Evaluation of User Response to Reliable Shortest Path Information

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Abstract
Travel times of link in urban networks are governed by stochastic process because of random traffic congestion. Therefore, a traffic model cannot compute accurately travel times on link in terms of deterministic value. In order to calculate more realistic travel times, a traffic model should incorporate random travel time variations into the deterministic one. In this paper we introduce the concept of reliability for reflecting travel time variations and driver's travel time uncertainty and build the reliability assignment model. Reliability assignment model can reflect driver's behavior, try to minimize travel time variations and their own uncertainty in terms of the perceived travel time.

1. INTRODUCTION

Every driver attempts to use his/her shortest travel time path when traveling from his/her origin to destination. However, drivers do not know exactly the shortest path due to daily variations and uncertain nature of traffic conditions, and their bounded knowledge and information. In an emerging new era, these uncertainty can be reduced and thus bounded knowledge can be improved using a reliable information processed and distributed by advanced technologies. Even more, driver's needs on information tend to be differentiated and distinguished according to their desire, situation and characteristics. For example, some drivers need a traffic information mainly based on the shortest travel time irrespective of its reliability. On the other hand, others prefer more reliable path information to only the shortest in order to arrive their destination with higher reliability.

A typical example of latter case is a trip to airport, in which the most important factor of the trip is not to miss their plane with highest reliability.

It is in urban road networks that travel times of each link show a random process. Therefore, a traffic model cannot compute accurately travel times on each link in terms of only deterministic values. In order to calculate more realistic travel times, a traffic model should incorporate random travel time variations into the deterministic one. The random effects of traffic conditions can be expressed in the form of a reliability function. In other words, if a road is a section of the shortest distance path from one place to CBD, it can be categorized a low reliable road.

In this paper, the concept of reliability for reflecting travel time variations and driver's uncertainty about travel time is introduced. In particular, it is used that the reliability is a probability that a driver can pass on links within an expected travel time. As a result, reliability assignment utilizes a perceived travel time, which includes both congestion effects and reliable measures on each link, instead of deterministic travel time. Thus, it attempts to produce a realistic assignment considering travel time variations. Consequently, this model reflects these realistic phenomena on driver's behavior, which states that drivers try to minimize their travel time variations and their own uncertainty in terms of the perceived travel time.

In the next section we review several studies on the shortest path problem in stochastic networks. In section 3 we define the perceived travel time and the reliability, and then build a reliability assignment model. In section 4 numerical examination is conducted using a simple urban network in order to analyze driver's path choice behavior according to travel time variations. Some concluding remarks are made in section 5.

2. BACKGROUND

Several approaches finding stochastic shortest path in probabilistic networks have already been proposed in literature. Frank(1) and Mirchandani(2) assume that travel time of link is nonnegative and random variable with a given probability distribution. And they consider the problem of finding the expected shortest path travel time through a probabilistic network.

Frank(1) assumes that each link has an independent continuous probability distribution for travel times. He derives expressions for the exact probability distribution, in terms of characteristic functions, for the travel times of the shortest paths. But, when the link travel times can be approximated as having discrete probability distribution, with finite number of values, his method becomes unnecessarily tedious.

Mirchandani(2) assumes that the travel time on each link has a independent discrete probability distribution. He proposes an algorithm to compute the expected shortest travel time between two nodes in the transportation networks. And in reliability computations, associated with each link is a probability of failure and a probability of success. The concept of failure implies infinite travel time. he assumes that therein lies the connection between reliability and the expected shortest path travel time and then first proposes a new method which computes these two quantities simultaneously.

But, his method is not precise in reliability computations and merely calculates the expected shortest path travel time by product reliability and path travel time. Above all, because of computing the expected shortest path travel time of an entire system, this approach is not utilized in traffic model, requiring travel time of each path.

Eiger et al.(3) and Mirchandani and Soroush(4) define preferences among candidate paths in stochastic networks through particular utility functions. Eiger et al.(3) express that when the traveler's utility function for travel time is linear, the optimal path problem reduces to the simple shortest route problem in a deterministic network. And they also express that when the utility function is exponential, permanent preferences prevail, then by appropriate transformation of the link travel time probability density functions, the optimal path problem again reduces to the deterministic shortest route problem. Mirchandani and Soroush(4) propose the optimal path finding method when the traveler's utility function is quadratic in travel time.

We will propose the shortest path finding method through travel time and reliability, as being the method of Mirchandani(2). We define the perceived travel time and the reliability. Mirchandani(2) calculates travel time and reliability separately. But, by including reliability in perceived travel time we will reflect a driver's behavior, attempting to minimize his perceived travel time.

3. RELIABILITY ASSIGNMENT MODEL

3.1 Perceived Travel Time

The perceived travel time is a driver's expected travel time on link before departing the origin. When travel time varies and can not be predicted exactly, a driver might choose the path considering mean travel time and travel time variation caused by traffic congestion. In other words, a driver thinks over delay travel time as well as deterministic travel time on each link. To make an analysis of the perceived travel time, we separate the perceived travel time into mean travel time and delay travel time. The mean travel time is travel time needed for passing on link. The delay travel time is additional travel time caused by traffic congestion.

$$\text{[Perceived Travel Time]} = \text{[Mean Travel Time]} + \text{[Delay Travel Time]} \qquad (1)$$

The concept of perceived travel time is also utilized in stochastic user equilibrium. In stochastic user equilibrium the perceived travel time is a driver's subjective evaluation in travel time. This concept is the same as we will utilize in this study. But, the delay travel time is different from error term in stochastic user equilibrium. Error term is only a random variable. The delay travel time is safety margin learned from driver's own experience and travel information. In other words, the delay travel time is a function expressing driver's uncertainty in travel time and has a functional relation with mean travel time.

The reliability assignment model differs from the stochastic user equilibrium model, as though the perceived travel time is utilized. In stochastic user equilibrium traffic is assigned in proportion to logit or probit function. Strictly speaking, the discrete choice models can not express truly the path choice behavior of drivers, try to minimize the perceived travel time.

But, we enclose the delay travel time in perceived travel time, and then we assign the traffic volume by using the perceived travel time. Therefore, the reliability assignment evaluates the variations of respective routes' travel time and assigns the traffic so as to minimize time virtually used for travel.

3.2 **Reliability**

In this study we introduce the concept of reliability for computing delay travel time, driver's travel time uncertainty. The reliability is primary utilized in Operation Research. In Operation Research the reliability is a probability that a device performs adequately over the interval [0,t]. It is assumed that unless failure or replacement occurs, adequate performance at time t implies adequate performance during the interval [0,t].

To introduce the reliability in traffic model, we define the failure an event being happened traffic congestion. Therefore, the reliability in this study is a probability that travel time variation caused traffic congestion is not happened over the travel time interval [0,t] on link.

We assume that each link has an independent continuous Weibull distribution for delay travel times. The Weibull is the most complex and useful of the distributions generally used in reliability analysis. We assume that failure rates are increasing in proportion to time. That is, assume that a probability being happened travel time variation is increasing to mean travel time. If failure rates are increasing to time, the failure probability density function conforms to a normal distribution.

Equation (2) is the failure probability density function of weibull distribution and the reliability is calculated by using equation (3).

$$f(t) = \frac{m}{\eta}(t-\gamma)^{m-1} e^{-(\frac{t-\gamma}{\eta})^m} \qquad (2)$$

$$R(t) = e^{-(\frac{t-\gamma}{\eta})^m} \qquad (3)$$

where, m : shape parameter (m=3)

η : scale parameter

γ : location parameter(γ=0)

t : time interval [0,t]

An equation (4), transformation form of equation (3), is the equation for computing delay travel time and is a increasing or monotonically increasing function. It satisfies the assumption that a probability being happened delay travel time is increasing according to mean travel time.

$$D(t) = \alpha(1 - e^{-(\frac{t}{\eta})^3}) \qquad (4)$$

where, α : driver's subjective evaluation of delay travel time

α is a parameter expressing a driver's subjective evaluation of delay travel time on link. When travel time variation is hardly happened, in general, α has a low value because the effect of delay travel time on total travel time is small. In contrast to, the link, an effect of delay travel time on total travel time being large, has a high value of α.

3.3 **Reliability User Equilibrium Assignment**

Reliability assignment is formulated as a problem of minimizing the travel cost :

$$\min\ Z(x) = \sum_a \int_0^{x_a} [(1-\delta)t_{a0}(1+0.15(\frac{\omega}{C_a})^4) + \delta\alpha(1-e^{-(\frac{t_m}{\eta})^3})]d\omega \qquad (5)$$

$$\sum_k f_k^{rs} = q_{rs} \qquad \forall r,s$$

$$f_k^{rs} \geq 0 \qquad \forall k,r,s$$

$$X_a = \sum_r \sum_s \sum_k f_k^{rs} \rho_{a,k}^{rs} \qquad \forall a$$

where, t_{a0} : free-flow travel time on link a

x_a : traffic flow on link a

C_a : capacity of link a

t_m : mean travel time on link a ($t_m = t_{a0}(1+0.15(\frac{\omega}{C_a})^4)$)

δ : parameter reflecting driver's behavior

δ is a parameter reflecting driver's behavior. If δ is 1, a driver chooses the path according to the reliability only. And if δ is 0, a driver chooses the path through deterministic travel time computed by BPR function.

The first term of equation (5) is mean travel time computed by BPR function. The second term is delay travel time caused by traffic congestion. Because two terms of equation (5) are a monotonically increasing function, cost function is convex and then an unique solution is existed. The Frank-Wolfe algorithm is utilized as solution algorithm.

4. NUMERICAL EXAMINATION

4.1 Test Network

A numerical examination is presented to illustrate the application and assessment of the developed reliability assignment model. The example network is shown in Figure 1. The input data such as link capacity, free-flow cost, parameter is shown as in Table 1. It is assumed that there is one origin-destination pair from node 1 to node 4.

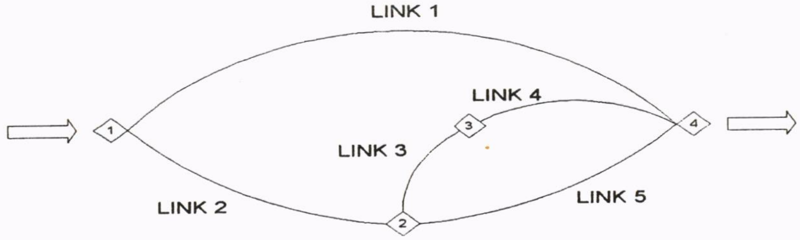

Figure 1. Example Network

Table 1. Input Data

link number	1	2	3	4	5
free-flow cost	200	80	40	100	40
capacity	800	600	400	600	400
α	100	800	1000	400	600
η = 300					

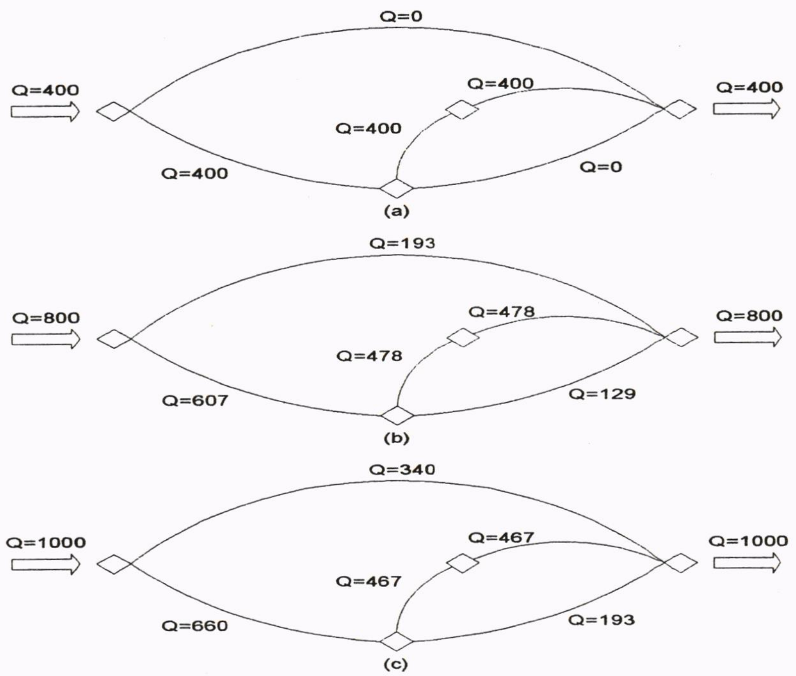

Figure 2. Traffic volume of link with varied travel demand (δ =0.5)

4.2 **Results**

Figure 2. show that when δ is 0.5, traffic volume of each link is different according to varied travel demand. In the case of Figure 2. (a), travel demand less than capacity, traffic is assigned on only one path, the deterministic shortest path. In the case of (b) or (c), travel demand more than capacity, traffic is assigned on all links. In the case of (b) and (c), traffic volume of link 3 and 5 are different. This phenomenon results from not difference of travel demand but that of reliability.

In Figure 3., when travel demand is constant(Q=800), traffic volume of each link is different according to δ. The result of case (a) is the same as that of deterministic user equilibrium because the reliability of each link is not considered(δ=0.0). In the case of (c), when travel cost is composed of only reliability, traffic volume of link 5 is zero. From this result we can infer that in emergency conditions a driver prefers the deterministic shortest path or the reliable shortest path. Link 1 is the reliable shortest path. This phenomenon is identical to driver's behavior in real life. But, in urban networks a driver usually considers both the deterministic travel time and the reliability on link, as in the case of (b).

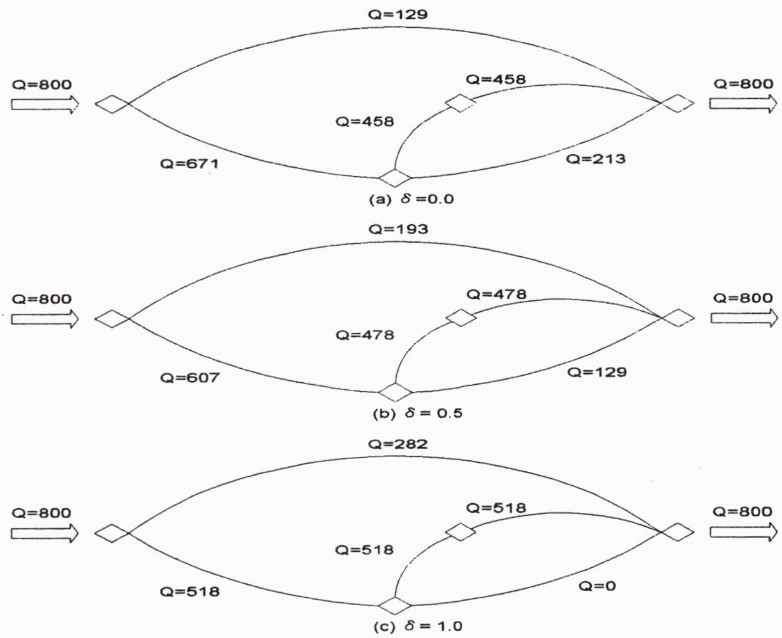

Figure 3. Traffic volume of link with varied δ

5. CONCLUDING REMARKS

The purpose of this paper is the construct of traffic model in which driver's travel time uncertainty could be considered. For the purpose, we define the perceived travel time and introduce the reliability.

In general, a driver attempts to minimize his perceived travel time, sum of mean travel time and delay travel time. The delay travel time is additional travel time caused by traffic congestion. In this study the delay travel time, the random effects of traffic congestion, is expressed in forms of reliability function and enclosed in perceived travel time. Therefore, reliability assignment model can reflect a driver's behavior, try to minimize the perceived travel time.

The model presented in this paper attempts to the following : 1) to establish the perceived travel time, 2) to deal with probable uncertainty in traffic conditions. Various issues remain to be investigated. The model should also be extended in order that it may be applied to more complex road networks.

REFERENCES

1. H. Frank, Shortest paths in probabilistic graphs. Oper. Res. 17(1969) 583-599.
2. P. B. Mirchandani, Shortest distance and reliability of probabilistic networks. Comp. Oper. Res. 3(1976) 347-355.
3. A. Eiger, P. B. Mirchandani and H. Soroush, Path preferences and optimal paths in probabilistic networks. Transportation Sci. 19(1985) 75-84.
4. P. B. Mirchandani and H. Soroush, Optimal paths in probabilistic networks : a case with temporary preferences. Comp. Oper. Res. 12(1985) 365-381.
5. C. E. Sigal, A. A. B. Pritsker and J. J. Solberg, The stochastic shortest route problem. Oper. Res. 14(1983) 371-385.
6. T. Uchida and Y. Iida, Risk Assignment : a new traffic assignment model considering the risk of travel time variation. Proceedings of the 12th International Symposium on Transportation and Traffic Theory(1993) 89-105.
7. F. S. Hillier, G. J. Lieberman, Introduction to operations research(fourth edition), Holden-Day, Inc. 1986.