DYNAMIC TRAFFIC ASSIGNMENT USING GENETIC ALGORITHM UNDER DYNAMIC TRAFFIC CONDITIONS

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Abstract: Dynamic traffic assignment(DTA) has been a topic of substantial research in the last two decades and it has recently received growing attention, with the news that it will utilize Advanced Traffic Information Systems(ATIS) applications. However DTA has many mathematical difficulties in searching its solution due to the complexity of spatial and temporal variables. Although many solution algorithms have been developed, conventional methods cannot find a solution when an objective function or constraints is not convex. In this paper, we provide a new method using a genetic algorithm(GA) to solve the DTA model. To apply this new method, we formulated the DTA model based on Merchant-Nemhauser's model (1978), which has a nonconvex constraint set. To handle the nonconvex constraint set, the GENOCOP III system, which is one of the GAs, is used in this study. Results for the sample network have been compared with the results of conventional method.

Key Words: GENOCOP III system, Genetic Algorithm, Dynamic Traffic Assignment, Dynamic System Optimal, Dynamic User Optimal

1. INTRODUCTION

We try to provide a new method using the GA to solve the DTA model. To apply this new method, we formulated the DTA model based on Merchant-Nemhauser's model (1978)(4). Unfortunately original Merchant-Nemhauser's model cannot present dynamic user optimal(DUO) state, since it only present dynamic system optimal(DSO) state. To formulate the objective function for DUO, we need to consider users' travel time(10)(11).

Thus, we drove a travel time function, which is monotonically nondecreasing and convex with respect to density, using the modified Greenshied's speed-density relationship(1)(3). Using the above travel time function, we formulated the DTA model, which can present the objective functions for DUO and DSO. However, the above model has nonconvex constraints set, similar to the original model. Therefore we may not be able to find a global solution, while many local solutions may be available(5).

To overcome nonconvexiy, we used the GONOCOP III system, which is one of the GAs(Genetic Algorithms)(7)(8). GONOCOP III system can handle nonconvex constraints set. It is based on concepts of co-evolution and repair algorithm and avoids many disadvantages of other GA systems. The results for the sample network have been compared with the results of conventional method.

2. GONOCOP III SYSTEM

2.1 Genetic Algorithm

GAs are search algorithms based on the mechanics of natural selection and natural genetics. They combine survival of the fittest among string structures with a structured, yet randomized, information exchange to form a search algorithm with some of the innovative flair of human search. In every generation, a new set of the artificial creatures(strings) is created using bits and pieces of the fittest of the old. An occasional new part is tried for good measure. Although effective, GAs can be quite simple in their application, they efficiently exploit historical information to speculate on new search points with expected improved performance(9).

GAs demonstrate several distinct advantages. First, they employ an efficient optimal solution searching technique, which can be described as multi-hill climbing. The global solutions can be easily found for both linear and nonlinear formulations. Second, the optimal solution searching process is independent of the form of the objective function. Unlike conventional techniques, in which the algorithms usually rely on the structure of the formulation such as the conditions for the decomposition algorithms. GA models can be implemented without such considerations. Third, conventional algorithms are often sensitive to the input patterns such as the conditions set forth by Monte Carlo techniques.

2.2 GENOCOP III System

Introduction

GAs have many distinct advantages as mentioned above, however, they have also some problems such as inefficiency of computation process and constraints handling. So many methods have been studied to handle constraints efficiently by GAs for numerical optimization problems.
One of these methods is the GENOCOP(for GEnetic algorithm for Numerical Optimization of COnstrained Problems) system that can handle linear constraints only(6)(9). In this paper we use GENOCOP III system, which is advanced version of GENOCOP system and able to handle any set of constraints included nonlinear and nonconvex constraints.

This method doesn't need additional turning of several parameters in contrast to other methods so it has efficiency in computation. It can be applied to various optimization problems because it is independent of the form of the constraints set.
In this paper we try to search the solution of DTA, which has many spatial and temporal variables and constraints. Though it is possible to search the solution using exiting GAs, it is very inefficiently and complicated. So we try to apply GENOCOP III system to DTA model.

Process

The general nonlinear programming problem is to find $\overline{X}$ so as to

$$\text{Optimize } f(\overline{x}),\ \overline{X} = (x_1, \ldots, x_n) \in R^n \quad (1)$$
$$\text{s.t. } \overline{X} \in F \subseteq S$$

The set $S \subseteq R$ defines the search space and the set $F \subseteq S$ defines as feasible search space. Usually, the search space S is defined as a n-dimensional rectangle in R:

$$l(i) \leq x_i \leq u(i),\ 1 \leq i \leq n \quad (2)$$

Whereas the feasible set $F \subseteq S$ is defined by a set of additional m$\geq$0 constraints:

$$g_j(\overline{X}) \leq 0, \text{ for } j = 1, \ldots, q \text{ and } h_j(\overline{X}) = 0 \text{ for } j = q+1, \ldots, m \quad (3)$$

It is also convenient to divide all constraints into four subsets: linear equations LE, linear inequalities LI, nonlinear equations NE, and nonlinear inequalities NI. Of course, $g_j \in$ LI $\cup$ NI and $h_j \in$ LE $\cup$ NE. In fact, we need not consider linear equation LE, since we can remove them by expressing values of some variables as linear functions of remaining variables and marking appropriate substitutions.

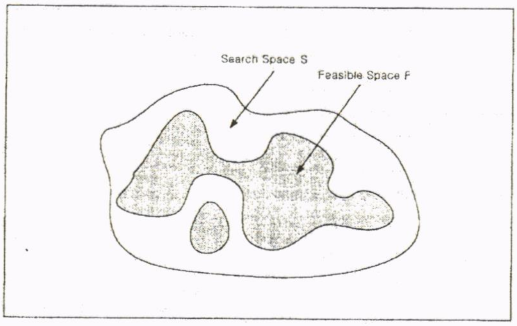

Figure 1. A search space and its feasible parts

The GENOCOP III system incorporates the original GENOCOP system, but also extends it by maintaining two separate populations, where a development in one population influences evaluations of individuals in the other populations(6).

The first population consists of so-called search points from S, which satisfy linear constraints of the problem. The feasibility(in the sense of linear constraints) of these points is maintained by specialized operators. The second population consists of so-called reference points form F; these points are fully feasible, i.e., they satisfy all constraints.

Reference points $\overline{R}$, being feasible, are evaluated directly by the objective function(i.e., $eval(\overline{R}) = f(\overline{R})$). On the other hand, unfeasible search points are 'repaired' for evaluation and the repair process works as follows. Assume that there is a search points $\bar{S} \notin F$. In such a case the system selects one of the reference points, say $\bar{R}$, and creates random points $\bar{Z}$ from a segment between S and $\bar{R}$ by generating random number a from the range <0, 1>: $\bar{Z} = a\bar{S} + (1-a)\bar{R}$.

Once a feasible $\bar{Z}$ is found, $eval(\bar{S}) = eval(\bar{Z}) = f(\bar{Z})$. Additionally, if $f(\bar{Z})$ is better than $f(\bar{R})$, then the points $\bar{Z}$ replaces $\bar{R}$ as a new reference point. Also, $\bar{Z}$ replaces $\bar{S}$ with some probability of replacement P_r.

The GENOCOP III system avoids many disadvantages of other GAs. It introduces few additional parameters(the population size of reference points, probability of replace) only. It always returns a feasible solution. A feasible search space F is searched by making references from the search points. The neighborhoods of better reference points are explored more often. Some reference points are moved into the population of search-points, where they undergo transformation by specialized operations, which preserve linear constraints(6)(7).

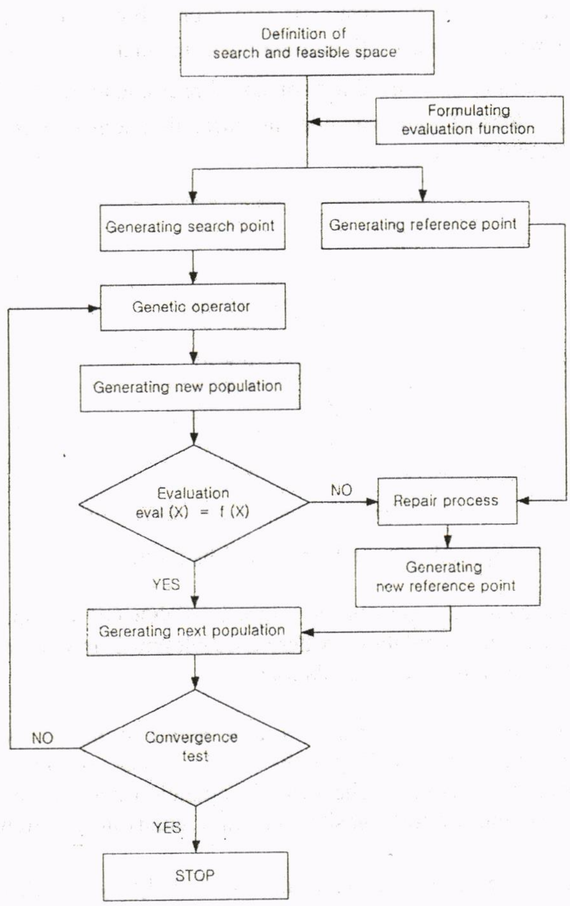

Figure 2. Process of GENOCOP Ⅲ System

3. MODEL FORMULATION

We formulate the DTA model based on Merchant-Nemhauser's model. To present the objective function of dynamic user optimal, we use the modified Greenshield's model(2)(3). The summary of notation in this paper is follows.

n = set of nods
a = set of links
t = time slices
x_a = number of vehicles on link a at the beginning of the *tth* period
$g(x)$ = exit function
l_a = length of link a
u = travel speed on link a at time t
$u_{\max}$ = maximum travel speed on link a at time t
$u_{\min}$ = minimum travel speed on link a at time t
k_j = jam density
T_a = travel time on link a at time t

3.1 Link Travel Time Function

In our model, we propose to use a modified Greenshild's equation to derive the link travel time function as shown below.

Modified Greenshild's Equation : $u'_a = u_{\min} + (u_{\max} - u_{\min})(1 - \frac{k'_a}{k_j})$ (4)

Link Travel Time Function : $T_a(x'_a) = \frac{l_a^2 k_j}{u_{\max} k_j l_a - (u_{\max} - u_{\min}) x'_a}$ (5)

This function is nonnegative, nondecreasing, continuous, convex function and present the travel cost on links.

3.2 Exit Function

The exit function represents a physical phenomenon, it is assumed not to depend on time. So we assume that the exit function is equal to the traffic volume on link. Using traffic-flow relationships, we can express the exit function as

$$g_a(x'_a) = u_{\max} \frac{x'_a}{l_a} - (u_{\max} - u_{\min}) \frac{(x'_a)^2}{l_a^2 k_j} \tag{6}$$

The above function depends on number of vehicles on link only. When first-oder condition is $\frac{dg_a}{dx_a} = 0$, exit flow is maximum. Thus maximum exit flow is as follows.

$$g_a(x_a)_{\max} = \frac{u_{\max}^2 k_j}{4(u_{\max} - u_{\min})} \tag{7}$$

3.3 Objective Function

We make two objective functions. One is dynamic user optimal(DUO) and the other is dynamic system optimal(DSO). In DUS we determine vehicle flow on each link at each time resulting from drivers using minimal-time route under the currently prevailing travel times. In DSO drivers act so as to minimize total travel cost at each time(10)(11)(12). These objective functions can be expressed as follows.

$$\text{DUO}: \quad \sum_{a=1}^{A} \sum_{t=1}^{T} \int_{0}^{x_a^t} T_a(x)dx \tag{8}$$

$$\text{DSO}: \quad \sum_{a=1}^{A} \sum_{t=1}^{T} \int_{0}^{x_a^t} x_a^t T_a(x_a^t) \tag{9}$$

3.4 Constraints

The fundamental transformation equation known as state equation is

$$x_a^{t+1} = x_a^t - g_a(x_a^t) + d_a^t, \quad t = 0,1,\dots,T-1 \tag{10}$$

where decision variables d_a^t is the umber of vehicles that admitted onto link a during the tth time period.
We have the conservation equation as follows.

$$\sum_{a \in A(n)} d_a^t = F^t(n) + \sum_{a \in B(n)} g_a(x_a^t), \quad t = 0,1,\dots,T-1, \quad \forall n \in N - \{n\} \tag{11}$$

Where $A(n)$ is the set of links, which point out of node n and $B(n)$ is the set of links which point into node n. $F^t(n)$ denotes the external input at node n for the tth time period. Exit flow has to satisfy two constraints, which are nonnegatively constraints and maximum flow constraints as shown.

$$0 \le d_a \le \frac{u_{\max}^2 k_j}{4(u_{\max} - u_{\min})} \tag{12}$$

The vehicles on link have upper bound. The maximum vehicles on link are equal to traffic volume under jam density.

$$0 \le x_a \le k_j \times l_a \tag{13}$$

3.5 Model

Considering all constraints, we make DTA model as follows.

Objective Function

$$\text{DUO}: \sum_{a=1}^{A}\sum_{t=1}^{T}\int_{x_a^t} T_a(x)dx$$

$$\text{DSO}: \sum_{a=1}^{A}\sum_{t=1}^{T}\int_{x_a^t} x_a^t T_a(x_a^t)$$

s.t.

$$x_a^{t+1} = x_a^t - g_a(x_a^t) + d_a^t, \quad t = 0,1,...,T-1$$

$$\sum_{a\in A(n)} d_a^t = F^t(n) + \sum_{a\in B(n)} g_a(x_a^t), \; t = 0,1,...,T-1, \; \forall n \in N - \{n\}$$

$$0 \le d_a \le \frac{u_{\max}^2 k_j}{4(u_{\max} - u_{\min})}$$

$$0 \le x_a \le k_j \times l_a$$

$$x_a^0 = R_a(given)$$

4. APPLICATION OF GONOCOP III SYSTEM

4.1 Sample Network

To demonstrate the application of GENOCOP III system to DTA model, we make the sample network, which has 4-centroids, 9-nodes and 24-links given in Figure 3. We classify links to two categories, which are highway and arterial roads.

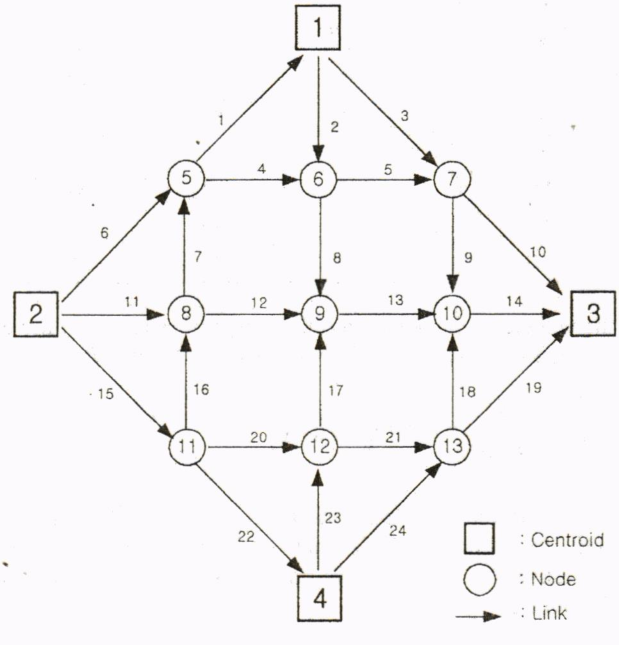

Figure 3. Sample Network

The physical characteristics of each links are shown in Table 1.

Table 1. Physical Characteristics of Links

Link	Classification	Length (km)	Num. Of Lanes	Max. Velocity (km/hr)	Min. Velocity (km/hr)	Jam Density (veh/lane-km)	Max. Exit Vehicles (veh/lane-5min)
1	Arterial	12	1	80	10	80	152
2	Arterial	8	1	60	10	80	120
3	Arterial	12	1	80	10	80	152
4	Arterial	8	1	60	10	80	120
5	Arterial	8	1	60	10	80	120
6	Arterial	12	1	80	10	80	152
7	Arterial	8	1	60	10	80	120
8	Arterial	8	1	60	10	80	120
9	Arterial	8	1	60	10	80	120
10	Arterial	12	1	80	10	80	152
11	Highway	15	2	100	10	80	185
12	Highway	15	2	100	10	80	185
13	Highway	15	3	100	10	80	185
14	Highway	15	4	100	10	80	185
15	Arterial	12	1	80	10	80	152
16	Arterial	8	1	60	10	80	120
17	Arterial	8	1	60	10	80	120
18	Arterial	8	1	60	10	80	120
19	Arterial	12	1	80	10	80	152
20	Arterial	8	1	60	10	80	120
21	Arterial	8	1	60	10	80	120
22	Arterial	12	1	80	10	80	152
23	Arterial	8	1	60	10	80	120
24	Arterial	12	1	80	10	80	152

We assume that the vehicles, which enter a link at any time slice, can't traverse the link at the same time slice. So time slices have to satisfy below upper bound(10).

$$T < \min\{\frac{l_a}{u_{\max}}, \quad \forall a\} = \frac{8km}{60km/hr} = 0.13hr = 8\min$$

Thus in our model, we decide that time periods are 5-min which is smaller than 8-min and total time slices are 10(i.e. total analysis time is 50-min). In sample network, there are 3-originations and 1-destination, so all drivers travel toward node 4. For each time slices, OD flows are shown in Table 2.

Initial link flows are assumed that highway links have vehicles at 40% of maximum link volume and arterial links have vehicles at 30%.

Table 2. OD flows (veh/5min)

Interval	Node 1	Node 2	Node 3
1	120	390	100
2	130	450	150
3	135	510	185
4	130	480	150
5	115	560	180
6	130	590	140
7	100	615	130
8	110	620	150
9	130	590	130
10	125	570	140

4.2 Solution Process

In sample network, we have 24-state equations, 12-conservation equations, 24-maximum exit flow inequalities, 48-nonnegatively inequalities and 24-maximum link flows inequalities. The search space S is composed of 96-inequalities, the remains of constraints compose feasible space F.

There are 12-decision variables, which are exit flows from link at each time periods. We can express decision variables as follows for GENOCOP III system.

$$\overline{D} = (d_1^t, d_2^t, d_5^t, d_6^t, d_7^t, d_{11}^t, d_{16}^t, d_{17}^t, d_{18}^t, d_{20}^t, d_{23}^t)$$

The GENOCOP III system process for solving our DTA model is summarized as follows.

Step 0: Find initial exit flow d_a^0 using given initial link flows.

Step 1: Formulate constraints related with x_a^t using state equations.

Step 2: Formulate constraints related with d_a^t using conservation equations.

Step 3: Find search space and feasible space considering constraints.

Step 4: Generate search points and reference points(i.e. initialize $\overline{D}$).

Step 5: Generate new population using search points by genetic operators.

Step 6: Repair unfeasible population of new population and make new reference points and search points.

Step 7: Select the best reference point and test convergence.

4.3 Results

As the results for the sample network, the solutions for two objective functions converge(see Table 3). In particular, the results for DSO satisfy user equilibrium state: i.e. used routes have equal travel time in each interval.

To demonstrate the performance of GENOCOP III system, we compared the results with those of Quasi-Newton method, which is a conventional method to solve NLP. In the comparative analysis, we found that both algorithms have the same objective function values. However, we found that the GENOCOP III system is more efficient in terms of execution time results(see Table 4 and 5).

Table 3. Exit Flow From Link (Unit: vehicle)

Classification		Time Periods									
		1	2	3	4	5	6	7	8	9	10
DUO	d1	125	109	108	116	114	115	116	119	121	122
	d2	120	120	120	120	120	120	107	108	116	114
	d5	107	109	111	112	112	112	112	111	111	111
	d6	50	101	115	108	126	133	139	140	133	129
	d7	117	107	102	100	98	99	101	103	105	105
	d9	120	120	120	120	119	114	114	113	112	112
	D11	290	247	280	263	307	324	337	340	324	312
	D16	69	57	50	51	51	53	54	55	56	56
	D17	107	104	98	92	88	84	80	76	75	73
	D18	120	120	120	117	107	105	99	93	89	85
	D20	70	56	50	51	51	53	54	55	56	56
	D23	120	120	120	108	101	83	78	86	79	84
DSO	d1	125	112	118	116	114	114	116	118	120	121
	d2	120	120	120	120	114	114	101	105	113	111
	d5	107	109	111	110	110	109	109	107	107	107
	d6	50	101	115	108	126	133	139	140	133	129
	d7	94	101	95	93	89	90	92	94	96	96
	d9	120	120	113	113	113	112	112	111	110	110
	D11	290	247	280	263	307	323	337	340	324	313
	D16	64	45	43	44	43	44	46	47	48	48
	D17	107	103	95	89	83	80	76	72	71	69
	D18	120	119	110	110	106	104	99	93	89	85
	D20	64	45	44	44	43	44	46	47	48	48
	D23	120	120	120	100	104	84	79	87	79	83

Table 4. Results for DUO

Classification		Objective Function Value (hr)	Execute Time (sec)
GENOCOP Ⅲ System		16,390.22	10.2
Quasi-Newton Method	Fist execute	16,385.41	178.9
	Second execute	16,389.76	46.2

Table 5. Results for DSO

Classification		Objective Function Value (hr)	Execute Time (sec)
GENOCOP Ⅲ System		19,511.94	9.4
Quasi-Newton Method	Fist execute	19,501.52	172.9
	Second execute	19,512.91	59.7

5. CONCLUSIONS

In this paper the GENOCOP III system is presented as the new solution algorithm for the DTA model. As the results for the sample network, we found that GENOCOP III system can be applied to the part of DTA.

Also we have demonstrated the performance and efficiency of the GENOCOP III system compared with conventional method. In the comparative analysis, GENOCOP III system showed more efficient than conventional method. Those results are evident that we will be able to expand the GENOCOP III system to other transportation parts.

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