OFFTRACKING MODEL ON HORIZONTAL CURVE SECTIONS

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Abstract: Pavement widening is needed for the reason that a vehicle or a truck occupies greater width within horizontal curve sections because rear wheels generally track inside the path traced by front wheels and the difference is called as offtracking. Offtracking is divided into two distinct types according to vehicle speeds: low-speed and high-speed offtracking. Low-speed offtracking is a simple phenomenon that the rear wheels track toward the inside of a horizontal curve. In this research earlier studies about low-speed offtracking were reviewed, and then a mathematical model for calculating the amount of low-speed offtracking of any type of vehicle developed at any point of the path that the vehicle following was established and tested. Finally, vehicle simulation program that calculates the path of vehicle and the low-speed offtracking was developed based on the calculation results from the model. Graphical solution was also provided.

Key Words: Horizontal Curves, Low-speed Offtracking Model, Vehicle Simulation Program.

1. INTRODUCTION

1.1 Background

Differently from tangent sections, horizontal curve sections have additional problems on geometric design of roads that must be considered in-depth while in design. Pavement widening is one of the problems. Pavement widening is needed for the reason that a vehicle or a truck occupies greater width within horizontal curve sections because rear wheels generally track inside the path traced by front wheels, and the difference is called as offtracking.

Offtracking is divided into two distinct types according to vehicle speeds: low-speed and high-speed offtracking. Low-speed offtracking is a simple phenomenon that the rear wheels track toward the inside of a horizontal curve. High-speed offtracking, on the other hand, is a dynamic phenomenon that the rear of the vehicles moves outward because of the lateral acceleration of the vehicle as it traverses a horizontal curve at high speeds. Low-speed offtracking of large trucks is more serious than that of passenger cars because low-speed offtracking increases with wheelbase of the vehicle.

Offtracking can pose problems whenever there is not enough space to accommodate both the width of the vehicle and the additional offtracking displacement. Thus sufficient consideration of offtracking is important when one determines the proper and reasonable value of widening in road design because trucks become larger and longer lately.

1.2 Objectives

The main objectives of this research were to develop a mathematical model for calculating the amount of low-speed offtracking of any type of vehicle developed at any point of the path that the vehicles following and a simulation program that calculates the path of vehicle and the low-speed offtracking based on the calculation results from the model.

2. LOW-SPEED OFFTRACKING

2.1 Low-speed Offtracking

When a vehicle makes a turn slowly such as parking lot maneuvers, the tires need not develop lateral forces. Thus they roll with no slip angle, and the vehicle must negotiate a turn as illustrated in Figure 1(Gillespie, 1992).

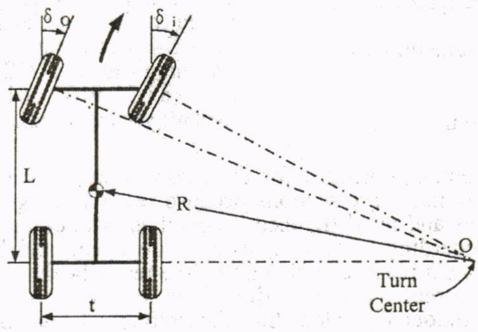

Figure 1. Geometry of a turning vehicle

In Figure 1, L is wheelbase of a vehicle, t is width of a vehicle, R is turning radius, and δ_o and δ_i are steer angles. If the rear wheels have no slip angle, the center of turn must lie on the projection of the rear axle. Likewise, the perpendicular from each of the front wheels should pass through the same point(the center of turn). Then the vehicle can negotiate a turn softly because the steer angle of outside wheel is smaller than that of inside wheel.

The other significant aspect of low-speed turning is the offtracking. When a vehicle turns, its rear wheels track inside the path traced by the front wheels. The magnitude of this difference in paths, known as offtracking, generally increases with the spacing between the axles of the vehicle and decreases for larger-radius turns(Glaus, W. D. *et al.*, 1991).

The AASHTO Green Book notes low-speed offtracking as follow: Low-speed offtracking is a purely geometrical phenomenon wherein the rear axles of a truck track toward the inside of a horizontal curve, relative to the front axle(AASHTO, 1994).

A typical and general formula that calculate low-speed offtracking was originally recommended by the Western Highway Institute(WHI)(Sayers, 1986). WHI offtracking formula is:

$$OT_{\max} = R - \sqrt{R^2 - \sum L^2} \qquad (1)$$

where,

OT_{max} = maxium offtracking;
R = radius of the path traced by the center of foremost axle;
L = wheelbase.

WHI formula used Pythagorean theorem and the sum of squares of wheelbases (Heald, 1986). Figure 2 shows low-speed offtracking of a tractor-semitrailer vehicle. From the definition of

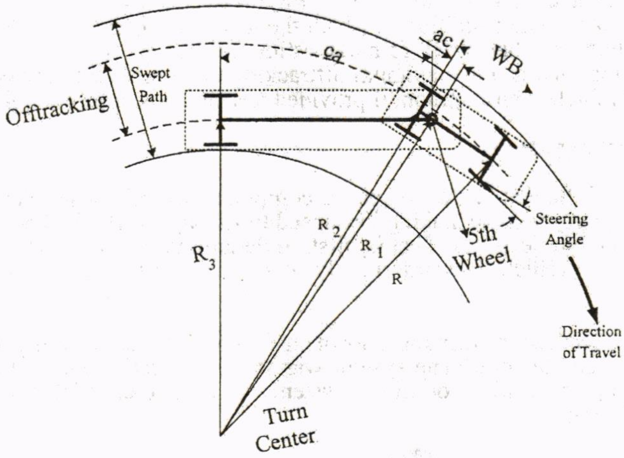

Figure 2. Geometry of a Turning Tractor-Semitrailer

offtracking, it is the difference between R and R_3 as shown in the Figure 2. Thus,

$$OT_{max} = R - R_3 \tag{2}$$

Pythagorean theorem can be used to calculate the radius of the path at the center of the tractor rear axle(R_2) as follows:

$$R_2^{\ 2} = R^2 - WB^2 \tag{3}$$

Also the following equation can be developed to calculate the radius of the path traced by the 5th wheel(R_1).

$$\begin{aligned} R_1^{\ 2} &= R_2^{\ 2} + ac^2 \\ &= R^2 - WB^2 + ac^2 \end{aligned} \tag{4}$$

Then the radius of the path taken by the center of the trailer axle(R_3) is :

$$\begin{aligned} R_3^{\ 2} &= R_1^{\ 2} - ca^2 \\ &= R^2 - WB^2 + ac^2 - ca^2 \end{aligned} \tag{5}$$

Equation 2 and 5 can be combined to yield:

$$\begin{aligned} OT_{max} &= R - R_3 \\ &= R - \sqrt{R^2 - (WB^2 - ac^2 + ca^2)} \end{aligned} \tag{6}$$

If $\sum L^2$ is substituted for (WB^2 - ac^2 + ca^2) in the equation 6, the following results.

$$OT_{max} = R - \sqrt{R^2 - \sum L^2}$$

where,

R	= radius of the path traced by the center of foremost axle;
WB	= wheelbase of tractor;
ac	= distance between tractor rear axle and hitch;
ca	= distance between trailer rear axle and hitch.

When a vehicle traverses a turn, the amount of low-speed offtracking increases and the vehicle reaches a steady state condition in which the low-speed offtracking don't increase any more. This condition is what WHI formula estimates and we call it a fully developed low-speed offtracking, that is, the maximum offtracking that will eventually occur for a given turning radius and vehicle configuration, if provided turn angle is large enough.

2.2 Vehicle Offtracking Model

Recently, Michael W. Sayers has developed a computer method for graphing the complete swept path of an arbitrary vehicle making low speed turns(Sayers, 1986). The method was one of stepping through the trajectories. That is, first, to determine the initial position of a vehicle and then to move the vehicle forward slightly to calculate the new position of it and the process repeats.

He used two x-y coordinate systems as shown in Figure 3 to describe the swept path of a vehicle. While an absolute coordinate system was used to describe the positions of vehicle units as they trace a path, relative coordinate system was used to calculate the position of any point on the vehicle unit.

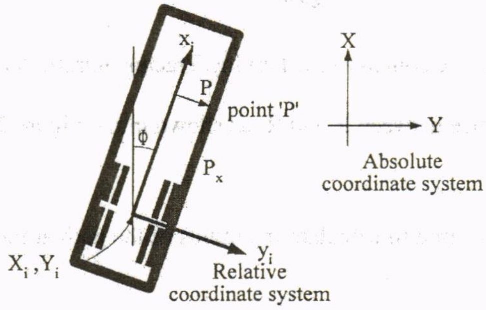

Figure 3. Coordinates of Arbitrary Point P in a Vehicle

He also assumed that for small movements, the paths of all points on the vehicle were approximately circular.

3. NEW OFFTRACKING MODEL

3.1 General Approach

In limitation, Sayers model involves lot of complicated equations that are so difficult to understand that engineers can't use it freely as they wish, mainly because of the model structure of calculating circular wheel paths. New model in this paper ignores the circular wheel paths and instead assumes that wheel paths form straight lines in horizontal curves. By doing this, lots of computational efforts seem reduced and the maximum use of model capability can be realized.

1) Coordinate System

The model applied two x-y coordinate systems including an absolute system and a relative coordinate system to calculate the swept path of a vehicle.

2) Bicycle Model

Most offtracking models neglect the effect of a vehicle width. In this paper, the model assumes that all nonsteered wheels are rigidly connected and represented by a single "equivalent wheel" located at the centroid of real wheel positions. Multiple-axle suspensions are similarly modeled as a single effective axle, usually located at the geometric center of nonsteered axles. Figure 4 shows how a tractor-semitrailer vehicle would be represented by two linked bicycle models.

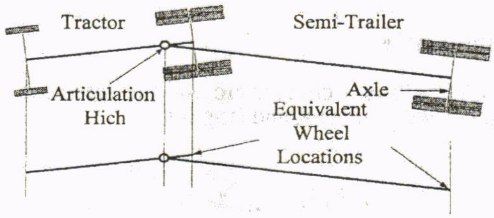

Figure 4. Bicycle Model

3) Moving Mechanism

This model uses one of stepping work. When calculation is performed for a specific point along the input path, the vehicle is moved forward slightly and calculation is repeated for a new position. In this model, it is assumed that if the increment(Δ s) is very small, the path of rear axle forms an straight line. Thus the movement of a vehicle on a circular input path can be illustrated as in Figure 5.

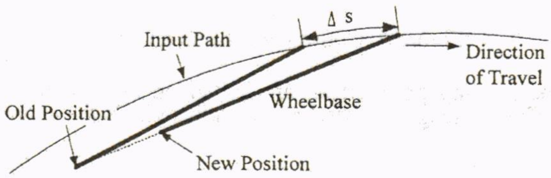

Figure 5. Moving Mechanism

4) Algorithm

Figure 6 shows the algorithm used in the new model.

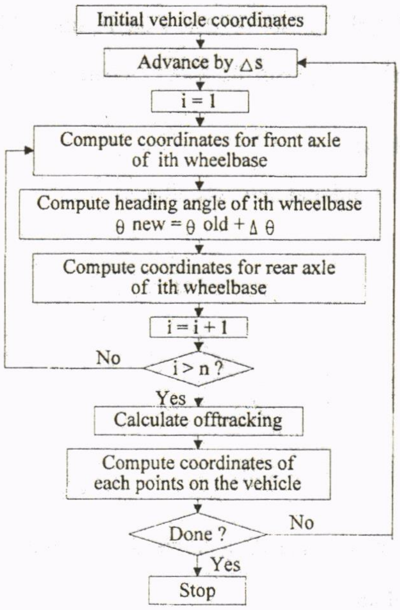

Figure 6. Algorithm of the New Model

3.2 Derivation

1) Coordinates of Foremost Axle

It is assumed that the input path is a circular arc and the foremost axle moves on this arc. Figure 7 shows a circular arc with radius R and length s.

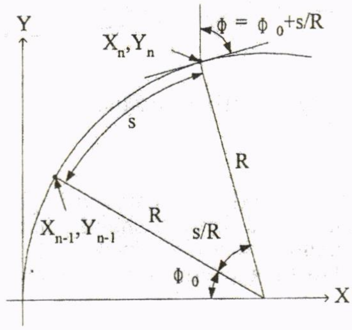

Figure 7. Coordinates of on an Arc

The coordinates of the starting point of an arc are given as X_{n-1}, and Y_{n-1}, and the initial heading angle is Φ_0. The arc length divided by the radius yields the central angle. Thus,

$$\Phi = \Phi_0 + \frac{s}{R} \tag{7}$$

$$X_n = X_{n-1} + R\{\cos\Phi_0 - \cos(\Phi_0 + s/R)\} \tag{8}$$

$$Y_n = Y_{n-1} + R\{\sin(\Phi_0 + s/R) - \sin\Phi_0\} \tag{9}$$

But these equations are not applicable to a straight line. Thus, using trigonometric identities, equation 8, and 9 can be manipulated to yield:

$$X_n = X_{n-1} + s\{\cos\Phi_0 \sin(s/2R)\sin(s/2R)/(s/2R) + \sin\Phi_0 \sin(s/R)/(s/R)\} \tag{10}$$

$$Y_n = Y_{n-1} + s\{\cos\Phi_0 \sin(s/R)\sin(s/R) - \sin\Phi_0 \sin(s/2R)\sin(s/2R)/(s/2R)\} \tag{11}$$

2) Heading Angle of Wheelbase

Figure 8 illustrates geometry of old and new wheelbases.

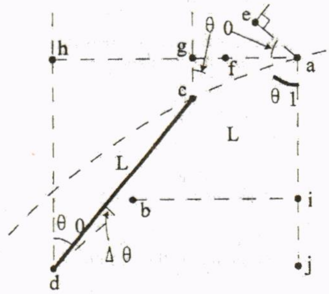

Figure 8. Heading Angle of Wheelbase

The meanings of each symbol are:

a = point of new front axle, (X^f_n, Y^f_n),

b = point of new rear axle, (X^r_n, Y^r_n),
c = point of old front axle, (X^f_{n-1}, Y^f_{n-1}), and
d = point of old rear axle, (X^r_{n-1}, Y^r_{n-1})

When moving a short distance, Δ s, heading angle of the wheelbase can be written as:

$$\theta_1 = \theta_0 + \Delta\theta \tag{12}$$

where,

$$\Delta\theta = \sin^{-1}(\overline{ae} / \overline{ad}) \tag{13}$$

$$\overline{ae} = \overline{af} \cos\theta_0 \tag{14}$$

$$\overline{af} = \overline{ah} - (\overline{gh} + \overline{fg}) \tag{15}$$

$$\overline{ah} = X^f_n - X^r_{n-1} \tag{16}$$

$$\overline{gh} = X^f_{n-1} - X^r_{n-1} \tag{17}$$

$$\begin{aligned} \overline{fg} &= \overline{cg} \tan\theta_0 \\ &= (Y^f_n - Y^f_{n-1}) \tan\theta_0 \end{aligned} \tag{18}$$

$\overline{ad}$ in the equation 13 is obtained by using the Pythagorean theorem:

$$\begin{aligned} \overline{ad} &= \sqrt{\overline{aj}^2 + \overline{dj}^2} \\ &= \sqrt{(Y^f_n - Y^r_{n-1})^2 + (X^f_n - X^r_{n-1})^2} \end{aligned} \tag{19}$$

3) Coordinates of Rear Axle

In Figure 8, the point of new rear axle b can be obtained by:

$$X^r_n = X^f_n - L\sin\theta_1 \tag{20}$$

$$Y^r_n = Y^f_n - L\cos\theta_1 \tag{21}$$

4) Offtracking

Figure 9 illustrates the swept path of a vehicle that travels a curve section subtended by an angle Φ and a tangent section. Point a is on the curve section and point b is on the tangent section of the path of rear axle. Then offtracking at point a, Off_{curve}, is as follows:

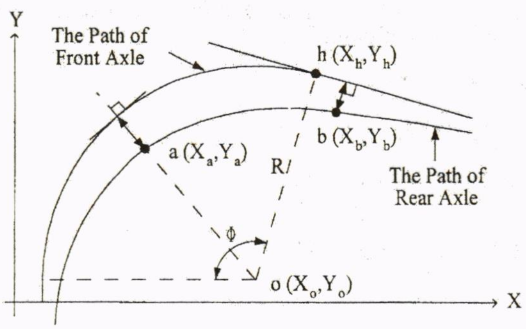

Figure 9. Offtracking

$$Off_{curve} = R - \sqrt{(X_a - X_o)^2 + (Y_a - Y_o)^2} \tag{22}$$

Offtracking at point b is distance between point b and the tangent line. Thus, offtracking at the point b is:

$$Off_{tangent} = \frac{|mX_b - Y_b + Y_h - mX_h|}{\sqrt{m^2 + (-1)^2}} \tag{23}$$

where,

m = tangent line slope

5) Coordinates of a Point in a Vehicle

Figure 10 illustrates the absolute coordinates and relative coordinates of a point in a vehicle.

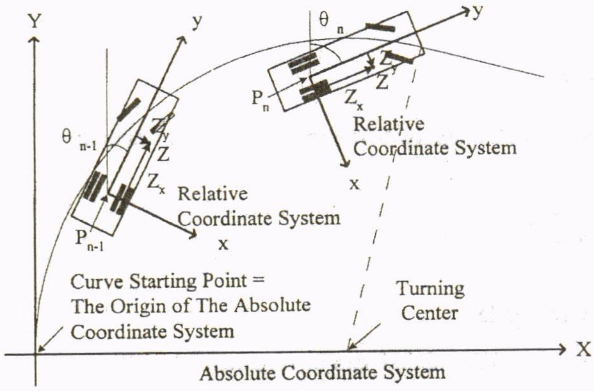

Figure 10. Coordinates of Points in a Vehicle

In Figure 10, if absolute coordinates of point P_{n-1} is $(X_{P,n-1}, Y_{P,n-1})$, absolute coordinates of point Z is:

$$X_Z = X_{P,n-1} + Z_y \sin\Phi_{n-1} + Z_X \cos\Phi_{n-1} \tag{24}$$

$$Y_Z = Y_{P,n-1} + Z_y \cos\Phi_{n-1} - Z_X \sin\Phi_{n-1} \tag{25}$$

where Φ_{n-1} is the heading angle of wheelbase. Similarly, if absolute coordinates of point P_n is $(X_{P,n}, Y_{P,n})$, absolute coordinates of point Z is:

$$X_Z = X_{P,n} + Z_y \sin\Phi_n + Z_X \cos\Phi_n \tag{26}$$

$$Y_Z = Y_{P,n} + Z_y \cos\Phi_n - Z_X \sin\Phi_n \tag{27}$$

where Φ_n is the heading angle of wheelbase.

4. VEHICLE SIMULATION PROGAM

4.1 Program Design

A computer program was developed based on the new offtracking model. Using the computer program one can graph the complete swept path of any type of vehicle making a turn at low speed. This program calculates the magnitude of both transient offtracking and maximum offtracking and also animates the moving vehicle, so that designers can estimate the offtracking of large vehicles in an easier, faster, and accurate manner. This program consists of one main program and three processors.

1) Input Processor

This processor inputs travel condition(the type of a vehicle, the radius and angle of travel input path et al.) specified by the user and transmits it to the compute processor.

2) Compute Processor

This processor computes the offtrackings using the model developed in the paper and the results of computation are transmitted to the output processor.

3) Output Processor

This processor provides the outputs by the Compute Processor in numeric forms and simulates moving vehicles.

4.2 Configuration

The GUI(Graphic User Interface) of this program is window type. This program has three windows. One is a main window controlling all works in the program. Another is an input dialogue box providing travel conditions. The third is an output window for plotting the results of simulation.

Figure 11 illustrates the output window plotting the computation results and animation for the situation when a tractor-semitrailer negotiates a curve(radius is 15m and angle is 140°) followed by a 30 meter long tangent.

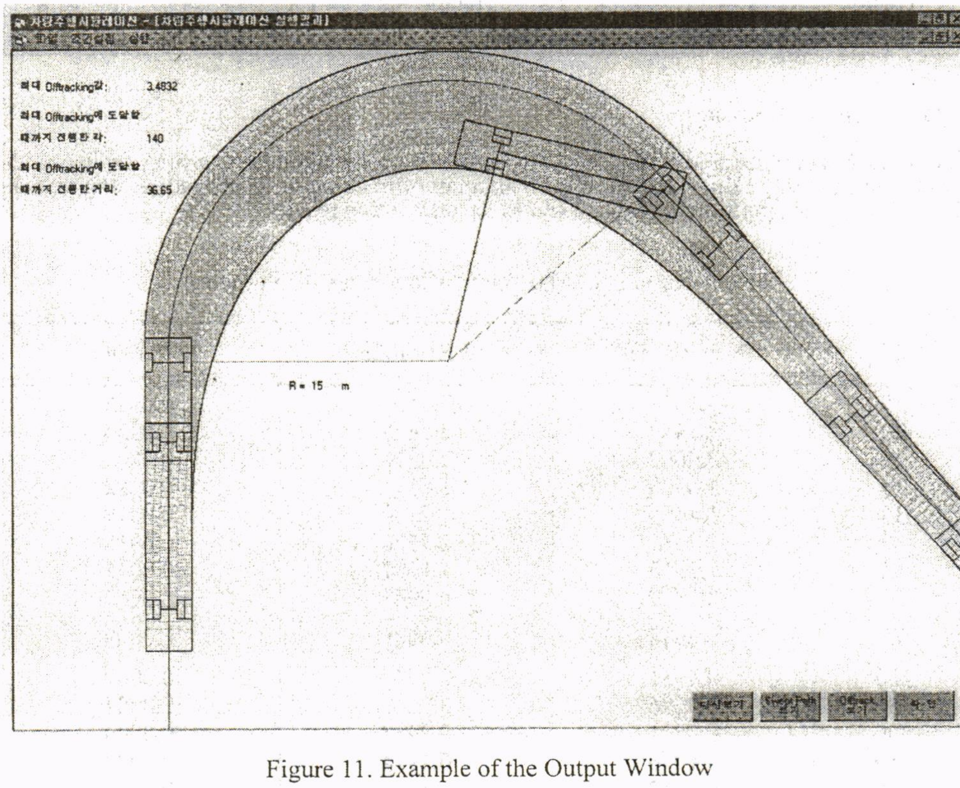

Figure 11. Example of the Output Window

5. VALIDATIONS AND APPLICATION OF THE PROGRAM

5.1 Comparisons with Other Model

For validation of the new offtracking model, computation results through this model and other model were compared. Because this simulation program intended for analyzing the complete swept path of a vehicle making any type of turn, two comparisons(maximum offtracking and transient offtracking) were performed. The calculation results through this model for maximum offtracking were compared with that through WHI method. Calculation results for transient offtracking were compared with that through Sayers model.

The simulating conditions were as follows:

- Vehicle: tractor-semitrailer
- Input path: one circular curve with two tangents on both sides
- Front axle of the vehicle travels on the input path.
- Length of curve is long enough to yield a maximum offtracking
- A simulating interval length is 0.01m(1cm).

The dimension of a tractor-semitrailer is shown in Tabel 1.

Table 1. Dimension of a Tractor-Semitrailer (Unit: meter)

Dimension / Vehicle	Length	Width	Height	Wheelbase	Front Overhang	Rear Overhang	Min. Turning Radius
Tractor - Semitrailer	16.7	2.5	4.0	Tractor: 4.2 Trailer : 9.0	1.3	2.2	12.0

1) Maximum Offtracking

New model was compared with the WHI model. Because minimum turning radius of the tractor-semitrailer is 12m, calculations of maximum offtracking for curve radius ranging from 15m to 300m were performed. Table 2 shows the calculation results.

Table 2. Results for Maximum Offtrackings (Unit: meter)

Radius	New Model	WHI Model	Differences
15	3.7638	3.7590	0.0048
20	2.6437	2.6403	0.0034
25	2.0602	2.0575	0.0027
30	1.6939	1.6917	0.0022
35	1.4406	1.4387	0.0019
40	1.2543	1.2526	0.0017
45	1.1112	1.1097	0.0015
50	0.9977	0.9963	0.0014
75	0.6614	0.6605	0.0009
100	0.4951	0.4944	0.0007
150	0.3296	0.3292	0.0004
200	0.2471	0.2468	0.0003
250	0.1976	0.1974	0.0002
300	0.1647	0.1644	0.0003

The differences between two models decreased as radius increased, and the maximum difference was 0.48cm for 15m curve radius.

2) Transient Offtracking

Again, new model was compared with Sayers model. Table 3 shows the calculation results for transient offtrackings by two models for a 100m radius curve. In new model, maximum offtracking was reached at a point where the vehicle running the curve for 97.11m on arc(55.64° subtending the arc). In Sayers model, it was 94.88m and 54.36° .

Table 3. Results for Transient Offtrackings (Unit: meter)

Travel Distance on Curve	New Model	Sayers Model	Differences
10	0.0587	0.0587	0.0001
20	0.3303	0.3297	0.0006
30	0.4402	0.4392	0.0010
40	0.4769	0.4757	0.0012
50	0.4891	0.4878	0.0013
60	0.4931	0.4918	0.0013
70	0.4944	0.4931	0.0013
80	0.4949	0.4936	0.0013
90	0.4950	0.4937	0.0013
100	0.4951	0.4938	0.0013
110	0.4951	0.4938	0.0013

Looking into two types of offtracking produced by new models, it is ascertained that comparison results were satisfactory as the differences between two model outputs were only in millimeter order. In stead using Sayers model which is so difficult to be used freely, engineers can now utilize the advantage of new model which produces the same degree accuracy in the outputs with a much easier fashion.

Although validation was not made based on field survey offtracking data for a variety of vehicles currently operating on roads, it is certain that new model is very helpful to engineers in understanding pavement widening needs within horizontal curves.

5.2 Application of Program

This program calculates the swept path of a vehicle making all type of turns at low speed. It can be used to estimate the pavement widenings required for horizontal curve sections.
In addition, engineers can use this program in designing at-grade intersections. For example, the suitability of curb radii, the size of channelization islands, and lane width can be reviewed by the new model.

Figure 12 shows a situation where the program output is overlapped with an intersection sharp drawing. The radius of curve was 12m. One can say that it would be better to modify the intersection design.

In designing a new intersection, engineers can decide the minimum curve radius of a intersection corner by calculating the minimum turning paths of design vehicles using the program.

Figure 13 shows an example in designing the oblique-angle(50.5°) turns with corner islands.

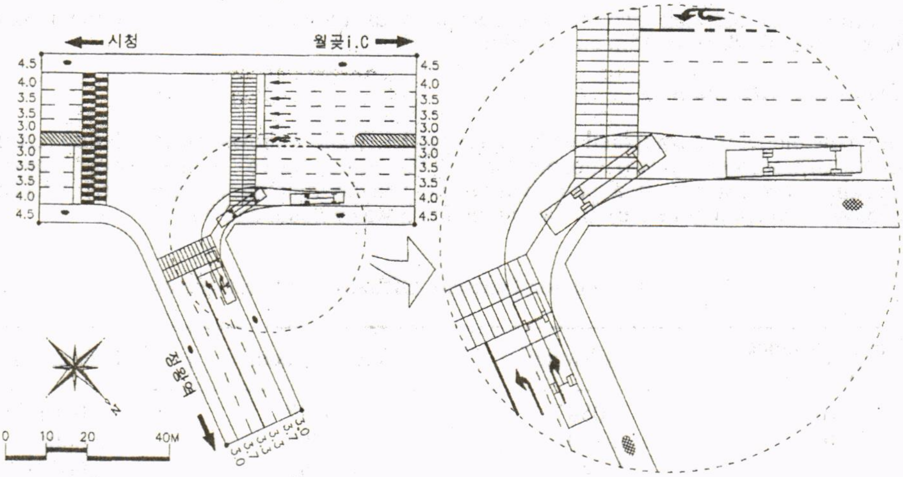

Figure 12. Example Case 1

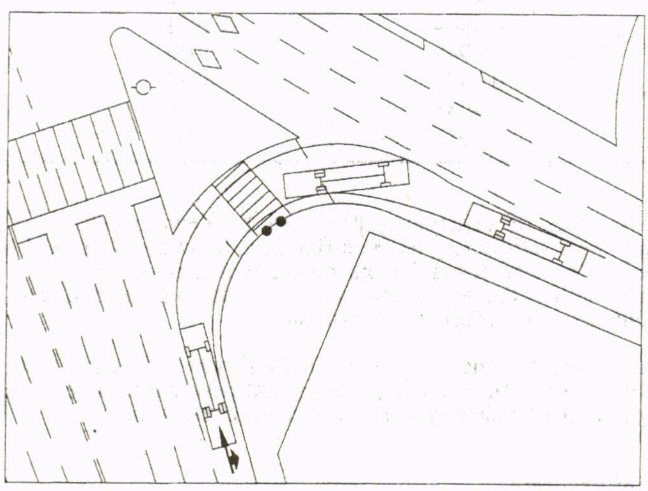

Figure 13. Example Case 2

6. CONCLUSIONS

Following conclusions were made in this research:

- A mathematical model for calculating the amount of low-speed offtracking of any type of vehicle developed at any point of the path at low speed was developed.
- A vehicle simulation program that calculates the path of vehicle and the low-speed offtracking and plots it graphically using animation method was developed based on the calculation results from the model.
- For validation of the new offtracking model, computation results through this model and other model were compared. Differences between the models were insignificant.
- Engineers can use this program in designing horizontal curve sections, and at-grade intersections in an easier, faster, and accurate manner.

The following topics are required to be studied in the future research:

- Developments for the offtracking model that consider the effects of speed and superelevation on large vehicle offtracking.
- Reflection in the model structure of the vehicle driver's tendency of precontrol steering wheels prior to entering a horizontal curve to make a natural vehicle path.

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