APPLICATION OF TRIP ASSIGNMENT MODELS TO THE PASSENGER'S CHOICE OF TRAIN SERVICE

Chi-Kang LEE
Professor
Department of Transportation and Communication Management
National Cheng Kung University
1, University Road
Tainan, 70101, TAIWAN
Fax:+886-6-275 3882
Email:cklee@mail.ncku.edu.tw

Abstract: The paper presents a modeling framework for solving the service planning problem of a high speed rail. This framework explicitly considers both the Passenger's and the operator's viewpoints in a bi-level program. The passenger's choice of train service is formulated as a trip assignment model on the base of user optimal behavior, with a service network and a generalized cost function. Various issues of the model building and model estimation for the passenger's choice and the service planning are discussed.

1. INTRODUCTION

The construction of Taiwan high speed rail will be started in the near future. However, some political representatives have been trying to add a station for their hometown and/or to increase the train frequency to stop at their hometown station. In order to operate the rail line effectively and make the planning result reasonable, a problem oriented scientific method has to be developed. The objective of this paper is to present a modeling framework for discussing and solving the service planning problem of Taiwan high speed rail.

In practice, a sequential process is used to design the train service as well as to construct a timetable. For an example of the regular interval problem, the Dutch railway considers the following four steps in the process: (1) design of train service, (2) construction of a regular interval timetable, (3) check for the usage of station track and platform, and (4) expansion of the hourly timetable (Hoogiemstra, et. al. 1996, 1998). However, the passenger's choice behavior has not been explicitly represented in the process, and there still exists ambiguity to integrate the four steps. In this paper, a bi-level program is proposed for the service planning process, in which the relationship between the operator and the passenger is represented explicitly.

The structure of the paper is the following. The second section states the meaning of the service planning problem for a rail company, from the viewpoints of the passenger and the operator respectively. Then, the passenger's choice behavior of train service is discussed in the third section, using a network for the choice of alternative and a generalized cost function for the choice criteria. The route choice problem with the network and the generalized cost function will result in the rider-ship pattern of train service. After that, the issues for the route choice problem and the service planning problem are discussed in section four. They are (1) model building for the passenger's choice of train service, (2) model estimation for the coefficients in the generalized cost function, and (3) model building for the service design problem. Finally, concluding remarks are made in section five.

2. PROBLEM STRUCTURE

A train service plan of Taiwan high speed rail will consist of several types of stop pattern/service class and associated train frequency/train length. For example, a direct express train stops only two stations, a limited express train stops three to five stations, and a rapid train stops all the ten stations. Since Taiwan high speed rail is only a line, the complexity of her service plan is not as difficult as the network case of Taiwan Railway Administration. However, the first step in service planning is to develop a reasonable structure for discussing and finding a set of stop pattern and train frequency, with a satisfactory value of the operator's objective. In order to simplify our discussion in the static modeling area firstly, the service planning is considered for designing a regular interval timetable. As the problem [P1] written as follows, the problem decision variables are stop pattern (S) and train frequency (F), and the decision maker of the problem is the railway operator. The patronage or rider-ship of the service plan (X) is an important factor for the operator's objective, such as profit or market share, but it is not directly controlled by the operator.

[P1] Maximize Operator's Objective (S, F, X)
s.t. Operational constraints
Resource constraints

The selection of a service plan is centrally dependent on the demand side or the passenger's choice. In practice, a service plan will not be utilized, if its passenger rider-ship is not good. Therefore, the key sub-problem in the planning process is a model of passenger's choice behavior of train service. As the problem [P2] illustrated below, it generates a reasonable rider-ship pattern (X) with a given service plan (S, F), on the base of the passenger's point of view. That is, the purpose of the model [P2] is to reflect the passenger's choice behavior of train service.

[P2] Maximize Passenger's Objective (X)
s.t. a given service plan (S, F)

With the passenger's choice model [P2], we can solve the rider-ship of a given service plan, i.e. (X|S, F). As stated before, this solution will be used frequently for computing the operator's objective value in the service planning model [P1]. Hence, as illustrated in the problem [P], the choice model [P2] is formulated as a constraint of problem[P1]. That is, for each alternative service plan (S, F), which is feasible in the upper level problem [P1]; there may exist one corresponding result of its rider-ship (X|S, F), which is the result of the lower level problem [P2]. Therefore, in the service design problem [P], the optimal service plan is dependent on both the operation/resource constraints and the passenger's responsive behavior. Furthermore, the relationship between the operator and the passenger is explicitly represented in the bi-level program [P]. In other words, the operator takes into account of the passenger's response to train service in his planning process, and the passenger is responsive to the train service in accordance to his/her choice criteria.

[P] Maximize Operator's Objective (S, F, X)
(decision variable: S, F)
s.t. operational constraints
resource constraints
Maximize Passenger's Objective (X, S, F)
(decision variable: X)

3. PASSENGER'S CHOICE OF TRAIN SERVICE

3.1. A Network Representation for the Passenger's Choice of Train Service

The objective of this section is to explain the network representation for the choice problem [P2]. With the network concept, the passenger's choice of train service can be described as a route choice problem, which has been widely studied and utilized in transportation planning.

Consider the example network for 5 stations and 4 service types, which illustrated in Figure 1. There are two types of node in the network: origin/destination (O/D) node a, b, c, d, and e; and station/service node a1, a2, and so on. The second digit number of a station/service node represents the service type. Each service type represents a stop pattern. Given a service plan (S, F), there is a corresponding network structure. For example, the path of a1->c1` represents a direct express train from station a to station c; the path a2->b2`->b2->c2`->c2->d2`->d2->e2` represents a rapid train from station a to station e with a stop for every intermediate station; and so on. There are two types of link: the movement link, e.g. link a2->b2`, and the dwell link for boarding and alighting, e.g. b2`->b2. The other links are utilized to represents the relationship between different types of train service, or the relationship between train service and trip O/D. For example, link c2`->c represents an egress link, link c->c2 represents an access link, link c1`->c4 represents a transfer link. In summary, there are 2 types of node and 5 types of link in the network.

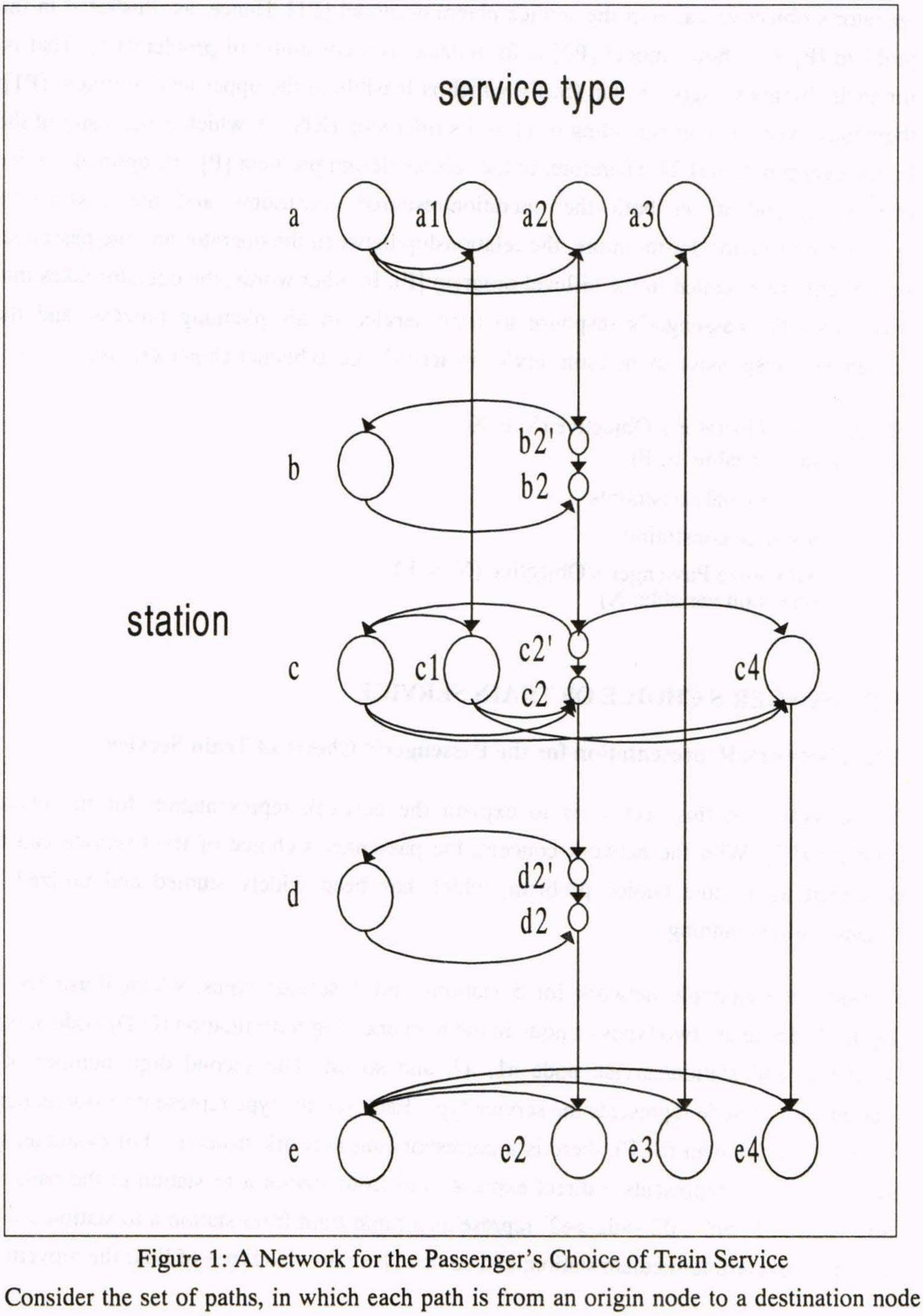

Figure 1: A Network for the Passenger's Choice of Train Service

Consider the set of paths, in which each path is from an origin node to a destination node for the route choice problem of the network. Any node in the network can be an intermediate node for a path expect the O/D node. Then, a path in the set represents an alternative of train service for the passenger from the origin to destination. For example, there are 5 paths or alternatives for O/D pair (a,e). They are the following.

Path1: a->a1->c1`->c4->e4` ->e. (Use the 1st and 4th types of train service.)

Path2: a->a1->c1`->c2->d2`->d2->e2`->e. (Use the 1st and 2nd types of train service.)

Path3: a->a2->b2`->b2->c2`->c2->d2`->d2->e2`->e. (Use the 2nd type of train service.)

Path4: a->a2->b2`->b2->c2`->c 4->e4`->e. (Use the 2nd and 4th types of train service.)

Path5: a->a3->e3`->e. (Use the 3rd type of train service.)

Therefore, the route choice problem of the network can represent the passenger's choice of train service. In other words, the solution of the route choice problem will result in the rider-ship of the given service plan. That is, a path flow represents the number of trips for the path O/D using the train service, and a link flow represents the number of trips using the train service on the link section.

3.2. Choice Criteria

We assume the passenger is a cost minimizer, and his/her choice criteria is based on a generalized cost function. This section demonstrates how the cost function can be defined on the network, link by link, so that the cost associated with a path is exactly the generalized cost for that alternative, where the path cost is the sum of the cost associated with each link in the path.

As an example, we assume that the passenger consider 4 types of cost, and they are: in vehicle travel time (IVTT), out of vehicle travel time (OVTT), out of pocket cost (OPCT), and crowd or discomfort condition (CDC). IVTT equals to the sum of the running time of train between two stations and the dwell time at a station. OVTT equals to the sum of access time, waiting time, transfer time, and egress time. OPCT is in general calculated as the product of distance and fare rate, and each type of train service has a specific fare rate. For example, the fare rate for the direct express train equal to the sum of the basic fare rate, which is the one for the rapid train, and an extra rate. CDC is an index used to reflect the passenger's behavior of escaping from crowded train service. For a static situation, the passenger can choose the type of train service with a low load factor; or for a dynamic situation, the passenger can choose not only the type of train service with a low load factor, but also the train service in an off peak period. Then, a linear generalized travel cost function, GC, can be defined as follows for the following discussion.

$$GC = c_0 + c_1\ IVTT + c_2\ OVTT + c_3\ OPCT + c_4\ CDC$$

The relationship between the link cost on the network and the costs described above is discussed in the following. Firstly, the cost of an access link, e.g. a->a2, consists of the access time/cost and waiting time for using the train service at the origin node. The access time/cost is usually dependent on the transportation and land use condition at the origin node/station. The waiting time may be dependent on the station's characteristics, e.g. the passenger information system in the station, and the frequency of train service. The latter factor seems not very important for scheduled train service, since most passengers know the departure time of their train in advance. However, the train service of high frequency is convenient and has high probability to be used. Secondly, the cost of an egress link, e.g. e2`->e, is the egress time/cost from the train to the destination. Similar to the access time/cost, the egress time/cost depends on the transportation and land use condition at the destination node/station. Thirdly, the cost of a transfer link is only transfer time. The transfer time may be different for different service pairs at a transfer station. For example, the average transfer time for service pair (1,4) may be shorter than that for service pair (1,2) at station c, if the headway of train service 4 is shorter than that of train service 2. Fourthly, the cost of a movement link, e.g. a2->b2`, consists of the running time of train, the distant fare, and the discomfort for that section. In practice, the running of a train between two station is not sensitive to the number of passengers in the train (Hsieh, et al. 1997). The scheduled running time is usually the time with minimum energy consumption, and it is longer than the minimum running time. Hence, there is a float for the train to catch up the schedule. In brief, the running time may not be flow dependent but the energy cost may be flow dependent. Only the running time is considered by the passenger, but the operating cost is important to the operator. Moreover, the fare rate is fixed in problem [P2], but it may be a service design variable in the upper level problem [P1]. Besides, it is evident that the crowd or discomfort condition is flow dependent. On average, the discomfort condition depends on the average load factor for a vehicle, and its relationship may be similar to the congestion curve used in the highway system. For example, assume the CDC index is defined as follows.

Where, CDC: crowd and discomfort condition of link l

$$\mathbf{CDC(l) = IVTT(l)[0.15(\frac{X(l)}{K(l)})^4]}$$

IVTT: travel time of link l
X: flow on link l
K: practical capacity of link l

The practical capacity of the train, K, can be defined as the 75% of the maximum capacity. Then, the discomfort when the train loaded 75% is only 15% higher than the condition of almost no passenger. But, the discomfort condition will increase very fast when the train is very much loaded. Fifthly, the cost associated with the dwell link primarily consists of boarding time and alighting time. As stated in the literature, the boarding/alighting time is dependent on the average number of boarding/alighting passengers (Harris, et al., 1992). For example, the dwell time for link (b2`->b2) is dependent on the sum of the flow on egress link (b2`->b) and the flow on access link (b->b2). If the required dwell time is longer than the scheduled time, the departure time of the train will delay at that station. Therefore, if the dwell time is not fixed in the model, it is flow dependent as an asymmetric problem.

In summary, consider 'Path1' as an example, the cost of the path is the sum of the following link costs. The cost of access link (a->a1) is the access time/cost and waiting time at the origin a. The cost of movement link (a1->c1`) is the train running time, the fare, and the discomfort condition for the section (a1->c1`). The cost of link (c1`->c4) is the transfer time at station c. The cost of link (c4->e4`) is the train running time, the fare, and the discomfort condition for the section (c4->e4`). The cost of link (e4`->e) is the egress time/cost at the destination. Therefore, the cost associated with 'Path1', which calculated as the sum of link costs, can fully reflect the generalized cost for the alternative of using service type 1 and service type 4 from origin a to destination e.

4. MODELING DISCUSSION

4.1. Modeling for the Passenger's Choice of Train Service

As stated in section 3.2, user optimal is utilized for the passenger's choice behavior of train service. The user optimal trip assignment model can be written as a linear programming problem if the generalized cost of an alternative is flow independent and the passenger does not have stochastic behavior. For example, the dwell time of a train at a station is a constant, the discomfort condition is not considered in the cost function, and so on. Then, only the path or alternative with the lowest cost will be utilized for each O/D pair. In other words, the all-or-nothing solution will be the result for the linear programming problem. It seems not the case in practice, because the passenger does not have complete information so that he/she does not choose the lowest cost alternative all the time. If the passenger's random behavior is considered, the trip assignment model will be a stochastic user optimal route choice problem. In general, logit model and probit model are widely used in practice. Due to the I.I.A. property of logit model, probit model seems much appropriate for the passenger's choice of train service (Ben-Akiva, et al., 1985; Sheffi, 1985). The probit model is capable to reflect the similarity or difference between service types, especially for the flow independent problem.

If the generalized cost is flow dependent but not asymmetric, the trip assignment model can be formulated and solved as a nonlinear programming problem. For example, if the discomfort condition CDC is flow dependent as described in section 3.2, the deterministic user optimal model can be solved by the Frank-Wolfe algorithm (Sheffi, 1985; Patriksson, 1994). Then, there are three extension directions for the flow dependent problem. One is the stochastic user optimal model. It considers the passenger's random behavior. The second is the user optimal model with elastic demand. It considers the competition between the high speed rail and other modes. The third is the stochastic user optimal with elastic demand. Note that, the access/egress time/cost discussed in section 3.2 is usually O/D dependent. That is, it is fixed with a pair of O/D for different alternatives of train service. Therefore, it can be ignored for the models with fixed O/D demand. Moreover, if the dwell time is flow dependent as discussed in section 3.2, the route choice model will become an asymmetric trip assignment problem. However, the dwell time in practice is carefully kept as the scheduled dwell time or the train will catch up the station delay in the next following section, because the dwell time is an important factor for controlling headway, train delay, or line capacity (Lee, 1989, 1999). Therefore, only symmetric assignment models are tested in this study.

If we further extend our modeling framework with a dimension of time, then we can describe the passenger's behavior much more accurately. An one-hour O/D demand table is usually generated for designing a regular interval timetable, using the daily O/D table and a peak factor. Hence, the demand fluctuation in time and the different demand pattern in direction are not explicitly formulated in the static models. For example, the passenger may choose his/her departure time and train service simultaneously, in accordance with his/her appointment at the destination. Since many modeling and solution techniques for the dynamic route choice problem have been developing in recent years, the passenger's choice model of train service may have a similar model structure. This extension will not further discuss in this paper (e.g. refer to Ran, et al., 1996; Chen, 1999).

4.2. Model Estimation for the Coefficients in the Generalized Cost Function

In the previous discussion, we assume we have a generalized cost function with a set of function coefficients. Hence, we have to choose appropriate numbers for the coefficients from practical transportation studies, such as the study of travel time value, the study of perceived and actual waiting time, etc. However, if we want to calibrate the generalized cost function, the estimation problem can be performed as a bi-level program (Lee, 1987). The upper level problem is to solve the maximum likelihood estimator for the generalized cost function with observed rider-ship pattern, and the lower level problem is the passenger's choice model to solve the estimated rider-ship pattern with a given set of coefficients. There is no way for us to test the estimation problem in this study, because there is no observed rider-ship pattern yet for Taiwan high speed rail.

4.3. Modeling for the Service Planning Problem

As described previously, the service design problem [P] is a bi-level program. That is, the operator take into account of the passenger's choice behavior in his service design process, and the passenger makes his/her choice in response to the provided service. The first decision variable S, stop pattern/service class, in the upper level problem [p1] can be defined as an integer variable, which has an effect on the network structure on the lower level route choice problem [p2]. The second decision variable F, train frequency/train length, in the upper level problem [P1] can be defined as a real variable, which has an effect on the parameters in the lower level route choice problem [P2]. For example, if the train length in a service plan is changed, the practical capacity which used to compute the load factor for the discomfort index CDC has to be updated; where the train capacity is a constant parameter in the lower level problem [P2]. Therefore, the decision variable of the lower level problem, X, rider-ship of train service, is sensitive to the decision variable in the upper level problem, (S, F), because the flow pattern X is dependent on the network structure associated with the route choice problem and the parameters in the generalized cost function. The modeling and solution algorithm for the bi-level problem will not further discussed in this paper (e.g. refer to Falk, et al. 1995).

5. CONCLUDING REMARKS

The paper presents a bi-level framework for modeling the relationship between the railway operator and the passenger. The decision maker of the upper level problem is the operator for designing train service, and the decision maker of the lower level problem is the passenger for choosing provided train service. The passenger's choice problem is formulated as a route choice model, with a service network and a generalized cost function, on the base of user optimal behavior. In the paper, various modeling issues associated with this structure are discussed, including the possible types of formulation for the passenger's choice model, the possible formulation for the model estimation problem, the bi-level formulation for the service design problem, and so on.

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