km}(t)$: part of the traffic volume flowing into Link a at time t which flows from Origin k to End m through Route r;

$v_{ar}^{km}(t)$: part of the traffic volume flowing out of Link a at time t which flows from Origin k to End m through Route r;

$w_r^{km}(t)$: the traffic volume arriving End m from Origin k through Route r at time t;

$W_r^{km}(t)$: the traffic volume arriving End m from Origin k through Route r before time t;

$q^{km}(t)$: the given traffic volume from Origin k to End m at time t;

Furthermore, we define $A(i)$ as the set of links beginning with Node i, $B(i)$ as the set of links ending with Node i. Then we get the following link traffic state equation:

$$\frac{dx_{ar}^{km}(t)}{dt} = u_{ar}^{km}(t) - v_{ar}^{km}(t), \forall k, m, r, a, \qquad (2)$$

$$\frac{dE_r^{km}(t)}{dt} = e_r^{km}(t), \forall k, m, r, k \neq m \qquad (3)$$

3.1.2 Constraints

- Traffic Conservation constraints:

$$q^{km}(t) = \sum_{a\in A(k)} \sum_{r} u_{ar}^{km}(t), \forall k, m; k \neq m \qquad (4)$$

$$\sum_{a\in B(i)} v_{ar}^{km}(t) = \sum_{a\in A(i)} u_{ar}^{km}(t), \forall k, m, r, i; i \neq k, m \qquad (5)$$

$$\sum_{a\in B(m)} \sum_{r} v_{ar}^{km}(t) = e^{km}(t), \forall k, m; k \neq m \qquad (6)$$

- Nonnegative constraints:

$$x_{ar}^{km}(t) \geq 0, u_{ar}^{km}(t) \geq 0, v_{ar}^{km}(t) \geq 0, \forall k, m, r, a; \qquad (7)$$

$$e_r^{km}(t) \geq 0, E_r^{km}(t) \geq 0, \forall k, m, r; \qquad (8)$$

- ISC constraints:

Consider the case with no right-turn traffic. If the intersection signal cycle time and the split at Origin y of Link a are T_y and G_y respectively, those at End z are T_z and G_z respectively, then ISC constraints can be expressed as follows:

$$u_{ar}^{km}(t)=0,\quad t\in[(s-1)T_y, G_y sT_y \quad \forall k,m,r,a; s=1,2,\ldots,N_y \tag{9}$$

$$v_{ar}^{km}(t)=0,\quad t\in[(s-1)T_z, G_z sT_z \quad \forall k,m,r,a; s=1,2,\ldots,N_z \tag{10}$$

Where:

$$N_y=\frac{T}{T_y},\quad N_z=\frac{T}{T_z}$$

- Original constraints:

$$x_{ar}^{km}(0)=0, E_r^{km}(0)=0, \forall k,m,r,a; \tag{11}$$

- General constraints on symbol definition:

$$\sum_{kmr}u_{ar}^{km}(t)=u_a(t), \sum_{kmr}v_{ar}^{km}(t)=v_a(t),$$
$$\sum_{kmr}x_{ar}^{km}(t)=x_a(t), \sum_r e_r^{km}(t)=e^{km}(t), \sum_r E_r^{km}(t)=E^{km}(t), \forall k,m \tag{12}$$

- FIFO (First-in-first-out) constraints:

$$x_{ar}^{km}(t)=\sum_{b\in \bar{r}}\left\{x_{ar}^{km}[t+\tau_a(t)]-x_{br}^{km}(t)\right\}+\left\{E_r^{km}[t+\tau_a(t)]-E_r^{km}(t)\right\},$$
$$\forall k,m,r,i,a\in B(i); i\neq k \tag{13}$$

3.2 Determining The DUO Solution With ISC Constraints

Without ISC constraints, DUO problem can be converted into an optimal-control problem with its object function below. The optimal numerical solution can be got using Frank Wolfe Arithmetic.

$$\min_{u,v,x,e,E} J=\int_0^T\sum_a\left\{\int_0^{u_a(t)}f_{1a}[x_a(t),\gamma]d\gamma+\int_0^{v_a(t)}f_{2a}[x_a(t),\gamma]d\gamma\right\} \tag{14}$$

The constraints are listed above, from Equation 1 to Equation 13.

Consider traffic signal controls at all the intermediate intersections on Route r (excluding Origin k and End m). Now Equation 9 and Equation 10 are included in the constraints. To get the proper solution, the original solution $u_{ar}^{km}(t)_n, v_{ar}^{km}(t)_n$ (solution without considering ISC constraints) must be corrected without changing the limit values and object function value in a signal cycle. This method is discussed in details below.

- ISC at Origin y of Link a

According to constraints in Equation 9 and Equation 10, the solution is determined by two integral equations listed below:

$$\int_{G_y sT_y}^{sT_y} u_{ar}^{km}(t)dt - \int_{(s-1)T_y}^{sT_y} u_{ar}^{km}(t)dt = \int_{(s-1)T_y}^{sT_y} [u_{ar}^{km}(t)_n - v_{ar}^{km}(t)_n]dt \qquad (15)$$

$$\begin{aligned} &\int_{G_y sT_y}^{sT_y} \left\{ \int_0^{u_{ar}^{km}(t)} f_{1a}(x_a(t),\gamma)d\gamma \right\} dt + \int_{(s-1)T_y}^{sT_y} \left\{ \int_0^{v_{ar}^{km}(t)} f_{2a}(x_a(t),\gamma)d\gamma \right\} dt = \\ &\int_{(s-1)T_p}^{sT_p} \left\{ \int_0^{u_{ar}^{km}(t)_n} f_{1a}(x_a(t),\gamma)d\gamma + \int_0^{v_{ar}^{km}(t)_n} f_{2a}(x_a(t),\gamma)d\gamma \right\} dt \end{aligned} \qquad (16)$$

Where Equation 15 is got from $x_{ar}^{km}(sT_y) = x_{ar}^{km}(sT_y)_n$, Equation 16 is got from the equivalency of object functions in a signal cycle.

By this method, we get DUO solution with ISC at End z of Link a. As for the case with ISC at both Origin y and End z of Link a, the original DUO solution need to be corrected twice to get the final solution $u_{ar}^{km}(t), v_{ar}^{km}(t), x_{ar}^{km}(t)$, considering ISC at Origin y first and ISC at End z next.

Therefore, to get proper DUO solution with ISC constraints, the two integral equations above need to be solved after the original solution $u_{ar}^{km}(t)_n, v_{ar}^{km}(t)_n, x_{ar}^{km}(t)_n$ is got using Frank-Wolfe Arithmetic. The solutions of the two equations are not unique. By defining $u_{ar}^{km}(t) = \alpha_1 u_{ar}^{km}(t)_n, v_{ar}^{km}(t) = \alpha_2 v_{ar}^{km}(t)_n$ and working out α_1, α_2 through the two equations, we can get the final numerical DUO solutions with ISC constraints.

4. DISCUSSION ON THE IMPROVED METHOD OF DUO MODEL WITH ISC CONSTRAINTS

Due to its abstraction and simplification of intersections and the indeterminacy of solutions, the model discussed above has some shortages in traffic control optimal simulation and assistant decision-making:

- An intersection is simplified into a split in this model without considering the signal control diversity on all entrance lanes, such as multiple phases, left-turn ban and free right-turn.
- Because the corrected solutions are not unique and using the equivalency of limit values as a correctional principle can not properly simulate the actual traffic flow assignment state, certain errors may be created through this method.

Therefore, we propose an improve method of DUO model with ISC constraints in the following part of this paper. In this method, an intersection with signal controls is simulated into multiple links according to the different flow directions of entrance lanes (displayed in Figure 1), the links are integrated into the network with their capacities determined by the saturated traffic volumes of the three directions of any entrance. In this way, the signal controls at intersections are converted into constraints of certain virtual links.

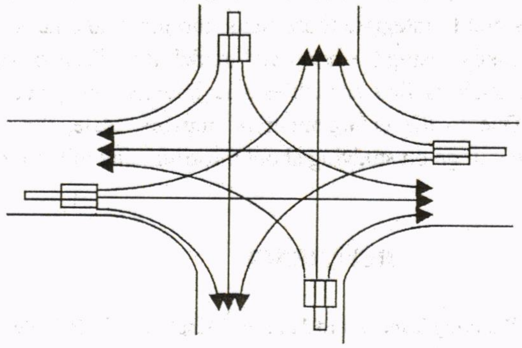

Figure 1 Virtual Links At An Intersection

Given a certain link c_i in the set of virtual links C_i of an intersection i with signal controls ($i \in$ Intersection Set I) and T_i defined as the signal cycle time of Intersection i, the traffic volumes $u_{c,r}^{km}(t)$, $v_{c,r}^{km}(t)$, $x_{c,r}^{km}(t)$ on c_i at different times in a signal periodicity can be worked out, if the time assignments of all phases are also known.

$$x_{c_i r}^{km}(t) = 0, u_{c_i r}^{km}(t) = v_{c_i r}^{km}(t)$$

$$u_{c_i r}^{km}(t) = v_{c_i r}^{km}(t) = 0, t \notin P_{gc_i}, \forall c_i \in C_i \quad (17)$$

Where

P_{gc_i} is the green time in a signal cycle on virtual link c_i of Intersection i

By integrating the constraints into DUO model, we can use Frank-Wolfe Arithmetic to directly get the optimal DUO solution with intersection signal controls and thus make up shortages of the previous model. We can also analyze and evaluate the effects of different signal controls on the whole network, such as link capacities and intersection queue lengths. It is relatively a reasonable way to simulate traffic flow. Because the network scale is enlarged to a certain extent, the solution scale is enlarged accordingly.

5. CONCLUSION

This paper firstly analyzes the effects of traffic management on dynamic traffic assignment models. Then the dynamic user optimal (DUO) model considering intersection signal control (ISC) is discussed and proposed in this paper. Finding some disadvantages of this new model in simulating intersection, we propose an improved scheme, which is to transfer the ISC mode into a series of virtual links and to integrate these links into the whole network. This method can simulate ISC well and easily. Using the improved model, the effects of signal controls on the whole traffic network, such as link capacities and intersection queue lengths, can be discussed and evaluated. Due to the enlargement of network scale, the solution scale is enlarged accordingly. So we will go on studying about the arithmetic of the model.

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